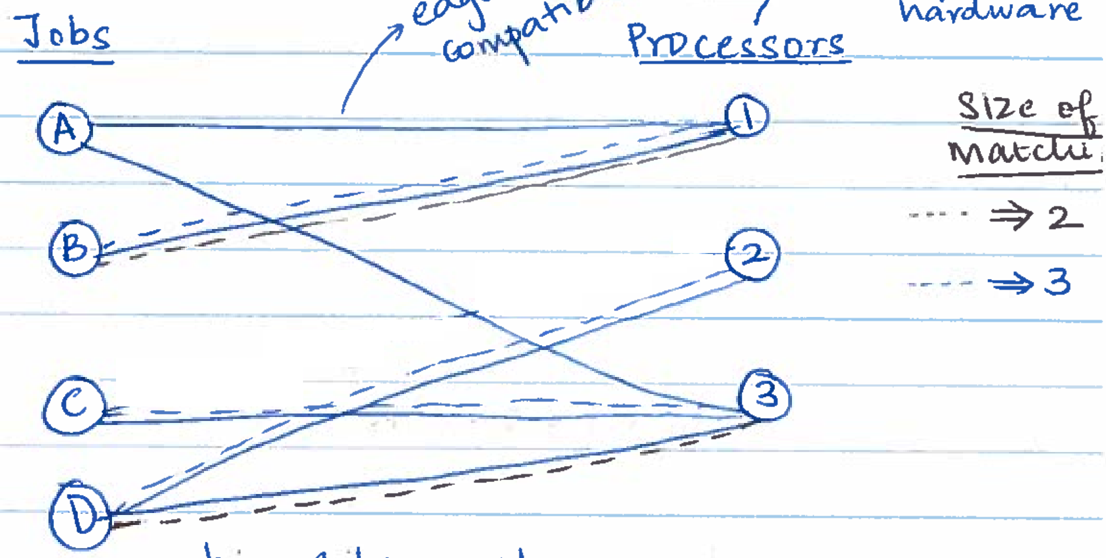
11/27/18

⇒ BIPARTITE MATCHING:

edge indicates job compatible with processor

with particular hardware

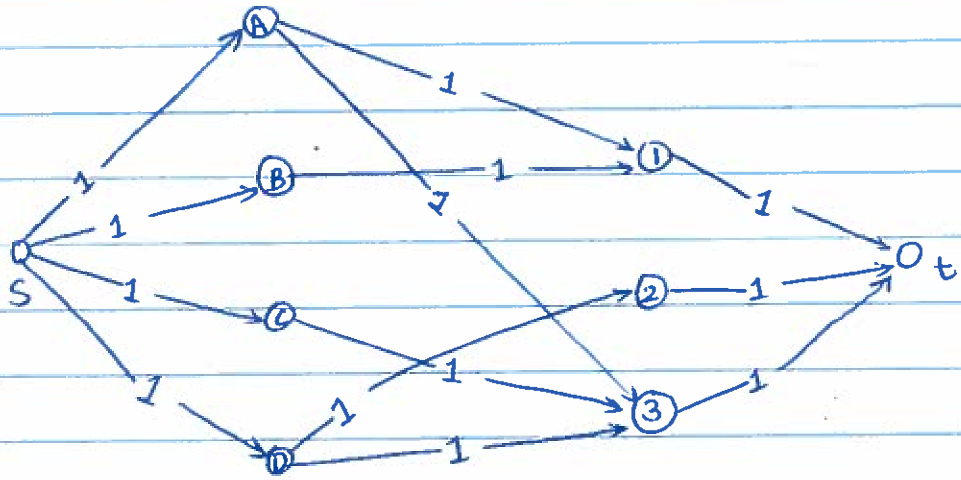


bipartite graph

- Valid Matching: ≤ 1 job per processor
  ≤ 1 processor per job
- Goal: Find the largest set of jobs that can be ru
  (Maximum Matching)

An application of network flow!

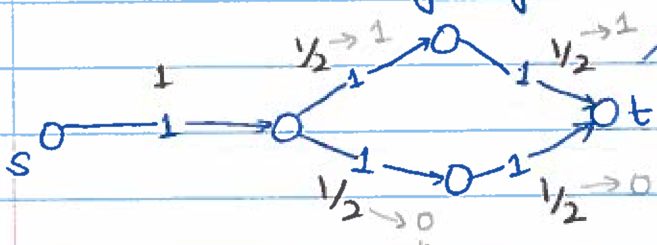
- Convert to a max-flow problem and run Ford-Fulkerson



Max flow network constructed!

