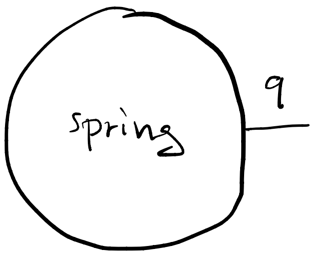
Syntax



box spring = $\{q : (\text{Velocity}, \text{Force})\}$

box is a typed set of ports

semantics

system

types correspond to the nature of physical interface linguistics Frege compositionality principle

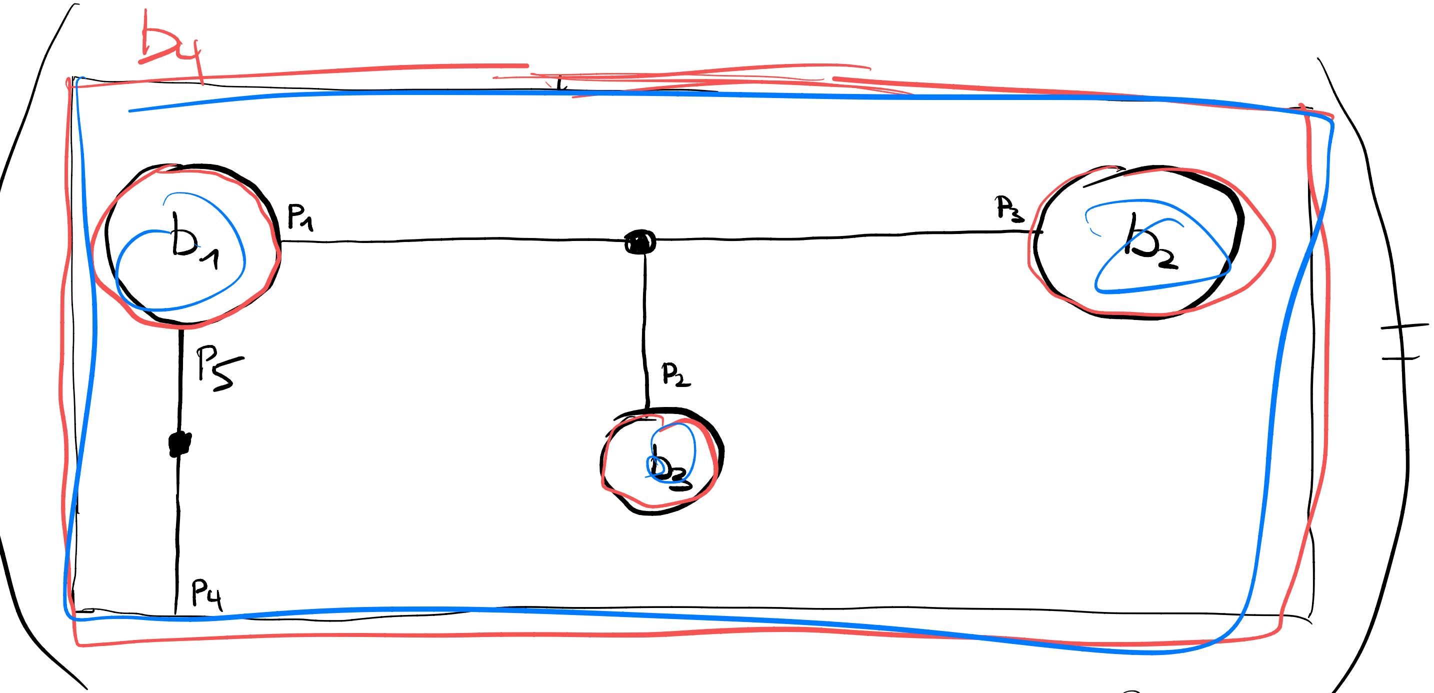
semantics of composite system

composite system

$\overbrace{\text{the meaning of the } \overbrace{\text{whole}}^{\text{composite system}}}^{\text{semantics of composite system}}$ is determined

