

01204211 Discrete Mathematics

Lecture 9: Counting 1

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September 8, 2015

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- ▶ How to find these 2 representatives? One of your friends suggests that to be fair to everyone, you have to look at every possible pair and see how the 2 members of the pair play together as a team.
- ▶ It might take a very long time, you think. How many pairs are there?

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- ▶ The correct number of pairs is 780; too many possibilities to consider, you conclude.

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- ▶ Your friend argue that 15 is too small. Because the number of members is 40 and we will miss someone there.
- ▶ So you ask, how many pairs one have to randomly choosing a pair from 40 members so that it is very likely that every member is picked once?
- ▶ You try to calculate the number, but your friend starts writing a program to simulate.

Club representatives (2)

- ▶ Here's the table of the simulation. For each value of number of random pairs, 2,000 simulations has been conducted.

Number of pairs to random	% of choosing everyone once
20	0.00
30	0.00
40	0.15
50	2.45
60	12.05
80	51.65
100	78.00
120	91.25
140	97.10

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- ▶ You end up choosing randomly 100 pairs, as it has about 80% chance. You feel so tired, but you keep wondering if you can calculate the number without having to write a program.

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- ▶ To find the best team organization, you ask them to try all possible configurations of race choices against AI players. How many games do you have to watch?

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- ▶ Each member has 3 choices and this member's choice is independent of the other. Therefore, there are $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$ possible ways.

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- ▶ Each member has 3 choices and this member's choice is independent of the other. Therefore, there are $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$ possible ways.
- ▶ You are still tired from watching 100 pairs of players. So you change your mind and ask them to try only configurations that contain all the three races. How many are there?