Homework #9

You do not need to turn in these problems. The goal is to reinforce what we learned in class, as well as to cover material we didn't have time to cover in class. The in-class quiz that will cover the same or similar problems. Material on homework can also appear on exams.

Some known $\mathcal{NP}$-complete problems
You can assume the following problems are $\mathcal{NP}$-complete, and reduce them to the problems below to show that the problems below are also $\mathcal{NP}$-complete. Remember, to prove that problem X is $\mathcal{NP}$-complete, you must: (1) show that it is in $\mathcal{NP}$ (show that a solution can be verified efficiently) and (2) show that a known $\mathcal{NP}$-complete problem polynomial-time reduces to problem X.

3-**colorability.** *Given a graph G, can the set of vertices be partitioned into 3 sets such that no two vertices within the same set have an edge between them? In other words, can the graph be colored with three colors such that no adjacent vertices share a color?*

Vertex Cover. *Given a graph G and a number k, does G contain a vertex cover of size at most k? (Recall that a vertex cover $V' \subseteq V$ is a set of vertices such that every edge $e \in E$ has at least one of its endpoints in V'.)*

Subset Sum. *Given a set of n integers $S = \{x_1, x_2, \ldots, x_n\}$ and a target sum T, does there exist a subset $S' \subseteq S$ such that $\sum_{x \in S'} x = T$?)*

Problem 1: Approximate Subset Sum
Show that the following problem is $\mathcal{NP}-$Complete:
Given set S of n integers, $S = \{x_1, ..., x_n\}$, and two integers T, e such that $T > e \geq 1$ is there a subset $S' \subseteq S$ such that $T - e \leq \sum_{x \in S'} x \leq T$?

Problem 2: Efficient Recruiting
Suppose you're helping to organize a summer sports camp, and the following problem comes up. The camp is supposed to have at least one counselor who is skilled at each of the n sports covered by the camp (baseball, volleyball, etc.). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is: For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports? We'll call this the **Efficient Recruiting** Problem. Show that **Efficient Recruiting** is $\mathcal{NP}$-Complete.

Problem 3: Feedback Vertex Set, Directed
A set of vertices $S \subseteq V$ of a graph $G(E, V)$ is a feedback vertex set if the induced subgraph after deleting the vertices of S (and the respective edges) has no cycle. Show that the

following problem is $\mathcal{NP}-$Complete:
FVS. *Given a directed graph $G(V,E)$ and non negative integer k, is there a Feedback Vertex Set(FVS) $S \subseteq V$ with k or less vertices?*

Problem 4: 2018-colorability
We can extend the definition of 3**-colorability** to the following:

k**-colorability**: *Given a graph G, can the set of vertices be partitioned into k sets such that no two vertices within the same set have an edge between them?*

Show that 2018**-colorability** is $\mathcal{NP}$-complete.