by the $\underbrace{\text{meaning of the } \overbrace{\text{parts}}^{\text{subsystems}}}_{\text{semantics of the subsystems}}$

and $\underbrace{\text{how the parts are combined}}_{\text{syntax}}$



$= \text{expr}$

expression in the EPHS syntax

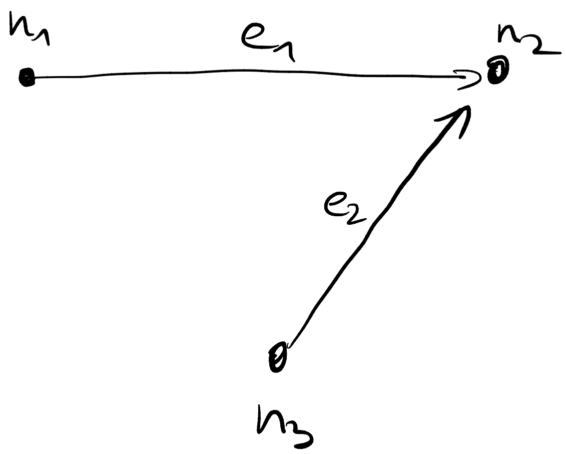
"$b_4 = \text{expr}(b_1, b_2, b_3)$" Julia syntax

$$C = \left(\begin{tikzcd} \text{Port} \arrow[r, "\text{ij}"] \arrow[d, "\text{box}"] \arrow[dr, "\text{ipt}"'] & \text{Junction} \arrow[d, "\text{jpt}"] & \text{BoundaryPort} \arrow[l, "\text{bj}"'] \arrow[dl, "\text{bpt}"] \\ \text{Box} & \text{PortType} & \end{tikzcd}\right)$$

$$\text{jpt} \circ \text{ij} = \text{ipt}$$

$$\text{jpt} \circ \text{bj} = \text{bpt}$$

directed graph



graphical / diagramatic representation

instance

```
g = Graph(
  N=[1,2,3],
  E=[(1,2),(3,2)]
)
```

data structure / data type

```
struct Graph
  N:: Vector{Int}
  E:: Vector{Pair{Int}}
end
```

$E = \{e_1, e_2\}$

$V = \{n_1, n_2, n_3\}$

$src(e_1) = n_1$

$src(e_2) = n_3$

$tgt(e_1) = n_2$

$tgt(e_2) = n_2$

mathematical structure

general

abstract

schema

$$\begin{tikzcd} E \arrow[r, shift left, "source"] \arrow[r, shift right, "target"'] & N \end{tikzcd}$$

Syntax (mulicategory)

objects: all boxes $\qquad b = \{ \underbrace{p_1 : P_1,\ p_2 : P_2, \ldots}_{\text{finite set of typed ports}} \}$

port $\quad$ port type

morphisms: all composition patterns

$$\text{expr} : (b_1, b_2) \to b_3$$

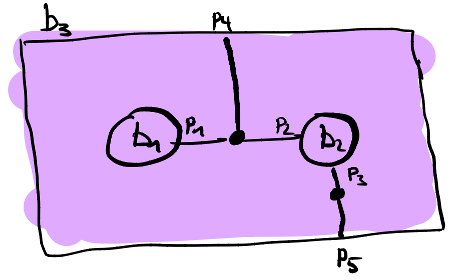
$b_1 = \{ p_1 : P_1 \}$

$b_2 = \{ p_2 : P_2,\ p_3 : P_3 \}$

$b_3 = \{ p_4 : P_4,\ p_5 : P_5 \}$

constraints: $P_1 = P_2 = P_4$

$P_3 = P_5$



expr: $\mathcal{C} \to \text{Set}$

functor sends objects in $\mathcal{C}$ (e.g. Port, Box, ...) to sets (e.g. of ports, boxes, ...)

