EE 360C ALGORITHMS - December 4 - FALL 2018 - Evening

TRAVELLING SALESMAN: Given a set of distances on n cities, and a bound D, is there a tour of length $\leq$ D?

tour → Start, visit every node, come back

To show problem X is NP-complete:

1. $X \in NP$ i.e. There is efficient verifier
2. $\underbrace{\text{[some Known NP-Complete problem]}}_{Y} \leq_P X$

i.e. can use X to solve Y

Intuition: I know that problem Y is hard.
If I show that Y is "easier" than X
i.e. $Y \leq_P X$,
then I learn that X is at least as hard as Y, so if Y NP-complete X also NP-complete

$Y \leq_P X$ means: $\exists$ poly-time algo R that transforms instances of Y to instances of X s.t.:

- S_Y is a yes-instance of Y $\Leftrightarrow$ $R(S_Y)$ is yes-instance of X.
- S_Y is a no-instance of Y $\Leftrightarrow$ $R(S_Y)$ is a no-instance of X.

Set Cover: Given a set U of N elements
a collection $S_1, S_2, \ldots, S_m$ of subsets of U,
and a number k, does there exist at most k of these subsets s.t. their union is all of U?

ex. $U = \{a, b, c, d, e\}$
$S_1 = \{a, b\}$
$S_2 = \{b, c, d\}$
$S_3 = \{b, d, e\}$
$S_4 = \{a, b, c\}$
$k = 2$

Yes: $S_3 \cup S_4 = U$.

$\boxed{\text{To show SET COVER is NP-complete}}$

1. Set Cover $\in$ NP: The collection of $\leq k$ subset whose union is U. is the certificate that can be checked in polynomial time.

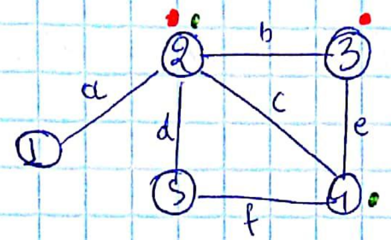
FOR EXAM
- You dont need to memorize what the NP-complete problems mentioned in class are.
- "Available" NP-complete problems will be given to use for reduction, and the tricky part will be for you to choose which one to use to show NP-completeness.

Assume NP-complete
2. $\boxed{\text{VERTEX-COVER}} \leq_p$ SET-COVER

VERTEX COVER: Given an undirected graph G and integer Z, is there a subset M of at ~~least~~ most Z nodes such that every edge has at least one endpoint in M?

M: "set of vertices that cover all the edges".

ex.



$z = 2$

does not cover $\{4,5\}$

yes: $M = \{2,4\}$

Given instance (G, z) of VERTEX-COVER,
construct instance of Set Cover:

- $U :=$ set of all edges
- for every node i, create S_i of edges incident to i.
- $k := z$.

ex. We would convert the above instance of VERTEX COVER to the following instance of SET-COVER: (in poly-time)

Set M that is the vertex cover corresponds to a set of subset whose union is all of

$U = \{a, b, c, d, e, f\}$

$S_1 = \{a\}$

$S_2 = \{a, b, c, d\}$

$S_3 = \{b, e\}$ $\qquad k = z = 2.$

$S_4 = \{c, e, f\}$

$S_5 = \{d, f\}$

Then (G, z) is a yes-instance of VERTEX COVER
$\Longleftrightarrow (U, \{S_1, \ldots, S_m\}, k)$ constructed above is a yes-instance of SET-COVER.

Does this construction also show that Set-Cover $\leq_p$ VERTEX-COVER.

No! For example, assume there is an instance of Set-Cover where an element is part of more than 3 sets: e.g. $S_1 = \{a, b\}$

$S_2 = \{a, \ldots\}$

$S_3 = \{a, \ldots\}$

The construction we created can't be used to express this as a graph: we can't have an edge (a), be incident to more than two vertices (S_1, S_2, S_3).