01204211 Discrete Mathematics
Lecture 3: Logical proofs

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How to prove a mathematical statement

Given propositions P and Q, these are a very useful logical equivalences (referred to as the De Morgan's Laws).

- $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
- $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

(Note that $\neg$ takes precedence over $\vee$ or $\wedge$.)

How can we prove that the first statement is true?

Proof by exhaustion

> For any proposition P and Q, $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.

Proof.
We will prove by exhaustion. There are 4 cases as in the truth table below.

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg Q \wedge \neg P$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Note that for all possible truth values of P and Q, $\neg(P \vee Q)$ equals $\neg P \wedge \neg Q$. Thus, the statement is true. $\qquad\square$