

《计算机辅助几何设计》第五次作业

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思考题 1

1.

证明. 由 k 阶非均匀B样条基函数的定义,

$$\begin{aligned} N_{i,3}(t) &= (t_{i+3} - t_i) \cdot [t_i, \dots, t_{i+3}](\cdot - t)_+^2 \\ &= [t_{i+1}, t_{i+2}, t_{i+3}](\cdot - t)_+^2 - [t_i, t_{i+1}, t_{i+2}](\cdot - t)_+^2 \\ &= \frac{[t_{i+2}, t_{i+3}](\cdot - t)_+^2 - [t_{i+1}, t_{i+2}](\cdot - t)_+^2}{t_{i+3} - t_{i+1}} - \frac{[t_{i+1}, t_{i+2}](\cdot - t)_+^2 - [t_i, t_{i+1}](\cdot - t)_+^2}{t_{i+2} - t_i} \\ &= \frac{\frac{(t_{i+3}-t)_+^2 - (t_{i+2}-t)_+^2}{t_{i+3}-t_{i+2}} - \frac{(t_{i+2}-t)_+^2 - (t_{i+1}-t)_+^2}{t_{i+2}-t_{i+1}}}{t_{i+3} - t_{i+1}} - \frac{\frac{(t_{i+2}-t)_+^2 - (t_{i+1}-t)_+^2}{t_{i+2}-t_{i+1}} - \frac{(t_{i+1}-t)_+^2 - (t_i-t)_+^2}{t_{i+1}-t_i}}{t_{i+2} - t_i} \\ &= \begin{cases} 1 - \frac{t_{i+2} - t_{i+1} - 2t - \frac{(t_{i+1}-t)^2}{t_{i+1}-t_i}}{t_{i+2} - t_i}, & t_i \leq t < t_{i+1} \\ \frac{t_{i+3} + t_{i+2} - 2t - \frac{(t_{i+2}-t)^2}{t_{i+2}-t_{i+1}}}{t_{i+3} - t_{i+1}} - \frac{\frac{(t_{i+2}-t)^2}{t_{i+2}-t_{i+1}}}{t_{i+2} - t_i}, & t_{i+1} \leq t < t_{i+2} \\ \frac{(t_{i+3} - t)^2}{(t_{i+3} - t_{i+2})(t_{i+3} - t_{i+1})}, & t_{i+2} \leq t < t_{i+3} \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

QED

2.

证明. 由非均匀B样条基函数的局部支撑性以及B样条基函数的差商定义可得,

$$\begin{aligned} \frac{1}{t_{i+k} - t_i} \int_{-\infty}^{\infty} N_{i,k}(x) dx &= \frac{1}{t_{i+k} - t_i} \int_{t_i}^{t_{i+k}} N_{i,k}(x) dx \\ &= \int_{t_i}^{t_{i+k}} [t_i, \dots, t_{i+k}](t - x)_+^{k-1} dx \\ &= [t_i, \dots, t_{i+k}] \int_{t_i}^{t_{i+k}} (t - x)_+^{k-1} dx \\ &= [t_i, \dots, t_{i+k}] \frac{(t - t_i)^k}{k} \\ &= \frac{1}{k}. \end{aligned} \tag{1}$$

QED

思考题 2

思考题 3

1.

- 次数 $p = 3$, 节点向量 $u = [0, 0, 0, 0, 1/2, 1, 1, 1, 1]$

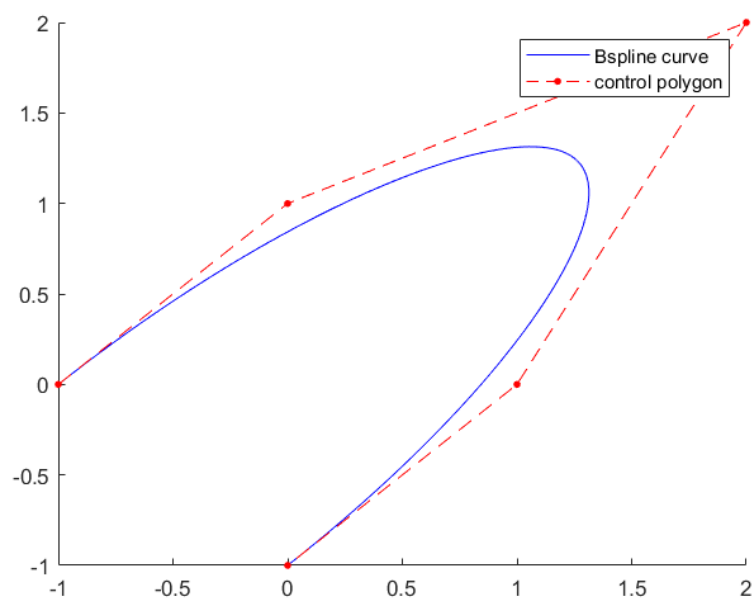


图 1: 控制顶点 $p_0 = (-1, 0)'$, $p_1 = (0, 1)'$, $p_2 = (2, 2)'$, $p_3 = (1, 0)'$, $p_4 = (0, -1)'$

