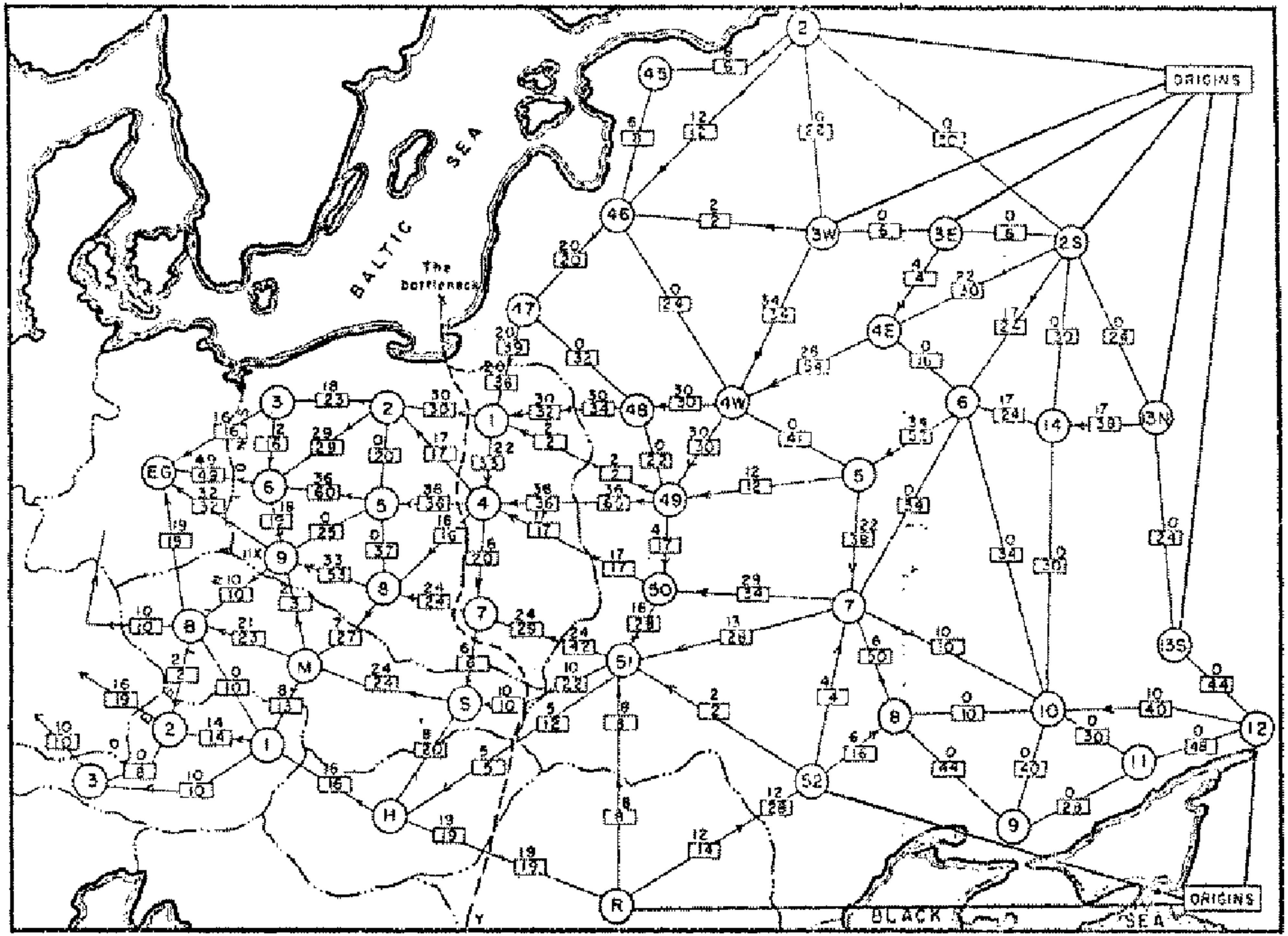
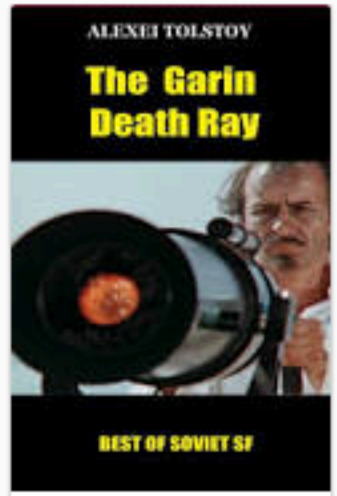
The Soviet Rail Network



