

附录 部分习题参考答案

习题 5.1

- $D = \{(x, y) | x^2 + y^2 \leq 1\}$, 是闭集, 闭区域, 有界集;
 - $D = \{(x, y) | |x| + |y| < 2\}$, 是开集, 开区域, 有界集;
 - $D = \{(x, y) | -1 \leq x \leq 1, |y| \geq 2\}$, 是闭集, 不是闭区域, 无界集;
 - $D = \{(x, y) | -y^2 \leq x \leq y^2, y \neq 0\}$, 不是开集, 不是闭集, 无界集;
 - $D = \{(x, y, z) | x^2 + y^2 + z^2 < 9\}$, 是开集, 开区域, 有界集;
 - $D = \{(x, y, z) | x < \sqrt{z}, 0 \leq z \leq 1, -2 < y < 2\}$, 不是开集, 不是闭集, 无界集.

$$2. (1) f\left(2x, \frac{1}{y}\right) = \frac{y}{2x} + 2xy; \quad (2) f(x, y) = x^2 + 2xy + 2.$$

$$4. (1) \frac{5}{4}; \quad (2) 0; \quad (3) 0; \quad (4) \frac{1}{2}; \quad (5) 0; \quad (6) 0; \quad (7) e;$$

$$(8) 1; \quad (9) 0; \quad (10) e^2; \quad (11) 1; \quad (12) 0; \quad (13) e^{-2}; \quad (14) \frac{1}{2}.$$

$$7. (1) \lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y) = 1, \quad \lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y) = -1;$$

$$(2) \lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y) = +\infty, \quad \lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y) = 1;$$

$$(3) \lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y) = 0, \quad \lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y) = 0.$$

习题 5.2

- $z'_x = 2xy^3 + \frac{1}{2\sqrt{x}}, \quad z'_y = 3x^2y^2 + 2;$
 - $z'_x = \frac{y}{x^2 + y^2} + \frac{x}{\sqrt{x^2 + y^2}}, \quad z'_y = -\frac{x}{x^2 + y^2} + \frac{y}{\sqrt{x^2 + y^2}};$
 - $z'_x = 2^x \ln 2 + yx^{y-1}, \quad z'_y = x^y \ln x + 3y^2;$
 - $z'_x = -ye^{-xy} + e^{-y} - ye^{-x}, \quad z'_y = -xe^{-xy} - xe^{-y} + e^{-x};$
 - $u'_x = yz(1 + xy)^{z-1} + yz \cos(xyz), \quad u'_y = xz(1 + xy)^{z-1} + xz \cos(xyz),$
 $u'_z = (1 + xy)^z \ln(1 + xy) + xy \cos(xyz);$
 - $u'_x = -\frac{x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}, \quad u'_y = \frac{y}{y^2 + z^2} - \frac{y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}},$
 $u'_z = \frac{z}{y^2 + z^2} - \frac{z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}};$
 - $u'_x = y^z x^{y^z-1} + x^{z-1} y^z z + yz x^{y^z-1}, \quad u'_y = x^{y^z} y^{z-1} z \ln x + x^z y^{z-1} z + x^{y^z} z \ln x,$
 $u'_z = x^{y^z} y^z \ln x \cdot \ln y + (xy)^z \ln(xy) + x^{y^z} y \ln x;$
 - $u'_x = \frac{\operatorname{sgn} y}{2\sqrt{x(y-x)}}, \quad u'_y = -\frac{x}{2\sqrt{xy^2(y-x)}} - \frac{\operatorname{sgn} z}{2\sqrt{y(z-y)}}, \quad u'_z = \frac{y}{2\sqrt{yz^2(z-y)}}.$

2. $f'_x(0, 0) = 0$.

3. (1) $\frac{\partial^2 z}{\partial x^2} = -y^2(\sin xy + \cos xy)$, $\frac{\partial^2 z}{\partial x \partial y} = (1 - xy) \cos xy - (1 + xy) \sin xy$;

(2) $\frac{\partial^2 u}{\partial x^2} = yz(2 + xyz)e^{xyz}$, $\frac{\partial^3 u}{\partial x \partial y \partial z} = (2x + 4x^2 yz + x^3 y^2 z^2)e^{xyz}$;

(3) $\frac{\partial^2 u}{\partial x^2} = \frac{y^2 + z^2}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$, $\frac{\partial^3 u}{\partial x \partial y \partial z} = \frac{3xyz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}}$;

(4) $\frac{\partial^2 u}{\partial y \partial z} = x^{yz} \ln x + zyx^{yz} \ln^2 x + xy^{xz-1} + x^2 zy^{xz-1} \ln y$;

(5) $\frac{\partial^2 z}{\partial x^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2}$, $\frac{\partial^2 z}{\partial y^2} = \frac{x^2 - y^2}{(x^2 + y^2)^2}$;

(6) $\frac{\partial^2 z}{\partial x^2} = \frac{-2xy}{(x^2 + y^2)^2}$, $\frac{\partial^2 z}{\partial y^2} = \frac{2xy}{(x^2 + y^2)^2}$.