and sends morphisms in $\mathcal{C}$ (e.g. box) to functions (e.g. from the set of ports to the set of boxes)

$\mathcal{C}$ is a free category generated by this graph

$$\begin{tikzcd}
\text{Port} \arrow[r, "\text{ij}"] \arrow[d, "\text{box}"] \arrow[dr, "\text{ipt}"] & \text{Junction} \arrow[d, "\text{jpt}"] & \text{BoundaryPort} \arrow[l, "\text{bj}"'] \arrow[dl, "\text{bpt}"] \\
\text{Box} & \text{PortType} &
\end{tikzcd}$$

Vect

objects: all vector spaces

morphism: all linear functions

$f : V_1 \to V_2$

V_1 as above
$V_2 = \mathbb{R}^1$

$V_1 = \{ x = (a, b) \mid a \in \mathbb{R}, b \in \mathbb{R} \} = \mathbb{R}^2$
and structure $(+, \cdot)$

$\forall v_1 = (a, b) \in V_1$:

$f(v_1) = \underbrace{a + 2b}_{\text{implementation of } f}$

FinSet category of finite sets

objects: finite sets

morphisms: functions between finite sets

$$\{\underbrace{\overset{1}{a},\ \overset{2}{b},\ \overset{3}{c}}_{\text{names}}\} \cong \underbrace{\{\underbrace{1,\ 2,\ 3}_{\text{ids}}\}}_{\text{skeleton of FinSet}} = \underline{3}$$

every finite set with n elements is isomorphic to $\underline{n}$

the isomorphism is defined by assigning id numbers

$\coprod$: coproduct in FinSet (called disjoint union)

def. by universal property in general

we define it directly in FinSet

$\coprod$ is a functor from FinSet $\times$ FinSet $\to$ FinSet

$$\{1,2\} \sqcup \{1,2,3\} = \{(1,\ell),(2,\ell),(1,r),(2,r),(3,r)\}$$

$\{1,2\} \cong \underline{2}$, $\{1,2,3\} \cong \underline{3}$, $\{(1,\ell),\dots,(3,r)\} \cong \underline{5}$

"labels" make elemts disjoint

$f : \{1,2\} \to \{1\}$ $\quad f(1) = 1$, $f(2) = 1$

$g : \{1,2,3\} \to \{1,2\}$ $\quad g(1) = 1$, $g(2) = (1)$, $g(3) = 2$

$$f \sqcup g : \underbrace{\{1,2\} \sqcup \{1,2,3\}}_{\{(1,\ell),(2,\ell),(1,r),(2,r),(3,r)\}} \to \underbrace{\{1\} \sqcup \{1,2\}}_{\{(1,\ell),(1,r),(2,r)\}}$$

the above definition of coproduct is biased towards arity 2

ℓ, r

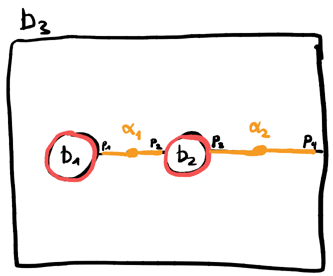
left summand

right summand

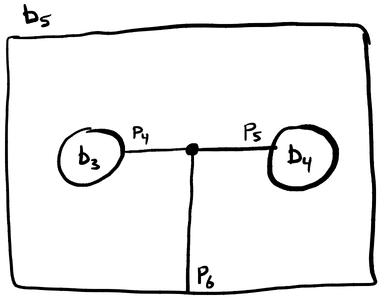
the C-Set approach is unbiased with regards to arity

Exercise

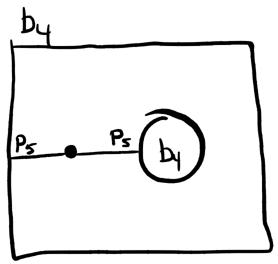
$f : (b_1, b_2) \to b_3 \quad =$



$g : (b_3, b_4) \to b_5 \quad =$



$id_{b_4} : b_4 \to b_4 \quad =$



Task: • form the composite $g \circ (f, id_{b_1}) = g \circ_1 f$

(maybe use your human intuition first)

