12/4/18

NP-COMPLETE

→ Travelling Salesman: Given a set of distances on 'n' cities and a bound D, is there a ~~touur~~ tour of length $\leq D$

undirected weight graph

Start, visit all cities, and come back.

→ To show your problem X is NP-Complete:

(1) $X \in NP$ Argue that it has efficient verifies

(2) [some known NP-Complete Problem] $\leq_p X$

i.e., your problem can be used to solve a known NP-Complete problem.

Y

∘ Common Mistake: Instead of [some known NP-Complete problem] $\leq_p X$

people do $X \leq_p$ [some known NP-Complete problem] → THIS IS WRONG!

→ For problem X and Y, we write $Y \leq_p X$ if $\exists$ poly-time algo R taking instance of Y to instances of X such that if:

(1) S_Y is a yes-instance of Y $\Leftrightarrow$ $R(S_Y)$ is a yes-instance of X

input to Y

(2) S_Y is a no-instance of Y $\Leftrightarrow$ $R(S_Y)$ is a no-instance of X

logically redundant to prove both parts, we can prove only for first part.

→ Set Cover: Give a set U of n elements, and a collection $S_1, S_2, \ldots, S_m$ of subsets of U, and a number k, does there exist at most k of these sets whose union is equal to all of U?

Example of Set Cover:

$U = \{a, b, c, d, e\}$

$S_1 = \{a, b\}$

$S_2 = \{b, c, d\}$

$S_3 = \{b, d, e\}$ →

$S_4 = \{a, b, c\}$ → $S_3 \cup S_4 = U$

$k = 2$

→ Claim: Set Cover is NP-Complete.

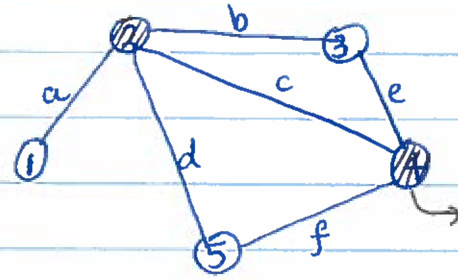
1. Set Cover $\in$ NP because the collection of subsets that covers U is the certificate which can be checked in poly time

$\leq k$

a.k.a "witness" "t"

2. $\underbrace{\text{[Known NP-Complete problem]}}_{\text{VERTEX COVER}} \leq_p$ [Set Cover]

∘ Vertex Cover: Given undirected graph G and integer k, is there a subset M of at most k nodes such that every edge has at least one endpoint in M?



→ Example of Vertex Cover

$K = 2$

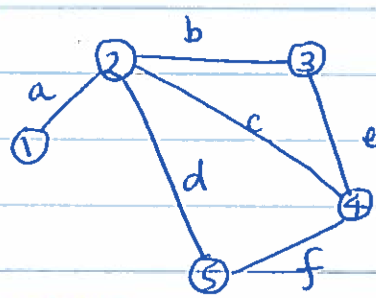
→ Solution → witness = $\{2, 4\}$

- Given instance (G, k) of vertex cover, construct instance of set cover as follows:

  U = set of all edges

  for each node i, let S_i = all edges incident to i.

  Number k for set cover = k for vertex cover.



~~EXAM~~ INSTANCE OF VERTEX COVER

→

INSTANCE OF SET COVER:

$U = \{a, b, c, d, e, f\}$

$S_1 = \{a\}$

→ $S_2 = \{a, b, c, d\}$

$S_3 = \{b, e\}$

→ $S_4 = \{c, e, f\}$

$S_5 = \{d, f\}$

Then (G, k) is a yes-instance of vertex Cover

$\Leftrightarrow$ $(U, \{S_1, \ldots, S_n\}, k)$ constructed as above is a yes-instance of Set Cover.

★ You don't have to map instances the other way around [i.e, set cover to vertex cover] → not clear how to do so.