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METHODOLOGIES AND APPLICATION

An improved sobel edge detection method based on generalized type-2 fuzzy logic

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Abstract Edge detectors have traditionally been an essential part of many computer vision systems. There are different methods that have been proposed for improving edge detection in real images. This paper proposes an edge detection method based on the Sobel technique and generalized type-2 fuzzy logic systems. To limit the complexity of handling generalized type-2 fuzzy logic, the theory of α-planes is used. Simulation results are obtained with the Sobel operator (without fuzzy logic), then with a type-1 fuzzy logic system (T1FLS), an interval type-2 fuzzy logic system (IT2FLS) and with a generalized type-2 fuzzy logic system (GT2FLS). The proposed generalized type-2 fuzzy edge detection method is tested with synthetic images with promising results. To illustrate the advantages of using generalized type-2 fuzzy logic in combination with the Sobel operator, the figure of merit of Pratt measure is applied to measure the accuracy of the edge detection process.

Keywords α-planes · General type-2 fuzzy logic system · Generalized type-2 fuzzy logic system · Interval type-2 fuzzy logic system · Image processing · Edge detection

1 Introduction

Early stages of vision processing try to identify features in images that are relevant to estimating the structure and properties of objects in a scene. Edges are one such feature. Edges are significant local changes in the image and are important features for analyzing images. Edges typically occur on the boundary between two different regions in an image. Edge detection is frequently the first step in recovering information from images, and it is an important topic and continues to be an active research area. For this reason, there is a great interest in developing new techniques for edge detection (Torre and Poggio 1986; Ramesh et al. 1995). The idea of this work is to develop a methodology of fuzzy edge detection to contribute to the state of the art of current works related to edge detectors (Canny 1986; Sobel 1970; Teruhisa et al. 2005; Tao et al. 1993; Hu et al. 2007; Mendoza et al. 2007; Verma and Sharma 2011; Biswas and Sil 2012). The objective of the systems based on fuzzy logic is to achieve better edge detection when image processing is performed under high noise levels (Melin et al. 2010; Bustince et al. 2009; Setayesh et al. 2012; Mendoza et al. 2009).

In the last years, great progress has been made in transitioning from traditional type-1 fuzzy logic systems (T1FLS) to interval type-2 fuzzy logic systems (IT2FLS) and, most recently, to generalized type-2 fuzzy logic systems (GT2FLS). Of course, the idea of going into higher orders or types of fuzzy logic is to achieve better models of uncertainty. In this sense, it is theoretically expected that using IT2FLS can provide better performance for a fuzzy logic system (FLS) than using T1FLS. Therefore, the GT2FLS has the potential to provide better performance than the IT2FLS (Melin et al. 2010; Castillo and Melin 2008; Castro et al. 2007; Pagola et al. 2013), which is why there has been so much interest in interval type-2 and generalized type-2 fuzzy logic systems (Greenfield and Chiclana 2013; Jeon et al. 2009; Zhai and Mendel 2011; Wagner and Hagras 2010, 2011). However, several efforts have been made to limit the complexity of generalized type-2 fuzzy logic; while Wagner

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and Hagras (2010, 2011) have introduced the zSlices-based representation, Mendel and Liu (Mendel et al. 2009; Mendel 2010; Liu 2008) have put forward a representation based on α-planes. In Sect. 3, we offer a description of α-planes, which enables the computation with generalized type-2 fuzzy sets.

The contribution of this paper is the proposed new method for edge detection based on the Sobel technique and generalized type-2 fuzzy logic, which shows the advantages that exist in using generalized type-2 fuzzy logic systems over interval type-2 fuzzy logic systems. For this paper, it is important to mention the Sobel operator since it is used as a basis for our investigation; therefore, in Sect. 2 a brief description of the Sobel edge detector is presented (Sobel 1970). Generalized type-2 fuzzy logic theory and α-planes are new research areas and for this reason, in Sect. 3 some basic concepts of this theory are defined. Section 4 describes the methodology used for developing both edge detectors: GT2FLS and IT2FLS. To evaluate the quality of the detected edges, the metric of Pratt is used, which is presented in Sect. 5. The rest of the paper is organized as follows. Section 6 presents simulation results with benchmark images to illustrate the advantages of the proposed generalized type-2 fuzzy edge detection method. Finally, Sect. 7 offers some conclusions about the proposed methods and results.

2 Sobel edge detector

The Sobel operator when applied to gray-scale images calculates the gradient of the brightness intensity of each pixel, giving the direction of the greater possible increase of black to white, and in addition calculates the amount of change of that direction (Sobel 1970).

The Sobel operator performs a 2-D spatial gradient measurement on an image. Typically, it is used to find the approximate absolute gradient magnitude at each point in an input gray scale image.

The Sobel edge detector uses a pair of 3×3 convolution masks, one estimating the gradient in the x-direction (columns) and the other estimating the gradient in the y-direction (rows).

A convolution mask is usually much smaller than the actual image. As a result, the mask is slid over the image, manipulating a square of pixels at a time. The Sobel masks are shown in (1, 2) (Gonzalez et al. 2004; Jacques and Bauchspiess 2001; Mendoza et al. 2007, 2009a):

$$\mathrm{Sobel}_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \tag{1}$$

$$\mathrm{Sobel}_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}, \tag{2}$$

where Sobel_x and Sobel_y are the Sobel Operators throughout the *x-axis* and *y-axis*.

If we define I as the source image, g_x and g_y are two images which at each point contain the horizontal and vertical derivative approximations, the latter are computed as (3) and (4).

$$g_x = \mathrm{Sobel}_x * I \tag{3}$$

$$g_y = \mathrm{Sobel}_y * I, \tag{4}$$

where g_x and g_y are the gradients along *x-axis* and *y-axis*. The gradient magnitude g is calculated with (5) (Gonzalez et al. 2004; Fan et al. 2004).

$$g = \sqrt{g_x^2 + g_y^2} \tag{5}$$

3 Fuzzy logic systems

Fuzzy set theory is an important tool to solve problems in which objects are associated with imprecise information. A key problem in representing knowledge by means of fuzzy sets is to choose a membership function, which best represents such information. Fuzzy logic was originally conceived by Zadeh (1965), based on a theory of fuzzy sets that differ from traditional sets in considering the degree of membership to a set.

