

Final is cumulative!

Dynamic Programming

- Optimal Substructure
- Memoization
- Weighted Interval Scheduling
- Knapsack
- LCS
- Bellman-Ford
- Misc

Network Flow

- Relationship between max flow and min-cut.
- Ford-Fulkerson algorithm
  - ↳ integer max flow (integer flow on every edge)
  - capacities = 1  $\Rightarrow$  0/1 max flow.
- Using network flow to solve other problems

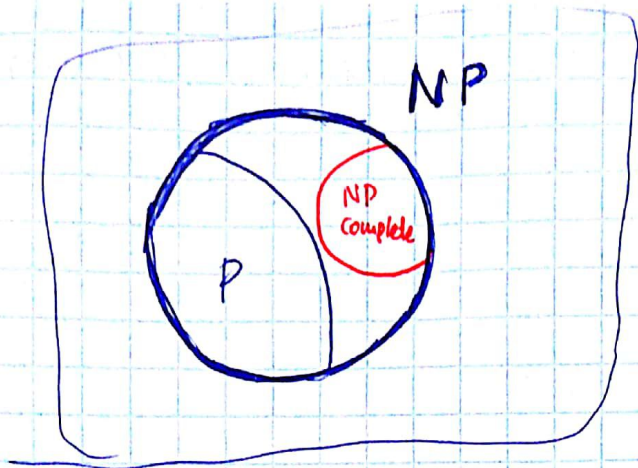
NP-Completeness

- What is P, NP, NP-Complete?
- How to prove your problem is NP-Complete
  - ↳ poly-time reduction.

Subset Sum: Given positive integers  $w_1, w_2, \dots, w_n$  and a target  $T$ , is there a subset of the integers that sums to precisely  $T$ ?

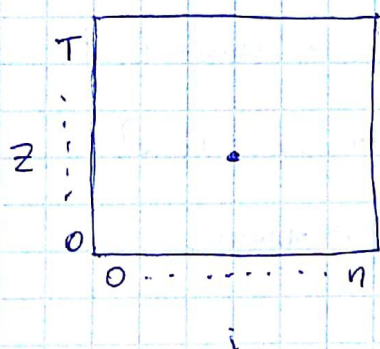
$$\text{OPT}(i, z) = \begin{cases} 1, & \text{if } \exists \text{ subset of } w_1, \dots, w_i \text{ that sums to } z \\ 0, & \text{else} \end{cases}$$





(Subset-Sum, cont.)

$$\text{OPT}(i, z) = \begin{cases} \text{base cases} \\ \max\{\text{OPT}(i-1, w), \text{OPT}(i-1, z-w_i)\} \end{cases}$$



$$O(n \cdot T)$$

if  $T$  is  $2^n$ ,  
it won't be poly-time

pseudo-polynomial time

↑  
polynomial in numeric  
value of the input

Subset-Sum is an NP-Complete problem.

$$\text{C-SAT} \leq_p \text{3-CNF-SAT} \leq_p \text{SUBSET-SUM}$$

conjunctive normal form

(OR OR OR) AND (OR OR OR) ...

usual way in  
which it is shown  
that SUBSET-SUM  
is NP-Complete.