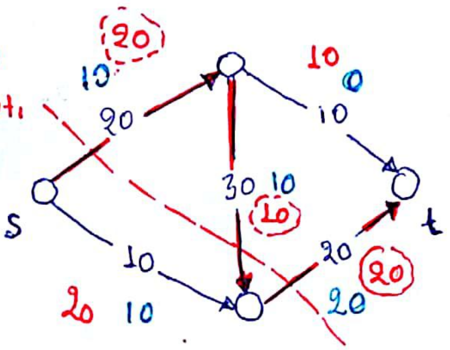
NETWORK FLOW

- Directed graph G
- Two distinguished vertices s and t ← assume no edge leaves t.
- $C(e) \geq 0$ is the capacity of edge e
  ↳ i.e. how much flow we can send through e.

assume no edge enters s

flow leaving s: 30

net flow through cut: 20 - 10 + 20 = 30



example of a valid flow in light blue

example of another valid flow in red

Def: s-t flow is a function assigning a non-negative real number to every edge, s.t.:

- flow on e doesn't exceed capacity of e: $f(e) \leq c(e)$.
- except for s,t flow is conserved $\forall v \in V \setminus \{s,t\}$

$$\sum_{e \text{ into } v} f(e) = \sum_{e \text{ out of } v} f(e)$$

Def: value of flow f $\quad v(f) = \sum_{e \text{ out of } s} f(e)$

$= \text{net flow through any } \underbrace{\text{s-t cut}}$

there is a lemma that proves this

partition of nodes s.t. s is on one side, t on the other

$\boxed{\text{Max-flow problem}}$

Find s-t flow of max value.

Def: <u>capacity</u> of s-t cut (A,B) is $cap(A,B) = \sum_{e \text{ from } A \text{ to } B} c(e)$

sum of the capacities of edges leaving A.

<u>Lemma</u>: Let f be any s-t flow, and (A,B) be any s-t cut, then $v(f) \le cap(A,B)$.

$\boxed{\text{MIN-CUT problem}}$

Find the s-t cut of minimum capacity.

Suppose we find a flow f and a cut (A,B) s.t.

$$V(f) = cap(A,B).$$

Then f is a max flow and (A,B) is the min cut.

"residual graph of G w.r.t. a flow f":

Construct "residual" graph G_f ← depends on particular flow f.

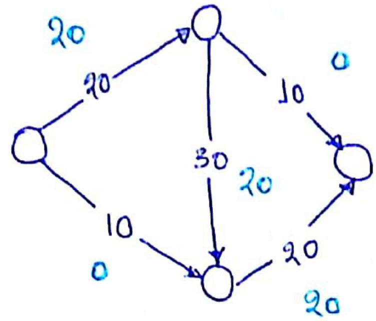
edge of G: $u \xrightarrow[6 = f(e)]{17 = c(e)} v$

edge of G_f: $u \underset{6 = f(e)}{\overset{11 = c(e) - f(e)}{\rightleftarrows}} v$ "forward edge" / "backward edge"

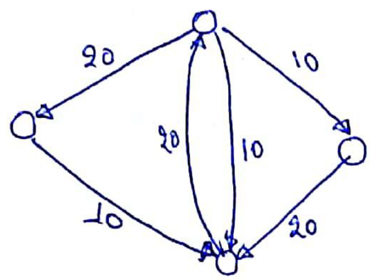
We call an edge (a,b) "forward" if it also exist in G, and backwards otherwise (i.e. if (b,a) exists in G).

* Don't put edge if it is zero.

Example:



G



G_f