

Africast-Time Series Analysis & Forecasting Using R

5. Basic modeling and forecasting



Outline

1 Statistical forecasting

2 Benchmark methods

Outline

1 Statistical forecasting

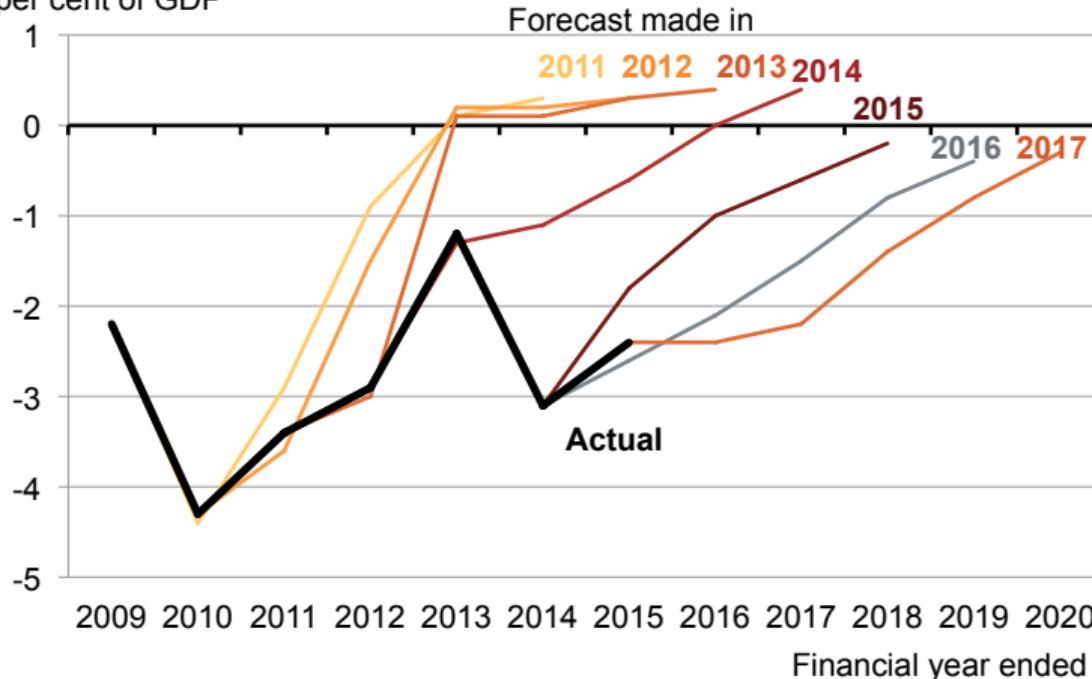
2 Benchmark methods

Forecasting is difficult

Commonwealth plans to drift back to surplus
show the triumph of experience over hope

GRATTAN
Institute

Actual and forecast Commonwealth underlying cash balance
per cent of GDP



What can we forecast?



What can we forecast?



What can we forecast?

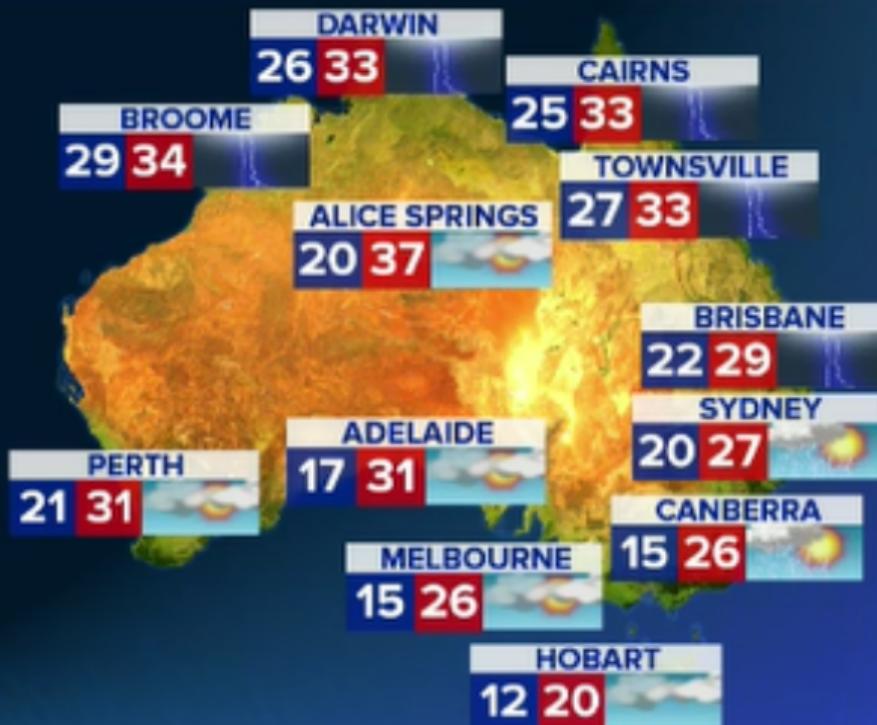


What can we forecast?



What can we forecast?

TOMORROW



What can we forecast?



What can we forecast?



Which is easiest to forecast?

- 1 daily electricity demand in 3 days time
- 2 timing of next Halley's comet appearance
- 3 time of sunrise this day next year
- 4 Google stock price tomorrow
- 5 Google stock price in 6 months time
- 6 maximum temperature tomorrow
- 7 exchange rate of \$US/AUS next week
- 8 total sales of drugs in Australian pharmacies next month

Which is easiest to forecast?

- 1 daily electricity demand in 3 days time
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-
- how do we measure “easiest”?
 - what makes something easy/difficult to forecast?

Factors affecting forecastability

Something is easier to forecast if:

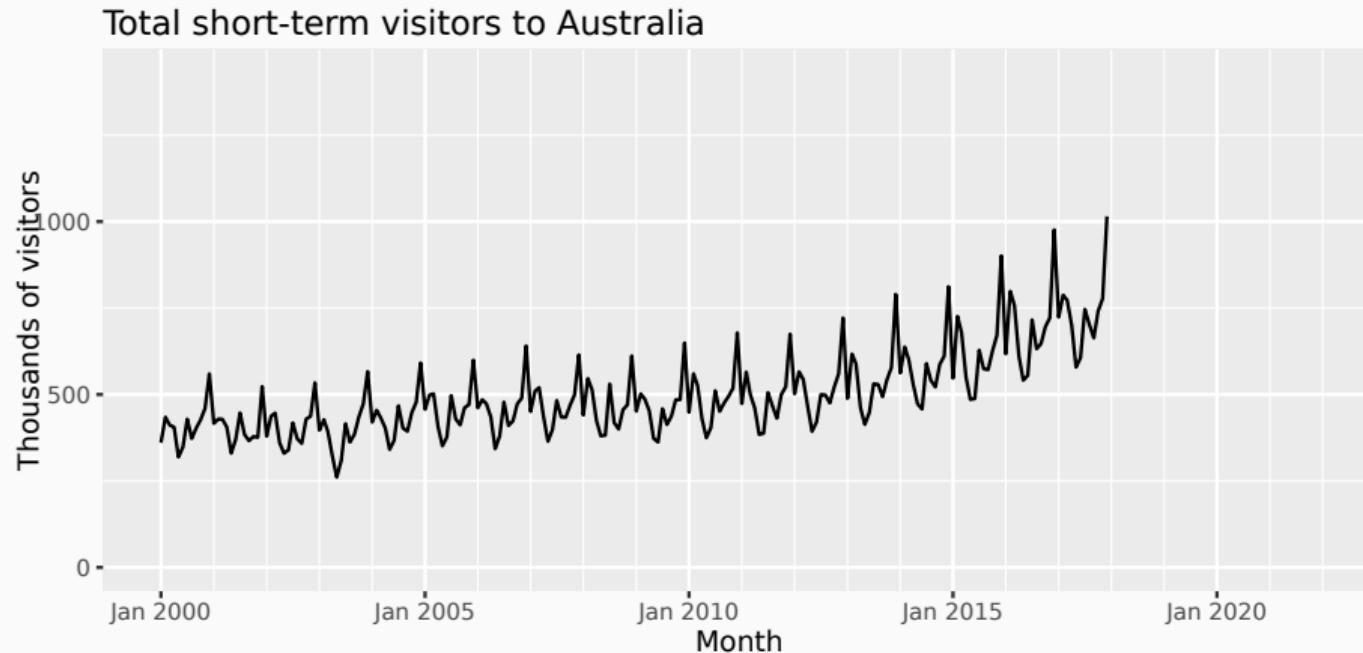
- we have a good understanding of the factors that contribute to it
- there is lots of data available;
- the forecasts cannot affect the thing we are trying to forecast.
- there is relatively low natural/unexplainable random variation.
- the future is somewhat similar to the past

Random futures

A forecast is an estimate of the probabilities of possible futures.

