11/15/18

CHAPTER 7 : NETWORK FLOWS

- Maximize flow of material through network

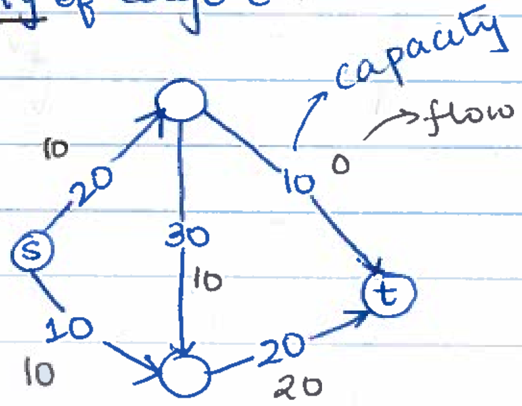
FLOW NETWORK:

→ Directed graph G, two distinguished vertices → s (source) a

t (sink) (move things from source to sink)

Assumption: No edges enter s and no edges leave t

→ c(e) is capacity of edge e (non-negative integer)



non-negat
re

- Def: s-t flow is a function f assigning a numb
  to every edge such that:
  - flow on e does not exceed capacity of e

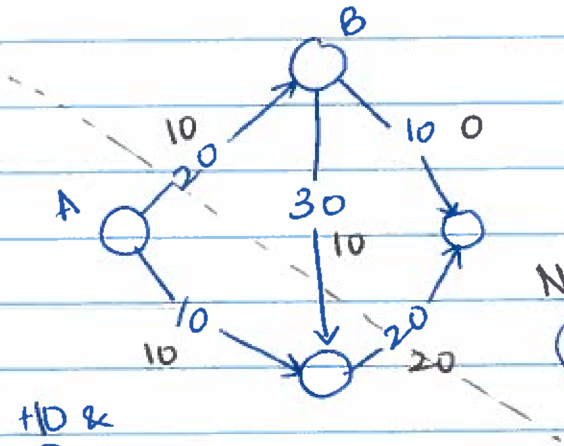
$$\boxed{f(e) \leq c(e)}$$

  - except for s, t flow is conserved:

$$\boxed{\forall\ v \text{ except } s,t : \sum_{e \text{ into } v} f(e) = \sum_{e \text{ out of } v} f(e)}$$

- Def: Value of flow f:

$$v(f) = \sum_{e \text{ out of } s} f(e)$$



flow leaving $s = 20$

Net flow through cut $= +10 - 10 + 20$
$= 20$

~~defined~~ such that s on one side and t on the other side.

?
What is +10 & what −10?

→ +10 from A to B

→ −10 from B to A

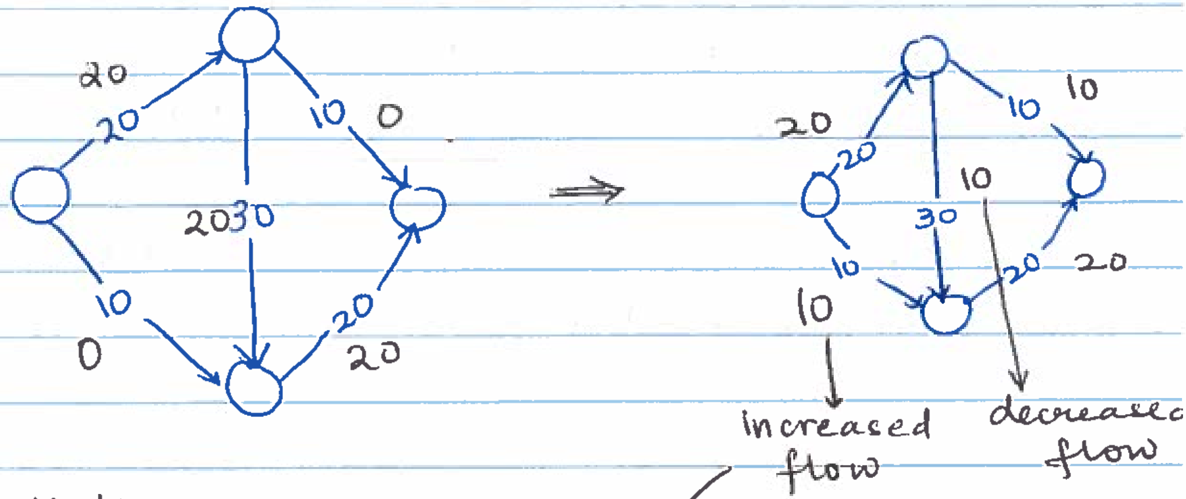
- Lemma: $v(f) = \sum_{e \text{ out of } s} f(e)$ = net flow through any s-t cut

Value of flow f

- For any $\overbrace{\text{s-t cut}}^{\text{A contains s and B contains t}}$ (A, B)

$$\sum_{\substack{e \text{ from} \\ A \text{ to } B}} f(e) - \sum_{\substack{e \text{ from} \\ B \text{ to } A}} f(e) = \sum_{\substack{e \text{ out} \\ \text{of } s}} f(e)$$

→ **MAX-FLOW PROBLEM**: Find s-t flow of maximum value.



value = 20

Not optimal!

Need to figure what can be increased/decreased.

value = 30
Optimal!

Make it more difficult!

Do not only look for increasing flows on edges!

→ How do you know what is the optimal value
[Can't just look at capacities of edges out of s!]

◦ **Def**: Capacity of s-t cut (A, B) is the sum of the capacities of edges from A to B:

$$cap(A,B) = \sum_{e \text{ out of } A} c(e)$$

Lemma: Let f be any s-t flow and (A, B) be an s-t cut, then $v(f) \leq cap(A,B)$

→ <u>Min-cut problem</u>: find s-t cut of minimum capacity

Suppose we find an s-t flow and s-t cut (A, B) such that $\underbrace{v(f)}_{\text{max-flow}} = \underbrace{cap(A,B)}_{\text{min-cut}}$

then $v(f)$ is the max flow value and A-B is min-cut (got from previous lemma).