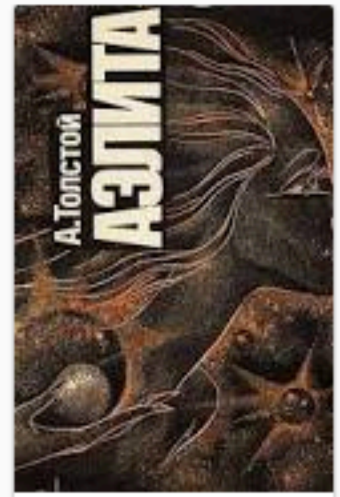
On the history of the transportation and maximum flow problems. Alexander Schrijver, Math Programming, 2002.

Aleksey Nikolayevich Tolstoy:
Science fiction writer

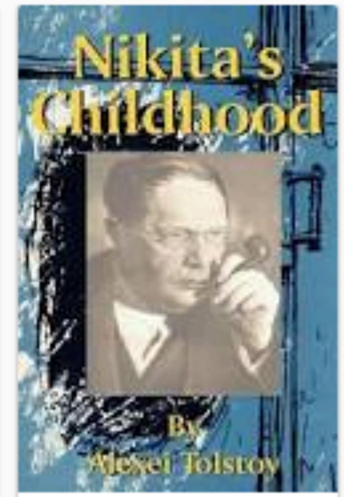
The Golden Key, or the Ad...
1936