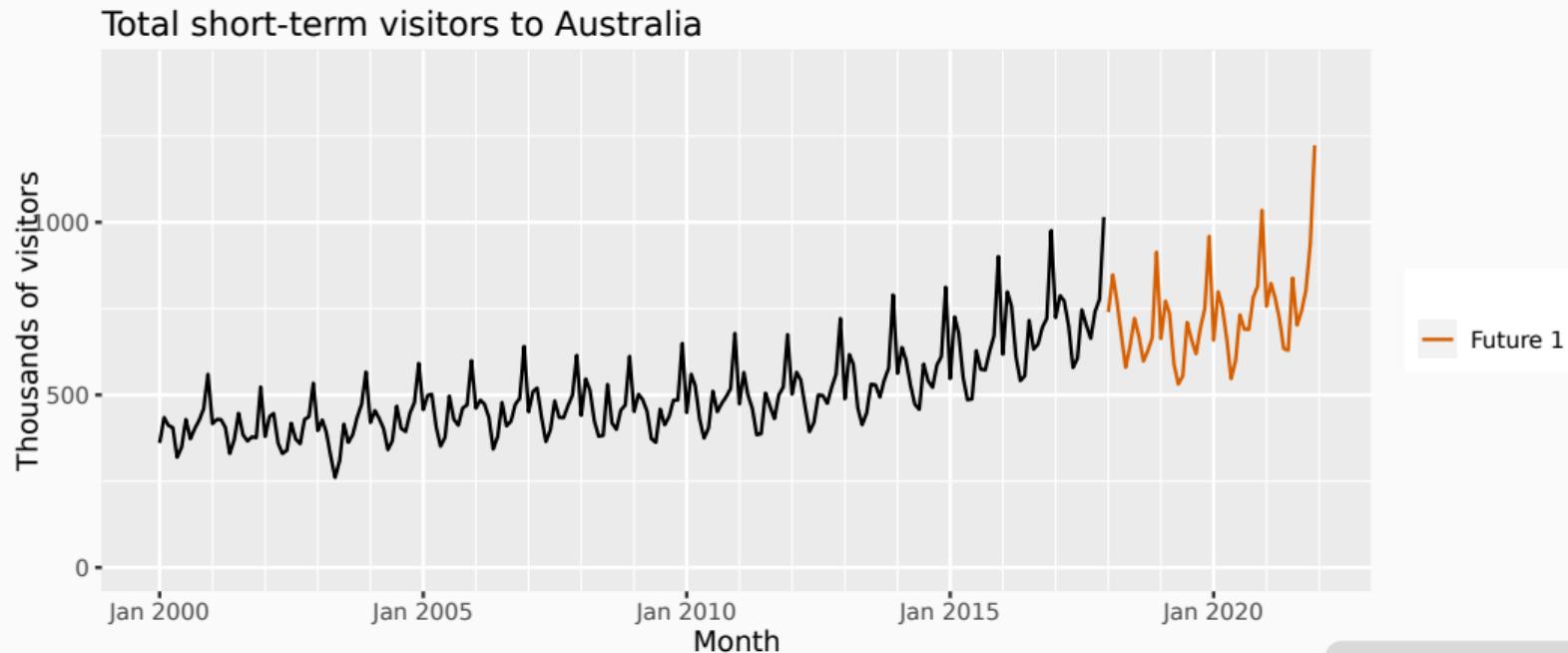
Random futures

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Random futures

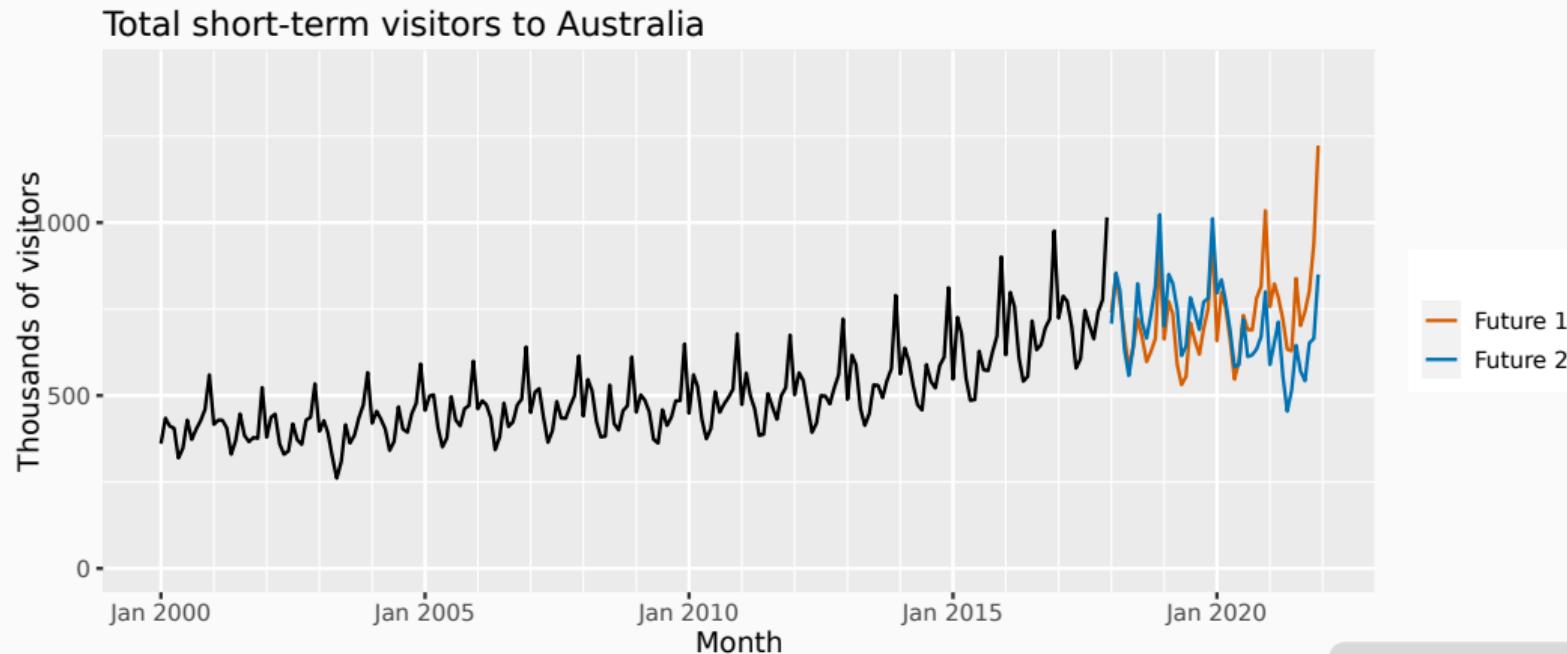
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Simulated futures
from an ETS

