ECON 409 Project Proposal

Idalee Vargas, Lora Yovcheva, Mauricio Vargas, Santiago Naranjo

Model

As suggested by Molodtsova and Papell, the model is outlined as follows:

$$egin{aligned} \Delta s_{t+1} &= eta_0 + eta_1 (ilde{\pi}_t - \pi_t) + eta_2 (ilde{y}_t - y_t) + \epsilon_t, \quad \epsilon_t \sim_{i.i.d.} N(0, \sigma^2) \ \Delta s_{t+1} &= s_{t+1} - s_t \ s_t &= \ln S_t \end{aligned}$$

This model is delineated as symmetric, non-smoothed, and homogeneous, with:

- S_t representing the price of 1 pound sterling in US dollars.
- $ilde{\pi}_t$ being the UK inflation rate.
- π_t being the US inflation rate.
- $ilde{y}_t$ indicating the UK output gap.
- y_t indicating the US output gap.

The output gaps are determined through:

$$egin{align} & \ln Y_t = y_t + ar{y}_t \ ar{y}_t = lpha_0 + ar{y}_{t-1} +
u_t, \quad
u_t \sim_{i.i.d.} N(0, \sigma_
u^2) \ y_t = lpha_1 y_{t-1} + \eta_t, \quad \eta_t \sim_{i.i.d.} N(0, \sigma_\eta^2) \ \end{matrix}$$

Introducing a stochastic trend for potential output and an autoregressive model for the output gap. It assumes that $\ln Y_t$ and $ar y_t$ are I(1) variables, and y_t is I(0).

For the UK output gap, we have:

$$egin{align} & \ln ilde{Y}_t = ilde{y}_t + ar{ ilde{y}}_t \ ar{ ilde{y}}_t = \gamma_0 + ar{ ilde{y}}_{t-1} + ilde{
u}_t, \quad ilde{
u}_t \sim_{i.i.d.} N(0, \sigma^2_{ ilde{
u}}) \ & ilde{y}_t = \gamma_1 ilde{y}_{t-1} + ilde{\eta}_t, \quad ilde{\eta}_t \sim_{i.i.d.} N(0, \sigma^2_{ ilde{\eta}}) \ \end{gathered}$$

This approach assumes $\ln ilde{Y}_t$ and $ar{ ilde{y}}_t$ are I(1) variables and $ilde{y}_t$ is I(0).

Data

The model utilizes monthly data, with updates occurring on the last business day of each month:

- π_t : US CPI index, released with a one-month delay.
- $\tilde{\pi}_t$: UK CPI index, released with a one-month delay.
- Y_t : US Industrial Production Index, released with a one-month delay.
- ullet $ilde{Y}_t$: UK Industrial Production Index, released with a two-month delay.

Given the inherent lags within the data, we propose the following autoregressive models for inflation rates, treating them as random walks:

$$egin{aligned} \pi_t &= \pi_{t-1} + e^\pi_t, & e^\pi_t \sim_{i.i.d.} N(0, \sigma^2_{e^\pi}) \ & ilde{\pi}_t &= ilde{\pi}_{t-1} + e^{ ilde{\pi}}_t, & e^{ ilde{\pi}}_t \sim_{i.i.d.} N(0, \sigma^2_{e^{ ilde{\pi}}}) \end{aligned}$$

Both models encapsulate the stochastic nature of inflation rates in their respective economies.

To account for the delays in data availability, we adjust our original exchange rate model as follows:

$$egin{aligned} \Delta s_{t+1} &= eta_0 + eta_1(\mathbb{E}_{t-1}[ilde{\pi}_t] - \mathbb{E}_{t-1}[\pi_t]) + eta_2(\mathbb{E}_{t-2}[ilde{y}_t] - \mathbb{E}_{t-1}[y_t]) + \epsilon_t \ \Delta s_{t+1} &= s_{t+1} - s_t \ s_t &= \ln S_t \end{aligned}$$

The expectations are determined based on the information available at time t-1 and t-2:

$$egin{aligned} \mathbb{E}_{t-1}[\pi_t] &= \pi_{t-1} \ \mathbb{E}_{t-1}[ilde{\pi}_t] &= ilde{\pi}_{t-1} \ \mathbb{E}_{t-1}[y_t] &= lpha_1 y_{t-1} \ \mathbb{E}_{t-2}[ilde{y}_t] &= \gamma_1^2 ilde{y}_{t-2} \end{aligned}$$

Integrating these expectations, the revised model becomes:

$$\Delta s_{t+1} = eta_0 + eta_1 (ilde{\pi}_{t-1} - \pi_{t-1}) + eta_2 (\gamma_1^2 ilde{y}_{t-2} - lpha_1 y_{t-1}) + \epsilon_t$$

Ridge Regression

We define the loss function as:

$$L(\widehat{\Delta s}_{t+1}|eta_0,eta_1,eta_2) = MSE(\widehat{\Delta s}_{t+1}) + \lambda \sum_{i=0}^2 eta_i^2.$$

where:

$$eta^* = (eta_0^*, eta_1^*, eta_2^*)' = rg\min_{eta \in \mathbb{R}^3} L(\widehat{\Delta s}_{t+1} | eta)$$

Tunning Parameters

- 1. λ : Regularization term in *Ridge* regression.
- 2. h: Rolling window size for model estimation and the output gaps, to avoid including structural breaks that could affect forecasts.

Estimating y_t and $ilde{y}_t$

To estimate $\Delta \ln Y_t$:

$$\Delta \ln Y_t = \Delta y_t + \Delta ar{y}_t$$

Knowing y_t is I(0):

$$egin{aligned} \Delta y_t &= y_t - y_{t-1} \ \mathbb{E}[\Delta y_t] &= \mathbb{E}[y_t - y_{t-1}] \ \mathbb{E}[\Delta y_t] &= 0 \end{aligned}$$

Since:

$$\mathbb{E}(y_{t+s}) = 0, orall s \in \mathbb{Z}$$

For $\Delta ar{y}_t$:

$$egin{aligned} \Deltaar{y}_t &= ar{y}_t - ar{y}_{t-1} \ \Deltaar{y}_t &= lpha_0 + ar{y}_{t-1} +
u_{t-1} - ar{y}_{t-1} \ \mathbb{E}[\Deltaar{y}_t] &= lpha_0 + \mathbb{E}[ar{y}_{t-1}] + \mathbb{E}[
u_{t-1}] - \mathbb{E}[ar{y}_{t-1}] \ \mathbb{E}[\Deltaar{y}_t] &= lpha_0 \end{aligned}$$

Thus:

$$egin{aligned} \mathbb{E}[\Delta \ln Y_t] &= \mathbb{E}[\Delta y_t] + \mathbb{E}[\Delta ar{y}_t] \ \mathbb{E}[\Delta \ln Y_t] &= lpha_0 \end{aligned}$$

We estimate the drift $lpha_0$ of $ar{y}_{t-1}$ as:

$$\hat{lpha}_0 = rac{1}{h} \sum_{i=t-1-h}^{t-1} \Delta \ln Y_i$$

h is the previously mentioned second tuning parameter.

Similarly, for $ilde{y}_{t-2}$:

$$\hat{\gamma}_0 = rac{1}{h} \sum_{i=t-2-h}^{t-2} \Delta \ln ilde{Y}_i$$