01204211 Discrete Mathematics
Lecture 3: Logical proofs

Jittat Fakcharoenphol

August 18, 2015

How to prove a mathematical statement

Given propositions P and Q, these are a very useful logical equivalences (referred to as the De Morgan's Laws).

- $\neg(P \lor Q) \equiv \neg P \land \neg Q$
- $\neg(P \land Q) \equiv \neg P \lor \neg Q$

(Note that $\neg$ takes precedence over $\lor$ or $\land$.)

How can we prove that the first statement is true?

Proof by exhaustion

For any proposition P and Q, $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.

Proof.

We will prove by exhaustion. There are 4 cases as in the truth table below.

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg Q \wedge \neg P$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Note that for all possible truth values of P and Q, $\neg(P \vee Q)$ equals $\neg P \wedge \neg Q$. Thus, the statement is true. $\qquad \square$