In recent years, great progress has been made in transitioning from traditional type-1 fuzzy logic systems to interval type-2 fuzzy logic systems and, most recently, to generalized type-2 fuzzy logic systems. The idea of going into higher orders or types of fuzzy logic is to achieve better models of uncertainty and in this case they can be applied in edge detection systems. Edge detection is very difficult when an image is corrupted by noise; the objective of the systems based on computational intelligence techniques, fuzzy logic systems in this case, is to achieve better edge detection when working in image processing with high noise levels.

In this section, we define the interval type-2 fuzzy logic, generalized type-2 fuzzy logic and some important associated concepts that are used throughout this paper.

3.1 Interval type-2 fuzzy logic

An interval type-2 fuzzy set $\tilde{A}$ is characterized by a membership function in the form given by (6):

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u) J_x \subseteq [0,1], \tag{6}$$

where $\int\int$ denotes the union over all admissible input variables x and u. In fact $J_x \subseteq [0,1]$ represents the primary membership of x, and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set, known as the secondary set. Hence, a type-2 membership grade can

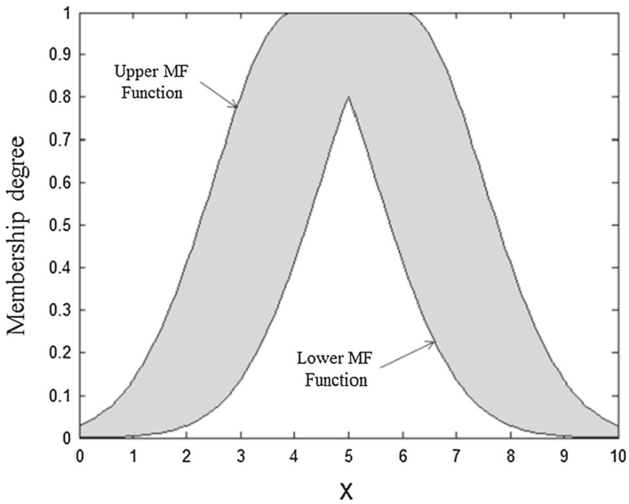


Fig. 1 Interval type-2 membership function

be any subset in [0, 1], the primary membership and corresponding to each primary membership there is a secondary membership (which can also be in [0, 1]) that defines the possibilities for the primary membership. Uncertainty is represented by a region, which is called the footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x,u)=1, \forall u \in J_x \subseteq [0,1]$, we have an interval type-2 membership function, as shown in Fig. 1. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\overline{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

3.2 Generalized type-2 fuzzy logic

The main contribution of this work is based on the use of generalized type-2 fuzzy logic and the α-planes theory; therefore, in this section some important concepts and the methodology used to develop the fuzzy inference system are presented. The general methodology that is used in this work is illustrated in Fig. 2.

3.3 Definition of generalized type-2 fuzzy sets

A generalized type-2 fuzzy set, denoted by $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$, $u \in J_x \subseteq [0,1]$ and $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$, and can be represented by (7) (Mendel et al. 2009; Liu 2008; Mendel and John 2002; Zhai and Mendel 2011, 2010).

$$\tilde{A}=\left\{((x,u),\mu_{\tilde{A}}(x,u))|\forall x \in X, \forall u \in J_x \subseteq [0,1]\right\}, \quad (7)$$

If $\tilde{A}$ is continuous, it can be denoted by (8):

$$\begin{aligned}\tilde{A} &= \left\{\int_{x\in X} \mu_{\tilde{A}}(x)/x\right\} \\ &= \left\{\int_{x\in X}\int_{u\in J^u_{x\subseteq[0,1]}} \mu_{\tilde{A}}(x,u)/(x,u)\right\} \\ &= \left\{\int_{x\in X}\left[\int_{u\in J^u_{x\subseteq[0,1]}} f_x(u)/u\right]/x\right\},\end{aligned} \quad (8)$$

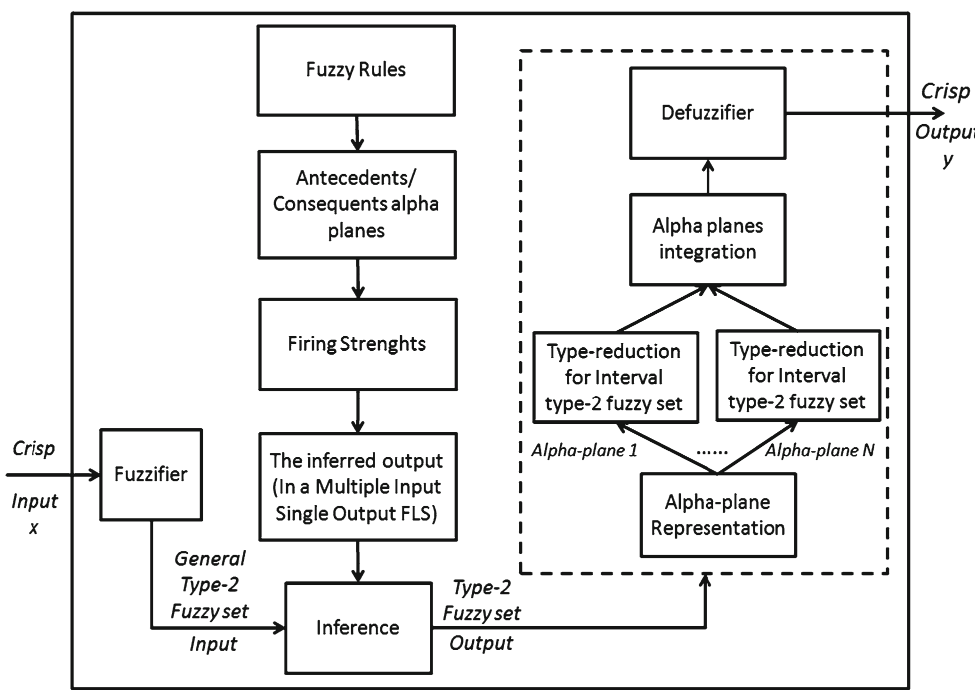


Fig. 2 Generalized type-2 fuzzy logic system with α-planes

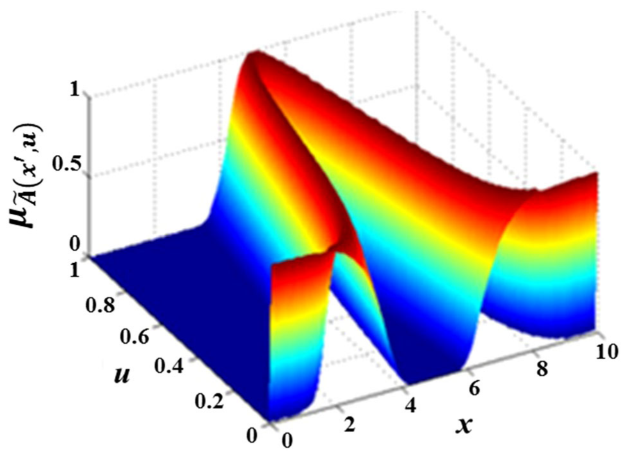


Fig. 3 Generalized type-2 membership function

where $\int\int$ denotes the union for x and u.

