Color Edge Detection Using Orthogonal Polynomials

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Abstract

An orthogonal polynomials based maximizing signal-to-noise ratio (SNR) scheme for edge detection in 2-D color image is proposed in this paper. The proposed framework takes into account not only the spatial interaction within each of the three color planes (R, G, B) but also the interaction between the color planes. The edge detected output by the proposed scheme is also compared with two existing color edge detection schemes.

1 Introduction

An edge in color image is defined by discontinuity in a three dimensional color space. The well known R, G, B color space has been chosen here in which the discontinuity can be assumed to be in the form of Euclidean distance. Traditionally, color edge detection has been treated as a simple extension of the monochrome edge detection in separable color space. The use of the Hueckel operator in the luminance-chrominance color space has been proposed in [1] and an application of the compass gradient edge operators to color images has been highlighted in [2] . However, the existing color edge detection methods are based on extension of the well known gradient based techniques proposed for monochrome images. In these methods [3], edges are computed in three chosen color components separately and are merged finally by using some specified procedures. However, this approach may be very unsatisfactory in certain cases where the image gradients show the same strength but in opposite direction. Moreover, these edge detection methods are mainly based on a single criterion, namely, maximization of amplitude response of some derivative operators to the image. However, a better criterion, namely, "maximization of the operators' responses towards edges compared to the operators' responses towards noise" that has been proposed in [4] for monochrome images is difficult to extend for composite color images. Consequently, the problem of color edge detection in vector space has been proposed [5]. Here, color images are treated as vector fields and an edge detection method has been suggested. The "entropy operator" as edge detector is proposed in [6] . Here the entropy of brightness is defined in a local region in the picture. For color images, entropy of brightness as well as color is defined in the local region.

In this paper, a computational framework for facilitating color edge detection, in RGB color space is presented. Using a set of orthogonal polynomials difference operators are configured and employed to represent a color image region. A color edge detector based on maximizing SNR is then devised in presence of Guassian noise. The proposed color edge detector is also compared experimentally with two existing color edge detection schemes.

2 Orthogonal Polynomials Based Framework

Since an edge can be detected based on the local properties of the image, a local point-spread operator is required to be devised such that it is a cartezian as well as color coordinate separable and deblurring operator. The three dimensional point-spread function $M(x,y,z)$ can be considered to be a real valued function defined for $(x,y,z) \in X \times Y \times Z$ where X, Y and Z are ordered subsets of real values. In the case of a color image of size $(n \times n \times n)$ where X (rows) consists of a finite set, which for convenience can be labeled as $\{0,1,\ldots,n-1\}$, the functions $M(x,y,z)$ reduces to a sequence of functions

$$M(i,t) = u_i(t), i = 0,1,\ldots,n-1 \tag{1}$$

As shown in equation 2 the process of image analysis can be viewed as the linear transformation defined by the point-spread operator $M(x,y)(M(i,t) = u_i(t))$,

$$\beta'(\zeta,\eta,s) = \int_{x\in X}\int_{y\in Y}\int_{z\in Z} M(\zeta,x)M(\eta,y)M(s,z)I(x,y,z)dxdydz \tag{2}$$

where ζ,η,s are coordinates in the 3-D transformed space and $I(x,y,z)$ is a color image region wherein x and y are two spatial coordinates and z indicates the color space. Considering both X, Y and Z to be finite set of values $\{0,1,2\ldots,n-1\}$ equation 2 can be written in matrix notation as follows

$$|\beta'_{ijk}| = (|M| \otimes |M| \otimes |M|)^t |I| \tag{3}$$

where the point-spread operator $|M|$ is

$$|M| = \begin{vmatrix} u_0(t_1) & u_1(t_1) & \ldots & u_{n-1}(t_1) \\ u_0(t_2) & u_1(t_2) & \ldots & u_{n-1}(t_2) \\ & & \vdots & \\ u_0(t_n) & u_1(t_n) & \ldots & u_{n-1}(t_n) \end{vmatrix} \tag{4}$$

