01204211 Discrete Mathematics
Lecture 4: Proof techniques 1

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Proof techniques

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In this lecture, we will focus on two general proof techniques that originate from two simple inference rules.

- Direct proofs
- Indirect proofs

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- A **conjecture** is a statement which we do not know if it is true or false.

Fermat's Last Theorem

Theorem: No three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ when $n > 2$.

This theorem has been conjectured by Pierre de Fermat in 1637. It remained a conjecture until Andrew Wiles proved it in 1994.

Goldbach's conjecture

Conjecture: Every even integer greater than 2 can be expressed as the sum of two primes.

In 1742, Christian Goldbach proposed this cojecture to Leonhard Euler. It remains unsolved.

Euclid's axioms

Euclidean geometry is defined by the following 5 postulates (axioms).

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. (The parallel postulate) If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

References: Weisstein, Eric W. "Euclid's Postulates." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/EuclidsPostulates.html

The triangle postulate

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Is this statement always true?
There are other geometries where Euclid's 5$^{\text{th}}$ postulate is not true; then the triagle postulate may not be true in those cases.
Can you imagine one?

Direct proofs

- When we want to prove a fact