01204211 Discrete Mathematics
Lecture 6: Mathematical Induction 1

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or "We can pay any integer amount $x \geq 4$ baht with 2-baht coins and 5-baht coins."

Informal arguments

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- Try $n = 2$:

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- Try $n = 3$: LHS: $1 + 2 + 3 = 6$,

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- With this approach, we can't actually prove this statement.