$\otimes$ is the outer product and $|\beta'_{ijk}|$ be the n^3 matrices arranged in the dictionary sequence. $|I|$ is the image and $|\beta'_{ijk}|$ be the coefficients of transformation. We consider a set of orthogonal polynomials $u_o(t), u_1(t), ..., u_{n-1}(t)$ of degrees 0, 1, 2, ..., n-1, respectively. The generating formula for the polynomials is as follows.

$$u_{i+1}(t) = (t-\mu)u_i(t) - b_i(n)u_{i-1}(t) \ for \ i \geq 1, \tag{5}$$

$$u_1(t) = t - \mu, \quad and \quad u_0(t) = 1,$$

where

$$b_i(n) = \frac{< u_i, u_i >}{< u_{i-1}, u_{i-1} >} = \frac{\sum_{t=1}^{n} u_i^2(t)}{\sum_{t=1}^{n} u_{i-1}^2(t)}$$

and

$$\mu = \frac{1}{n}\sum_{j=1}^{n} t_j$$

Considering the range of values of t to be $t_j = j, j = 1, 2, 3, ..., n$, we get

$$b_i(n) = \frac{i^2(n^2 - i^2)}{4(4i^2 - 1)}, \quad \mu = \frac{1}{n}\sum_{j=1}^{n} t_j = \frac{n+1}{2}$$

Next, we construct point-spread operators $|M|$ s of different width from the above orthogonal polynomials using equation 4.

2.1 Complete set of difference operators for color images

In case of R-G-B color space, the elements of finite set X, Y and Z are labeled as $\{1, 2, 3\}$. The point - spread operator in (4) that defines the linear transformation of color images can be obtained as $|M| \otimes |M| \otimes |M|$. The set of 27 three dimensional basis operators $O_{ijk}, (0 \leq i, j, k \leq 2)$ can be computed as follows.

$$O_{ijk} = \hat{u}_i \otimes \hat{u}_j \otimes \hat{u}_k$$

where $\hat{u}_i$ is the (i+1) st column vector of $|M|$. It can be shown easily that O_{ijk}s (except O_{000}, because β_{ooo} is the DC component) are symmetric finite difference operators. β'_{ijk}s are the coefficients of the linear transformations defined as follows.

$$|\beta'_{ijk}| = |\mathcal{M}|^t |I| \tag{6}$$

where $|\mathcal{M}|$ is the 3-D point-spread operator defined as $|\mathcal{M}| = |M| \bigotimes |M| \bigotimes |M|$. Now it can be shown easily that the orthogonal transformation (6) defined by the orthogonal system $|\mathcal{M}|$ is complete.

2.2 Responses Towards Color Edge and Noise

The coefficients $|\varsigma|$ of the complete 3-D orthogonal transformation defined by the set of polynomials are mean squared amplitude responses per unit volume of the n^3 (n = 3) basis operators. In general, $|\beta'|$ ($= |\mathcal{M}|^t |I|$) are mean squared amplitude responses of the operators, where

$$|\varsigma| = (|\mathcal{M}|^t |\mathcal{M}|)^{-\frac{1}{2}} |\beta'| \tag{7}$$

Division of each component of $|\beta'|$ by the corresponding element of $(|\mathcal{M}|^t |\mathcal{M}|)^{-\frac{1}{2}}$ results in unity noise gain during convolution with each of the 26 finite difference operators O_{ijk}s. In the presence of random noise only each of the 26 $|\varsigma|$ values which gives an estimate of the standard deviation of a component is the same as the standard deviation of the noise.

3 Separation of Responses towards color edges from responses towards noise

In presence of additive noise the proposed polynomials based model for the color image is

$$I(x, y, z) = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} \beta_{ijk}\ u_i(x)u_j(y)u_k(z) + \eta(x, y, z) \tag{8}$$

where the ηs are the additive noise which are normally distributed with zero mean and constant variance σ^2 and are uncorrelated. Since ς^2_{ijk}s are the corresponding mean squares of β_{ijk}s, each ς^2_{ijk} is a $\chi^2\sigma^2$ variate with one degree of freedom. In other words, due to completeness criterion, the local variance of the color image region is decomposed into the $(n^3 - 1)$ number of non-negative quadratic forms ς^2_{ijk} as follows.

