01204211 Discrete Mathematics
Lecture 6: Mathematical Induction 1

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September 1, 2015

Mathematical Induction

- In this lecture, we will focus on how to prove properties on natural numbers.
- For example, we may want to prove that for any integer $n \geq 1$,

  $$\sum_{i=1}^{n} i = i(i+1)/2,$$

  or for any integer $n \geq 1$,

  $$\sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1),$$

  or "We can pay any integer amount $x \geq 4$ baht with 2-baht coins and 5-baht coins."

A review of the summation notation (by examples)

- $\displaystyle\sum_{i=1}^{10} i = 1 + 2 + \cdots + 10.$
  (reads "sum from $i = 1$ to 10 of i" or "sum of i from $i = 1$ to 10")
- $\displaystyle\sum_{i=7}^{9} (i^2 + i) = (7^2 + 7) + (8^2 + 8) + (9^2 + 9.$
  (reads "sum from $i = 7$ to 9 of $i^2 + i$" or "sum of $i^2 + i$ from $i = 7$ to 9")
- The range of the index may be sets. For example, let $A = \{1, 2, 4, 15\}$, we have that $\displaystyle\sum_{i\in A} i^2 = 1^2 + 2^2 + 4^2 + 15^2$.
- What is $\sum_{i=5}^{2} i$? Note that in this case, the range is empty. This sum is called an **empty sum**. By convention, we define it to be zero.

Informal arguments

- Let's try to check that $\sum_{i=1}^{n} i = i(i+1)/2$, for any integer $n \geq 1$, by experimentation.
- Try $n = 1$: LHS: 1, RHS: $1(1+1)/2 = 1$, OK
- Try $n = 2$: LHS: $1 + 2 = 3$, RHS: $2(2+1)/2 = 3$, OK
- Try $n = 3$: LHS: $1 + 2 + 3 = 6$, RHS: $3(3+1)/2 = 6$, OK
- Try ...
- With this approach, we can't actually prove this statement.