01204211 Discrete Mathematics
Lecture 6: Mathematical Induction 1

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or "We can pay any integer amount $x \geq 4$ baht with 2-baht coins and 5-baht coins."

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- The range of the index may be sets. For example, let $A = \{1, 2, 4, 15\}$, we have that $\sum_{i \in A} i^2 = 1^2 + 2^2 + 4^2 + 15^2$.
- What is $\sum_{i=5}^{2} i$? Note that in this case, the range is empty. This sum is called an **empty sum**. By convention, we define it to be zero.

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- Try $n = 3$: LHS: $1 + 2 + 3 = 6$,

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- With this approach, we can't actually prove this statement.