In (6) J_x is called the primary membership of x in $\tilde{A}$. At each value of x say $x = x'$, the two-dimensional (2-D) plane, whose axes are u and $\mu_{\tilde{A}}(x', \mathrm{u})$, is called a vertical slice of $\tilde{A}$ (Zhai and Mendel 2011; Mendel and John 2002). A secondary membership function is a vertical slice of $\tilde{A}$. This is $\mu_{\tilde{A}}(x = x', u)$, for $x' \in X$ and $\forall_u \in J_{x'} \subseteq [0, 1]$, and it is described as:

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x') = \int_{u \in J_{x'}} f_{x'}(u)/u J_{x'} \subseteq [0, 1] \tag{9}$$

in which $0 \leq f_{x'}(u) \leq 1$.

In Fig. 3, we can find a representation of a generalized type-2 membership function.

3.3.1 Fuzzifier

The fuzzifier maps crisp inputs into generalized type-2 fuzzy sets to process within the fuzzy logic system. In this paper, we focus on the type-2 singleton fuzzifier as it is fast to compute and, thus, suitable for the generalized type-2 FLS real-time operation (Wagner and Hagras 2010).

3.3.2 Inference

3.3.2.1 Fuzzy rules The structure of the rules in the generalized type-2 FLS is the standard Mamdani-type FLS rule structure used in the type-1 FLS and in interval type-2 FLS, but in the paper, we assume that the antecedents and the consequents sets are represented by generalized type-2 fuzzy sets. So for a type-2 FLS with p inputs $x_1 \in X_1, \ldots, x_p \in X_P$ and one output $y \in Y$, Multiple Input Single Output (MISO), if we assume there are M rules, the kth rule in the generalized type-2 FLS can be written as follows (Mendel 2001):

$$R^k : IF\ x_1\ is\ \tilde{F}_1^k \text{ and} \ldots \text{and } x_p\ is\ \tilde{F}_p^k, THEN\ y\ is\ \tilde{G}^k \tag{10}$$

3.3.2.2 Antecedents/consequents α-planes For performing the inference in the fuzzy system, the α-planes representation was used. An α-plane for the generalized type-2 fuzzy system $\tilde{A}$, denoted by $\tilde{A}_\alpha$, is the union of all primary membership functions of $\tilde{A}$ whose secondary grades are greater than or equal to $\alpha (0 \leq \alpha \leq 1)$, (Wagner and Hagras 2010, 2011; Mendel et al. 2009; Mendel 2010). The equation of the α-plane is represented by (11):

$$\tilde{A}_\alpha = \left\{(x, u), \mu_{\tilde{A}}(x, u) \geq \alpha | \forall x \in X, \forall u \in [0, 1]\right\} = \int_{\forall x \in X} \int_{\forall u \in J_x} \{(x, u) | f_x(u) \geq \alpha\} \tag{11}$$

The α-planes are obtained in the secondary membership functions of the antecedents $\tilde{F}_i^k$ and consequents $\tilde{G}_i^k$ of the ith input, kth rule. The α-planes of the $\tilde{F}_i^k$ create an interval type-2 fuzzy set, (Mendel et al. 2009; Liu 2008; Zhai and Mendel 2011), which is defined by expression (12)

$$(\tilde{F}_i^k)_\propto = \left\{ \int_{x_i' \in X_i} \left[\int_{\mu^{\propto}_{\tilde{F}_i^k}(x_i') \in \left[\underline{\mu}^{\propto}_{\tilde{F}_i^k}(x_i'), \overline{\mu}^{\propto}_{\tilde{F}_i^k}(x_i')\right]} 1/\mu^{\propto}_{\tilde{F}_i^k}(x_i') \right] /x \right\}, \tag{12}$$

where $(\tilde{F}_i^k)_\propto$ can be written as

$$(\tilde{F}_i^k)_\propto = \left\{ \int_{x_i' \in X_i} \left[\underline{\mu}^{\propto}_{\tilde{F}_i^k}(x_i'), \quad \overline{\mu}^{\propto}_{\tilde{F}_i^k}(x_i')\right] /x_i' \right\} \tag{13}$$

The α-planes of the consequents $\tilde{G}_i^k$, are defined by (14)

$$(\tilde{G}_i^k)_\propto = \left\{ \int_{y_i \in Y_i} \left[\int_{\mu^{\propto}_{\tilde{G}_i^k}(y_i) \in \left[\underline{\mu}^{\propto}_{\tilde{G}_i^k}(y_j), \overline{\mu}^{\propto}_{\tilde{G}_i^k}(y_j)\right]} 1/\mu^{\propto}_{\tilde{G}_i^k}(y_i) \right] /y_j \right\} \tag{14}$$

Another expression for $(\tilde{G}_i^k)_\propto$ is

$$(\tilde{G}_i^k)_\propto = \left\{ \int_{x_i' \in X_i} \left[\underline{\mu}^{\propto}_{\tilde{G}_i^k}(y_j), \quad \overline{\mu}^{\propto}_{\tilde{G}_i^k}(y_j)\right] /y_j \right\} \tag{15}$$

3.3.2.3 Firing strengths The firing strengths of the rules are calculated, where the firing sets $\mu^{\alpha}_{\tilde{F}^k_i}(x'_i)$ for each α-plane α of the ith input and kth rule of a singleton type-2 FLS are represented by (16):

$$\begin{aligned}\underline{\Omega}^k_{\propto}(x') &= \sqcap^n_{i=1}\left\{\underline{\mu}^{\alpha}_{\tilde{F}^k_i}(x'_i)\right\}\\ \overline{\Omega}^k_{\propto}(x') &= \sqcap^n_{i=1}\left\{\overline{\mu}^{\alpha}_{\tilde{F}^k_i}(x'_i)\right\}\end{aligned} \tag{16}$$

