01204211 Discrete Mathematics
Lecture 9: Counting 1

Jittat Fakcharoenphol

September 8, 2015

Let's count

Club representatives: You are a second year student. Your board game club has 40 members which are in the first year. There is a big competition very soon, so the club has to find exactly 2 representatives (from the first-year students) for the competition.

- How to find these 2 representatives? One of your friends suggests that to be fair to everyone, you have to look at every possible pair and see how the 2 members of the pair play together as a team.
- It might take a very long time, you think. How many pairs are there?

Club representatives (1)

- To choose the member of the pair, you pick the first member and then pick the second member.
- There are 40 ways to choose the first member. For every person you pick as the first member, there are exactly 39 left to pick as the second one. Therefore, there are $40 \cdot 39 = 1,560$ ways.
- Wait.. This is over counting. Picking a as the first member and b as the second member results in the same pair as picking b first and a second. Thus, every pair is counted twice.
- The correct number of pairs is 780; too many possibilities to consider, you conclude.

Club representatives (2)

- Since 780 is too many, you decide to randomly choose 15 pairs of representatives and observe how each pair plays.
- Your friend argue that 15 is too small. Because the number of members is 40 and we will miss someone there.
- So you ask, how many pairs one have to randomly choosing a pair from 40 members so that it is very likely that every member is picked once?
- You try to calculate the number, but your friend starts writing a program to simulate.

Club representatives (2)

- Here's the table of the simulation. For each value of number of random pairs, 2,000 simulations has been conducted.

Number of pairs to random	% of choosing everyone once
20	0.00
30	0.00
40	0.15
50	2.45
60	12.05
80	51.65
100	78.00
120	91.25
140	97.10

- You end up choosing randomly 100 pairs, as it has about 80% chance. You feel so tired, but you keep wondering if you can calculate the number without having to write a program.

Club representative again

A team: Another team competition is coming up. It requires a team of 5 players. In the team, each player can play either as Protoss, Terrans, or Zerg. Luckily, only one team of 5 members volunteers to participate.

- To find the best team organization, you ask them to try all possible configurations of race choices against AI players. How many games do you have to watch?
- Each member has 3 choices and this member's choice is independent of the other. Therefore, there are $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$ possible ways.
- You are still tired from watching 100 pairs of players. So you change your mind and ask them to try only configurations that contain all the three races. How many are there?