Question 14 - 9231/Oct/Nov/20/12 Q2

Prove by mathematical induction that $7^{2n} - 1$ is divisible by 12 for every positive integer n. [5]

let P_n be the statement that for some positive integer n, $7^{2n} - 1$ is divisible by 12.

For the base case $n=1$,

$7^2 - 1 = 48 = 12(4)$ is divisible by 12

$\therefore P_1$ is true

Assume that, for some positive integer $n=k$, $7^{2k} - 1$ is divisible by 12.

Let $f(k) = 7^{2k} - 1$

$\therefore$ For $n = k+1$,

$$f(k+1) - f(k) = 7^{2(k+1)} - 1 - 7^{2k} + 1$$
$$= 7^{2k}(7^2 - 1)$$
$$= 7^{2k}(49 - 1)$$
$$= 12(4 \times 7^{2k})$$ is divisible by 12

$\therefore$ This implies $f(k+1)$ is divisible by 12.

$\therefore P_k \Rightarrow P_{k+1}$

$\therefore$ As P_1 is true and $P_k \Rightarrow P_{k+1}$, by mathematical induction, it is true that $7^{2n} - 1$ is divisible by 12 for all positive integers n