In a multiple input single output (MISO) FLS, the inferred outputs $\underline{\boldsymbol{\mu}}^{\boldsymbol{\alpha}}_{\tilde{\boldsymbol{B}}_j}(\boldsymbol{y}_j)$ and $\overline{\boldsymbol{\mu}}^{\boldsymbol{\alpha}}_{\tilde{\boldsymbol{B}}_j}(\boldsymbol{y}_j)$ of each rule k are represented by (17):

$$\begin{aligned}\underline{\mu}^{\alpha}_{\tilde{B}_j}(y_j) &= \underline{\Omega}^k_{\propto}(x') \sqcap \underline{\mu}^{\propto}_{\tilde{G}^k_j}(y_j)\\ \overline{\mu}^{\alpha}_{\tilde{B}_j}(y_j) &= \overline{\Omega}^k_{\propto}(x') \sqcap \overline{\mu}^{\propto}_{\tilde{G}^k_j}(y_j),\end{aligned} \tag{17}$$

where $\mu^{\alpha}_{\tilde{G}^k_j}$ is the type-2 fuzzy MF that represents the $\boldsymbol{\alpha}$th α-plane, kth rule, jth input of the consequents.

The outputs of the fired rules ($\boldsymbol{M}$) are combined using the join operation to produce the overall output set, which can be written as follows:

$$\begin{aligned}\underline{\mu}^{\alpha}_{\tilde{B}_j}(y_j) &= \sqcup^r_{k=1}\left\{\underline{\mu}^{\propto}_{\tilde{B}^k_j}(y_j)\right\}\\ \overline{\mu}^{\alpha}_{\tilde{B}_j}(y_j) &= \sqcup^r_{k=1}\left\{\overline{\mu}^{\propto}_{\tilde{B}^k_j}(y_j)\right\}\end{aligned} \tag{18}$$

3.3.3 Type reducer

To perform the defuzzification process, the centroid method is used. The centroid of a generalized type-2 fuzzy set $\tilde{A}$ can be obtained by taking the union of the centroids of all the α-planes of $\tilde{A}$, then the Karnik–Mendel algorithm is used for computing the centroid of each α-plane. The centroid of a generalized type-2 fuzzy system, introduced by Karnik and Mendel (Liu 2008; Liu et al. 2012; Mendel and Fellow 2013), uses the following definition of the centroid (19) (Mendel and John 2002):

$$Y_C(\propto) = \text{Centroid}(\tilde{A}(\propto)) = \int_{u_1\in{}^{\propto}J_{x_1}} \cdots \int_{u_N\in{}^{\propto}J_{x_N}} \propto \Big/ \frac{\sum_{i=1}^N x_i\mu_A(x_i)}{\sum_{i=1}^N \mu_A(x_i)} = \propto / \left[{}^{\propto}y_l, {}^{\propto}y_r\right], \tag{19}$$

where $\left[{}^{\propto}y_l, {}^{\propto}y_r\right]$ is the domain of the centroid. Obviously, computing ${}^{\propto}y_l$ and ${}^{\propto}y_r$ is the same as computing y_l and y_r for an interval type-2 fuzzy set; therefore, centroid-type reduction is performed by the Karnik–Mendel algorithms to compute ${}^{\propto}y_l$ and ${}^{\propto}y_r$ (Mendel and Fellow 2013; Wu 2013).

3.3.4 Alpha-plane integration

The results of the α-planes are integrated by (20, 21) (Mendel 2001).

$$\hat{y}^l_j(x') = \frac{\sum_{i=1}^N \propto_i \ {}^{\propto_i}y^l_j(x')}{\sum_{i=1}^N \propto_i} \tag{20}$$

$$\hat{y}^r_j(x') = \frac{\sum_{i=1}^N \propto_i \ {}^{\propto_i}y^r_j(x')}{\sum_{i=1}^N \propto_i} \tag{21}$$

3.3.5 Defuzzification

After realizing the type reduction and integrating the results of all the α-planes, defuzzification is performed using the average of y^l and y^r to obtain the defuzzified output (Mendel 2010; Liu 2008).

$$\hat{y}_j(x') = \frac{\hat{y}^l_j(x') + \hat{y}^r_j(x')}{2} \tag{22}$$

4 Edge detector using fuzzy logic

In this section, the edge detectors based on interval and generalized type-2 fuzzy logic are described. The fuzzy approach for edge detection based on the Sobel operator consists in using (3) and (4) to obtain the gradients in the two directions, and then use them as inputs for an appropriately defined fuzzy inference system.

4.1 Type-2 fuzzy inference system for edge detection

For the interval and generalized type-2 fuzzy inference system, two inputs are required, which are the gradients with respect to *x-axis* and *y-axis*, calculated with (3) and (4); for this case study, we call them DH and DV, respectively. For all the inputs and output fuzzy variables, the memberships are defined by Gaussian membership functions with uncertain mean. In the input membership functions, we used three linguistic variables: LOW, MIDDLE and HIGH; the parameters of the membership functions depend on the gradients of the image to use for the experiment. The output (EDGES) is represented by the linguistic values: BACKGROUND and EDGES. The EDGES output variable is used to find and normalize the edges to any range of required values which also has been adjusted to have a range between –4.5 and 5, since it is the better range of values to normalize the edges matrix. In Fig. 4, the general structure of the generalized type-2 fuzzy inference system is presented; the two models (Sobel + ITFLS and Sobel + GTFLS) are defined under the same conditions, which are required to be able to make a comparison of both results.