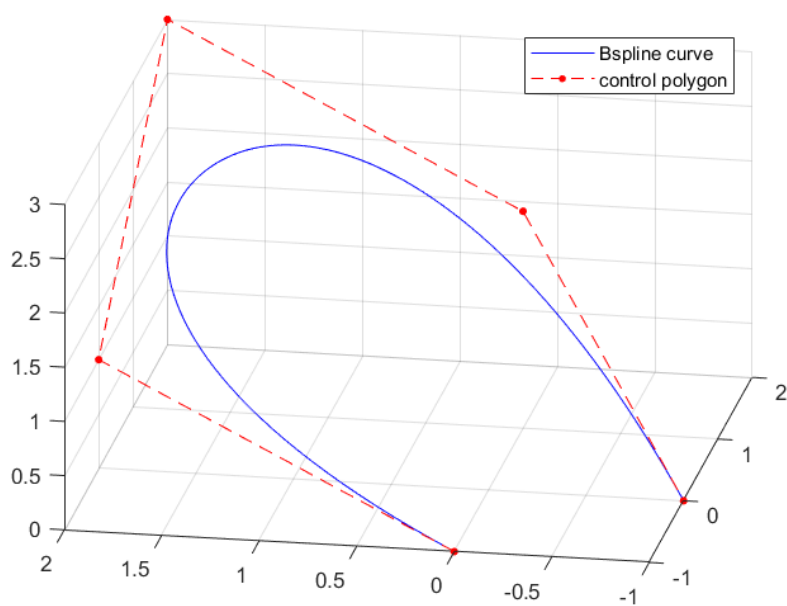


图 2: 控制顶点 $p_0 = (-1, 0, 0)'$, $p_1 = (0, 2, 1)'$, $p_2 = (2, 2, 3)'$, $p_3 = (1, 0, 2)'$, $p_4 = (0, -1, 0)'$

- 次数 $p = 2$, 节点向量 $u = [0, 0, 0, 1/3, 2/3, 1, 1, 1]$

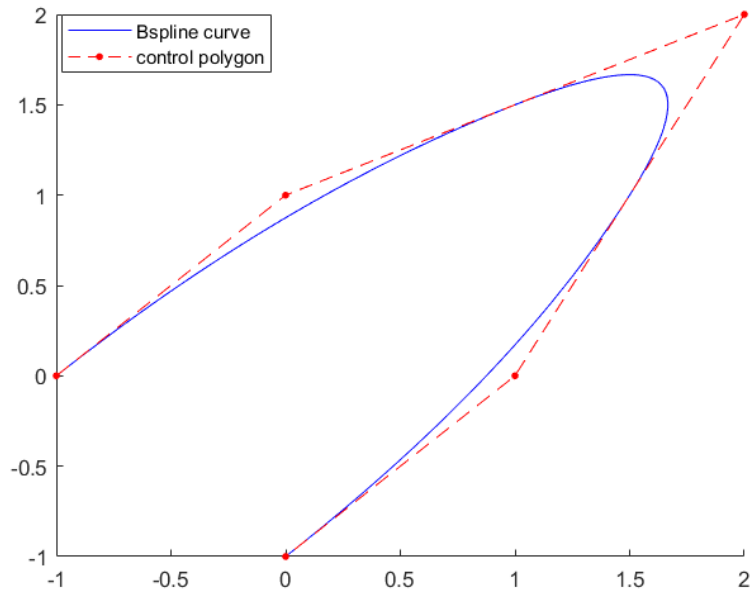


图 3: 控制顶点 $p_0 = (-1, 0)'$, $p_1 = (0, 1)'$, $p_2 = (2, 2)'$, $p_3 = (1, 0)'$, $p_4 = (0, -1)'$

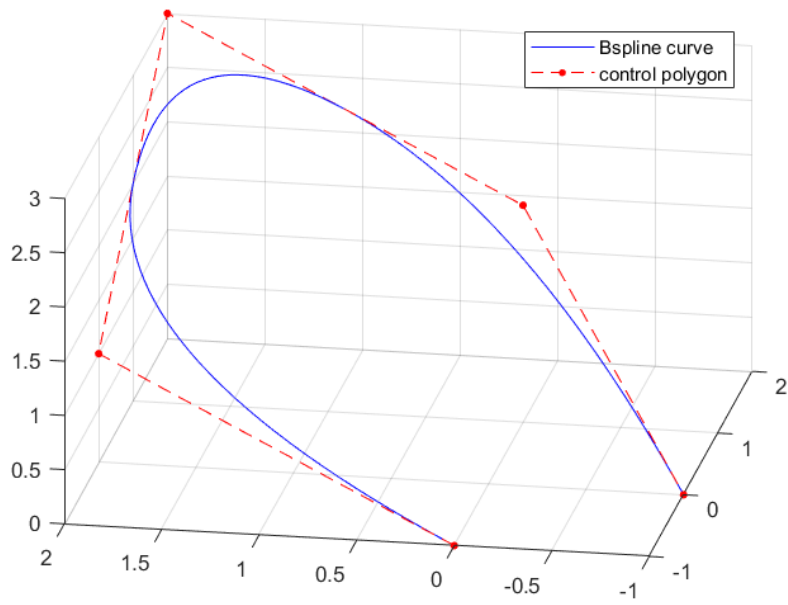


图 4: 控制顶点 $p_0 = (-1, 0, 0)'$, $p_1 = (0, 2, 1)'$, $p_2 = (2, 2, 3)'$, $p_3 = (1, 0, 2)'$, $p_4 = (0, -1, 0)'$