My Note

Jiandong Zhao

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Abstract

This is a note.

1 Category Theory

1.1 Structure of category

The fundamental structure in category are composition and identity.

Composition: For any pair of morphisms where the first ends where the second starts there exists a composition.
Identity: For any object there exists an identity morphism.

1.2 Functor

Functor is the morphisms from one category to another category that preserve the structure of category, i.e. the compositions and identities.
Natural transformation is a morphisms from one functor to another functor that preserve the structure of functor.

1.3 Acset

1.3.1 C-Set

For a functor $F : C \to Set$.
Algebra over an operad is a functor from multicategory to Sets, where C is a multicatoegory. Multicategory means multi input single output. The functor F gives concrete meaning to C, in the sense that it picks out some objects(Sets) and morphisms(n-ary Functions).

1.3.2 C-FinSet

For a functor $C - FinSet : C \to FinSet$, where C is a free category and FinSet is a finite Set.
C consists of objects and morphisms:

- objects: Box, Port, Junction, BoundaryPort, PortType
- morphisms: box, ij, bj, ipt, jpt, bpt

FinSet consists of objects and morphisms:

- objects: finite sets: set of boxes, set of ports, set of junctions, set of boundaryports, set of port types.
- morphisms: functions between finite sets (I will call it sets of mapping), e.g. $\{p_1 \to b_1, p_1 \to b_2, p_2 \to b_2 \dots\}$, $\{p_1 \to j_1, p_1 \to j_2, p_2 \to j_2 \dots\}$, $\dots$, where $S_f(Port)(p_1) = b_1$ means port 1 belongs to box 1.

C-Set is a functor:

- In terms of objects: it sends the objects in **C** to the objects in **FinSet**, e.g. $S_f(Box) = \{b_1, b_2, b_3 \dots\}$, $S_f(Port) = \{p_1, p_2, p_3 \dots\}$, $\dots$
- In terms of morphisms: it sends the morphisms in **C** to the morphisms in **FinSet**, e.g. $S_f(box) = \{p_1 \to b_1, p_1 \to b_2, p_2 \to b_2 \dots\}$, $S_f(ij) = \{p_1 \to j_1, p_1 \to j_2, p_2 \to j_2 \dots\}$, $\dots$

1.3.3 Coproduct

For a functor $\coprod : FinSet \times FinSet \rightarrow FinSet$
Coproduct in FinSet or in category is defined by the disjoint union of sets.

1.3.4 Acset

Attributed C-Set

Port	box	ij	ipt
p_1	b_1	α_1	P_1
p_2	b_2	α_1	P_1
p_3	b_2	α_2	P_2

Junction	jpt
α_1	P_1
α_2	P_2

BoundaryPort	bj	bpt
P_4	α_2	P_2

1.4 Syntax and Semantics

Principle of compositionality:
The meaning of the whole is determined by the meaning of the parts and how the parts are combined.

the whole: the composite system
the meaning of the whole: semantics of the composite system
the parts: subsystems
the meaning of the parts: semantics of the subsystems
how the parts are combined: syntax

conclusion: The semantics of the composite system is determined by the meaning of the semantics of the subsystems and syntax.

Syntax:

- objects: all boxes
  box is a typed set of ports, e.g. $b = \{p_1 : P_1, p_2 : P_2, \dots\}$
- morphisms: all composition patterns
  types correspond to the nature of physical interface

There are three types of expressions to represent syntax and semantics: diagrammatic representation(schema, e.g. free category C), mathematical structure(mapping or tables), data structure/data type(in julia code)

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