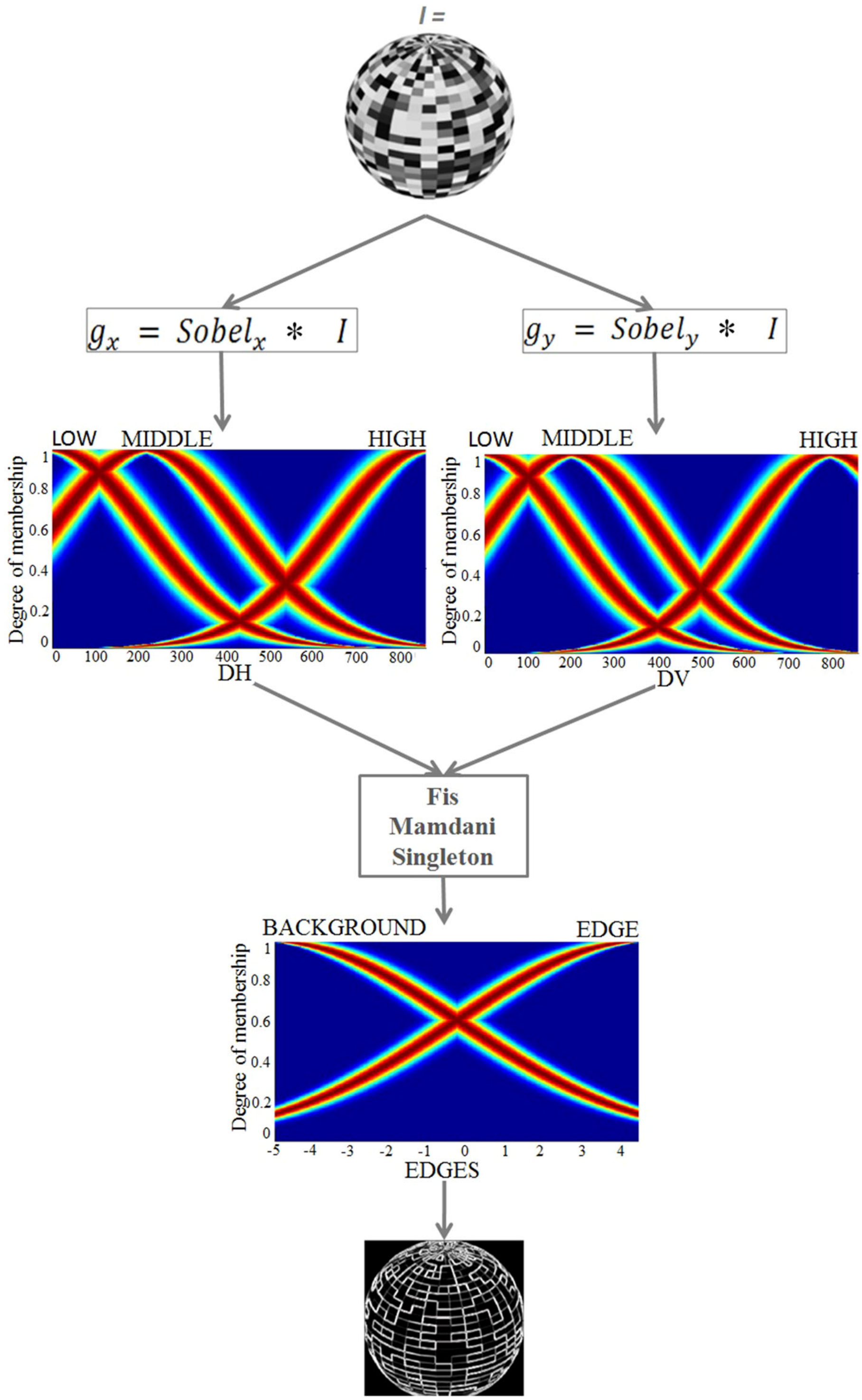


Fig. 4 Generalized type-2 fuzzy inference system for edge detection

4.2 Fuzzy inference rules

For modeling the process with the fuzzy system, we consider three fuzzy rules that allow to evaluate the input variables, so that the output image displays the edges of the image in color near white (high tone or EDGE), whereas the background was in tones near black (LOW tone or BACKGROUND); these rules were obtained based on expert knowledge and after several experiments. The fuzzy rules are the following.

(a) If (DH is HIGH) or (DV is HIGH) then (EDGES is EDGE)

(b) If (DH is MIDDLE) or (DV is MIDDLE) then (EDGES is EDGE)
(c) If (DH is LOW) and (DV is LOW) then (EDGES is BACKGROUND)

5 Quantitative evaluation: Pratt's figure of merit

Several quantifiable comparison methodologies exist for edge detection methods (Lopez-Molina et al. 2013; Bowyer et al. 2001; Go et al. 2001; Pratt 2007; Prieto and Allen 2003).One of the most frequently used techniques is the figure of merit of Pratt (Figure of Merit; FOM) (Ramesh et al. 1995; Go et al. 2001). This measure is well understood and it is possible to largely control what is being tested. Pratt's Figure of Merit (FOM) attempts to balance three types of errors that can produce erroneous edge maps: missing valid edge points, failure to localize edge points and classification of noise fluctuations as edge points (Ramesh et al. 1995; Pratt 2007). The Figure of Merit is defined in (23)

$$FOM = \frac{1}{\max(I_I, I_A)} \sum_{i=1}^{I_A} \frac{1}{1+ \propto d_i^2}, \tag{23}$$

where I_A represents the total number of actual edge pixels; i.e., those edge pixels that are found after applying any edge detector. The I_I represents the total number of ideal pixels in the image; i.e., the number of edge pixels in the reference image, and in this case the reference image (I_I) or ideal edge of Fig. 5 is the Fig. 6. The parameter α is a scaling constant

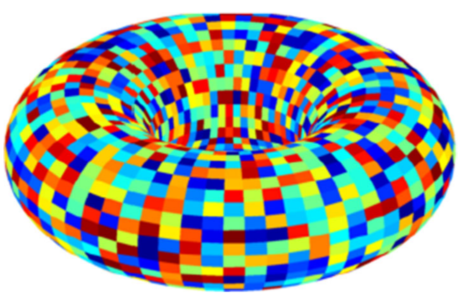


Fig. 5 Synthetic image donut

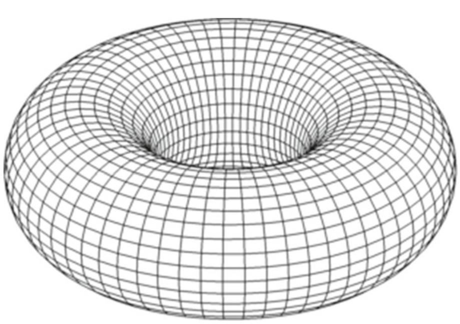
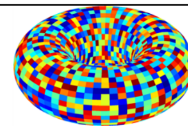
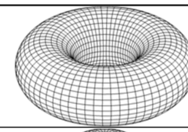
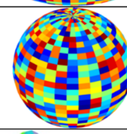
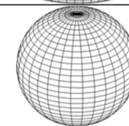
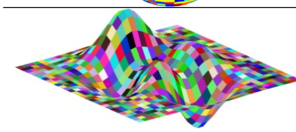
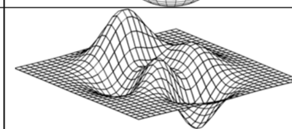
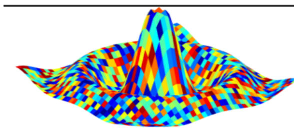
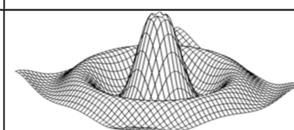
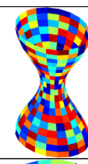
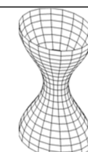
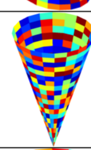
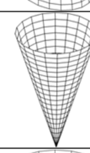
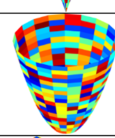
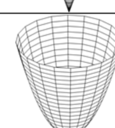
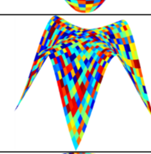
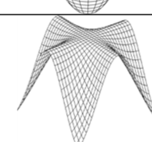
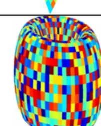
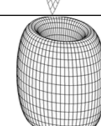
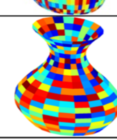
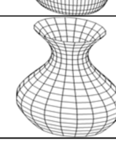