4. (1) $\cos x \cos y dx - \sin x \sin y dy$; (2) $\frac{2xy^2 + 1}{2y\sqrt{x^2 y + \frac{x}{y}}} dx + \frac{x^2 y^2 - x}{2y^2\sqrt{x^2 y + \frac{x}{y}}} dy$;

(3) $\frac{x dx + y dy + z dz}{x^2 + y^2 + z^2}$; (4) $y(1 - xyz)e^{-xyz} dx + x(1 - xyz)e^{-xyz} dy - x^2 y^2 e^{-xyz} dz$;

(5) dx ; (6) 0.2 .

7. 仅当 $\varphi(0, 0) = 0$ 时可微. 8. $f(x, y)$ 在 $(0, 0)$ 处连续, 可偏导, 可微, 连续可微.

9. (1) $f'_x(x, y) = \begin{cases} \frac{4}{3}x^{\frac{1}{3}} \sin \frac{y}{x} - yx^{-\frac{2}{3}} \cos \frac{y}{x}, & x \neq 0, \\ 0, & x = 0, \end{cases}$ $f'_y(x, y) = \begin{cases} x^{\frac{1}{3}} \cos \frac{y}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$

10. (1) $(2 - \frac{5}{x^2})dx^2 + 2dxdy + 6ydy^2$;

(2) $y(y-1)x^{y-2}dx^2 + 2x^{y-1}(1+y \ln x)dxdy + x^y \ln^2 x dy^2$;

(3) $e^x \sin y dx^2 + 2e^x \cos y dxdy - e^x \sin y dy^2$; (4) $-\frac{2}{y^2}dxdy + \frac{2x}{y^3}dy^2$.

11. (1) $f'_x(x, y)$; (2) $f'_y(x, y)$. 12. (1) $p!q!$; (2) $(-1)^m \cdot 2 \cdot \frac{(m+n-1)!(nx+my)}{(x-y)^{m+n+1}}$.

14. (1) 0.45046 ; (2) 9.00455 .

习题 5.3

1. (1) $\frac{du}{dt} = (\sin t)^{\cos t - 1}(\cos^2 t - \sin^2 t \ln \sin t)$; (2) $\frac{dy}{dx} = \frac{1}{e^x}(\frac{1}{x} - \ln x)$;

(3) $\frac{\partial z}{\partial x} = ye^{xy} + 2xy^2 - 4x + \frac{2x}{y^2}$, $\frac{\partial z}{\partial y} = xe^{xy} + 2x^2 y - \frac{2x^2}{y^3}$;

(4) $\frac{\partial z}{\partial x} = 2x + 2y^2 + \frac{1}{x+y^2} + \frac{2}{x} - \frac{y}{x^2}$, $\frac{\partial z}{\partial y} = 4xy + 4y^3 + \frac{2y}{x+y^2} + \frac{1}{x} - \frac{1}{y^2}$.

2. (1) $\frac{\partial z}{\partial x} = f'_1 + yf'_2$, $\frac{\partial z}{\partial y} = f'_1 + xf'_2$;

