

《计算机辅助几何设计》第四次作业

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2024 年 10 月 12 日

思考题1

1.

• a.

证明. 由于 $C_n^i = C_{n-1}^i + C_{n-1}^{i-1}$, 因此

$$M_{i,n}(t) = C_n^i t^i = (C_{n-1}^i + C_{n-1}^{i-1})t^i = M_{i,n-1}(t) + tM_{i-1,n-1}(t).$$

QED

• b.

Algorithm 1 类 de Casteljau 算法

Input: $c_i, i = 0, 1, \dots, n, \quad t \in [0, 1]$

Output: c_0^n

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1: for  $i = 0, 1, \dots, n$  do  
2:    $c_i^0 = c_i$   
3: end for  
4: for  $k = 1, 2, \dots, n$  do  
5:   for  $i = 0, 1, \dots, n - k$  do  
6:      $c_i^k = c_i^{k-1} + t c_{i+1}^{k-1}$   
7:   end for  
8: end for  
9: return  $c_0^n$ 
```

2.

证明. 令

$$B'_{i,n}(t) = C_n^i (1-t)^{n-i-1} t^{i-1} (i - nt) = 0,$$

可得 $t = \frac{i}{n}$, 因此 $B_{i,n}(t)$ 在 $t = \frac{i}{n}$ 处达到极值, 并且该极值是最大值。

QED

3.

证明. 设随机变量 ξ 服从二项分布, 即

$$\xi \sim B(n, t).$$

则其分布列为

$$P(\xi = i) = B_{i,n}(t), \quad i = 0, 1, \dots, n.$$

于是, ξ 的期望为

$$E\xi = \sum_{i=0}^n i B_{i,n}(t).$$

根据二项分布对参数 n 的可加性, 以及两点分布 η 的期望为 t , 可知

$$E\xi = nE\eta = nt.$$

因此, 我们有

$$\sum_{i=0}^n i B_{i,n}(t) = nt.$$

QED

思考题2

1.

证明.

$$\begin{aligned} \int_0^1 \|P'(t)\| dt &= n \int_0^1 \left\| \sum_{i=0}^{n-1} B_{i,n-1}(t)(P_{i+1} - P_i) \right\| dt \\ &\leq n \int_0^1 \sum_{i=0}^{n-1} \|B_{i,n-1}(t)(P_{i+1} - P_i)\| dt \\ &= n \sum_{i=0}^{n-1} \|P_{i+1} - P_i\| \int_0^1 B_{i,n-1}(t) dt \\ &= \sum_{i=0}^{n-1} \|P_{i+1} - P_i\|. \end{aligned}$$