Fig. 6 Reference image (II) of Fig. 5

Table 1 Synthetic images for simulation results

Synthetic Image	Reference Image
	
	
	
	
	
	
	
	
	
	

(usually 1/9) while $d(i)$ is the distance from an actual edge point to the nearest ideal edge point. The scaling factor is used to penalize edges that are localized, but offset from the true position.

If the result of the FOM given by (23) is 1 or very close to 1, this means that the detected edge I_A is the same or very similar to the ideal edge $I_{I.}$ Otherwise, when is closer to 0, there is a high difference between the detected edge and the ideal edge (Prieto and Allen 2003; Perez-Ornelas 2012).

Table 2 Simulation results with variation in the FOU for Sobel + IT2FLS

FOU factor	Pratt's (FOM) Mean
0	0.9321
0.2	0.9352
0.4	0.9413
0.6	0.9504
0.8	**0.9606**
1.0	0.9598

Bold value indicates the best result

6 Simulation results

For an objective evaluation of the proposed method, it is necessary to choose a ground truth image with known ideal edges because the method takes them as reference to compare with the detected edges. Some authors use synthetic images (Mendoza et al. 2009a, b) for validation of results, and this is a good method because with the computer we can generate the image edge with some mathematical functions and we can use it as the reference for the ideal edges (Fig. 6); then using the same function, the computer can generate a filled image and we can use it as a simulation of a realistic image (Fig. 5).

To objectively evaluate the performance of the proposed edge detectors, for the simulation we perform the experiments using synthetic images built plotting ten mathematical functions as original images and as ground truth reference for the ideal edges. In Table 1, the synthetic and reference images are presented.

For the interval type-2 fuzzy system and generalized type-2 fuzzy logic system (GT2FLS), the gradients are obtained with (3) and (4) and are used as inputs, and the fuzzy system was built using the membership functions presented in Fig. 4.

In the first test, we used a sample of ten synthetic images, and selected arbitrary values for the Footprint of Uncertainty (FOU); the FOU is varied in the range between 0 and 1. For a better analysis of the detected edges, the metric of Pratt (FOM) described in Sect. 5 is applied. The Mean of