- (2) $\frac{\partial u}{\partial x} = \frac{1}{y}f'_1$, $\frac{\partial u}{\partial y} = -\frac{x}{y^2}f'_1 + \frac{1}{z}f'_2$, $\frac{\partial u}{\partial z} = -\frac{y}{z^2}f'_2$;
- (3) $\frac{\partial u}{\partial x} = f'_1 + yf'_2 + yzf'_3 + 2\varphi'(2x - y)$, $\frac{\partial u}{\partial y} = xf'_2 + xzf'_3 - \varphi'(2x - y)$, $\frac{\partial u}{\partial z} = xyf'_3$;
- (4) $\frac{\partial u}{\partial x} = f(x^2 + y^2, \sqrt{x + y}) + 2x^2f'_1 + \frac{x}{2\sqrt{x + y}}f'_2$, $\frac{\partial u}{\partial y} = 2xyf'_1 + \frac{x}{2\sqrt{x + y}}f'_2 + 2y$.
3. (1) $\frac{\partial^2 z}{\partial x^2} = y^2f''_{11} + 2f''_{12} + \frac{1}{y^2}f''_{22}$, $\frac{\partial^2 z}{\partial x \partial y} = f'_1 - \frac{1}{y^2}f'_2 + xyf''_{11} - \frac{x}{y^3}f''_{22}$,
 $\frac{\partial^2 z}{\partial y^2} = x^2f''_{11} - \frac{2x^2}{y^2}f''_{12} + \frac{x^2}{y^4}f''_{22} + \frac{2x}{y^3}f'_2$;
- (2) $\frac{\partial^2 z}{\partial x^2} = f''_{11} + 2yf''_{12} + 2f''_{13} + y^2f''_{22} + 2yf''_{23} + f''_{33}$,
 $\frac{\partial^2 z}{\partial x \partial y} = xf''_{12} - f''_{13} + xyf''_{22} + (x - y)f''_{23} - f''_{33} + f'_2$,
 $\frac{\partial^2 z}{\partial y^2} = x^2f''_{22} - 2xf''_{23} + f''_{33}$.
4. (1) $f'_1 - \frac{1}{y^2}f'_2 - \frac{1}{x^2}f'_3 + xyf''_{11} - \frac{x}{y^3}f''_{22} + \frac{2}{xy}f''_{23} - \frac{y}{x^3}f''_{33}$;
- (2) $\frac{\partial z}{\partial x} = \frac{1}{y}f' + yg'_1 - \frac{y^2}{x^2}g'_2$, $\frac{\partial z}{\partial y} = g - \frac{x}{y^2}f' + \frac{y}{x}g'_2$, $\frac{\partial^2 z}{\partial y^2} = \frac{2x}{y^3}f' + \frac{x^2}{y^4}f'' + \frac{2}{x}g'_2 + \frac{y}{x^2}g''_{22}$;
- (3) $2(y - x)f''_{12} + (y^2 - x^2)f''_{22} + (y^2 - x^2)\varphi'(xy)$;
- (4) $3f''_{11} + 4(x + y + z)f''_{12} + 4(x^2 + y^2 + z^2)f''_{22} + 6f'_2$;
- (5) $f'' + \frac{2f'}{\sqrt{x^2 + y^2 + z^2}}$;
- (6) $3xy^2f(xy) + \frac{y^2}{x^3}f(\frac{y}{x}) + x^2y^3f'(xy) - y^3f'(xy)$.
5. (1) $-\frac{ye^{xy} + 2}{xe^{xy} + 2y}$; (2) $\frac{(x^2 + y^2)\sin 2x - x}{y - (x^2 + y^2)\sin 2y}$.
6. (1) $\frac{\partial z}{\partial x} = -\frac{z(1 + xyz)}{x(3 + xyz)}$, $\frac{\partial z}{\partial y} = -\frac{z(2 + xyz)}{y(3 + xyz)}$;
- (2) $\frac{\partial z}{\partial x} = -\frac{2x + y}{2z - y + 3}$, $\frac{\partial^2 z}{\partial x \partial y} = -\frac{(2z - y + 3)^2 + (2x + y)(2x + 3y + 3)}{(2z - y + 3)^3}$;
- (3) $\frac{\partial z}{\partial x} = -\frac{z}{x}$, $\frac{\partial^2 z}{\partial x^2} = \frac{2z}{x^2}$.
7. (1) $z - xy$; (2) $\frac{z}{y}$; (3) c ; (4) $\frac{7}{2}dx - \frac{1}{2}dy$; (5) $\frac{-(F'_1)^2F''_{22} + 2F'_1F'_2F''_{12} - (F'_2)^2F''_{11}}{y(F'_1)^3}$.
8. $f'_x - \frac{y}{x}f'_y + \left(1 - \frac{(x - z)e^x}{\sin(x - z)}\right)f'_z$.
9. (1) $\frac{dy}{dx} = \frac{y(z - x)}{x(y - z)}$, $\frac{dz}{dx} = \frac{z(x - y)}{x(y - z)}$;
- (2) $\frac{\partial u}{\partial x} = \frac{x - v}{v - u}$, $\frac{\partial v}{\partial x} = \frac{u - x}{v - u}$, $\frac{\partial u}{\partial y} = \frac{y - v}{v - u}$, $\frac{\partial v}{\partial y} = \frac{u - y}{v - u}$.

10. 52. 12. (1) $f'_x(x, y)$; (2) $f''_{xx}(x, y)$.

习题 5.4

1. $25 + (x-1) + 15(y-2) + (x-1)^2 + (x-1)(y-2) + 2(y-2)^2$.
 2. $(x+y) - \frac{1}{2}(x+y)^2 + \frac{1}{3}(x+y)^3 - \frac{1}{4}(x+y)^4 \frac{1}{(1+\theta x+\theta y)^4} \quad (0 < \theta < 1)$.
 3. $x^y = 1 + 3\Delta x + 3(\Delta x)^2 + \Delta x \Delta y + \cdots, \quad 1.04^{2.98} \approx 1.1240$.

习题 5.5

1. $\sqrt{2}\rho$. 2. $|\sin \varphi|$.

习题 5.6

1. (1) 切线 $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$, 法平面 $x+2y+3z-6=0$;
 (2) 切线 $\frac{x-\frac{\pi}{2}+1}{1} = \frac{y-1}{1} = \frac{z-2\sqrt{2}}{\sqrt{2}}$, 法平面 $x+y+\sqrt{2}z-4-\frac{\pi}{2}=0$;
 (3) 切线 $\frac{x-1}{1} = \frac{y}{-2} = \frac{z+1}{1}$, 法平面 $x-2y+z=0$;
 (4) 切线 $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-1}{-1}$, 法平面 $x-y-z+1=0$.
 2. (1) 切平面 $2x+4y-z-3=0$, 法线 $\frac{x-1}{2} = \frac{y-1}{4} = \frac{z-3}{-1}$;
 (2) 切平面 $y_0x+x_0y-2z_0z=0$, 法线 $\frac{x-x_0}{y_0} = \frac{y-y_0}{x_0} = \frac{z-z_0}{-2z_0}$;
 (3) 切平面 $x+y-\sqrt{2}z=0$, 法线 $\frac{x-\frac{\sqrt{2}}{2}}{1} = \frac{y-\frac{\sqrt{2}}{2}}{1} = \frac{z-1}{-\sqrt{2}}$.
 3. $\theta = \arccos \frac{b}{\sqrt{a^2+b^2}}$. 4. a . 5. $2x+2y-z=3$. 6. $\begin{cases} x=R\cos\theta, \\ y=R\sin\theta, \\ z=R\cos\theta-R. \end{cases}$
 7. $x+4y+6z=21$ 或 $x+2z=7$. 8. $2x+4y-z-5=0$.
 9. $x+y-z=2$ 或 $6x+3y-5z=9$.
 10. $\pm \left(\frac{a^2}{\sqrt{a^2+b^2+c^2}}, \frac{b^2}{\sqrt{a^2+b^2+c^2}}, \frac{c^2}{\sqrt{a^2+b^2+c^2}} \right)$.