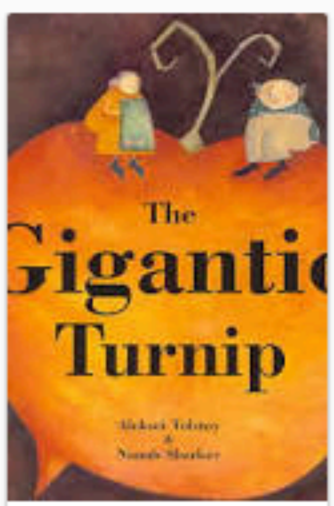
The Garin Death Ray
1927



Aelita
1923



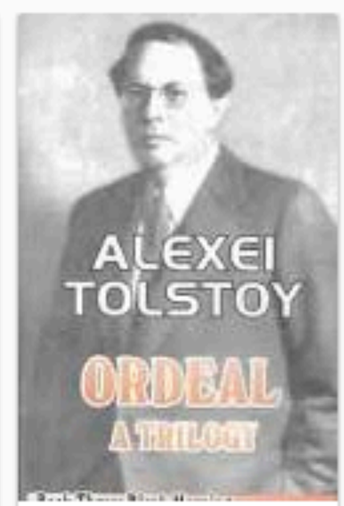
Nikita's childhood
1921



The Gigantic Turnip
1910

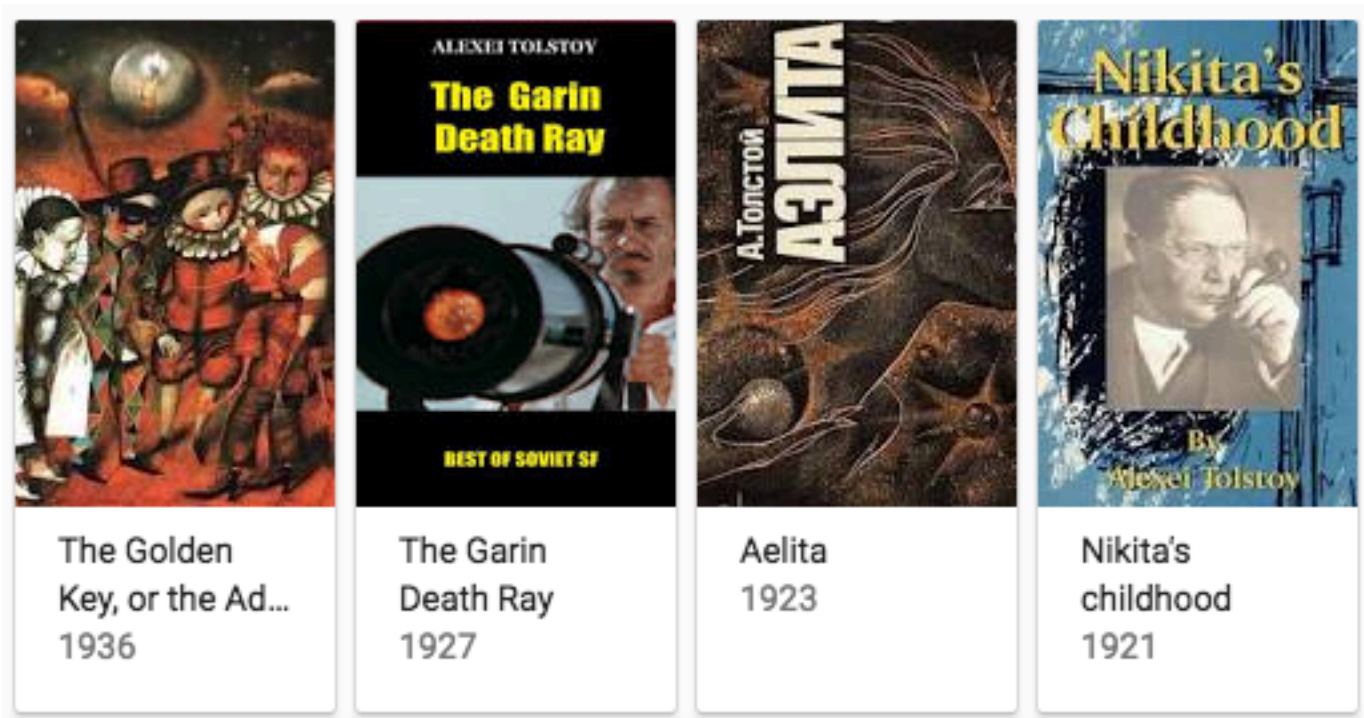
Пётр I
1930

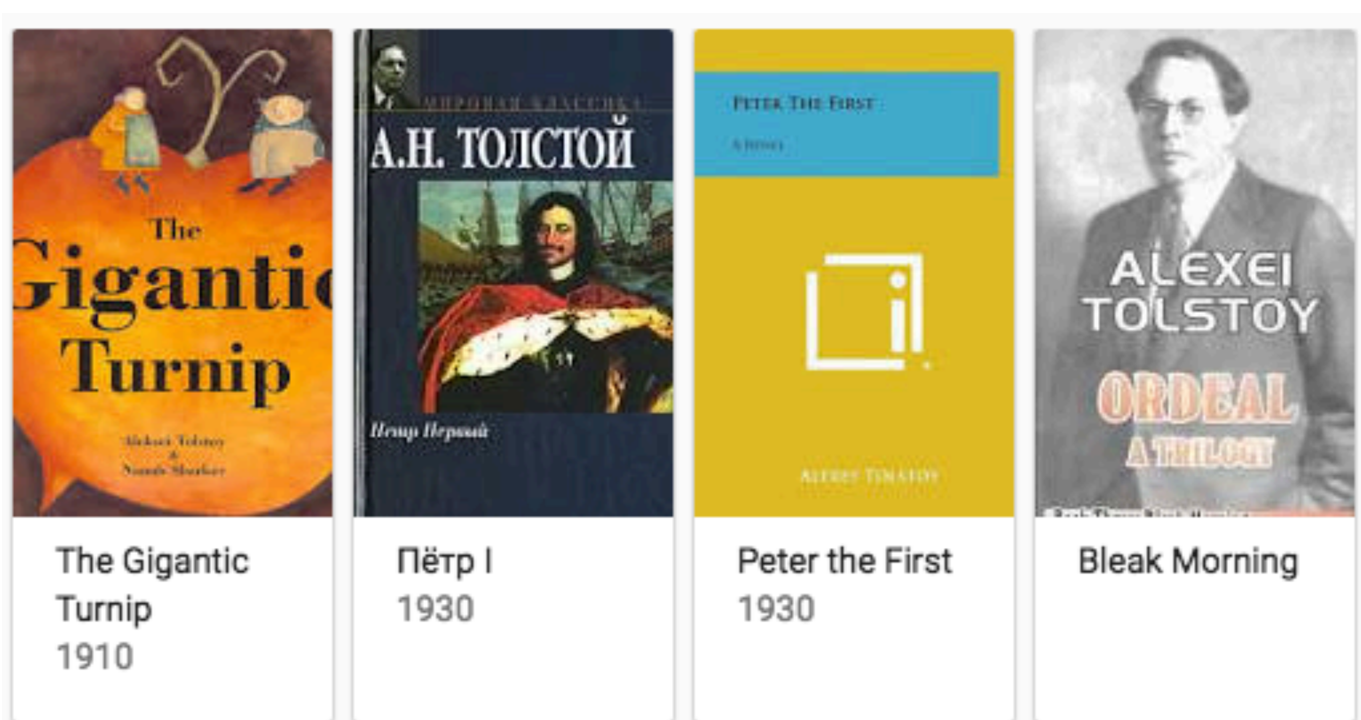
Peter the First
1930

