HW2 3(a)

$$E[\theta] = 0.8 = \frac{\alpha}{\alpha+\beta}$$

$$SE[\theta] = 0.02 = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}} = \frac{1}{\alpha+\beta}\sqrt{\frac{\alpha\beta}{\alpha+\beta+1}}$$

solve the system of two equations for alpha and beta...

2019 Midterm

1(b) $P(\theta) = Beta(\underset{\alpha}{1}, \underset{\beta}{1})$

1(c) $n=4, x=0$

1(i) $H_0: \theta \in [.49, .51]$ $\quad \delta = 1\%$

$H_a: \theta \notin [.49, .51]$

$$P(\theta|x) = Beta(\underset{\alpha+x}{1}, \underset{\beta+n-x}{5})$$

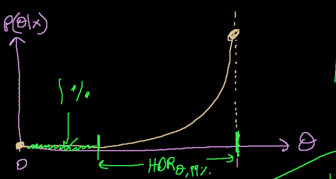
$$p_{val} = P(H_0|x) = P(\theta \in [.49, .51]) = \int_{.49}^{.51} \quad d\theta$$

$$= pbeta(.51, 1, 5) - pbeta(.49, 1, 5)$$

$$P(\theta \in HDR_{\theta, 1-\alpha} | x) = 1-\alpha$$

$$P(\theta \in CR_{\theta, 1-\alpha} | x) = 1-\alpha$$

HW3 2(j) $\quad x=6, n=6, \quad P(\theta) = Beta(1,1) \Rightarrow P(\theta|x) = Beta(7,1)$



$$HDR_{\theta, 99\%} = [qbeta(.01, 7, 1), 1]$$

$$\gamma = pbeta(qbeta(\gamma, 0.893, 1.876), 0.893, 1.876)$$

HW 3 2(f)

$H_a: \theta < 0.5$

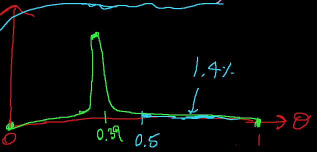
$H_0: \theta \geq 0.5$

You flip a coin 100 times and get 39 heads. Prove the coin is biased towards tails. Assume prior of indifference, P(θ) = U(0,1). $\alpha = 5\%$ $\quad \hat{\theta}_{MLE} = \frac{1}{7} = \bar{x}$ $\quad h_0 = 2$

$$\Rightarrow P(\theta|x) = Beta(40, 62)$$

$$p_{val} = P(H_0|x) = P(\theta \geq 0.5 | x)$$

$$= \int_{0.5}^{1} \quad d\theta = 1 - \int_{0}^{0.5} \quad d\theta$$

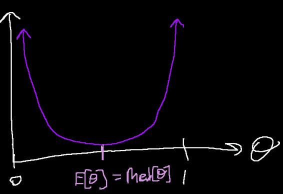


$$= \cancel{pbeta(1, 40, 62)} - pbeta(0.5, 40, 62) = 1.4\% < 5\% \Rightarrow \text{Reject } H_0$$

$$\mathcal{F}: Bin(n, \theta) \;\; n \text{ fixed}, \;\; P(\theta) = Beta(\alpha, \beta) \Rightarrow P(\theta|x) = Beta(\overset{0}{\alpha} + x, \overset{0}{\beta} + n - x)$$

Haldane's Idea

$$P(\theta) = Beta\left(\frac{1}{2}, \frac{1}{2}\right)$$



$$Mode[\theta]? \;\; \text{undefined}$$

$$= \frac{\alpha - 1}{\alpha+\beta-2} \quad \text{only if } \alpha \geq 1, \beta \geq 1$$

$$\frac{1}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

2020 2(a)

$$\frac{\cancel{\Gamma(1+\theta)}}{\cancel{\Gamma(1)}\cancel{\Gamma(\theta)}} \quad \theta\Gamma(\theta)$$

$$\mathcal{F}: X_1, \ldots, X_n \overset{iid}{\sim} Beta(1, \theta) = \frac{1}{B(1,\theta)} \cancel{x^{1-1}} (1-x)^{\theta-1} = \theta(1-x)^{\theta-1}$$

$$\mathcal{L}(\theta; x_1, \ldots, x_n) = \prod_{i=1}^{n} \theta(1-x_i)^{\theta-1} = \theta^n \prod (1-x_i)^{\theta-1}$$

2(b)

$$\ell(\theta; x_1, \ldots, x_n) = \ln(\quad) = n\ln(\theta) + (\theta - 1)\sum \ln(1-x_i)$$

2(c)

$$\ell' = \frac{d}{d\theta}[\quad] = \frac{n}{\theta} + \sum \ln(1-x_i) \overset{set}{=} 0 \Rightarrow \frac{n}{\theta} = -\sum \ln(1-x_i)$$

$$\Rightarrow \frac{\theta}{n} = -\frac{1}{\sum \ln(1-x_i)}$$

$$\Rightarrow \hat{\theta}_{MLE} = -\frac{n}{\sum \ln(1-x_i)}$$