习题 5.7

1. (1) $z(-3, 0) = -11$ 为极小值;
 (2) $z(1, 1) = z(1, -1) = z(-1, 1) = z(-1, -1) = -3$ 为极小值, $z(0, 0) = 0$ 为极大值;
 (3) $z(1, 1) = z(-1, -1) = -2$ 为极小值; (4) $z\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right) = -\frac{3}{2}$ 为极小值.
 2. (1) $z(-2, 0) = 1$ 为极小值, $z\left(\frac{16}{7}, 0\right) = -\frac{8}{7}$ 为极大值;

- (2) -6 为极小值, 2 为极大值, 极值点为 $(-1, 1)$.
3. $a = \frac{2p}{3}, b = \frac{p}{3}$, 绕 b 边旋转时体积最大.
4. $a = b = \frac{3p}{4}, c = \frac{p}{2}$ 时, 绕 c 边旋转时体积最大. 5. $\frac{3\sqrt{3}}{4}ab$. 6. $\frac{15\sqrt{2}}{8}$.
7. $z(1, -3) = -10$ 为最小值, $z\left(-\frac{\sqrt{10}}{2}, \frac{3\sqrt{10}}{2}\right) = 25 + 10\sqrt{10}$ 为最大值.
8. $z(-2, 3) = z(2, -3) = -50$ 为最小值, $z\left(\frac{3}{2}, 4\right) = z\left(-\frac{3}{2}, -4\right) = 106\frac{1}{4}$ 为最大值.
9. 最大值 $\frac{15}{2}$, 最小值 $-\frac{3}{2}$. 10. $z\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right) = -\frac{3}{2}$ 为最小值, $z(0, 0) = 3$ 为最大值.
12. $x + \sqrt{2}y + \sqrt{3}z = \sqrt{3}$.
13. 切点 $\left(\frac{1}{9a^2}, \frac{1}{9b^2}, \frac{1}{9c^2}\right)$, 最大体积 $\frac{1}{162a^2b^2c^2}$, 切平面 $3a^2x + 3b^2y + 3c^2z = 1$.
14. 体积最大值为 2 . 15. $\left(\frac{6a}{17}, \frac{4a}{17}, \frac{24a}{17}\right)$. 16. 最短距离 $\sqrt{9 - 5\sqrt{3}}$, 最长距离 $\sqrt{9 + 5\sqrt{3}}$.
17. 最近距离为 1 , 最远距离为 3 . 18. $AB = 4p, BC = 3p, CD = 2p$.

习题 5.8

1. $\frac{3}{10}\pi - \frac{7}{5}$. 2. $\frac{9}{\sqrt{6}}$. 3. $\frac{1}{3} + \frac{4}{3}e$. 4. $\frac{8\sqrt{3}}{39} + \frac{3}{13}$. 5. $\frac{10}{9\sqrt{17}}$.
6. 0 . 7. $\pm \frac{8\sqrt{2}-1}{5\sqrt{41}}$. 8. $\sqrt{2}$. 9. $3x_0 - 2y_0 + 5z_0$.

习题 6.1

1. (1) $\iint_{x^2+y^2 \leq 4} (2 - \sqrt{x^2+y^2})d\sigma$; (2) $\iint_{x^2+y^2 \leq 1} (x^2+y^2)d\sigma$. 2. (1) $\frac{1}{6}$; (2) $\frac{2}{3}\pi a^3$. 3. 1 .

习题 6.2

1. (1) $\frac{\pi}{12}$; (2) $5\ln 2 - 3\ln 3$; (3) $1 - \sin 1$; (4) $\frac{7}{6}$; (5) $\frac{16}{5}$;
(6) 4π ; (7) π ; (8) $\frac{1}{6}$; (9) $\frac{32}{21}$; (10) $1 - \frac{\sqrt{2}}{2}$.
2. (1) $\int_0^1 dy \int_y^1 f(x, y) dx$; (2) $\int_0^1 dx \int_0^1 f(x, y) dy + \int_1^2 dx \int_0^{2-x} f(x, y) dy$;
(3) $\int_{-2}^0 dx \int_0^{4-x^2} f(x, y) dy + \int_0^2 dx \int_{2-\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} f(x, y) dy$;
(4) $\int_0^1 dy \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y) dx + \int_1^4 dy \int_{y-2}^{\sqrt{y}} f(x, y) dx$; (5) $\int_0^1 dy \int_y^{1+\sqrt{1-y^2}} f(x, y) dx$.
3. (1) $\frac{1}{6}$; (2) $\frac{4(2+\pi)}{\pi^3}$; (3) $\frac{3\pi}{8}$.
4. (1) $\frac{\pi}{2}$; (2) 4π ; (3) $\frac{32}{9}$; (4) $\frac{3}{64}\pi^2$; (5) $\frac{\pi}{6a}$; (6) $\frac{3}{35} + \frac{3}{8}\pi - 1$.