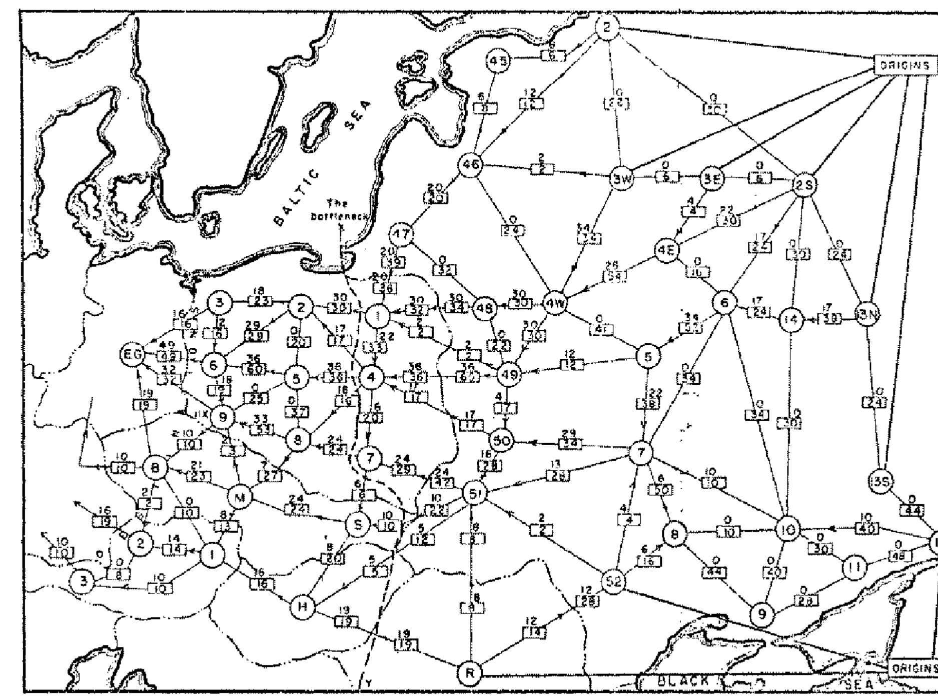


Bleak Morning

Aleksey Nikolayevich Tolstoy:
Science fiction writer







Maximum Flow and Minimum Cut

Max flow and min cut

- Two very rich algorithmic problems
- Cornerstone problems in combinatorial optimization
- Beautiful mathematical duality

