

01204211 Discrete Mathematics
Lecture 19: Modular arithmetic

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Review

In previous lectures, we studied various properties of integers and primes, and discussed primality testing algorithms. In this lecture, we will dive deeper into **modular arithmetic**, where we work with integers that “wrap around” when reaching a particular value, called the “modulus”.

An example

Alice was born on July. Betty was born in the next 7 months. Before Betty was born for 3 months, Cathy was born. Dave was born 10 months before Cathy. What is Dave's birth month?

We shall encode 12 months as numbers from 0 (for January) to 11 (for December). Let a be Alice's birth month. If we denote by b, c , and d birth months of Betty, Cathy, and Dave, we can write down the conditions as follows:

$$\begin{aligned}a \bmod 12 &= 6 \bmod 12 \\(a + 7) \bmod 12 &= b \bmod 12 \\(b - 3) \bmod 12 &= c \bmod 12 \\(c - 10) \bmod 12 &= d \bmod 12\end{aligned}$$

This is a familiar system of linear equations, but with a little twist: a “modulus” at the end.

Congruence

To deal with these equations, Carl Friedrich Gauss introduced a notation for them, called congruence. Instead of writing

$$(a + 7) \bmod 12 = b \bmod 12,$$

we write

$$a + 7 \equiv b \pmod{12}.$$

Formally, if

$$x \bmod m = y \bmod m,$$

we can write

$$x \equiv y \pmod{m}.$$

The system with the congruence notation

Let's rewrite our previous set of equations using this notation:

$$\begin{aligned}a &\equiv 6 \pmod{12} \\a + 7 &\equiv b \pmod{12} \\b - 3 &\equiv c \pmod{12} \\c - 10 &\equiv d \pmod{12}\end{aligned}$$

Now everything looks fairly much like normal equations. But do they behave the same?