5. (1) $\int_0^{\frac{\pi}{4}} d\theta \int_{\frac{1}{\cos \theta}}^{\frac{2}{\cos \theta}} f(\rho \cos \theta, \rho \sin \theta) \rho d\rho$; (2) $\int_0^{\frac{\pi}{4}} d\theta \int_0^{\frac{\sin \theta}{\cos^2 \theta}} f(\rho \cos \theta, \rho \sin \theta) \rho d\rho$.
6. (1) $\frac{\pi ab}{4}(a^2 + b^2)$; (2) $\frac{\pi}{2}$; (3) $\ln^2 2$; (4) $\frac{e-1}{2}$. 7. (1) $\frac{1}{3}$; (2) $\pi - 1$.
8. (1) 2π ; (2) $e - 1$; (3) $\frac{5}{24}$; (4) $\frac{2}{3}$; (5) 8 ; (6) $\frac{8}{3}$; (7) $\frac{\pi}{2} - 1$.
9. $2 \ln(\sqrt{2} + 1) - \sqrt{2}$. 10. $\frac{81}{5}\pi$.

习题 6.3

1. (1) $\int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_0^{10-x-y} f(x, y, z) dz$;
 (2) $\int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_0^{1-x^2-y^2} f(x, y, z) dz$;
 (3) $\int_{-a}^a dx \int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}} dy \int_{-c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}^{c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}} f(x, y, z) dz$;
 (4) $\int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} f(x, y, z) dz$.
2. (1) $\frac{1}{2880}$; (2) $\frac{1}{60}$; (3) $\frac{\pi}{4}$; (4) $\frac{\pi}{4} - \frac{1}{2}$; (5) 0 .
3. (1) $\frac{64\pi}{3}$; (2) $\frac{13\pi}{4}$; (3) $\pi \left(\ln 2 - 2 + \frac{\pi}{2} \right)$; (4) $\frac{3\pi}{4}$; (5) $\frac{4(4\sqrt{2}-1)}{15}\pi$;
 (6) $\frac{\pi}{8}$; (7) 2π ; (8) 0 ; (9) $\frac{8\pi}{5}$; (10) $\frac{\pi}{2}(e^{a^2} - 1)$; (11) $\frac{4}{15}R^5 \left(\frac{\pi}{2} - \frac{8}{15} \right)$.
4. $\frac{4}{15}\pi abc(a^2 + b^2 + c^2)$. 5. $\frac{\pi}{4}R^4$. 6. $\frac{59}{12}\pi$. 7. $f'(0)$.
9. $\frac{dF}{dt} = 2\pi th \left(\frac{h^2}{3} + f(t^2) \right)$, $\lim_{t \rightarrow 0^+} \frac{\int_0^1 F(xt) dx}{t^2} = \frac{\pi h}{3} \left(\frac{h^2}{3} + f(0) \right)$.

习题 6.4

1. (1) $\frac{\pi}{2}$; (2) $\frac{4}{3}R^3 \left(\frac{\pi}{2} - \frac{2}{3} \right)$; (3) $\frac{4\pi}{3}(\sqrt{2} - 1)$; (4) $8(2 - \sqrt{2})$; (5) $\frac{\pi}{16}$;
 (6) $\frac{17}{6}$; (7) $\frac{16}{7}$; (8) $\left(\frac{2}{3} - \frac{b}{4a} \right) b^3 \pi$; (9) $8\sqrt{2}\pi$.
2. (1) $\frac{\sqrt{14}\pi}{3}$; (2) $\frac{\pi}{6}(2\sqrt{2} - 1)$; (3) 16 ; (4) $24(2 - \sqrt{2})$; (5) $4\pi Rh$; (6) $2\sqrt{2}\pi$; (7) 16 ; (8) 16 .
3. $(2\sqrt{6} + \sqrt{15})\pi$. 4. $\frac{\pi^2 a^3}{4\sqrt{2}}$. 5. $F_x = 0, F_y = 0, F_z = \frac{2}{3}k\mu\pi(\sqrt{2} - 1)$.
6. $F_x = 0, F_y = -\frac{bk\pi}{3}[\sqrt{a^2 + b^2}(a^2 - 2b^2) + 2b^3]$.
7. (1) $\left(\frac{1}{5}, \frac{3}{5} \right)$; (2) $\left(\frac{5a}{6}, 0 \right)$; (3) $\left(\pi a, \frac{5a}{6} \right)$. 8. (1) $\left(0, 0, \frac{25}{64} \right)$; (2) $\left(0, 0, \frac{3}{5} \right)$; (3) $\left(\frac{2}{5}, \frac{1}{5}, \frac{1}{5} \right)$.

9. (1) $I_x = \frac{5}{12}$; (2) $I_0 = \frac{\pi}{8}$, $I_{y=-1} = \frac{5}{16}\pi + \frac{2}{3}$; (3) $I_x = \frac{35}{12}\pi a^4$; (4) $I_y = \frac{49}{32}\pi a^4$.

10. $I_x = I_y = \frac{28}{15}\pi\mu$, $I_z = \frac{8}{15}\pi\mu$. 11. $I_x = \frac{7\pi\mu}{12}$, $I_z = \frac{\pi\mu}{2}$.

习题 6.5

1. 1. 2. π . 3. $-\frac{\pi}{2}$.