Random futures

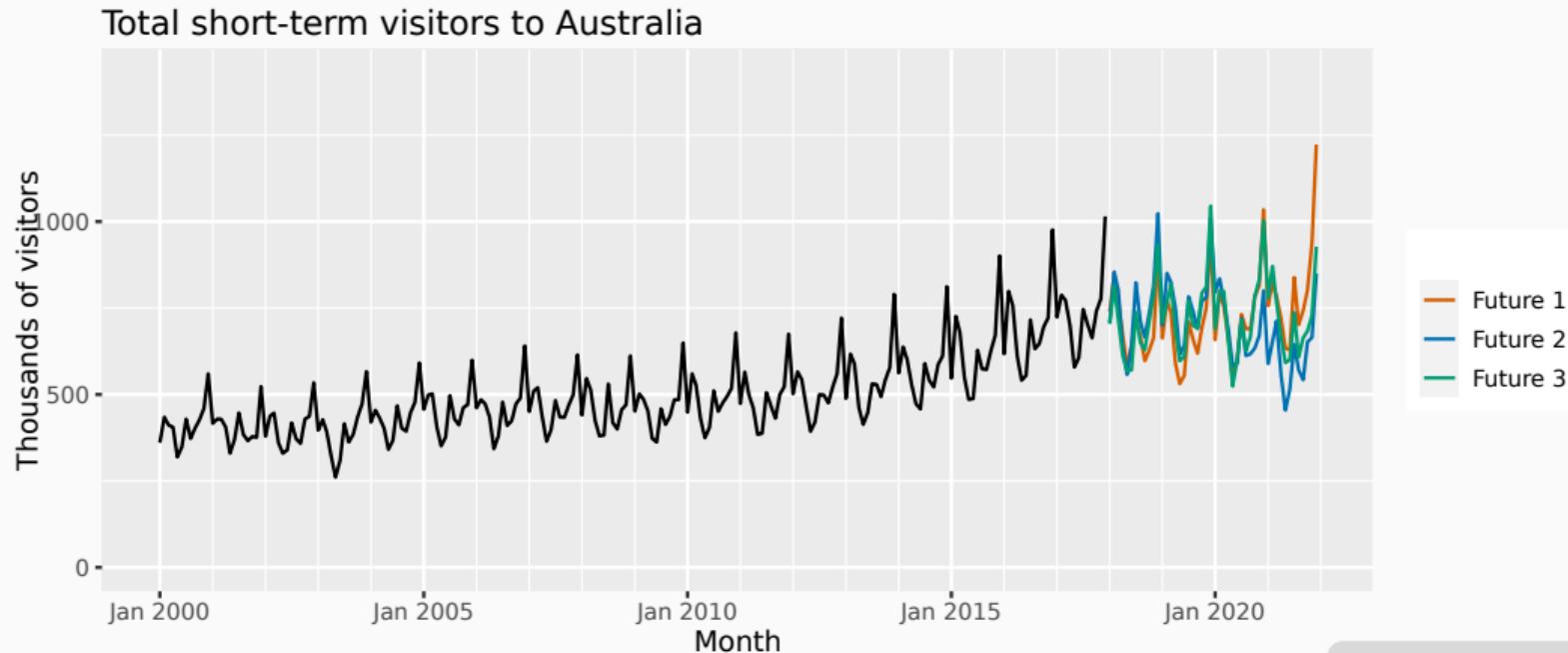
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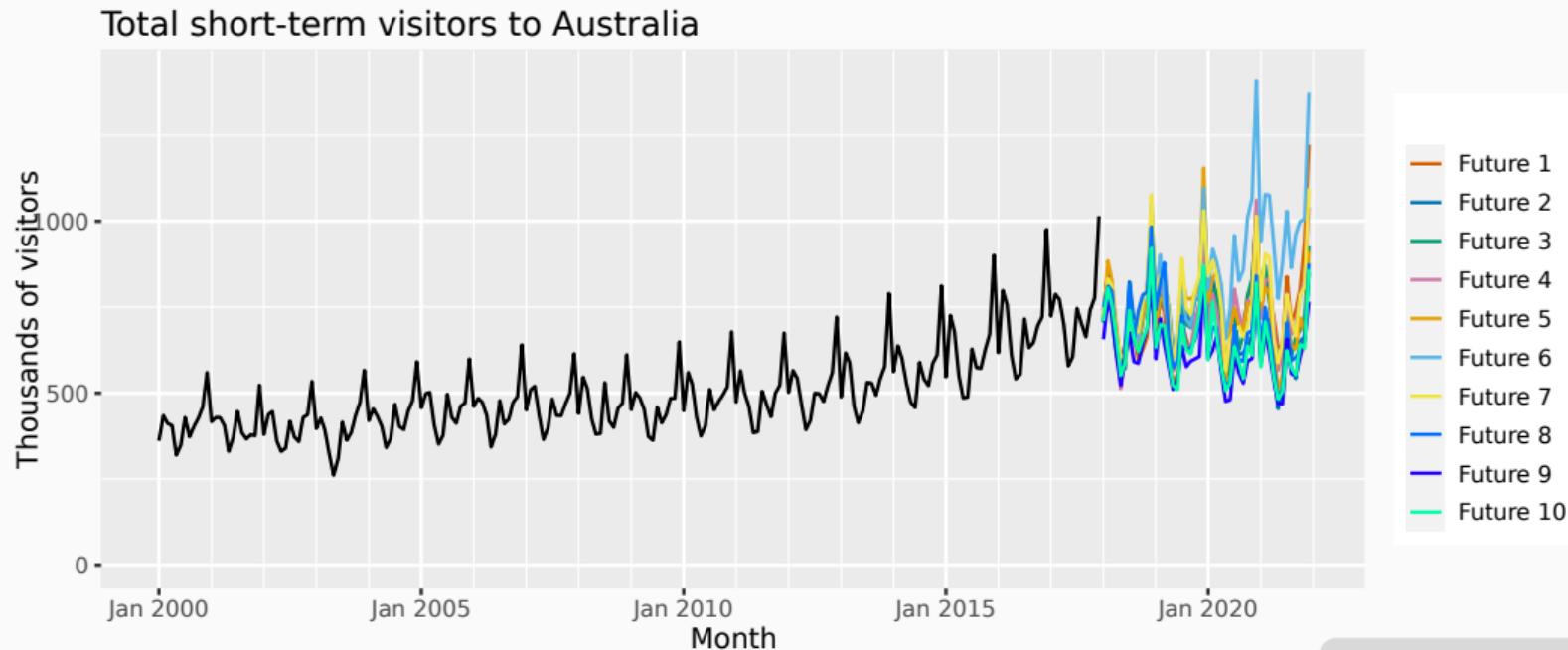
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Simulated futures
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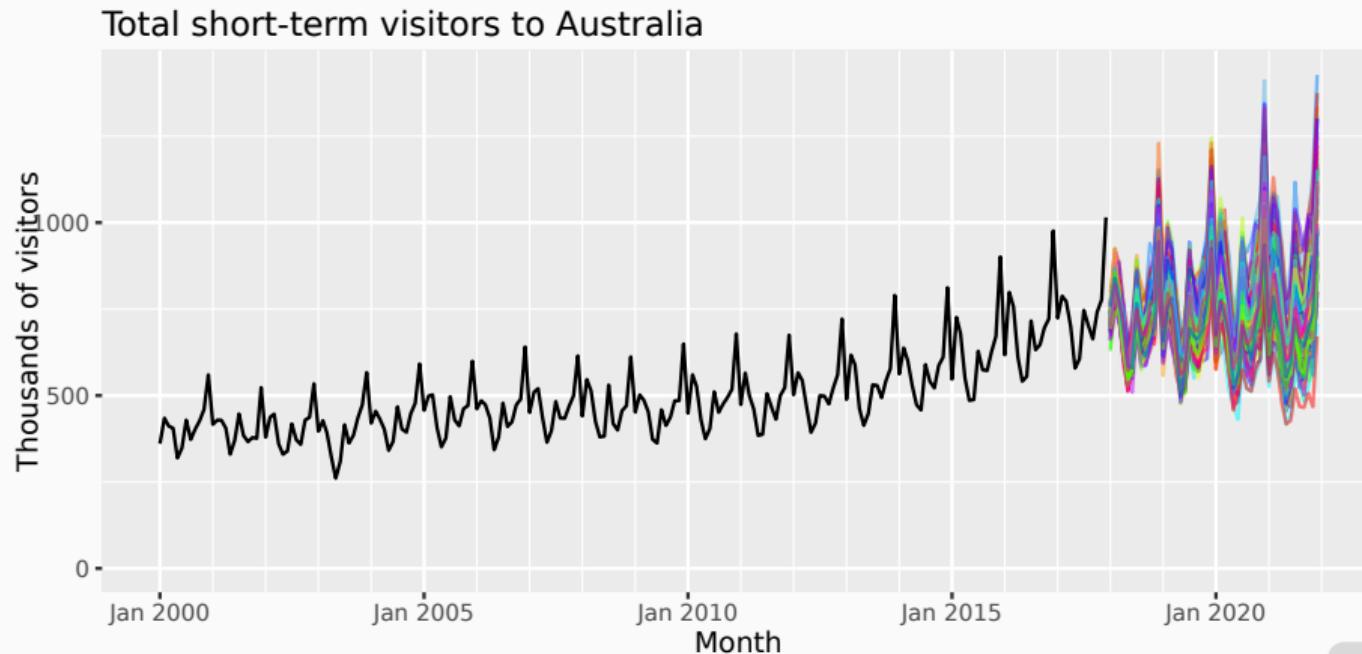
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Simulated futures
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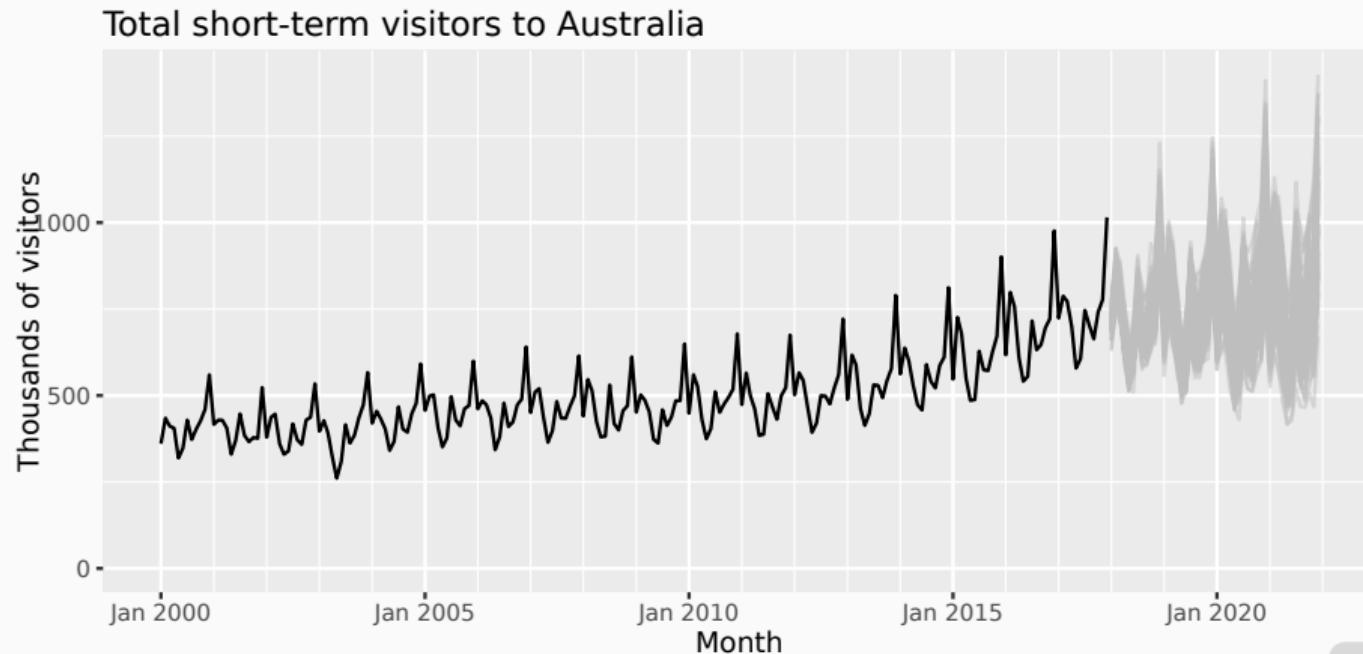
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Simulated futures
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Random futures

