01204211 Discrete Mathematics
Lecture 4: Proof techniques 1

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Proof techniques

Using inference rules, we can prove facts in propositional logic. However, in many cases, we want to prove wider range of mathematical facts. Inference rules play crucial parts in providing high-level structures for our proofs.

In this lecture, we will focus on two general proof techniques that originate from two simple inference rules.

- ▶ Direct proofs
- ▶ Indirect proofs

Terminologies

These are terminologies used when showing mathematical facts.

- A **theorem** is a statement that can be argued to be true.
- A **proof** is the sequence of statements forming that mathematical argument.
- An **axiom** is a statement that is assumed to be true. (Note that we do not prove an axiom; therefore, the validity of a theorem proved using an axiom relies of the validity of the axiom.)
- To prove a theorem, we may prove many simple lemmas to make our argument. A **lemma**, in this sense, is a smaller theorem (or a supportive one).
- A **corollary** is a theorem which is a "fairly" direct result of other theorems.
- A **conjecture** is a statement which we do not know if it is true or false.

Fermat's Last Theorem

> **Theorem:** No three positive integers a, b, and c can satisfy
> the equation $a^n + b^n = c^n$ when $n > 2$.

This theorem has been conjectured by Pierre de Fermat in 1637. It
remained a conjecture until Andrew Wiles proved it in 1994.

Goldbach's conjecture

> **Conjecture:** Every even integer greater than 2 can be expressed as the sum of two primes.

In 1742, Christian Goldbach proposed this cojecture to Leonhard Euler. It remains unsolved.

Euclid's axioms

Euclidean geometry is defined by the following 5 postulates (axioms).

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. (The parallel postulate) If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

References: Weisstein, Eric W. "Euclid's Postulates." From MathWorld–A Wolfram Web Resource.
http://mathworld.wolfram.com/EuclidsPostulates.html

The triangle postulate

The following statement is called the triangle postulate.

> The sum of the angles in every triangle is 180°.

The only way to prove this in Euclidean geometry is to use the parallel postulate.
Is this statement always true?
There are other geometries where Euclid's 5th postulate is not true; then the triagle postulate may not be true in those cases.
Can you imagine one?

Direct proofs

- When we want to prove a fact