EE230A Fall 2016

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Class Project

Due Wed. Nov. 30^{th} 11:50 am.

(4.5 Points)

In our class, we have studied the performances (e.g., P_es) of various digital communication systems in the presence of AGN. In wireless communications (e.g., in a cellular communication system), the dominant degrading factor is not just due to the GN effects, but the dominant ill-effect is due to the random fading of the received signal strength from the multipaths in the propagation of the transmitted waveform from the cell tower to the handset receiver.

For simplicity, consider the fading problem for the BPSK system. (BPSK modulation is not used in a modern digital cellular system; but the study of a BPSK system's degradation due to fading is easier to understand than those of other systems.) In a BPSK system, a sinusoidal waveform is modulated by a plus or minus value to represent one bit of data. For analysis purpose, we can consider the base-band equivalent model of the BPSK system can be expressed as a binary PAM system given by

$$X = \begin{cases} -W + N, H_0 \\ W + N, H_1 \end{cases}, \tag{1}$$

where W is a constant in the standard (non-fading) PAM model and N is a zero-mean Gaussian r.v. of variance σ^2. Now, we model the fading effect by modeling W to be a Weibull r.v. (The Weibull model is one of many fading models used in practice.) The Weibull pdf is defined in Matlab by

$$p(x,a,b) = a\,b\,x^{b-1}\,e^{-ax^b}, 0 < x < \infty, 0 < a, 0 < b. \tag{2}$$

Another version of the Weibull pdf (as given in Wikipedia) is defined by

$$p_1(x,\lambda,k) = (k/\lambda^k)\,x^{k-1}\,e^{-(1/\lambda^k)x^k}, 0 < x < \infty, 0 < \lambda, 0 < k. \tag{3}$$

The Weibull cdf (as given by Wikipedia) is defined by

$$F_1(x,\lambda,k) = 1 - e^{-(x/\lambda)^k}, 0 < x < \infty. \tag{4}$$

The Weibull cdf using the Matlab notation becomes

$$F(x,a,b) = 1 - e^{-a\,x^b}, 0 < x < \infty. \tag{5}$$

By comparing (2) with (3) and (4) with (5), it is clear that $b = k$ and $a = 1/\lambda^k$. (These parameters have physical interpretations, but we will not consider these issues here.) Then the mean $\mu = \lambda\Gamma(1 + 1/k) = a^{-1/b}\,\Gamma(1 + 1/b)$ and the second moment $m_2 = \lambda^2\Gamma(1 + 2/k) = a^{-2/b}\,\Gamma(1 + 2/b)$. In this Project, we will use the notation of (2) for the Weibull pdf and (5) for the Weibull cdf.

Then the SNR(dB) can be defined as

$$\text{SNR(dB)} = 10\log_{10}(m_2/\sigma^2) = 10\log_{10}((a^{-2/b}\Gamma(1+2/b))/\sigma^2)\ . \tag{6}$$

Problem 1. Find σ^2 in terms of a, b, and SNR(dB).

Next, we assume $P(H_0) = P(H_1) = 0.5$ and the decision threshold is set at 0. Then the average probability of error is given by

$$\begin{aligned} P_e &= (1/2)P(-W+N>0\,|\,H_0) + (1/2)P(W+N<0\,|\,H_1) \\ &= P(-W+N>0\,|\,H_0) \ = \ P(W<N) \ = \ E_{0\le N}\{P(W<N=n)\} \\ &= \frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty e^{-n^2/(2\sigma^2)}\left(1-e^{-an^b}\right)dn = \frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty e^{-n^2/(2\sigma^2)}dn - \frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty e^{-n^2/(2\sigma^2)}\left(e^{-an^b}\right)dn \\ &= \frac{1}{2} - \frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty e^{-n^2/(2\sigma^2)-an^b}dn \end{aligned} \quad . \tag{7}$$

Now, consider two special cases.

Ex. 1. Take $a = 1$ and $b = 2$. Then $m_2 = 1$. Thus this Weibull pdf becomes the Rayleigh pdf.

Problem 2. For $a = 1$ and $b = 2$, analytically evaluate the P_e expression in (7). (Hint: The integrand in (7) with these parameters is just a Gaussian 1-D pdf, so with some change of variable, you can obtain an explicit closed form expression for it.)

Problem 3. For SNR = 10 (or SNR(dB) = 10 dB) and SNR = 100 (or SNR(dB) = 20 dB), find their P_e values using the expression you found in Problem 2.

In Matlab, use the weibrnd(a,b,m,n) Weibull pseudo-random generator, where a and b are the Weibull parameters, and m is the number of rows and n is the number of column. Similarly, we use the normrnd($0,\sigma,m,n$) pseudo-random Gaussian random generator. We take $a = 1$ and $b = 2$.

Problem 4. For the SNR = 10 case, take $\sigma^2 = 0.10$. Using a MC simulation of 50,000 terms to find the simulated P_e value. This value should be very close to the value obtained by the analytical expression in Problem 3.

Problem 5. For the SNR = 100 case, take $\sigma^2 = 0.01$. Using a MC simulation of 500,000 terms to find the simulated P_e value. This value should be very close to the value obtained by the analytical expression in Problem 3.

Ex. 2. Take $a = 1$ and $b = 1$. Then $m_2 = 2$. Thus this Weibull pdf becomes the Exponential pdf and (7) now becomes

$$P_e = \frac{1}{2} - \frac{e^{\sigma^4/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \int_0^\infty e^{-\frac{(n+\sigma^2)^2}{2\sigma^2}} dn \quad . \tag{8}$$

Problem 6. With an appropriate change of variable in (8), evaluate the expression in (8). Then for SNR = 10 and SNR = 100 evaluate these two P_e's. . Hint: (This expression can be given explicitly in terms of the Q(.) function (i.e., the complimentary Gaussian cdf)).

Problem 7. Take $a = 1$ and $b = 1$. For the SNR = 10 case, take $\sigma^2 = 0.20$ Using a MC simulation of 50,000 terms to find the simulated P_e value. This value should be very close to the value obtained by the analytical expression in Problem 6.

Problem 8. Take $a = 1$ and $b = 1$. For the SNR = 100 case, take $\sigma^2 = 0.02$. Using a MC simulation of 500,000 terms to find the simulated P_e value. This value should be very close to the value obtained by the analytical expression in Problem 6.

For the class project, you need to turn in an informal report of 5-10 pages describing what the problem you are solving and the results. In other words, organize what was presented above and write it in your own words with conclusions. Any detailed information (including listing of your programming codes) should be placed in the Appendix. . Please attach a CD/memory stick with the hard copy of the report including your programming codes.

You need to give me a hard copy of the Project report by Wed. Nov. 30th 11:50 am (at the end of our class).