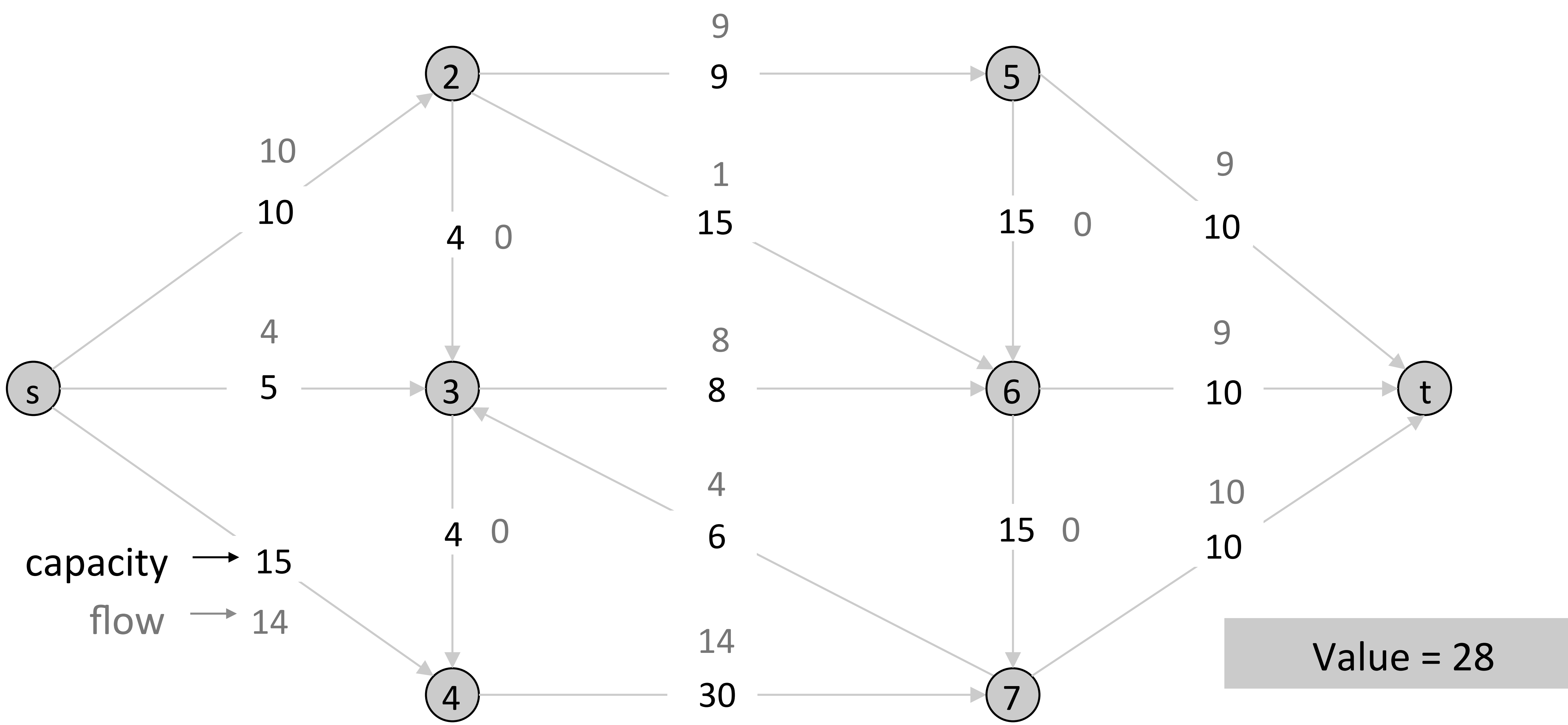
Nontrivial applications/reductions

- Data mining
- Open pit mining
- Airline scheduling
- Bipartite matching
- Baseball elimination
- Image segmentation
- Network connectivity
- Network reliability
- Distributed computing
- Egalitarian stable matching
- Security of statistical data
- Network intrusion detection
- Multi-camera scene reconstruction
- Many, many more...

THE MAXIMUM FLOW PROBLEM

The Max Flow Problem

Find the *s*-*t* flow of maximum value.



Towards a Max Flow Algorithm

Greedy Algorithm

- Start with $f(e) = 0$ for all edges $e \in E$
- Find an s-$t$path P where each edge has $f(e) < c(e)$.
- Augment flow along path P
- Repeat until you get stuck

Doesn't work!

The Ford-Fulkerson Algorithm

Max-Flow

  Initially $f(e) = 0$ for all e in G

  While there is an s–t path in the residual graph G_f

    Let P be a simple s–t path in G_f

    $f' = \mathtt{augment}(f, P)$

    Update f to be f'

    Update the residual graph G_f to be $G_{f'}$

  Endwhile

  Return f

```
augment(f, P)
  Let b = bottleneck(P, f)
  For each edge (u, v) ∈ P
    If e = (u, v) is a forward edge then
      increase f(e) in G by b
    Else ((u, v) is a backward edge, and let e = (v, u))
      decrease f(e) in G by b
    Endif
  Endfor
  Return(f)
```