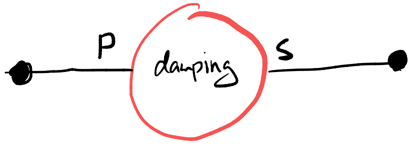

$s_e = \theta - \theta_0$, hence F_2 needs θ_0 to predict $\theta = \theta_0 + s.e$ (see below)

F_1 and F_2 are NN.

damping treated as a "black box" (without (E)PHS structure)

$$\begin{bmatrix} p.f \\ s.f \end{bmatrix} = \overset{NN}{F}\left(\theta_0, \begin{bmatrix} p.e \\ s.e \end{bmatrix}\right) = \begin{bmatrix} F_1(\theta_0, p.e, s.e) \\ F_2(\theta_0, p.e, s.e) \end{bmatrix}$$

$$\begin{bmatrix} p.f \\ s.f \end{bmatrix} = \frac{1}{\theta_0} d \begin{bmatrix} \theta_0 + s.e & -p.e \\ -p.e & \frac{(p.e)^2}{\theta_0 + s.e} \end{bmatrix} \begin{bmatrix} p.e \\ s.e \end{bmatrix} = \underbrace{\begin{bmatrix} d \cdot (p.e) \\ -d \frac{(p.e)^2}{\theta_0 + s.e} \end{bmatrix}}_{\text{difficult to see structural properties (related to thermodynamic consistency) here}}$$

in general $\underset{\text{n-dim}}{f} = \frac{1}{\theta_0} \underbrace{\underbrace{M(\theta_0, e)}_{\text{nxn matrix-valued function}} e}_{\substack{\text{non-linear function} \\ \text{(written as a matrix-vector product)}}}$

EPHS structure of a resistive component

structural properties:

- 1st law: $M(0, e) = 0$ for all e
- Onsager symmetry: symmetry of $M(\theta_0, e)$ for all θ_0 and e
- 2nd law: $M(\theta_0, e)$ is positive semi-definite

} $M(\theta_0, e)$ is s.p.s.d for all θ_0 and e

learning resistive models:

parametrize functions with input (θ_0, e) and s.p.s.d nxn matrices as output with an upper/lower triangular matrix D,

i.e. $M(\theta_0, e) = D^T(\theta_0, e) \underbrace{D(\theta_0, e)}_{\substack{\text{system of neural networks with} \\ \text{each component representing a non-zero entry in the} \\ \text{upper tridiagonal matrix}}}$

such that $\underbrace{D(0, e) e = 0}_{\text{How to include this constraint in the parametrization?}}$ (1st law)