《计算机辅助几何设计》第五次作业

姓名：殷文良　　学号：12435063

2024 年 10 月 31 日

思考题 1

1.

证明. 由k阶非均匀B样条基函数的定义，

$$
\begin{aligned}
N_{i,3}(t) &= (t_{i+3} - t_i) \cdot [t_i, \cdots, t_{i+3}](\cdot - t)_+^2 \\
&= [t_{i+1}, t_{i+2}, t_{i+3}](\cdot - t)_+^2 - [t_i, t_{i+1}, t_{i+2}](\cdot - t)_+^2 \\
&= \frac{[t_{i+2}, t_{i+3}](\cdot - t)_+^2 - [t_{i+1}, t_{i+2}](\cdot - t)_+^2}{t_{i+3} - t_{i+1}} - \frac{[t_{i+1}, t_{i+2}](\cdot - t)_+^2 - [t_i, t_{i+1}](\cdot - t)_+^2}{t_{i+2} - t_i} \\
&= \frac{\frac{(t_{i+3}-t)_+^2 - (t_{i+2}-t)_+^2}{t_{i+3}-t_{i+2}} - \frac{(t_{i+2}-t)_+^2 - (t_{i+1}-t)_+^2}{t_{i+2}-t_{i+1}}}{t_{i+3} - t_{i+1}} - \frac{\frac{(t_{i+2}-t)_+^2 - (t_{i+1}-t)_+^2}{t_{i+2}-t_{i+1}} - \frac{(t_{i+1}-t)_+^2 - (t_i-t)_+^2}{t_{i+1}-t_i}}{t_{i+2} - t_i} \\
&= \begin{cases}
1 - \dfrac{t_{i+2} - t_{i+1} - 2t - \frac{(t_{i+1}-t)^2}{t_{i+1}-t_i}}{t_{i+2} - t_i}, & t_i \le t < t_{i+1} \\[3mm]
\dfrac{t_{i+3} + t_{i+2} - 2t - \frac{(t_{i+2}-t)^2}{t_{i+2}-t_{i+1}}}{t_{i+3} - t_{i+1}} - \dfrac{\frac{(t_{i+2}-t)^2}{t_{i+2}-t_{i+1}}}{t_{i+2} - t_i}, & t_{i+1} \le t < t_{i+2} \\[3mm]
\dfrac{(t_{i+3} - t)^2}{(t_{i+3} - t_{i+2})(t_{i+3} - t_{i+1})}, & t_{i+2} \le t < t_{i+3} \\[2mm]
0, & \text{otherwise}
\end{cases}
\end{aligned}
$$

<div align="right">QED</div>

2.

证明. 由非均匀B样条基函数的局部支撑性以及B样条基函数的差商定义可得，

$$
\begin{aligned}
\frac{1}{t_{i+k} - t_i} \int_{-\infty}^{\infty} N_{i,k}(x) dx &= \frac{1}{t_{i+k} - t_i} \int_{t_i}^{t_{i+k}} N_{i,k}(x) dx \\
&= \int_{t_i}^{t_{i+k}} [t_i, \cdots, t_{i+k}](t - x)_+^{k-1} dx \\
&= [t_i, \cdots, t_{i+k}] \int_{t_i}^{t_{i+k}} (t - x)_+^{k-1} dx \\
&= [t_i, \cdots, t_{i+k}] \frac{(t - t_i)^k}{k} \\
&= \frac{1}{k}.
\end{aligned}
\tag{1}
$$

<div align="right">QED</div>

思考题 2

1.

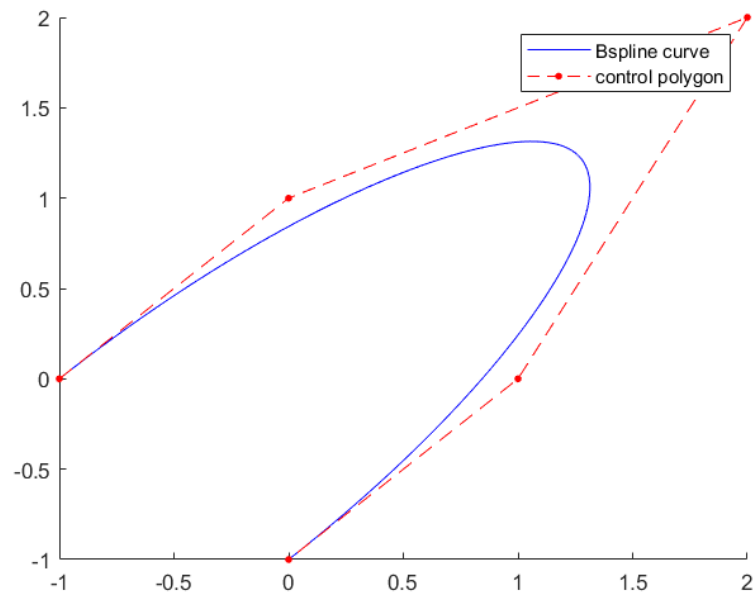- 次数$p = 3$，节点向量$u = [0, 0, 0, 0, 1/2, 1, 1, 1, 1]$

图 1: 控制顶点 $p_0 = (-1, 0)', p_1 = (0, 1)', p_2 = (2, 2)', p_3 = (1, 0)', p_4 = (0, -1)'$