习题 7.1

1. (1) $1 + \sqrt{2}$; (2) $2\pi a^{2n+1}$; (3) $\frac{256}{15}a^3$; (4) $\frac{\sqrt{2}}{2} + \frac{1}{12}(5\sqrt{5} - 1)$; (5) $(\frac{\pi}{4}a + 2)e^a - 2$;
 (6) $\sqrt{5}$; (7) $\frac{ab(a^2 + ab + b^2)}{3(a+b)}$; (8) $2a^2$; (9) 0; (10) $\frac{17\sqrt{17}-1}{12}$; (11) $2a^3\pi^2(1+2\pi^2)$;
 (12) $8\sqrt{2}\pi^3$; (13) $\frac{\sqrt{a^2+b^2}}{ab} \arctan \frac{2b\pi}{a}$; (14) $3\pi a^3$; (15) 9; (16) $\frac{2\pi a^3}{3}$.

2. 当 $a = b$ 时, 曲线的质量 $m = 4a^2$, 当 $a \neq b$ 时, $m = 2b^2 + \frac{2a^2b}{\sqrt{a^2-b^2}} \arcsin \frac{\sqrt{a^2-b^2}}{a}$.

3. $x_0 = \frac{4a}{3}$, $y_0 = \frac{4a}{3}$.

4. (1) $I_z = \frac{2}{3}\pi a^2 \sqrt{a^2 + k^2} (3a^2 + 4\pi^2 k^2)$;

(2) $\bar{x} = \frac{6ak^2}{3a^2 + 4\pi^2 k^2}$, $\bar{y} = \frac{-6\pi ak^2}{3a^2 + 4\pi^2 k^2}$, $\bar{z} = \frac{3k(\pi a^2 + 2\pi^3 k^2)}{3a^2 + 4\pi^2 k^2}$.

习题 7.2

1. (1) $-\frac{14}{15}$; (2) $-\frac{56}{15}$; (3) $-\frac{\pi}{2}a^3$; (4) 0; (5) -2π ; (6) $\frac{1}{30}$; (7) 0;
 (8) 13; (9) $\frac{1}{2}$; (10) $\frac{1}{35}$; (11) 0; (12) -2π ; (13) $-\sqrt{2}\pi a^2$; (14) 1.

2. (1) $\frac{1}{3}$; (2) 0; (3) -1 ; (4) 1; (5) $\pi - \frac{7}{3}$. 3. $\frac{29}{60}$.

习题 7.3

1. (1) $\frac{\pi a^4}{2}$; (2) $-2ab\pi$; (3) 12; (4) $\frac{1}{2}$; (5) $\frac{1}{5}(1 - e^\pi)$; (6) 0; (7) $\frac{20}{3}$; (8) 2.

2. (1) $\frac{3\pi a^2}{8}$; (2) 12π ; (3) $\frac{3\pi a^2}{2}$. 3. (1) 4; (2) 5; (3) $e^3 \cos 4 - 1$.

4. $yF'_y = xF'_x$.

5. (1) 坐标原点在围线 C 之外, $I = 0$; (2) 坐标原点在围线 C 之内, $I = 2\pi$.

6. (1) $\frac{1}{3}x^3 + x^2y - xy^2 - \frac{1}{3}y^3$; (2) $\frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2$; (3) x^2y ;

(4) $y^2 \cos x + x^2 \cos y$; (5) $x^3y + 4x^2y^2 + 12ye^y - 12e^y$.

7. (1) $\sqrt{1+a^2} - \sqrt{1+b^2}$; (2) π ; (3) $6\pi a^2 + 2\pi a(e^{2\pi a} - 1)$; (4) $\frac{m\pi a^2}{8}$; (5) $\frac{m\pi}{2}$.

12. $mS + e^{x_2}\varphi(y_2) - e^{x_1}\varphi(y_1) - m(y_2 - y_1) - \frac{m}{2}(x_2 - x_1)(y_2 + y_1)$.

14. 当点 A 在曲线 C 所围区域之外时, $I = 0$; 当点 A 在曲线 C 所围区域的内部时, $I = 2\pi$.

15. $Q(x, y) = x^3 + 3y^2 - 1$.

16. $f(x) = e^x$, 积分值为 $\ln 2 + \frac{\pi-1}{4} - \frac{e}{6} - \arctan(\cos 1)$.

习题 7.4

1. (1) πa^3 ; (2) $\frac{\sqrt{2}+1}{2}\pi$; (3) $\frac{3-\sqrt{3}}{2} + (\sqrt{3}-1)\ln 2$; (4) $4\sqrt{61}$; (5) $\frac{64\sqrt{2}a^4}{15}$; (6) $\frac{2\pi}{3}$; (7) 6π .

2. 4π . 3. $\frac{\pi}{60}(25\sqrt{5}+1)$. 4. $\frac{4}{3}\rho_0\pi a^4$. 5. πa^2 .

习题 7.5

1. $4\pi a^3$. 2. 0. 3. $\frac{2\pi a^7}{105}$. 4. 0. 5. $\frac{4\pi}{abc}(a^2b^2 + a^2c^2 + b^2c^2)$.

6. $4\pi R^3$. 7. π . 8. -8 . 9. $\frac{2\pi}{3}$. 10. $\frac{\pi R^4}{2}$. 11. $-\frac{2\pi}{3}$.