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}(I_{ijk} - \beta_{000})^2 = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} \varsigma^2_{ijk}\ \mathit{at}\ \mathit{not}(i = j = k = 0) \tag{9}$$

where β_{000} is the local mean $\left(\frac{1}{n^3}\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} I_{ijk}\right)$.

Without any ambiguity ς^2_{ijk} may be termed as variances corresponding to the responses of the last $(n^3 - 1)$ basis, difference operators. Our next task is to group these $(n^3 - 1)$ variances into two disjoint subsets, say ψ_s and ψ_e as the estimates of variances corresponding to the responses towards edges and noise, respectively.

The desired grouping can be accomplished by observing the fact that the polynomial finite difference operators $O^2_{01}, O^2_{10}, O^3_{01}, O^3_{10}$(the superscript denotes the width of the polynomial basis operator) can be used to represent some of the widely known enhancement/thresholding edge operators for monochrome images. In general, the set of 3-D polynomials based finite difference operators $\{O_{0j0}\}|_{0<j\leq n-1} \cup \{O_{i00}\}|_{0<i\leq n-1} \cup \{O_{00k}\}|_{0<k\leq 2}$ may be considered as the color edge operators. Thus the set of mean squared amplitude responses $\{\varsigma_{0j0}\}_{0<j\leq n-1} \cup \{\varsigma_{i00}\}_{0<i\leq n-1} \cup \{\varsigma_{00k}\}_{0<k\leq 2}$ of these color edge operators are basically the responses towards edges and the responses of the remaining operators are the responses towards noise.

3.1 The statistical Grouping criterion for Color edges

In order to frame the desired grouping criterion, we introduce here some statistical concepts and terminology. In statistics, a linear contrast A in the given input data sample $X(= \{x_1, x_2, ..., x_k\})$ is defined numerically shown as $A = \sum_{i=1}^{k} \lambda_i x_i$ where λ_is are some values such that $\sum_{i=1}^{k} \lambda_i = 0$. Suppose B is another linear contrast in X such that $B = \sum_{i=1}^{k} \mu_i x_i$ where $\sum_{i=1}^{k} \mu_i = 0$. Two contrasts A and B are said to be uncorrelated if $\hat{\lambda}(= \{\lambda_1, \lambda_2, \lambda_3, ...\lambda_k\})$

and $\hat{\mu}(=\{\mu_1,\mu_2,\mu_3,...,\mu_k\})$ are orthogonal. Now, it can be shown easily that uncorrelated linear contrasts are unbiased statistical estimates. The physical significance of which is that their effects are mutually independent to each other. Furthermore, if $A' = A/(\sum_i^k \lambda_i^2)^{\frac{1}{2}}$ then A' may be considered to be a linear contrast per unit length.

In case of the proposed polynomials based transformation the ς_{ijk}s (except ς_{000}), which are the mean squared amplitude responses of the finite difference operators O_{ijk}s (except O_{000}), can be shown as uncorrelated linear contrasts per unit length. ς_{ijk} may be called the linear contrast in $|I|$ due to jointly the i^{th} order finite difference along x axis, j^{th} order finite difference along y axis, and k^{th} order finite difference along the color coordinate. ς_{i00} is termed as the linear contrast due to i^{th} order finite difference along x axis only, whereas ς_{0j0} is the linear contrast due to the j^{th} order finite difference along y axis only and ς_{00k} is the linear contrast due to the k^{th} order finite difference along the color coordinate. As ς_{ijk}s are unbiased statistical estimates and ς_{ijk}^2 are estimates of variances corresponding to the proposed finite difference operators' amplitude responses the latter can be considered to be unbiased statistical estimates of variances. In fact each ς_{ij}^2 is a $\chi^2\sigma^2$ variate with one degree of freedom. Since $\{\varsigma_{i00}\} \cup \{\varsigma_{0j0}\} \cup \{\varsigma_{00k}\}$ are the responses towards color edges, $\psi_s = \{\varsigma_{i00}^2\} \cup \{\varsigma_{0j0}^2\} \cup \{\varsigma_{00k}^2\}$ are the unbiased statistical estimates of edge response and $\psi_e = \{\varsigma_{ijk}^2, 0 < i \leq n-1, 0 < j \leq n-1, 0 < k \leq 2\}$ are the unbiased statistical estimates for the noise present.