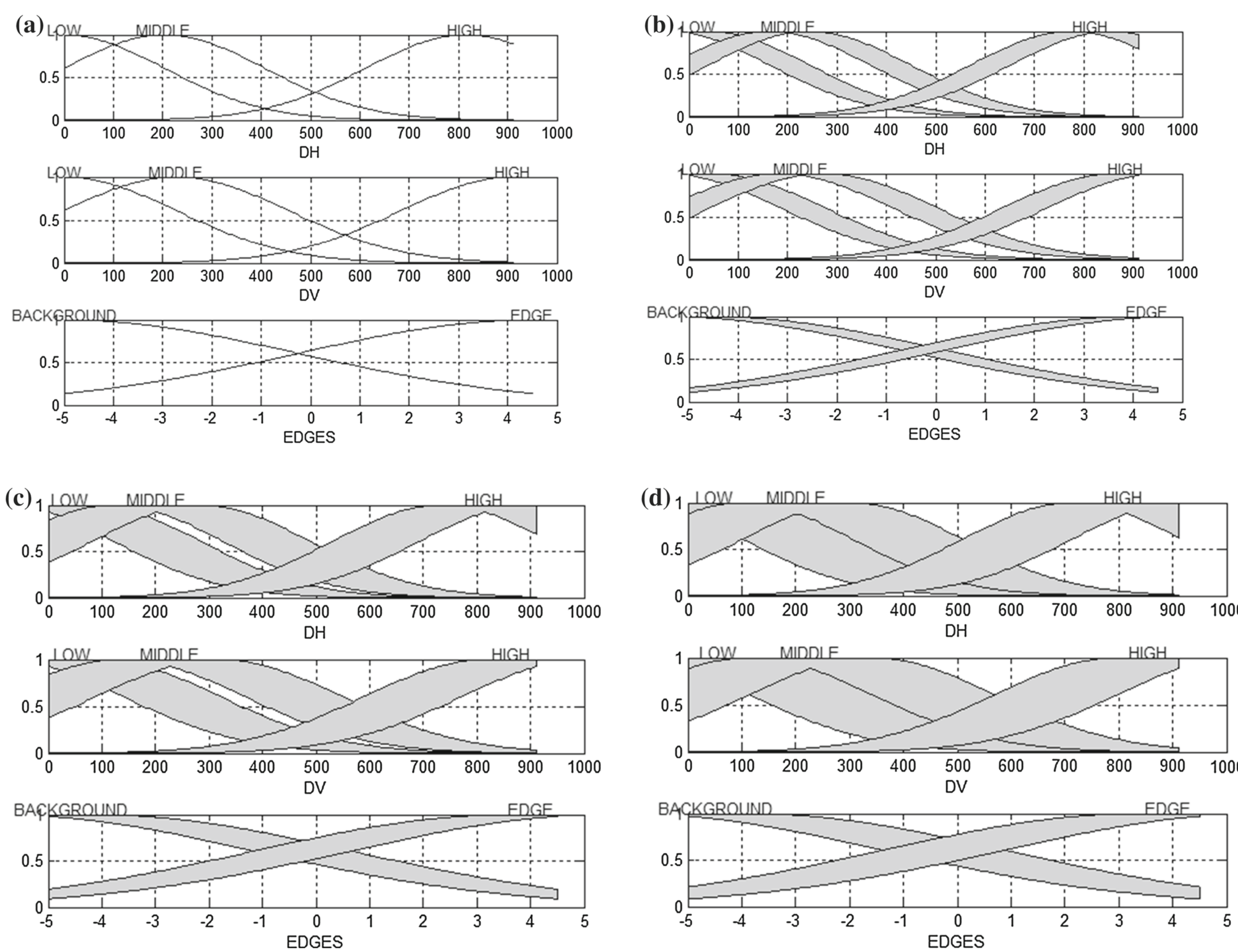


Fig. 7 IT2FLS membership functions with different values of the FOU

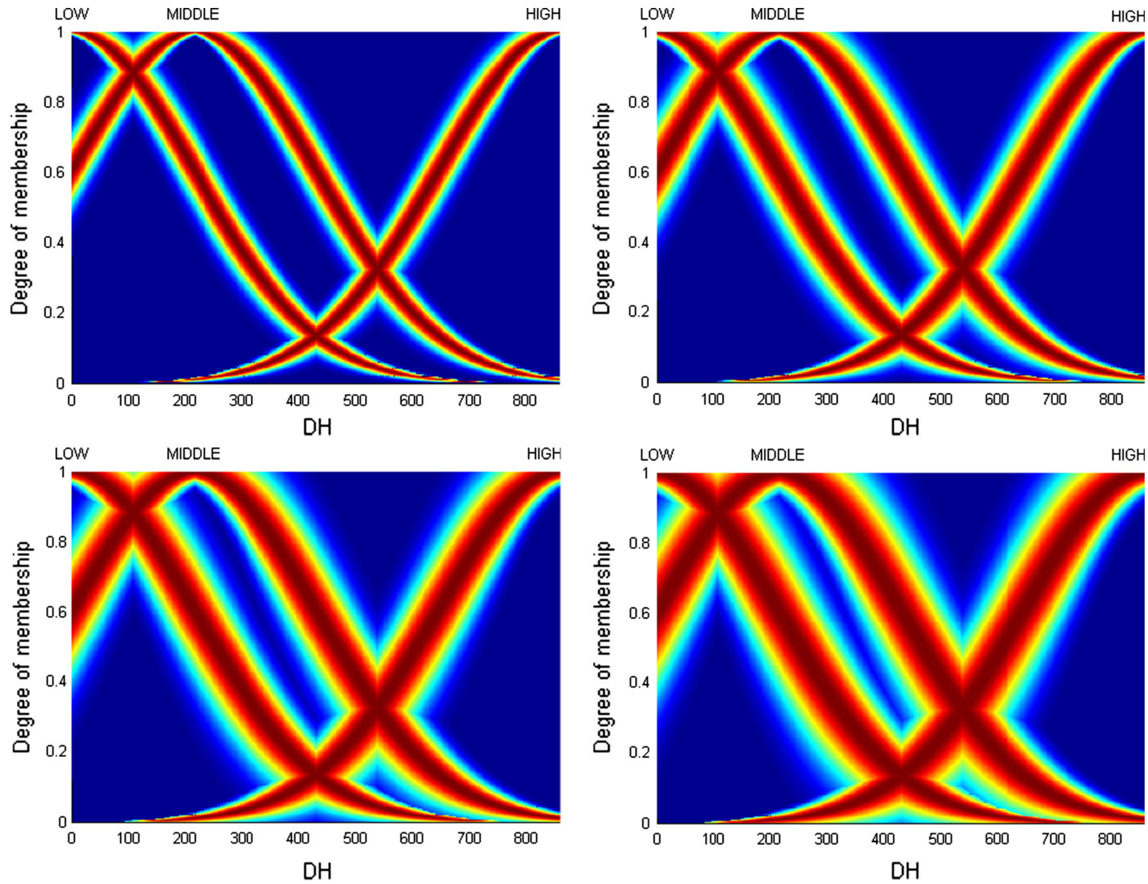


Fig. 8 DH Input for different generalized type-2 fuzzy membership functions

the metric values obtained after simulation of the ten synthetic images is presented in Table 2. It can be observed that the measurements obtained with the FOM given by (23) are better when using an FOU with a value of 0.8, where an FOM of 0.9606 is obtained. As mentioned previously, an FOM close to one means that the detected edge has good quality.

The trend (behavior) of the FOM values in Table 2 is such that as the FOU factor is increased, the values of the FOM also increase. These changes can be explained as follows: different FOUs represent different levels of uncertainty and there should be an optimal level of uncertainty for modeling the image; however, after the execution of the ten experiments, with an FOU factor of one, the FOM value was lower, and this means that the problem reaches its point of generalization and does not need more uncertainty level or has reached its optimum level.

In this paper, the methodology of the type-1 fuzzy inference system is not included; however in the simulation results when using an FOU value of 0, the type-1 fuzzy inference system is simulated, where an FOM of 0.9321 is obtained. Based on these results, it can be noted that the edge detector obtained using IT2FLS is better than the edge detector obtained with T1FLS. In the example of Fig. 7, the changes on the membership functions with different values of the FOU are shown.

Table 3 Simulation results with variation in the FOU for Sobel + GT2FLS

FOU factor	Pratt's (FOM) Mean
0.2	0.9617
0.4	**0.9618**
0.6	0.9617
0.8	0.9616
1.0	0.9615

Bold value indicates the best result

In another test, the proposed edge detector based on generalized type-2 fuzzy logic is now applied to the same synthetic images of Table 1. This method (GT2FLS) is designed with the same structure of the IT2FLS (which was described in Sect. 5), based on the same inputs, outputs, and number of fuzzy rules and also to generate the membership functions, the same parameters are used. In Fig. 8, the membership functions considered for this case study are shown.

The results of this simulation are shown in Table 3, where the best result is obtained with an FOU factor of 0.4 with an FOM value of 0.9618. The FOM values are very simi-

Table 4 Comparative analysis of GT2FLS and IT2FLS for images with white Gaussian noise

Edge detector	DBI's					
	20		30		50	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Sobel + IT2FLS	0.8515	0.0264	0.9114	0.0431	0.9371	0.0183
Sobel + GT2FLS	**0.8730**	**0.0472**	**0.9340**	**0.0138**	**0.9411**	**0.0185**

Bold values indicate the best result

Table 5 Simulation results using Sobel, type-1, interval type-2 and generalized type-2 fuzzy logic systems

Edge detector	Pratt's (FOM) Mean
Sobel	0.7875
Sobel + T1FLS	0.9321
Sobel + IT2FLS	0.9495
Sobel + GT2FLS	0.9617

lar; however, we noticed that when the FOU is increased, the quality of edge detector (FOM) is decreased; this means that the generalized type-2 memberships functions have more degrees of freedom in the footprint of uncertainty. For this case study, they reach the optimum level to have a support with an FOU factor of 0.4.