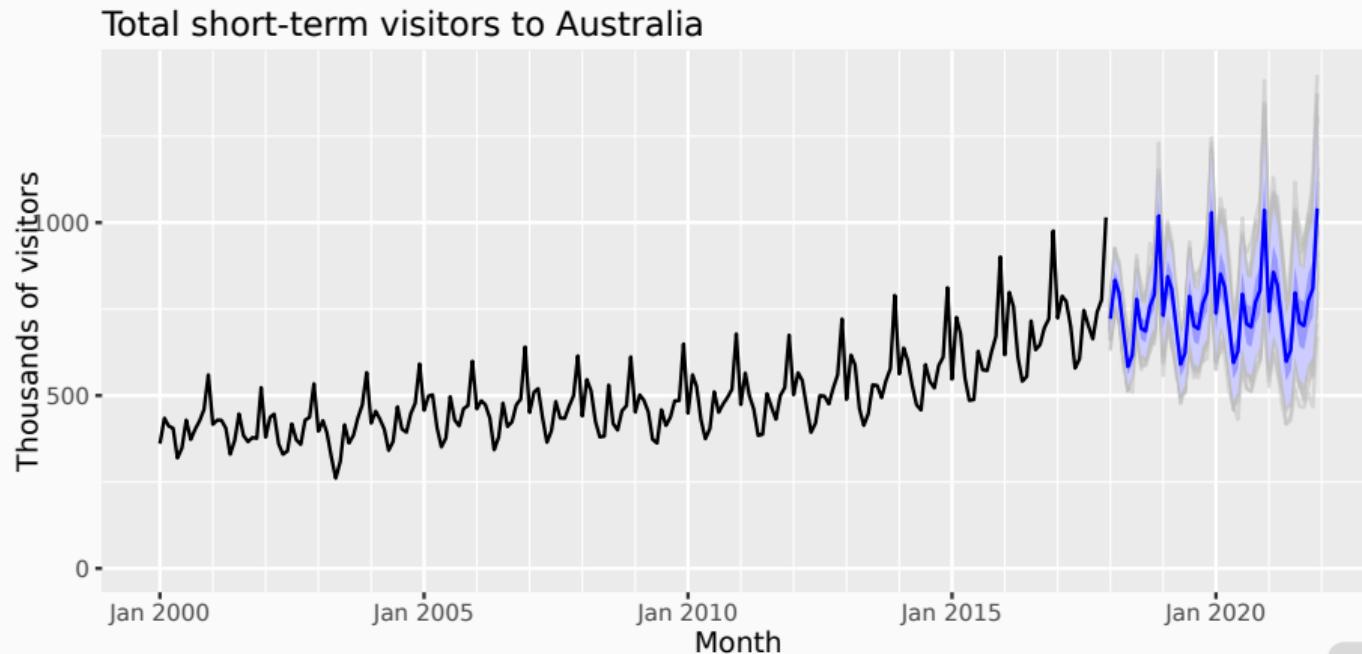
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Simulated futures
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Random futures

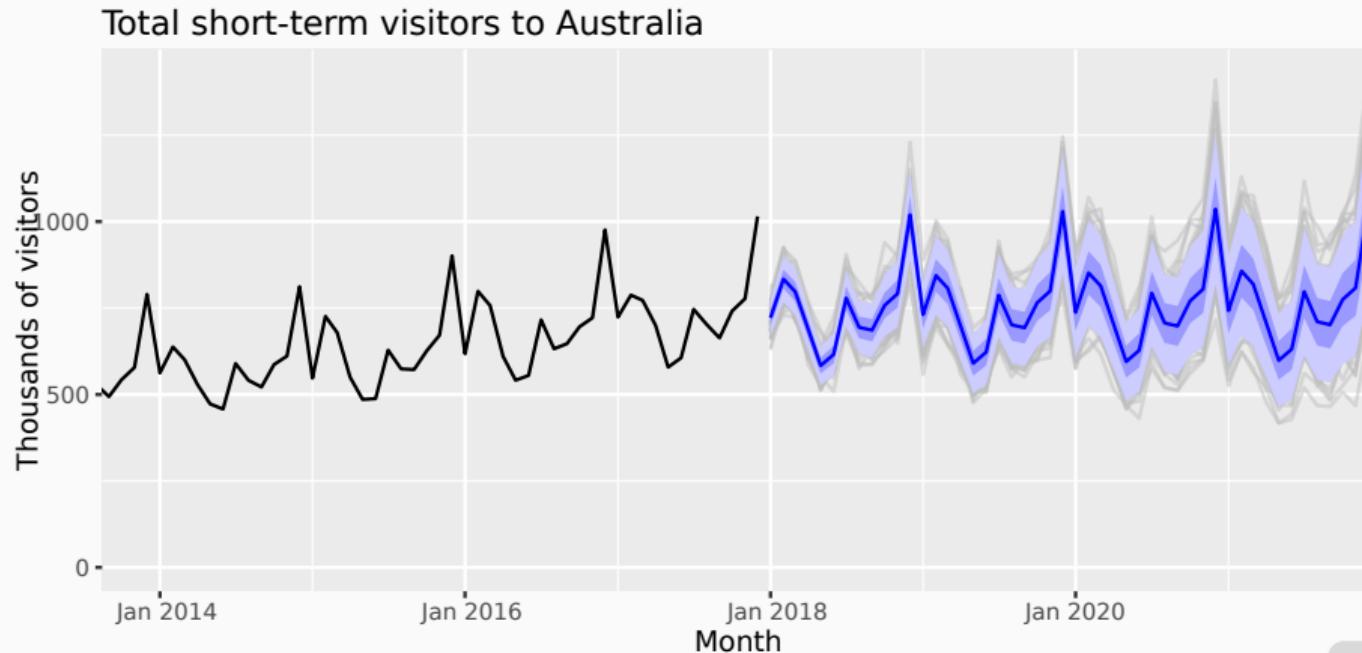
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Simulated futures
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Random futures

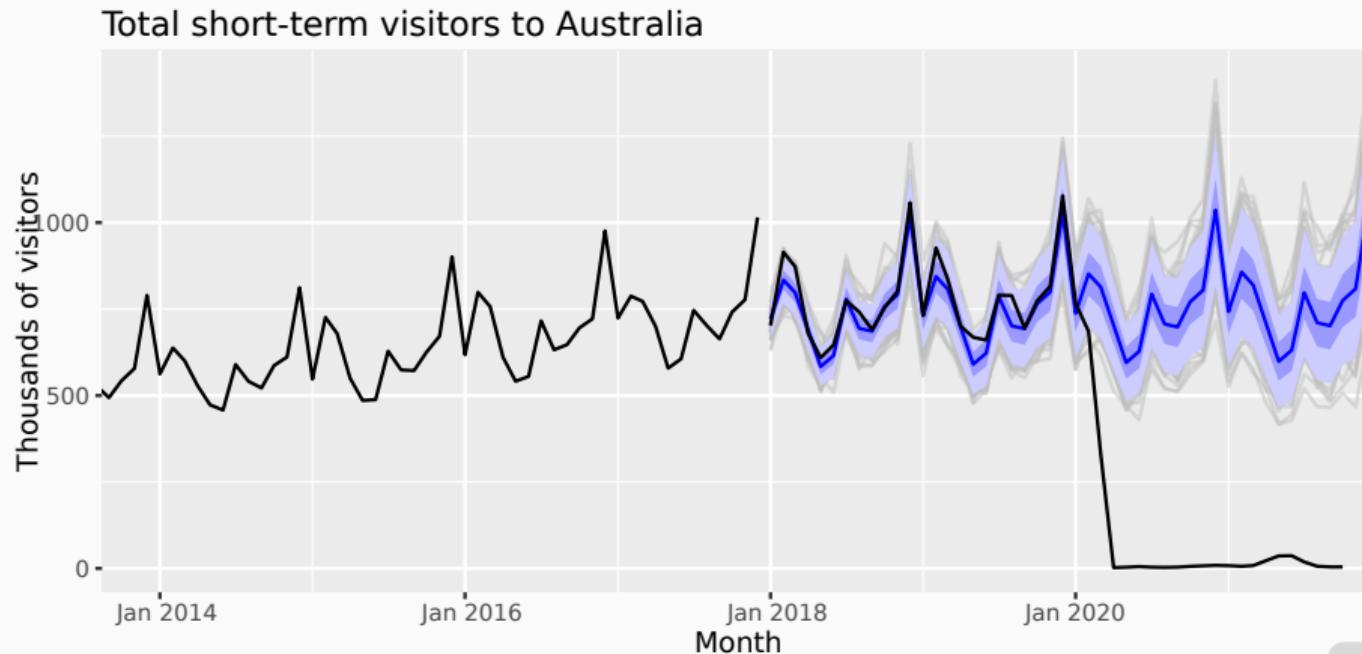
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Simulated futures
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Random futures

