Fall 2016 EE230A Class Project

Name: Dazhong Gu   UID:404588693

1. Introduction

In this project, we want to study the performance of a BPSK communication system. The system is different from what we learned in the class, since we take the degradation due to fading into consideration. The system is given by:

$$X = \begin{cases} -W + N, H_0 \\ W + N, H_1 \end{cases} \qquad (1)$$

The N is still a zero-mean Gaussian r.v. of variance σ^2. However, the W is no longer a constant. Instead, the W is now a Weibull r.v.. The pdf of Weibull r.v. is given by:

$$p(x,a,b) = abx^{b-1}e^{-ax^b}, 0 < x < \infty, 0 < a, 0 < b \qquad (2)$$

a and b are tunable parameters depending on the real-world condition. And the cdf of Weibull r.v. is given by

$$F(x,a,b) = 1 - e^{-ax^b}, 0 < x < \infty \qquad (3)$$

We define the SNR of this system as:

$$SNR(dB) = 10\log_{10}((a^{-2/b}\Gamma(1+2/b))/\sigma^2) \qquad (4)$$

1.1. Problem 1

Find σ^2 in terms of a, b and $SNR(dB)$.

Since we already have (4), it is an easy job to show σ^2 in form of a, b and $SNR(dB)$. We have:

$$\sigma^2 = \frac{a^{-2/b}\Gamma(1+2/b)}{10^{\frac{SNR(dB)}{10}}} \qquad (5)$$

Since we have all the details of W and N, if we set the decision threshold at 0, it is possible for us to calculate P_e. Luckily, the generous professor give the formula of P_e directly in the project description.

$$P_e = \frac{1}{2} - \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty e^{-n^2/(2\sigma^2) - an^b} dn \qquad (6)$$

Next we will study different conditions by fixing a and b to different values. First, we set $a = 1$ and $b = 2$. Second we set $a = 1$ and $b = 1$.

2. Ex1

Take $a = 1$ and $b = 2$. Thus this Weibull pdf becomes the Rayleigh pdf.

2.1. Problem 2

For $a = 1$ and $b = 2$, analytically evaluate the P_e expression

In this problem we can put $a = 1$ and $b = 2$ back into (6), do the integration of n, then we can get P_e only in term of σ^2. After calculation, we have:

$$P_e = \frac{1}{2} - \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty e^{-\left(1+\frac{1}{\sigma^2}\right)n^2} dn = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+2\sigma^2}}\right) \qquad (7)$$

2.2. Problem 3

For $SNR = 10$ (or $SNR(dB) = 10$ dB) and $SNR = 100$ (or $SNR(dB) = 20$ dB), find their P_e values using the expression you found in Problem 2

First, for $SNR(dB) = 10$ dB, we have:

$$\sigma^2 = \frac{a^{-2/b}\Gamma(1+2/b)}{10^{\frac{SNR(dB)}{10}}} = \frac{\Gamma(2)}{10} = 0.1 \qquad (8)$$

So P_e is:

$$P_e = 0.0436 \qquad (9)$$

Second, for $SNR(dB) = 20$ dB, we have:

$$\sigma^2 = \frac{a^{-2/b}\Gamma(1+2/b)}{10^{\frac{SNR(dB)}{10}}} = \frac{\Gamma(2)}{100} = 0.01 \qquad (10)$$

So P_e is:

$$P_e = 0.0049 \qquad (11)$$

Next, we will use Matlab to simulate the BPSK system. In Matlab, function wblrnd(a,b) is the Weibull pseudo-random generator and function normrnd($0,\sigma$) is the pseudo-random Gaussian random generator. **(Matlab says that the function weibrnd(a,b,n,m) will be removed in next version, so I choose wblrnd(*a,b*) instead as Matlab suggests.)**

2.3. Problem 4

For the SNR = 10 case, take σ^2 = 0.10. Using a MC simulation of 50,000 terms to find the simulated P_e value

My Matlab Code is in the Appendix at the end of the report. I generate the signal X for 50000 times. For each time, I compare the receiver's decision with the original signal to calculate the error rate. As the result, I get:

$$P_e = 0.0441 \quad (12)$$

2.4. Problem 5

For the SNR = 100 case, take σ^2 = 0.01. Using a MC simulation of 500,000 terms to find the simulated P_e value.

My Matlab Code is in the Appendix at the end of the report. I generate the signal X for 500000 times. For each time, I compare the receiver's decision with the original signal to calculate the error rate. As the result, I get:

$$P_e = 0.0045 \quad (13)$$

We can see in both SNR situations, the simulation result is very close to the analytical result.

3. Ex2

Take a = 1 and b = 1. Then m2 = 2. Thus this Weibull pdf becomes the Exponential pdf. The error rate becomes:

$$P_e = \frac{1}{2} - \frac{e^{\sigma^4/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \int_0^\infty e^{-\frac{(n+\sigma^2)^2}{2\sigma^2}}\, dn \qquad (14)$$

3.1. Problem 6

With an appropriate change of variable in (14), evaluate the expression in (14). Then for SNR = 10 and SNR = 100 evaluate these two P_e s.

First, for $SNR(dB)$ = 10 dB, we have:

$$\sigma^2 = \frac{a^{-2/b}\Gamma(1+2/b)}{10^{\frac{SNR(dB)}{10}}} = \frac{\Gamma(3)}{10} = 0.2 \qquad (15)$$

So P_e is:

$$P_e = \frac{1}{2} - \frac{e^{0.1}}{\sqrt{0.4\pi}}\int_0^\infty e^{-\frac{(n+0.2)^2}{0.4}}\,dn = 0.1382 \qquad (16)$$

Second, for $SNR(dB)$ = 20 dB, we have:

$$\sigma^2 = \frac{a^{-2/b}\Gamma(1+2/b)}{10^{\frac{SNR(dB)}{10}}} = \frac{\Gamma(3)}{100} = 0.02 \qquad (17)$$

So P_e is:

$$P_e = \frac{1}{2} - \frac{e^{0.01}}{\sqrt{0.04\pi}}\int_0^\infty e^{-\frac{(n+0.02)^2}{0.04}}\,dn = 0.0518 \qquad (18)$$

3.2. Problem 7

For the SNR = 10 case, take σ^2 = 0.20. Using a MC simulation of 50,000 terms to find the simulated P_e value

My Matlab Code is in the Appendix at the end of the report. I generate the signal X for 50000 times. For each time, I compare the receiver's decision with the original signal to calculate the error rate. As the result, I get:

$$P_e = 0.1392 \quad (19)$$

3.3. Problem 8

For the SNR = 100 case, take σ^2 = 0.02. Using a MC simulation of 50,000 terms to find the simulated P_e value

My Matlab Code is in the Appendix at the end of the report. I generate the signal X for 500000 times. For each time, I compare the receiver's decision with the original signal to calculate the error rate. As the result, I get:

$$P_e = 0.0519 \quad (20)$$

We can see in both SNR situations, the simulation result is very close to the analytical result.