In order to ensure that a set of $\chi^2\sigma^2$ variates with known degrees of freedom are basically the estimates of the same noise variance, Nair's test for homogeneity among variances [7] is used. The significance of the responses towards color edges compared to noise has then been measured by performing another statistical test viz. F-ratio test [8], after computing the mean square error variance, $\bar{\eta}_o^2$.

4 Edge detection by maximizing Signal-to-Noise Ratio

Algorithm

Input : `Color image with three components R, G and B of size 256 * 256 * 3`

Output : `Edge detected image of size ( 256 * 256)`

Step 1 `If( end of image), go to step 8.`
`Else extract a small color image region [I] of size (n*n*3).`

Step 2 `Compute the mean squared amplitude responses` ς_{ijk} `as follows:`
$|\varsigma_{ijk}| = (|\mathcal{M}|^t|\mathcal{M}|)^{-\frac{1}{2}}|\mathcal{M}|^t|I_{ijk}|$

Step 3 `Apply Nair's test to determine whether the variance` $\varsigma_{ijk}^2 \in \psi_e$ `are estimates of the same variance.`

If (yes) compute the mean squared error variance $\bar{\eta}_0^2$ from them.
Else go to step 1.

Step 4 Repeat the following for the set ψ_s of all the mean squared amplitude responses.

1. Perform variance ratio test (F test) with the error variance $\bar{\eta}_0^2$ as the denominator.
2. Add those mean squared variances for which the null hypothesis is rejected.

Step 5 Compute the RMS value of the sum of the mean squares obtained at step 4.

Step 6 If the RMS value $\geq$ a threshold T then the presence of an edge is detected at the center position of $|I|$
Else go to step 1.

Step 7 Go to step 1.

Step 8 Stop.

5 Experiments and Results

The orthogonal polynomials based framework for color edge detection has been experimented with different untextured color images. One such original untextured color image viz. bird image, is shown in figure 1. The presence of color edges is detected based on the fact that out of 26 mean-squared amplitude responses six, namely, $\varsigma_{001}, \varsigma_{002}, \varsigma_{010}, \varsigma_{020}, \varsigma_{100}, \varsigma_{200}$ are responses towards color edges whereas the remaining twenty mean squared amplitude responses $\{\varsigma_{ijk} | \varsigma_{ijk} \in \psi_e\}$ are responses towards noise. This has been verified by conducting experiments with a large number of randomly selected color image samples. The edge detection method using maximization of SNR has been applied on this image as stated in the previous section and the edge detected output is shown in figure 2. The edge detected output image by the proposed orthogonal polynomials based color edge detection scheme is also compared with (i) Color edge detection using vector order statistics [5] and (ii) Edge extraction using entropy operator [6] which are shown in figure 3 and 4 respectively. From these outputs, it is evident that the proposed color edge detection scheme shows either the same or better performance (for example see neck and head portions) than the other two color edge detection schemes.

6 Conclusion

An orthogonal polynomials based framework for color edge detection in RGB color space is presented in this paper. The framework takes into account not

Figure 1: Original Bird Image

Figure 3: Edge Detection Using Vector Order Statistics Scheme

Figure 2: Edge Detection Using Max. SNR Scheme

Figure 4: Edge Detection Using Entropy Scheme

only the spatial interaction within each of the three color planes but also the interaction between different planes. The proposed framework is based on a complete set of difference operators which are easily configurable from a set of orthogonal polynomials. The operators are employed to represent a color image region as a linear combination of the operator's responses towards color edge and noise. A simple statistical design of experiments paradigm is used for separating out the responses towards color edge from that of noise. The framework supports in devising a color edge detector based on maximizing signal-to-noise ratio (SNR). The proposed color edge detector is experimented on various untextured color images and are compared with two existing color edge detecting schemes.

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