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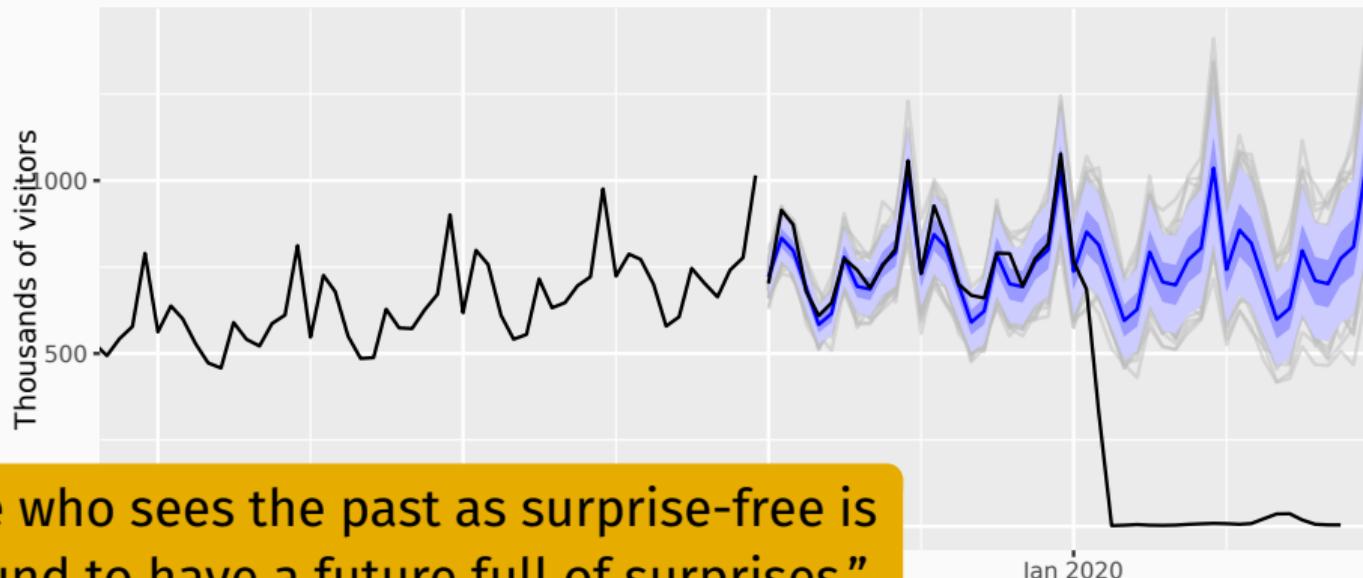


Simulated futures
from an ETS

Random futures

A forecast is an estimate of the probabilities of possible futures.

Total short-term visitors to Australia



“He who sees the past as surprise-free is bound to have a future full of surprises.”

(Amos Tversky)

Simulated futures
from an ETS

Statistical forecasting

- Thing to be forecast: y_{T+h} .
- What we know: y_1, \dots, y_T .
- Forecast distribution: $y_{T+h|t} = y_{T+h} \mid \{y_1, y_2, \dots, y_T\}$.
- Point forecast: $\hat{y}_{T+h|T} = E[y_{T+h} \mid y_1, \dots, y_T]$.
- Forecast variance: $\text{Var}[y_t \mid y_1, \dots, y_T]$
- Prediction interval is a range of values of y_{T+h} with high probability.

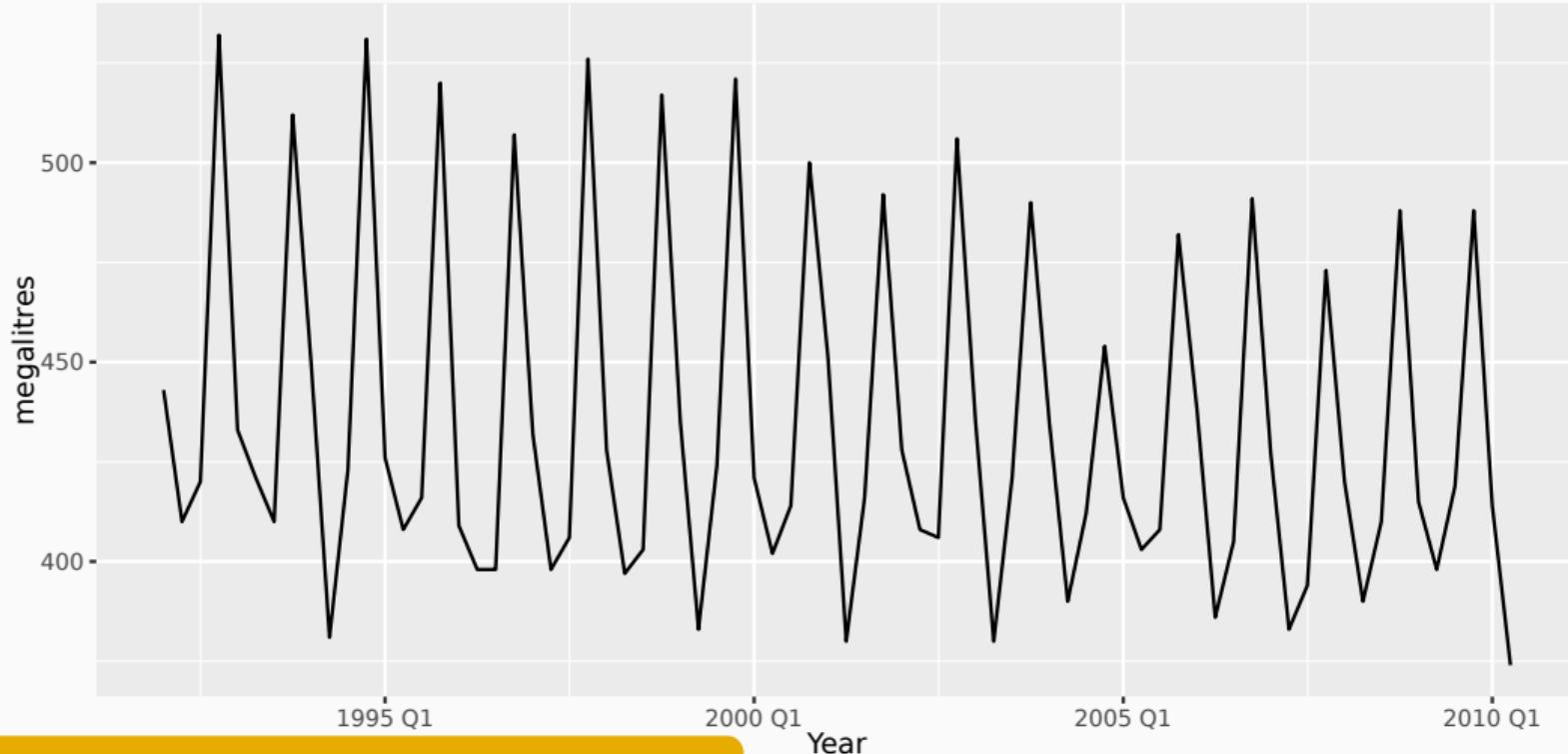
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2 Benchmark methods