习题 7.6

1. (1) $3a^4$; (2) 36π ; (3) $\frac{4abc\pi}{15}(a^2 + b^2 + c^2)$. 2. $\frac{5}{3}\pi a^3$. 3. 64π .

6. (1) 0; (2) 0; (3) $-\sqrt{3}\pi a^2$; (4) $-\frac{\pi}{2}a^3$; (5) -20π ; (6) 9π ; (7) $-\frac{9}{2}R^3$; (8) 0.

7. $-2\sqrt{2}\pi$. 8. $\frac{1}{3}h^3$. 9. $I_1 = 4\pi a^4, I_2 = 2\sqrt{3}a^4, I_1 - I_2 = 2(2\pi - \sqrt{3})a^4$.

10. $\frac{8-5\sqrt{2}}{6}\pi a^4$. 12. $\frac{32}{5}\pi$. 13. $2\pi^2 a^2 b$. 14. $k \left(\frac{1}{\sqrt{x_1^2 + y_1^2 + z_1^2}} - \frac{1}{\sqrt{x_2^2 + y_2^2 + z_2^2}} \right)$.

习题 7.7

1. $2\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}$. 5. $(x-a)^2 + (y-b)^2 + (z-c)^2 = 1$. 6. $2b^2\pi^2$.

7. (1) 2π ; (2) $2\pi a^2$. 8. (1) $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$; (2) $\mathbf{i} + \mathbf{j}$. 10. $\mathbf{0}$.

习题 8.1

1. 不收敛.

2. (1) $S_n = \frac{1}{5} \left[1 - \frac{1}{5n+1} \right]$, 原级数收敛, 和为 $\frac{1}{5}$;

(2) $S_n = \frac{1}{8} \left[1 - \frac{1}{(2n+1)^2} \right]$, 原级数收敛, 和为 $\frac{1}{8}$;

(3) $S_n = \sqrt{n+1} - 1$, 原级数发散;

(4) $S_n = \left[\cos \frac{\pi}{12} - \cos \left(n + \frac{1}{2} \right) \frac{\pi}{6} \right] / \left(2 \sin \frac{\pi}{12} \right)$, S_n 当 $n \rightarrow \infty$ 时极限不存在, 原级数发散.

3. (1) 收敛, 和为 $\frac{7}{16}$; (2) 发散; (3) 收敛, 和为 $\frac{1}{2}$; (4) 发散; (5) 发散; (6) 发散; (7) 发散.

4. (1) $-\frac{5}{18}$; (2) $\frac{1}{4}$; (3) $\frac{1}{m} \left(1 + \frac{1}{2} + \cdots + \frac{1}{m} \right)$; (4) $\frac{1}{8}$.

5. (1) 收敛; (2) 收敛; (3) 收敛; (4) 发散; (5) 发散.

习题 8.2

1. (1) 收敛; (2) 收敛; (3) 发散; (4) $a > 1$ 时收敛, $0 < a \leq 1$ 时发散; (5) 发散; (6) 收敛; (7) 收敛.
2. (1) 收敛; (2) 收敛; (3) 发散; (4) 发散; (5) 发散; (6) 收敛; (7) 收敛; (8) 收敛; (9) 收敛; (10) 收敛; (11) 收敛; (12) $0 < p \leq 1$ 时发散, $p > 1$ 时收敛; (13) 发散; (14) $p = 1, q > 1$, 及 $p > 1, q$ 任意时收敛; (15) $b < a$ 时收敛, $b > a$ 时发散, $b = a$ 时敛散性不能确定.
3. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ 收敛, 但 $\sum_{n=1}^{\infty} \frac{1}{n}$ 发散. 5. (1) 一定发散; (2), (3) 不一定.
7. $p > \frac{1}{3}$ 时收敛, $p \leq \frac{1}{3}$ 时发散. 8. $p > 1$ 时收敛, $p \leq 1$ 时发散.

习题 8.3

1. (1) 绝对收敛; (2) 发散; (3) 条件收敛; (4) 绝对收敛; (5) 条件收敛; (6) 条件收敛; (7) 条件收敛; (8) 条件收敛; (9) 发散; (10) $p \leq 0$ 时发散, $0 < p \leq 1$ 时条件收敛, $p > 1$ 时绝对收敛; (11) 条件收敛; (12) 绝对收敛; (13) 绝对收敛; (14) 条件收敛.
2. (1) 绝对收敛; (2) 条件收敛. 3. $a \neq 1$ 时绝对收敛, $a = 1$ 时发散.
4. $p > 1$ 时绝对收敛, $0 < p \leq 1$ 时, 条件收敛. 7. 不一定.

习题 8.4

1. (1) $\frac{1}{e} < x < e$; (2) $x > 0$; (3) $-4 < x < 4$.
2. (1) 一致收敛; (2) 非一致收敛; (3) 一致收敛, 非一致收敛; (4) 非一致收敛.
3. (1) 非一致收敛; (2) 一致收敛; (3) 一致收敛; (4) 一致收敛; (5) 一致收敛.

