Voluntary Exercises 5

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Exercise 1

The price of products sold by a company is a random variable with the density function:

$$f(x) = \theta x^{\theta-1}, \quad 0 < x < 1, \ \theta > 0$$

The company boss wants to test the hypotheses:

$$H_0 : \theta = 1 \quad \text{vs.} \quad H_1 : \theta = 2$$

A simple random sample of size 2 is taken, and the critical region is:

$$C = \{(x_1, x_2) : x_1 x_2 \leq 0.6\}$$

The level of significance α is the probability of rejecting H_0 when it is true, which is:

$$\alpha = P(x_1 x_2 \leq 0.6 \mid H_0 = 1)$$

Under H_0, the density function $f(x_i)$ is uniform on $(0,1)$, and the joint distribution of (x_1, x_2) is:

$$f(x_1, x_2) = f(x_1)f(x_2) = \theta^2 x_1^{\theta-1} x_2^{\theta-1}, \quad 0 < x_1, x_2 < 1$$

Under $H_0 = 1$ it's again a U(0,1).

$$1^2 x_1^{1-1} x_2^{1-1} = 1 x_1^0 x_2^0 = 1$$

In a U(0,1), F(0.6) = 0.6. So $\alpha = 0.6$

Exercise 2

A farmer tests the hypotheses:

$$H_0 : \mu = 0.4 \quad \text{vs.} \quad H_1 : \mu = 0.3$$

The sample mean is 0.33 kg, and the power of the test is 0.6406.

(a) What is the sample size employed by the farmer?

The power of the test is:

$$\text{Power} = P(1 - \beta)$$

$$\beta = P(Do \quad not \quad reject \quad H_0 \quad | \quad H_1) = 1 - 0.6406 = 0.3594$$

The statistic under H_1 is:

$$T_{H_1} = (0.33 - 0.3)\sqrt{n} = z_\beta = 0.36$$

We solve for n:

$$\frac{0.36}{0.03} = \sqrt{n} \to n = 144$$

(b) What hypothesis is accepted in this testing problem?

$$T_{H_0} = (0.33 - 0.4)12 = -0.84$$

It is a left-sided contrast as the distribution under H_1 is at the left side of the distribution under H_0. This means that we can't include the right tail as part of the error because we can't accept $H_1 = 0.3$ given this situation.

$$CR = \{(x_1, \ldots, x_n) \in \mathcal{R}^n : | \quad T_{H_0}(x_1, \ldots, x_n) < -Z_{0.95}\}$$

$$CR = (-\infty, -1.64485) \quad and \quad T_{H_0} \notin CR$$

So, We can't reject H_0 given with $\alpha = 0.05$ significance level.