QED

2.

证明. • 由升阶公式可知, $Q_i = \frac{i}{n+1}P_{i-1} + (1 - \frac{i}{n+1})P_i, i = 0, 1, \dots, n+1$, 其中 $P_{-1} = P_{n+1} = 0$ 。于是, $\forall i = 1, \dots, n-1$,

$$\begin{aligned} \|Q_{i+1} - Q_i\| &= \left\| \frac{i+1}{n+1}P_i + (1 - \frac{i+1}{n+1})P_{i+1} - \frac{i}{n+1}P_{i-1} - (1 - \frac{i}{n+1})P_i \right\| \\ &= \left\| \frac{i}{n+1}(P_i - P_{i-1}) + (1 - \frac{i+1}{n+1})(P_{i+1} - P_i) \right\| \\ &\leq \frac{i}{n+1}\|P_i - P_{i-1}\| + (1 - \frac{i+1}{n+1})\|P_{i+1} - P_i\| \\ &\leq \frac{i}{n+1}s_{\max}(P) + (1 - \frac{i+1}{n+1})s_{\max}(P) \\ &= \frac{n}{n+1}s_{\max}(P). \end{aligned}$$

当 $i = 0$ 时,

$$\|Q_1 - Q_0\| = \left\| \frac{1}{n+1}P_0 + \left(1 - \frac{1}{n+1}\right)P_1 - P_0 \right\| = \left\| \frac{n}{n+1}(P_1 - P_0) \right\| \leq \frac{n}{n+1}s_{\max}(P).$$

类似地, 当 $i = n$ 时, $\|Q_{n+1} - Q_n\| \leq \frac{n}{n+1}s_{\max}(P)$.

综上, $s_{\max}(Q) \leq \frac{n}{n+1}s_{\max}(P)$ 。

- 由a的结论可知, 升阶 r 次后控制多边形最长边

$$s_{\max}(P_r) \leq \left(\frac{n}{n+1}\right)^r s_{\max}(P) \rightarrow 0, \quad r \rightarrow \infty.$$

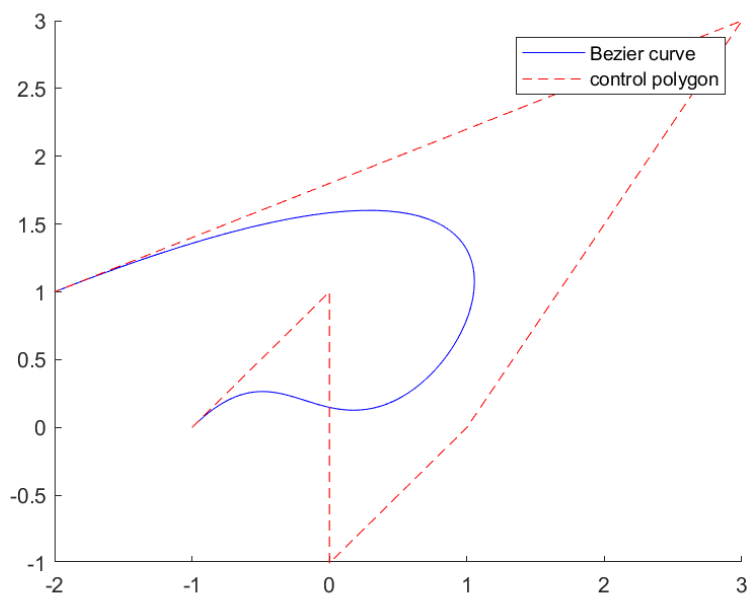
QED

思考题 3

思考题 4

1.

- 2维: $p_0 = (-1, 0)', p_1 = (0, 1)', p_2 = (0, -1)', p_3 = (1, 0)', p_4 = (3, 3)', p_5 = (-2, 1)'$.



2维Bézier曲线

- 3维: $p_0 = (-1, 0, 0)', p_1 = (0, 2, 1)', p_2 = (0, -1, 0)', p_3 = (1, 0, 2)', p_4 = (2, 2, 3)', p_5 = (2, 3, -1)'$.

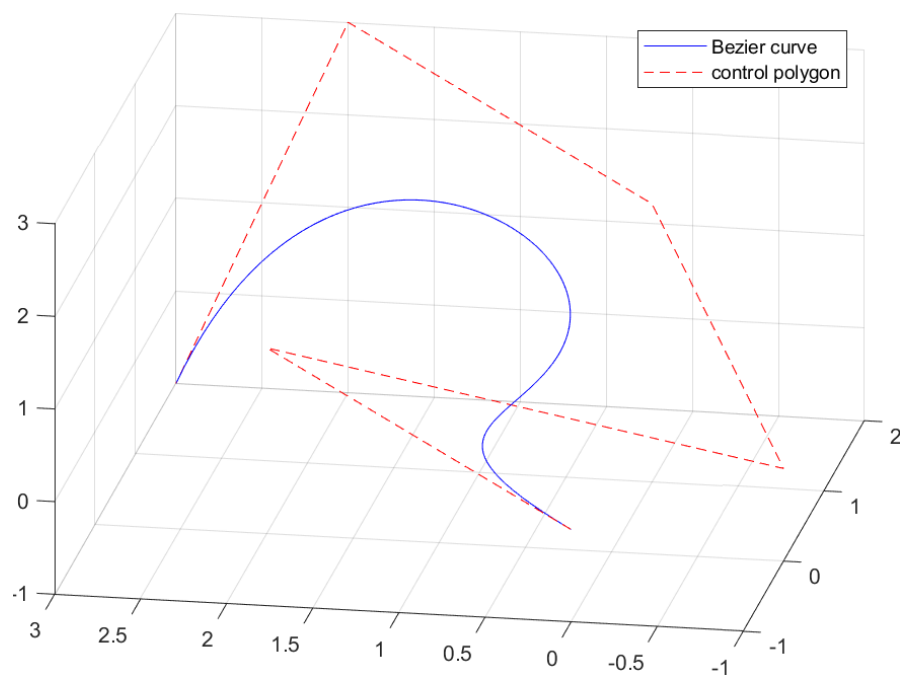


图 1: 3维Bézier曲线

2.