Problem set 1

Advanced Optimization and Decision Analytics 24/25

UC3M — *Master on Statistics for Data Science*

Due date: Sunday April 13 (end-of-day). Value: 45% of final grade.

Note: This is an individual assignment. Evidence of plagiarism will be penalized. Hand in the assignment as a pdf file through the Assignment module in Aula Global, with code & output printouts and all required explanations. Justify your answers.

Problem 1 (50 points). Consider optimization of the following function defined on $\mathbb{R}^3$,

$$f(\mathbf{x}) = 3x_1 + 5x_2 - 3x_3^2,$$

subject to the following constraints:

$$2x_1^2 - 37x_2 + 9x_3 = 18$$
$$5x_1 + x_2 + 5x_3^2 = 24.$$

(a, 15 puntos) Formulate the conditions that a feasible point must satisfy to be regular (not in general, but for this problem). Determine whether there is any non-regular feasible point, and identify all of them if there are.

(b, 10 puntos) Formulate the 1st-order NC for local optimum (NC1-Lop) (not in general, but for this problem) and obtain, using Matlab, all the points, if any, that satisfy it, with their multipliers.

(c, 15 puntos) Formulate and apply, using Matlab, the appropriate 2nd-order conditions to determine whether each point obtained in (b) is a local or global minimum or maximum.

(d, 10 puntos) Formulate and solve this problem with Gurobi–Python. Discuss the relation of the results obtained with those in (a, b, c).

Problem 2 (50 points). Consider maximization of the following function defined on $\mathbb{R}^3$,

$$f(\mathbf{x}) = x_1^2 + x_2^2 - x_3^2,$$

subject to the following constraints:

$$8x_1^2 + 24x_2 - 15x_3 \leqslant 129$$
$$x_1^2 + 2x_2^2 + 4x_3^2 \geqslant 15.$$

(a, 15 puntos) Formulate the 1st-order NC for local maximum (NC1-Lmax) (not in general, but for this problem) and obtain, using Matlab, all the points, if any, that satisfy it, with their multipliers. Note: you can assume that all feasible points are regular.

(b, 20 puntos) Formulate and apply, using Matlab, the appropriate 2nd-order conditions to determine whether each point obtained in (b) is a local or global maximum. Note: if you need to apply a tangent direction analysis for a certain point, proceed as for the case of equality constraints but considering only the contraint(s) that hold with equality at the point being analyzed.

(c, 15 puntos) Formulate and solve this problem with Gurobi–Python. Discuss the relation of the results obtained with those in (a, b).