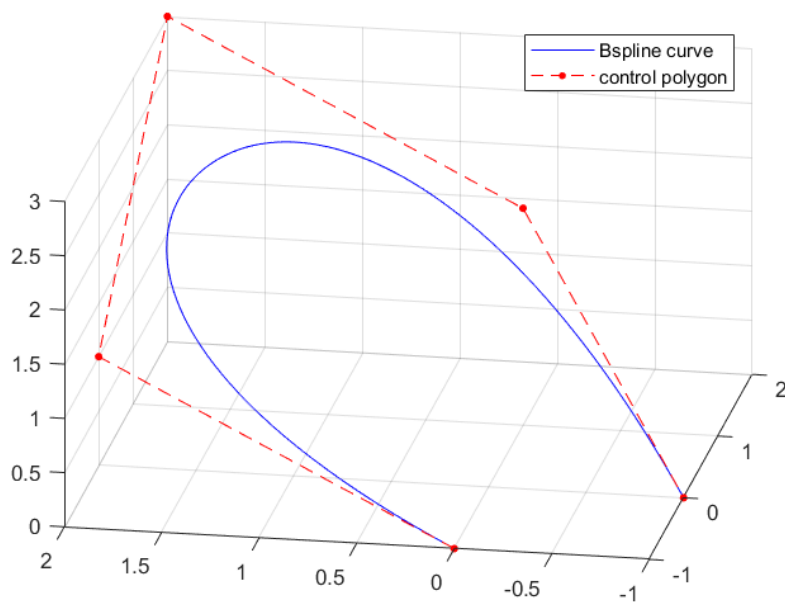图 2: 控制顶点 $p_0 = (-1, 0, 0)', p_1 = (0, 2, 1)', p_2 = (2, 2, 3)', p_3 = (1, 0, 2)', p_4 = (0, -1, 0)'$
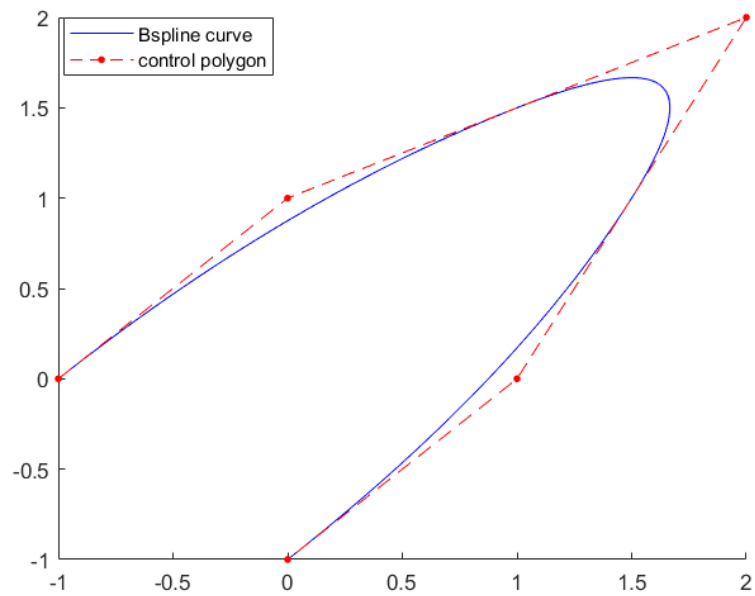
- 次数$p = 2$，节点向量$u = [0, 0, 0, 1/3, 2/3, 1, 1, 1]$

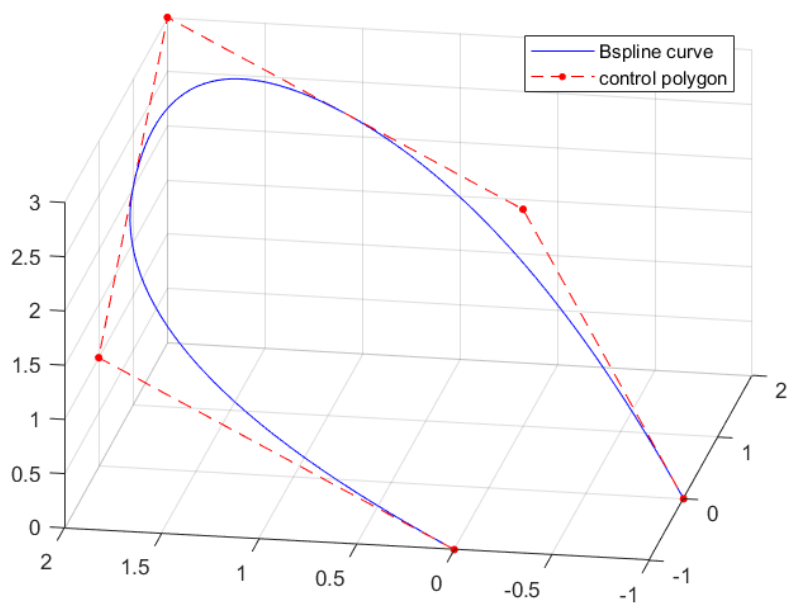图 3: 控制顶点 $p_0 = (-1, 0)', p_1 = (0, 1)', p_2 = (2, 2)', p_3 = (1, 0)', p_4 = (0, -1)'$

图 4: 控制顶点 $p_0 = (-1, 0, 0)', p_1 = (0, 2, 1)', p_2 = (2, 2, 3)', p_3 = (1, 0, 2)', p_4 = (0, -1, 0)'$