• explain the process in detail by concretizing the commuting diagram

$$\begin{tikzcd}
 & & b_{\mathrm{o}} \arrow[d, "\mathrm{j}_{\mathrm{o}}^{g}"] \\
 & \coprod_i b_i \arrow[r, "\mathrm{j}_{\mathrm{i}}^{g}"] \arrow[d, "\coprod_i \mathrm{j}_{\mathrm{o}}^{f_i}"] & J^g \arrow[d, "\mathrm{i}_{\mathrm{o}}"] \\
\coprod_i \coprod_k b_{i,k} \arrow[r, "\coprod_i \mathrm{j}_{\mathrm{i}}^{f_i}"] & \coprod_i J^{f_i} \arrow[r, "\mathrm{i}_{\mathrm{i}}"] & J
\end{tikzcd}$$

to the situation at hand

(your explanation should be (close to) an algorithm)

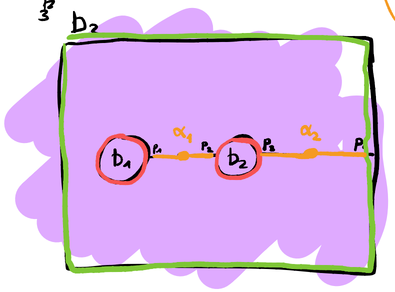
Exercise

$p_1 : P_1$

$$f : (\underbrace{\{p_1\}}_{b_1}, \underbrace{\{p_2, p_3\}}_{b_2}) \to (\underbrace{\{p_4\}}_{b_3})$$

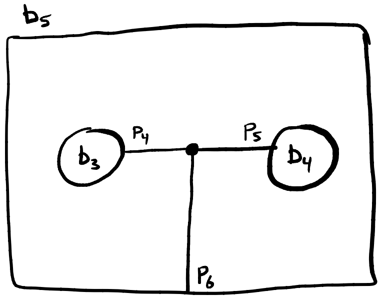
name ↓ Interface ↓ in, out

$$f : (b_1, b_2) \to b_3 \quad =$$

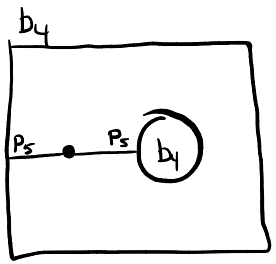


$$b_1 = \{p_1 : P_1, \; p_2 : P_2, \ldots\}$$

$$g : (b_3, b_4) \to b_5 \quad =$$



$$id_{b_4} : b_4 \to b_4 \quad =$$



$\coprod b_i$ → Junction ← BoundaryPort

Port ↓ Box ← label to make port distinct

a "C-Set" is a functor from C to Set/FinSet/skeleton of FinSet

$$C = \begin{tikzcd} \text{Port} \arrow[r, "ij"] \arrow[d, "box"] \arrow[dr, "ipt"] & \text{Junction} \arrow[d, "jpt"] & \text{BoundaryPort} \arrow[l, "bj"'] \arrow[dl, "bpt"] \\ \text{Box} & \text{PortType} & \end{tikzcd}$$