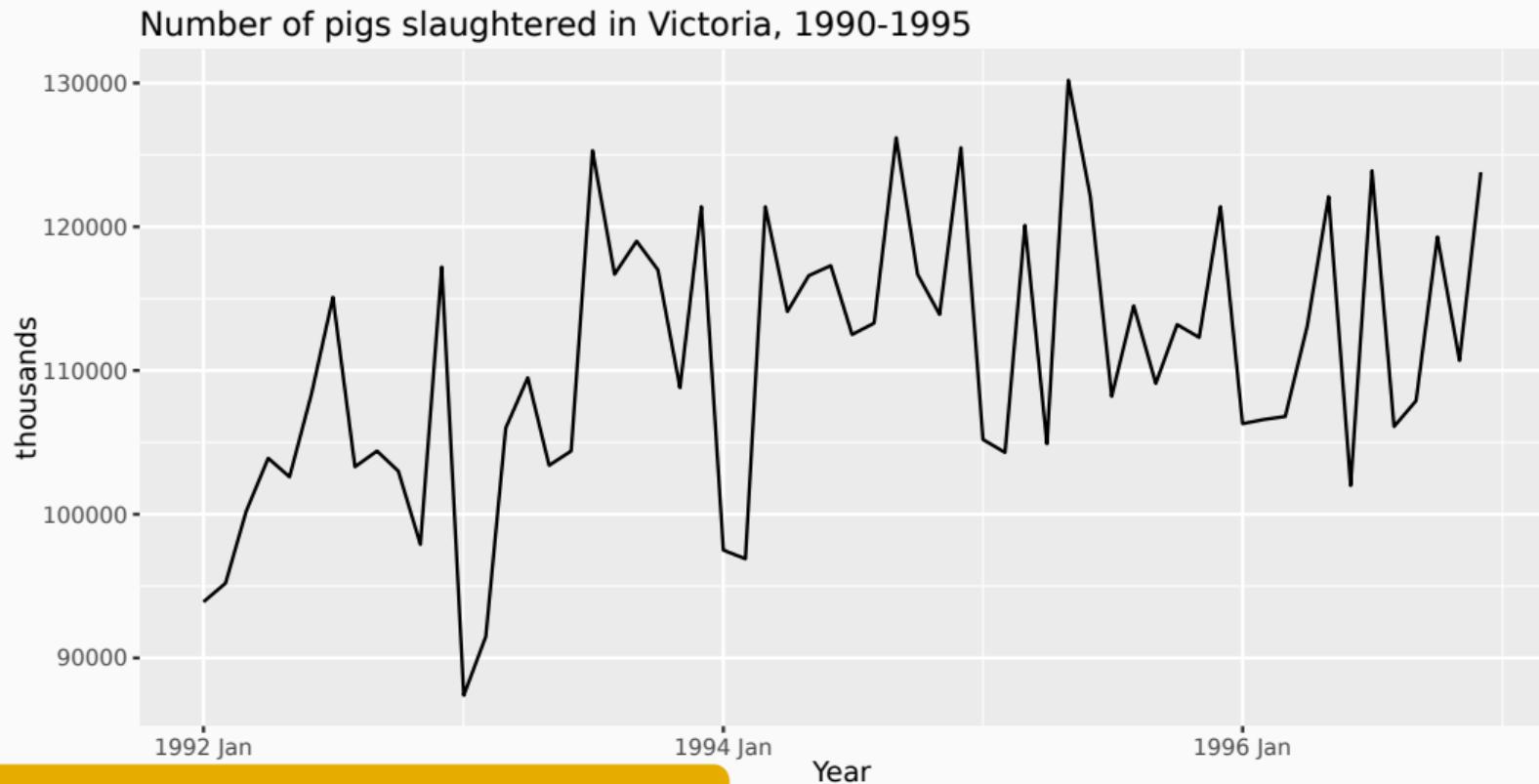
Some simple forecasting methods

Australian quarterly beer production



How would you forecast these
series?

Some simple forecasting methods



How would you forecast these
series?

Some simple forecasting methods

Facebook closing stock price in 2018

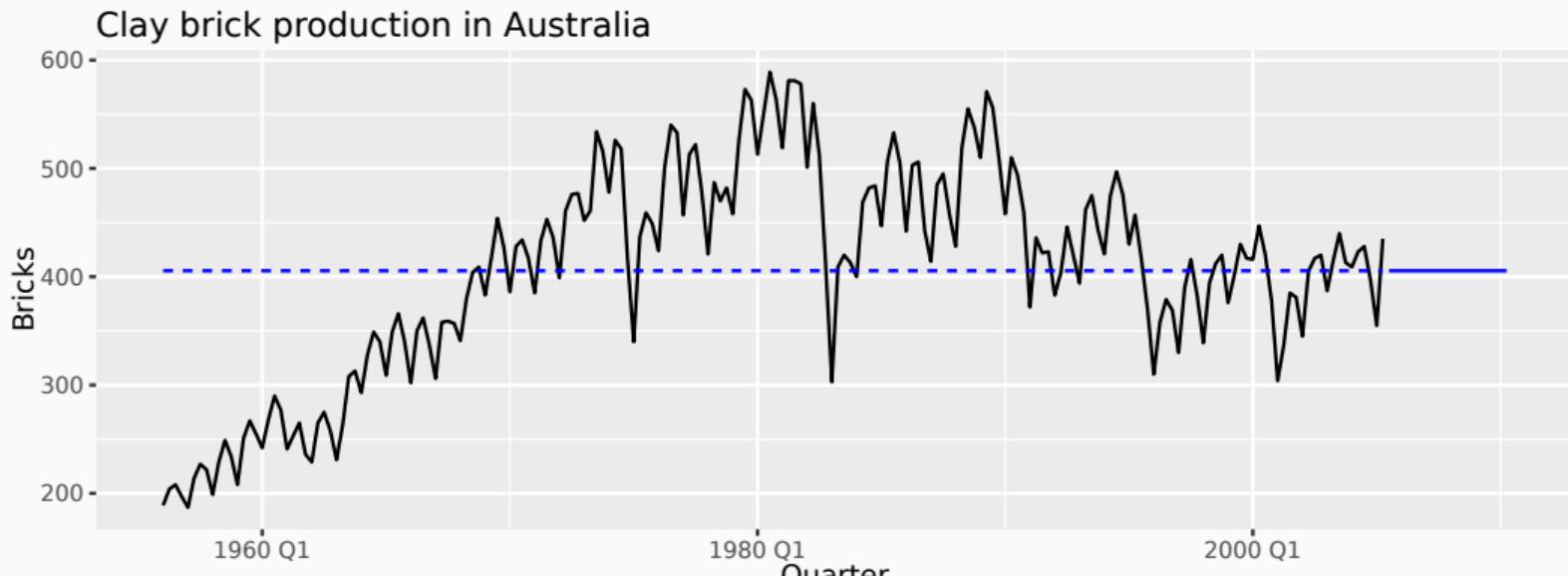


How would you forecast these
prices?

Some simple forecasting methods

MEAN(y): Average method

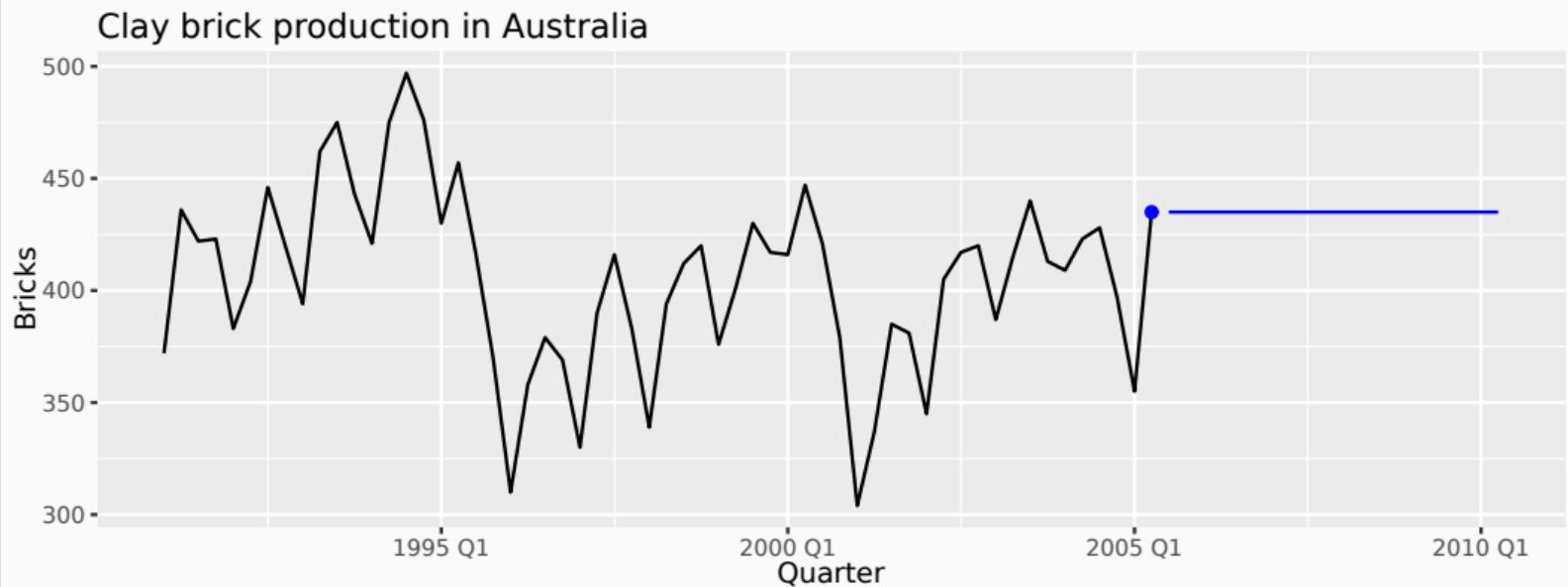
- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$



Some simple forecasting methods

NAIVE(y): Naïve method

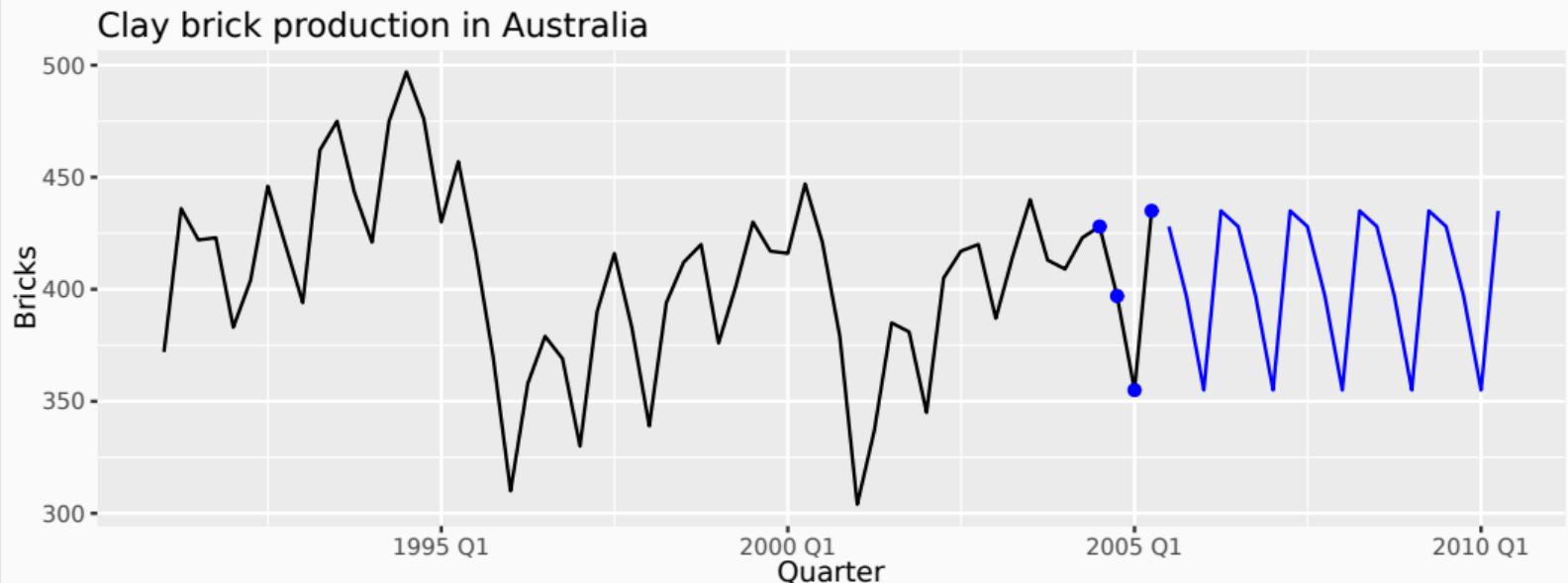
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

SNAIVE($y \sim \text{lag}(m)$): Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where $m = \text{seasonal period}$ and k is the integer part of $(h - 1)/m$.