Table 6 Simulation results using interval type-2 and generalized type-2 fuzzy logic systems

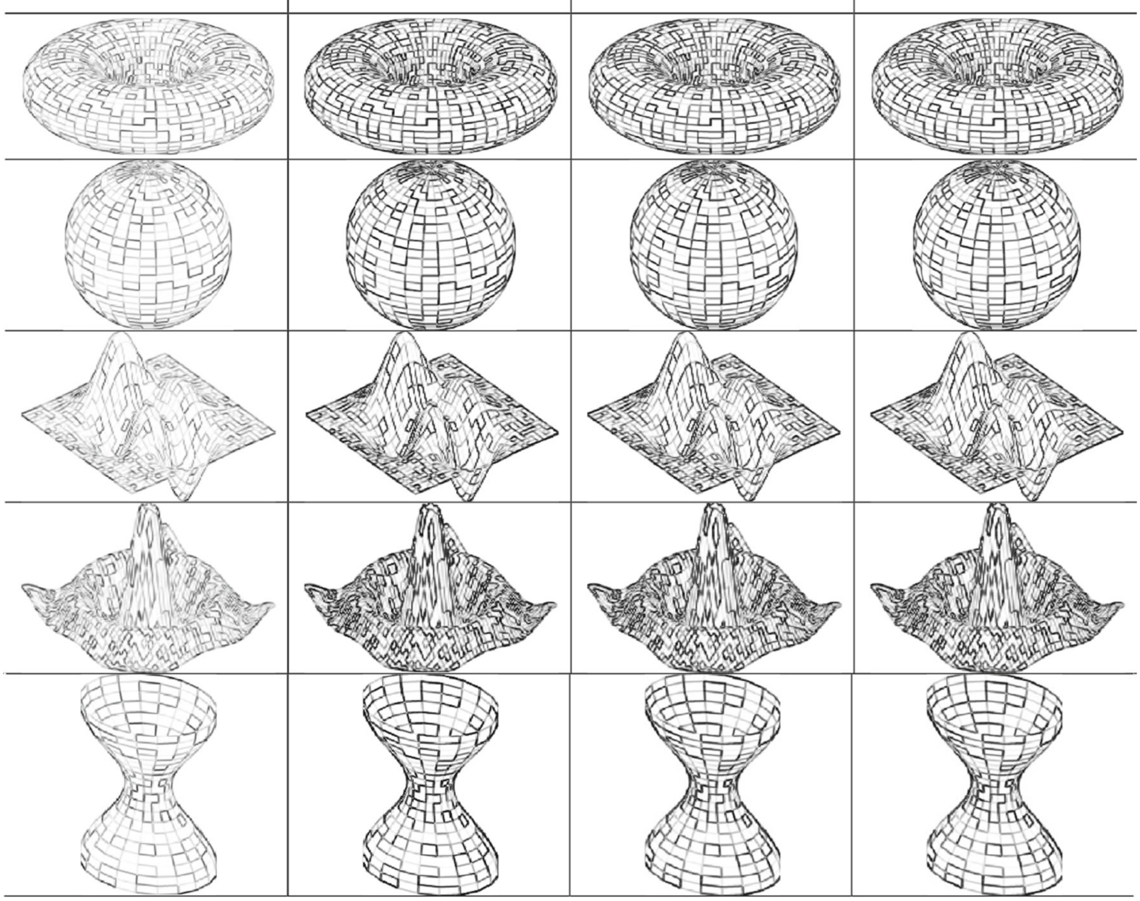
Edge detector	Pratt's (FOM) Mean	StDev
Sobel + IT2FLS	0.9495	0.0112
Sobel + GT2FLS	0.9617	0.000114

The tests were performed using images with white Gaussian noise; the noise level added was between 20 and 50 db. The test results of the 10 images with Gaussian noise for the IT2FLS and GT2FLS are shown in Table 4, and for all experiments the Sobel + GT2FLS was better than Sobel + IT2FLS.

Finally, the traditional Sobel edge detector is applied, and in the same manner the metric of Pratt (FOM) is implemented to evaluate the quality of edge detection; the results of this

Table 7 Detected edges using Sobel, with type-1, interval type-2 and generalized type-2 fuzzy logic systems

Sobel	Sobel + T1FLS	Sobel + IT2FLS	Sobel + GT2FLS



simulation are presented in Table 5. In Table 5, four edge detectors are shown: the traditional edge detector Sobel, the Sobel + T1FLS (obtained implicitly in simulating the edge detector IT2FLS with FOU factor of 0), the Sobel + IT2FLS and the proposed method Sobel + GT2FLS; for the IT2FLS and GT2FLS, the Mean is obtained from Tables 2 and 3, respectively. Tables 5 and 6 show that the proposed method based on GT2FLS is better than IT2FLS, obtaining an FOM of 0.9617, while IT2FLS was better than T1FLS with an FOM of 0.9495 and the traditional Sobel method is the worst with an FOM of 0.7875. In Table 7, examples of detected edges are shown after performing the simulation with the already mentioned methodologies (Sobel, T1FLS, IT2FLS, GT2FLS). Graphically, we cannot notice the difference in the detected edges and for this reason the metric (23) described in Sect. 5 is applied.

Based on the statistical values shown in Table 6, we can apply a t student test and the t value is of 2.44 and this has associated with a p value of 0.036, which means that there is sufficient statistical evidence to state that there is a significant advantage in using the generalized type 2 Sobel method. In other words, the difference in results shown in Table 6 is sufficient to say that generalized type 2 is better than interval type 2 for this process of edge detection.

7 Conclusions

In summary, the experimental results presented in Table 5 show that the use of generalized type-2 fuzzy inference systems can improve the performance in edge detection with respect to interval type-2 fuzzy systems. After evaluating the detected edges with Pratt's FOM, we can notice the advantages of the GT2FLS to model uncertainty in the gray tone values for the edges. The Sobel + GT2FLS method obtains better edges than when using the Sobel + IT2FLS, because it preserves more details of the original image; on the other hand, the Sobel + IT2FLS can improve the performance with respect to the Sobel+ T1FLS. In general, these results demonstrate that the use of fuzzy logic systems can improve traditional methods for image processing; specifically, the Sobel edge detector has been presented in this paper.

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