Category: C consists of class Objects $Ob(C)$, class Morphisms $hom(C)$. $f \in hom_C(a,b)$, f is a morphism from a to b

$hom_C(a,b)$ is all morphisms from a to b

associative law : $f\circ(g\circ h) = (f\circ g)\circ h$

identitive law : if $f \in Hom_C(a,b)$, $f: a\to b$, $f\circ id_a = id_b \circ f = f$

$$\begin{tikzcd} a \arrow[loop above, "id_a"] \arrow[r, "g"] & b \arrow[loop above, "id_b"] \end{tikzcd}$$

inverse mapping :

$$\begin{tikzcd} a \arrow[r, "id_a"] & a \arrow[dl, "f"] \\ b \arrow[u, "f^{-1}"] & \end{tikzcd}$$

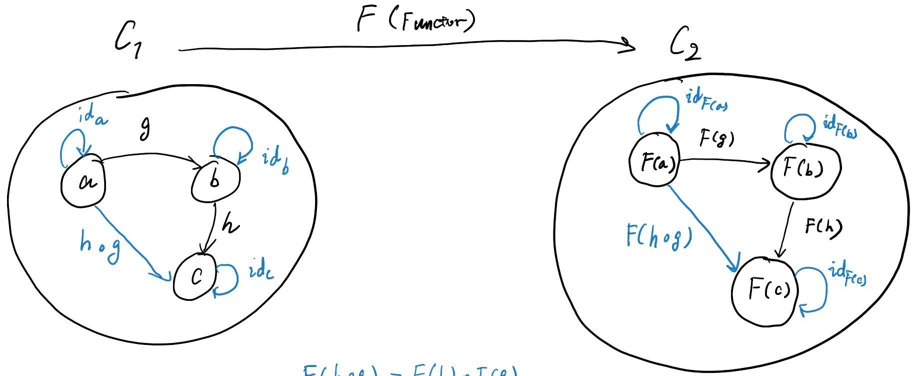
injective : If $x_1 \neq x_2$, then $f(x_1)\neq f(x_2)$, each $y\in Y$ has exactly one unique preimage $x \in X$

surjective : $\forall y \in Y$, $\exists x\in X$ such that $f(x) = y$

bijective : For $f: X\to Y$, each $x\in X$ maps to exactly one unique $y\in Y$

f^{-1} may not exist. But if $f: X\to Y$ is bijective, f^{-1} always exists and is an inverse mapping of f.

Functor :



$F(h\circ g) = F(h)\circ F(g)$

Bifunctor :

$u, v\in C \Rightarrow u\times v\in C$

$\because$ $u\xrightarrow{f} v$, $w \xrightarrow{g} z$

$\therefore$ $u\otimes w \xrightarrow{f\otimes g} v\otimes z$

$$\begin{tikzcd} u \arrow[r, "f"] & v \\ u\times w \arrow[u] \arrow[d] \arrow[r, "f\times g"] & v\times z \arrow[u] \arrow[d] \\ w \arrow[r, "g"] & z \end{tikzcd}$$

$\otimes$: $\underbrace{C\times C}_{(u,w)\ \to\ u\times w} \to C$

monoidal product / tensor product

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monoidal category : C have monoidal structure and monoidal structure consists of :

1) bifunctor $\otimes$: $C\times C\to C$

2) identity object $I\in C$

3) natural isomorphism $\alpha_{x,y,z}$: $x\otimes(y\otimes z) \cong (x\otimes y)\otimes z$

α is an associator

4) natural isomorphism λ_x : $I\otimes x\cong x$ (left unitor)

ρ_x : $x\otimes I\cong x$ (right unitor)

natural isomorphism is morphism of functors homeomorphic : $f: X \subset \mathbb{R}^m \to Y \subset \mathbb{R}^n$

if :

1. f is bijective
2. f is continuous
3. f^{-1} is continuous

then X and Y are homeomorphic

diffeomorphism fullfills the conditions 1, 2, 3 and 4. f and f^{-1} are smooth (means that f and f^{-1} are infinitely differentiable)

Multicategory : Multicategory is like a category, except that one allows multiple inputs and a single output.

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homo- : a map that preserves the structure

iso- : bijective & homo

endo- : a morphism from an object to itself

for instance : an endomorphism of a vector space V is a linear map $f: V \to V$

auto- : iso & endo

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epi- : surjective & homo

mono- : injective & homo, for example if $f \circ g_1 = f \circ g_2$, then $g_1 = g_2$

if $g_1 \neq g_2$, then $f \circ g_1 \neq f \circ g_2$

bi- : epi & mono, note that every isomorphism is bimorphism, but not every bimorphism is isomorphism.

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symmetric monoidal category :

bifunctor $\otimes : M \times M \to M$

unit object : $1 \in M$

associator : $a_{x,y,z} : (x \otimes y) \otimes z \to x \otimes (y \otimes z)$

left unitor : $\lambda_x : 1 \otimes x \to x$

right unitor : $\rho_x : x \otimes 1 \to x$

braiding : $B_{x,y} : x \otimes y \to y \otimes x$

the associator, unitor, braiding should obey triangle, pentagon, hexagon identity.