Some simple forecasting methods

RW(y ~ drift()): Drift method

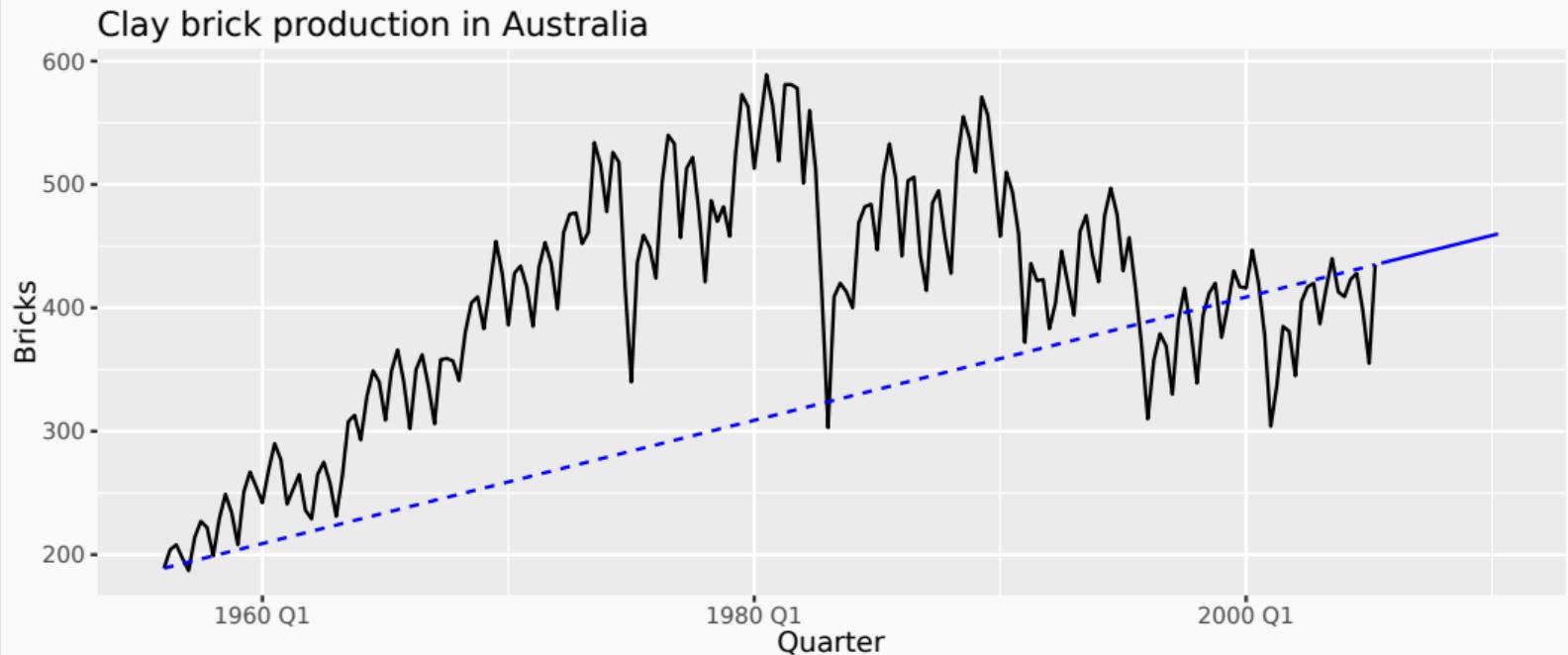
- Forecasts equal to last value plus average change.
- Forecasts:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    `Seasonal_naïve` = SNAIVE(Bricks),
    `Naïve` = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

```
# A mable: 1 x 4
  Seasonal_naïve    Naïve          Drift      Mean
              <model> <model>      <model> <model>
1       <SNAIVE> <NAIVE> <RW w/ drift> <MEAN>
```

A `mable` is a model table, each cell corresponds to a fitted model.

Producing forecasts

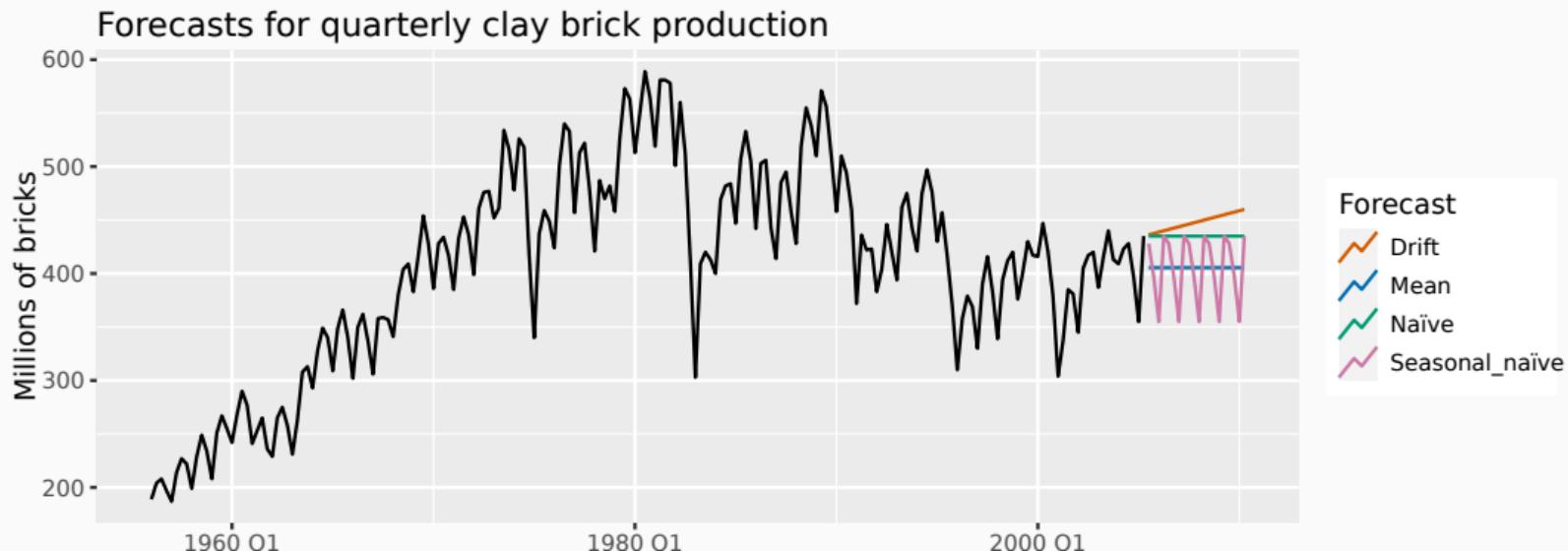
```
brick_fc <- brick_fit |>  
  forecast(h = "5 years")
```

```
# A fable: 80 x 4 [1Q]  
# Key:     .model [4]  
  
.model      Quarter      Bricks .mean  
<chr>       <qtr>       <dist> <dbl>  
1 Seasonal_naïve 2005 Q3 N(428, 2336)    428  
2 Seasonal_naïve 2005 Q4 N(397, 2336)    397  
3 Seasonal_naïve 2006 Q1 N(355, 2336)    355  
4 Seasonal_naïve 2006 Q2 N(435, 2336)    435  
# i 76 more rows
```

A fable is a forecast table with point forecasts and distributions.