$\coprod b_i$, $i = 1,2$

data defining f is encoded by a C-Set, $\underbrace{\text{a C-Set}}$ call it S_f

def. $S_f(\text{Port})$ defines $S_f(ij)$

Port	box	ij	ipt
p_1	b_1	α_1	P_1
p_2	b_2	α_1	P_1
p_3	b_2	α_2	P_2

Junction	jpt
α_1	P_1
α_2	P_2

BoundaryPort	bj	bpt
p_4	α_2	P_2

Box
b_1
b_2

PortType
P_1
P_2

$S_f : \text{Port} \mapsto \{p_1, p_2, p_3\}$

$S_f : \text{Box} \mapsto \{b_1, b_2\}$

my (anonymous) function syntax

$S_f : \text{box} \mapsto (p_1 \mapsto b_1, p_2 \mapsto b_2, p_3 \mapsto b_2)$

$S_f(\text{box})(p_1) = b_1$

$S_f(\text{box})(p_2) = b_2$

$S_f(\text{box})(p_3) = b_2$

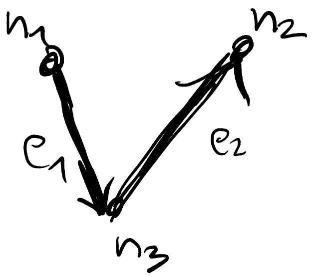
Catlab.jl

other example of $\underbrace{C\text{-Set}}_{C \to \mathrm{FinSet}}$ and $\underbrace{\text{attributed } C\text{-Set}}_{\text{"acset"}}$

$\downarrow$

"edge-weighted" directed graph

$$C = \left(\begin{tikzcd} \mathrm{Edge} \arrow[r, shift left, "src"] \arrow[r, shift right, "tgt"'] \arrow[d, "weight"'] & \mathrm{Node} \\ \mathrm{Weight} & \end{tikzcd} \right)$$



Node
n_1
n_2
n_3

Edge	src	tgt	weight
e_1	n_1	n_3	3.4
e_2	n_3	n_2	8.9

"edge list"

Task: • form the composite $g \circ (f, id_{b_1}) = g \circ_1 f$

$i = 1,2$

(maybe use your human intuition first)

• explain the process in detail by concretizing the commuting diagram

$$
\begin{tikzcd}
 & & b_o \arrow[d, "\mathrm{j}_{\mathrm{o}}^{g}"] \\
 & \coprod_i b_i \arrow[r, "\mathrm{j}_{\mathrm{i}}^{g}"] \arrow[d, "\coprod_i \mathrm{j}_{\mathrm{o}}^{f_i}"] & J^g \arrow[d, "\mathrm{i}_{\mathrm{o}}"] \\
\coprod_i \coprod_k b_{i,k} \arrow[r, "\coprod_i \mathrm{j}_{\mathrm{i}}^{f_i}"] & \coprod_i J^{f_i} \arrow[r, "\mathrm{i}_{\mathrm{i}}"] & J
\end{tikzcd}
$$

$i_o \circ j_i^g = i_i \circ \coprod j_o^{f_i}$

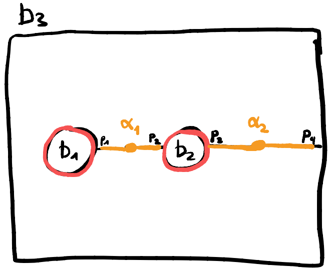
Equation defines the result

to the situation at hand

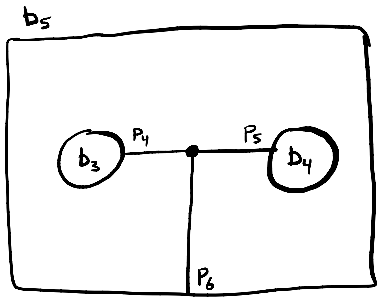
(your explanation should be (close to) an algorithm)