<u>Claim</u>: $\exists$ valid matching of size k

$\iff \exists$ $\underbrace{0/1}_{\text{flow that assigns 0 or 1 to every edge}}$ flow of value k



difficult problem if assigning fraction of a job.

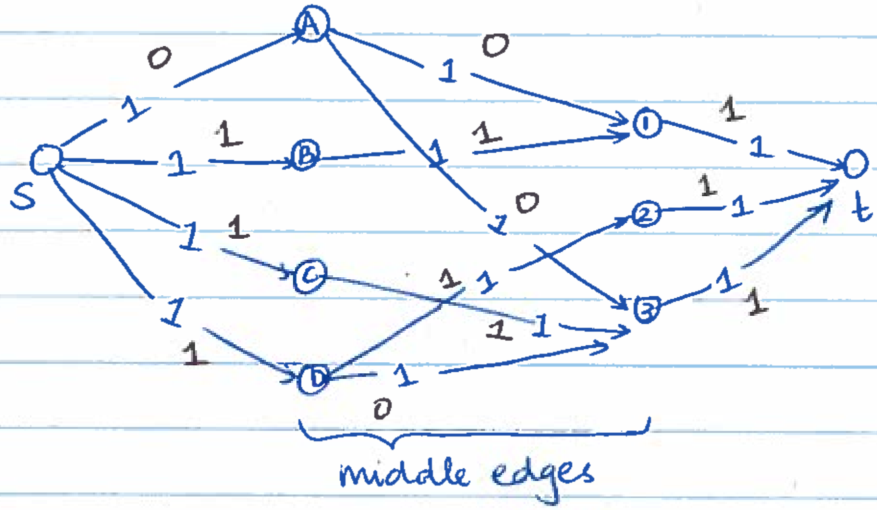
But we saw that FF algo on <u>integer capacities</u> returns maxflow that assigns <u>integers</u> to every edge.

$\Downarrow$

If capacity is 1 then FF returns 0/1 max flow

(⟹)

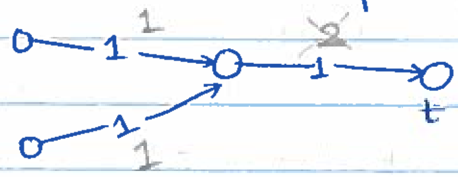
<u>Proof to Claim</u>: Assign flow 1 corresponding to the matching, 0 elsewhere.



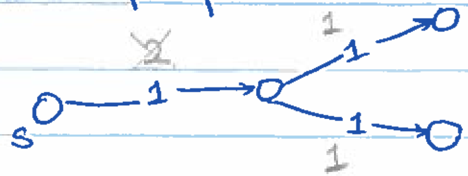
(⟸) If flow value is k, then k middle edges have flow 1. Assign the jobs to processors according to these edges.

? Is this a valid matching?

- Can multiple jobs be assigned to same processor?
  - Will exceed capacity so NO!



- Can multiple processors be assigned to same job?



NO!

Thus valid matching of size k!

Running Time:

FF: $O(m \cdot c) = O(m \cdot n)$ → # of jobs

m: # of edges

c: total capacity out of s

N-P COMPLETENESS

⇓ [chapter 8 in Textbook]

- Computational Intractibility

  polynomial v.s. exponential

  $O(n^c)$

  for some constant c

  Efficient | Not efficient / Not tractable

→ Context: Decision problems - yes/no answer.

- Does this bipartite graph have a matching of size $\geq k$?

- Given a Boolean formula, are there values of the variables that make it true?

BOOLEAN SATISFIABILITY PROBLEM (SAT) ⇒ $(x_1 \text{ OR NOT } x_2) \text{ AND } (x_2 \text{ OR } x_3)$

$x_1 = T$

$x_2 = F$

$x_3 = T$

Can assign 3 colors to nodes s.t. no two adjacent nodes hav the same color

- Is this undirected graph 3-colorable?
- (Subset Sum): Given positive integers $w_1, w_2, \ldots$ and a target W, is there a subset of $\{w_1, \ldots,$ that adds up to exactly W?

→ $\boxed{P:}$ Decision problem that can be ~~effectively~~ efficie[nt] poly-ti[me] computed

<u>Def:</u> Problem X is in P if there is poly time algo A such that

- if s is a "yes"-instance of X $\Leftrightarrow$ A(s) = Ye
- if S is a "no"- instance of X $\Leftrightarrow$ A(s) = n

All problems can be "<u>efficiently verified</u>". for yes answ
(the ones mentioned previously)
Asymmetric

- is this number composite?
not prime