Visualising forecasts

```
brick_fc |>  
  autoplot(aus_production, level = NULL) +  
  labs(title = "Forecasts for quarterly clay brick production",  
       x = "Year", y = "Millions of bricks") +  
  guides(colour = guide_legend(title = "Forecast"))
```



Prediction intervals

```
brick_fc |>  
  hilo(level = c(50, 75))
```

#	.model	Quarter	Bricks	.mean	`50%`	`75%`
	<chr>	<qtr>	<dist>	<dbl>	<hilo>	<hilo>
1	Seasonal_naïve	2005 Q3	N(428, 2336)	428	[395, 461]	[372, 484]
2	Seasonal_naïve	2005 Q4	N(397, 2336)	397	[364, 430]	[341, 453]
3	Seasonal_naïve	2006 Q1	N(355, 2336)	355	[322, 388]	[299, 411]
4	Seasonal_naïve	2006 Q2	N(435, 2336)	435	[402, 468]	[379, 491]
5	Seasonal_naïve	2006 Q3	N(428, 4672)	428	[382, 474]	[349, 507]
6	Seasonal_naïve	2006 Q4	N(397, 4672)	397	[351, 443]	[318, 476]
7	Seasonal_naïve	2007 Q1	N(355, 4672)	355	[309, 401]	[276, 434]
8	Seasonal_naïve	2007 Q2	N(435, 4672)	435	[389, 481]	[356, 514]
9	Seasonal_naïve	2007 Q3	N(428, 7008)	428	[372, 484]	[322, 524]

Prediction intervals

```
brick_fc |>
  hilo(level = c(50, 75)) |>
  mutate(lower = `50%`$lower, upper = `50%`$upper)
```

```
# A tsibble: 80 x 8 [1Q]
# Key:     .model [4]
#       .model   Quarter    Bricks .mean      `50%`      `75%` lower upper
#       <chr>     <qtr>    <dist> <dbl>      <hilo>      <hilo> <dbl> <dbl>
1 Seasonal_naïve 2005 Q3 N(428, 2336)  428 [395, 461]50 [372, 484]75  395. 461.
2 Seasonal_naïve 2005 Q4 N(397, 2336)  397 [364, 430]50 [341, 453]75  364. 430.
3 Seasonal_naïve 2006 Q1 N(355, 2336)  355 [322, 388]50 [299, 411]75  322. 388.
4 Seasonal_naïve 2006 Q2 N(435, 2336)  435 [402, 468]50 [379, 491]75  402. 468.
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```

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations y_1, \dots, y_t .
- We call these “fitted values”.
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Facebook closing stock price

```
fb_stock <- gafa_stock |>  
  filter(Symbol == "FB")  
fb_stock |> autoplot(Close)
```



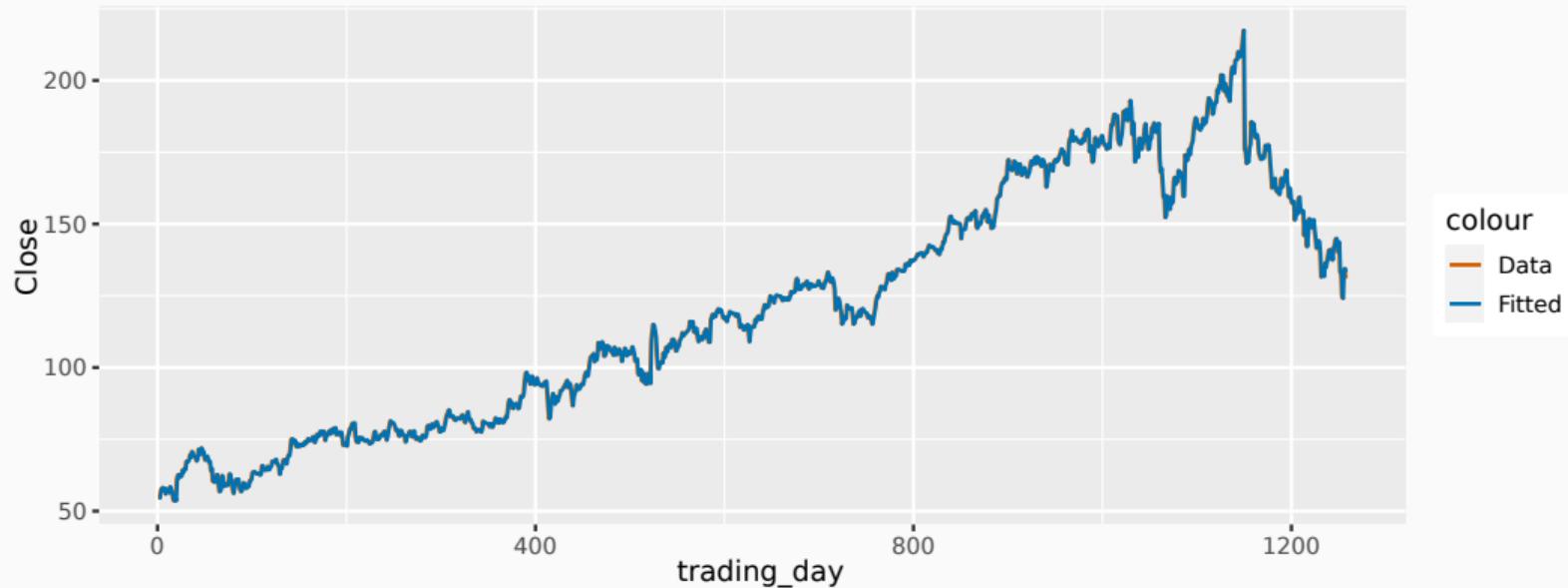
Facebook closing stock price

```
fb_stock <- fb_stock |>  
  mutate(trading_day = row_number()) |>  
  update_tsibble(index = trading_day, regular = TRUE)  
fit <- fb_stock |> model(NAIVE(Close))  
augment(fit)
```

```
# A tsibble: 1,258 x 7 [1]  
# Key:     Symbol, .model [1]  
  Symbol .model      trading_day Close .fitted .resid .innov  
  <chr>  <chr>        <int>  <dbl>   <dbl>   <dbl>   <dbl>  
1 FB     NAIVE(Close)      1  54.7    NA    NA    NA  
2 FB     NAIVE(Close)      2  54.6  54.7 -0.150 -0.150  
3 FB     NAIVE(Close)      3  57.2  54.6  2.64  2.64  
4 FB     NAIVE(Close)      4  57.9  57.2  0.720 0.720  
5 FB     NAIVE(Close)      5  58.2  57.9  0.310 0.310  
6 FB     NAIVE(Close)      6  57.2  58.2 -1.01 -1.01  
7 FB     NAIVE(Close)      7  57.9  57.2  0.720 0.720  
8 FB     NAIVE(Close)      8  55.9  57.9 -2.03 -2.03
```

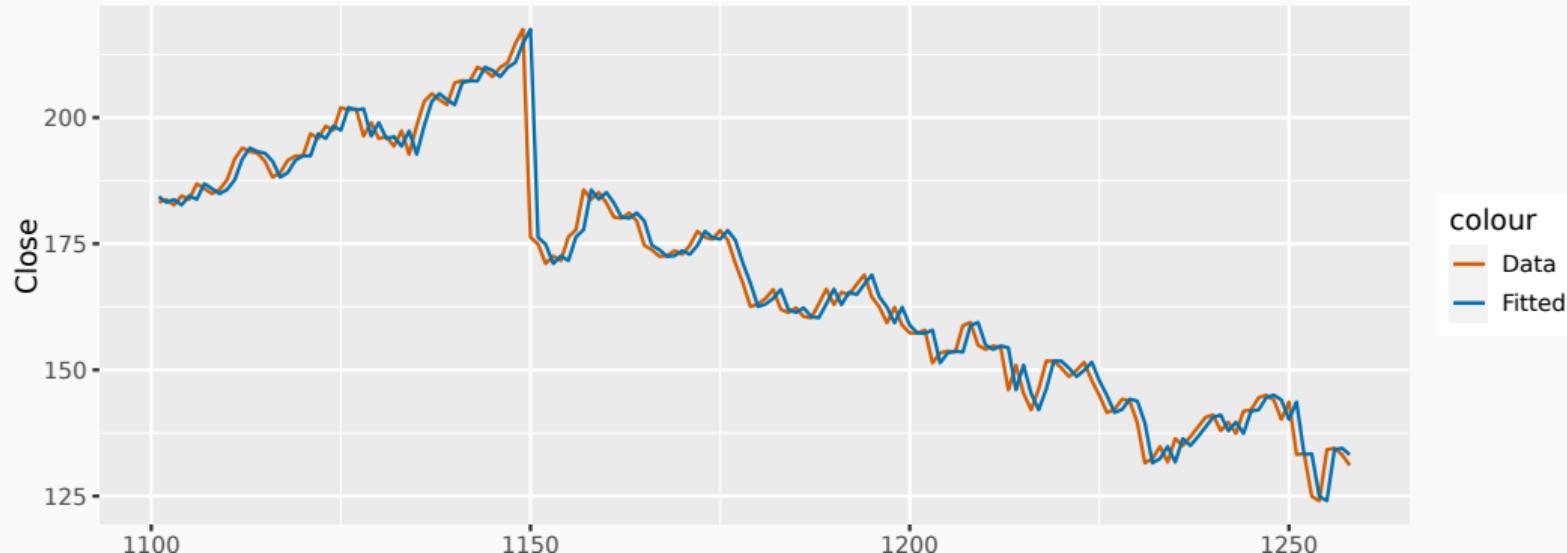
Facebook closing stock price

```
augment(fit) |>  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



Facebook closing stock price

```
augment(fit) |>  
filter(trading_day > 1100) |>  
ggplot(aes(x = trading_day)) +  
geom_line(aes(y = Close, colour = "Data")) +  
geom_line(aes(y = .fitted, colour = "Fitted"))
```



Facebook closing stock price

```
augment(fit) |>  
autoplot(.resid) +  
labs(x = "Day", y = "", title = "Residuals from naïve method")
```

Residuals from naïve method