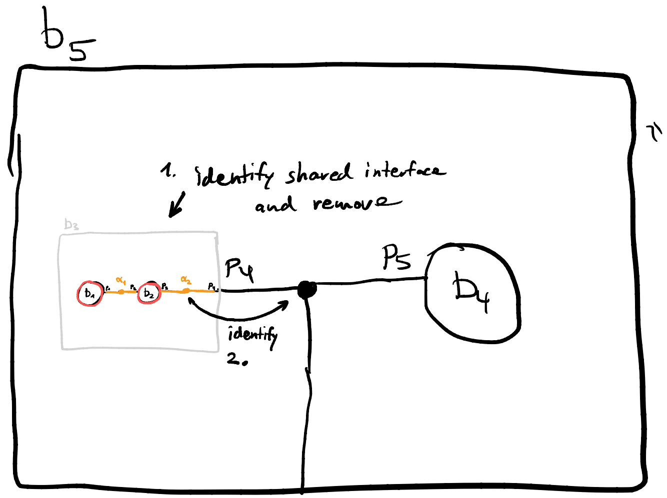
Solution

$f : (b_1, b_2) \to b_3$ =



$g : (b_3, b_4) \to b_5$ =

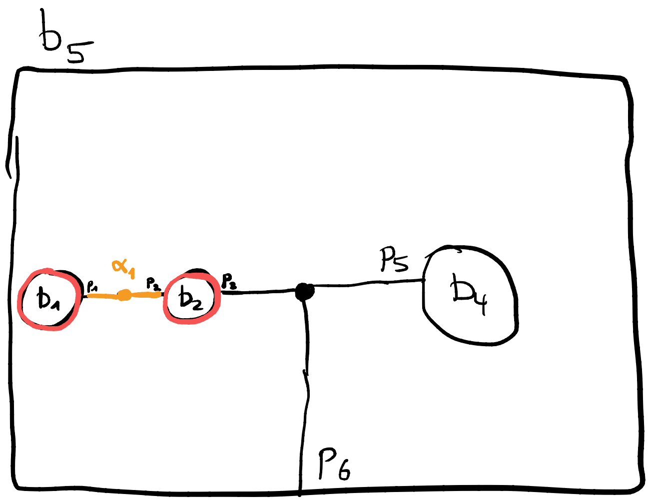




"immediate" step is not a C-set

a) form composite $g \circ_1 f$

"substitute def. of f into the first slot of g"



result

shared interfaces = inner boxes of g

$$\begin{tikzcd}
 & & b_o \arrow[d, "j_o^g"] \\
 & \coprod_i b_i \arrow[r, "j_i^g"] \arrow[d, "\coprod_i j_o^{f_i}"] & J^g \arrow[d, "i_o"] \\
\coprod_i \coprod_k b_{i,k} \arrow[r, "\coprod_i j_i^{f_i}"] & \coprod_i J^{f_i} \arrow[r, "i_i"] & J
\end{tikzcd}$$

$b_o = \{p_6\} = b_5$

$J^g = \{a_1, a_2\}$

$(\underbrace{\{p_4\}}_{b_3}, \underbrace{\{p_5\}}_{b_4}) = \coprod_i b_i$

commutativity of "pushout square"

$i_o \circ j_i^g = i_i \circ \coprod j_o^{f_i}$

does the identification of junctions

index i is for inner boxes of g

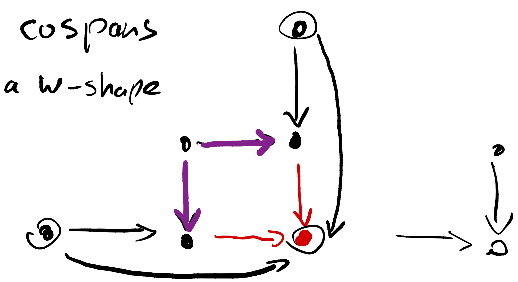
"cospan" shape/pattern

$\bullet \longrightarrow \bullet \longleftarrow \bullet$

$\bullet \longleftarrow \bullet \longrightarrow \bullet$

"span"

2 cospans in a w-shape



limit of a span diag. is called "pushout"

1. Complete example above

2. look at Catlab.jl (acsets)

- tell it about C (graphs, "bond-graph" syntax)

- create a Julia datatype for such acsets

- play: create some examples

  (understand again the connection between the visual and the table-based representation of an expression in the syntax)