习题 8.5

1. (1) $(-\infty, +\infty)$; (2) $(-1, 1)$; (3) $\left(-\frac{1}{e}, \frac{1}{e}\right)$; (4) $(-e, e)$; (5) $(-4, 4)$; (6) $(-\infty, +\infty)$; (7) $(-1, 1)$; (8) $\left(-\frac{1}{4}, \frac{1}{4}\right)$.
2. (1) $(-1, 1)$; (2) $(-1, 1)$; (3) $[-a, a]$; (4) $(-\sqrt{2}, \sqrt{2})$; (5) $(-2, 2)$; (6) $(-3, 3)$; (7) $\left(-\frac{1}{5}, \frac{1}{5}\right)$; (8) $\left[-\frac{1}{2}, \frac{1}{2}\right]$; (9) $[4, 6]$; (10) $(2 - e, 2 + e)$.
3. (1) $\frac{1}{(1-x)^2} (|x| < 1)$; (2) $\frac{1-x^2}{(1+x^2)^2} (|x| < 1)$; (3) $-x + \frac{1}{2} \arctan x + \frac{1}{4} \ln \left| \frac{1+x}{1-x} \right| (|x| < 1)$; (4) $\frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) (|x| < 1)$; (5) $\frac{2x}{(1-x)^3} (|x| < 1)$; (6) $\ln 3 - \ln(1-x), x \in [-5, 1)$.
4. (1) $\frac{\pi}{4}$; (2) 1.
5. (1) $x_0 = 3$; (2) $R = 8$. 6. (1) $R = \frac{\sqrt{5}-1}{2}$; (2) $f(x) = \frac{x}{1-x-x^2}$.

习题 8.6

1. (1) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ ($x \in \mathbb{R}$); (2) $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$ ($x \in \mathbb{R}$); (3) $\sum_{n=0}^{\infty} \frac{(-1)^n}{a^{n+1}} x^n$ ($|x| < |a|$);
 (4) $\frac{1}{2} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!} x^{2n}$ ($x \in \mathbb{R}$); (5) $\ln a + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{na^n} x^n$ ($-a < x \leq a$);
 (6) $x + \sum_{n=2}^{\infty} (-1)^{n-2} \frac{x^n}{(n-1)n}$ ($-1 < x \leq 1$);
 (7) $\sum_{n=1}^{\infty} \frac{1}{n} [(-1)^{n-1} 2^n - 1] x^n$ ($-\frac{1}{2} < x \leq \frac{1}{2}$); (8) $\sum_{n=0}^{\infty} \left[1 + \frac{(-1)^n}{6^n}\right] x^n$ ($|x| < 1$);
 (9) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ ($|x| \leq 1$); (10) $x + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!! x^{2n+1}}{(2n)!!(2n+1)}$ ($|x| \leq 1$);
 (11) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!(2n+1)}$ ($x \in \mathbb{R}$); (12) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n)!(4n+1)}$ ($x \in \mathbb{R}$).
 2. (1) $1 + \frac{3}{2}(x-1) + \frac{3}{8}(x-1)^2 + \sum_{n=3}^{\infty} 3 \frac{(-1)^n (2n-5)!!}{(2n)!!} (x-1)^n$ ($0 \leq x \leq 2$);
 (2) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x-1)^n$ ($0 < x \leq 2$); (3) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} (x-3)^n$ ($0 < x < 6$);
 (4) $\sum_{n=0}^{\infty} \left(\frac{1}{2^{n+1}} - \frac{1}{3^{n+1}}\right) (x+4)^n$ ($-6 < x < -2$).
 3. $f(x) = \sum_{n=0}^{\infty} \frac{1}{n!(2n+3)e} (x-1)^{2n+3}$, $x \in \mathbb{R}$. 4. (1) 4.1213; (2) 0.4861; (3) 1.0986.
 5. 令 $P(n) = a_0 + a_1 n + \cdots + a_m n^m \equiv b_0 + b_1 n + \cdots + b_m n(n-1) \cdots (n-m+1)$, 则
 $\sum_{n=1}^{\infty} \frac{P(n)}{n!} x^n = e^x (b_0 + b_1 x + \cdots + b_m x^m)$.
 6. (1) 当 $x \neq 0$ 时, $s(x) = e^{-x} \left(x^2 + 1 + \frac{1}{x}\right) - \frac{1}{x}$, 当 $x = 0$ 时, $s(x) = 0$;
 (2) $(1 + 2x^2)e^{x^2}$ ($x \in \mathbb{R}$); (3) 当 $-1 < x \leq 1$ 时, $s(x) = (x+1)\ln(1+x) - x$, 当 $x = -1$ 时, $s(x) = 1$; (4) 当 $-1 \leq x < 1, x \neq 0$ 时, $s(x) = 1 + \frac{1-x}{x} \ln(1-x)$, 当 $x = 0$ 时, $s(x) = 0$, 当 $x = 1$ 时, $s(x) = 1$.

习题 8.7

1. (1) 收敛; (2) 收敛; (3) 收敛; (4) 收敛; (5) 收敛; (6) 收敛; (7) 收敛; (8) 发散; (9) 收敛;
 (10) 仅当 $-2 < p < -1$ 时收敛; (11) 发散; (12) 收敛; (13) 收敛; (14) 发散; (15) 收敛;
 (16) 收敛; (17) $\alpha \geq 1$ 时收敛, $\alpha < 1$ 时发散; (18) 仅当 $p > 1, q < 1$ 时收敛;
 (19) 仅当 $-1 < p < 1$ 时收敛; (20) 仅当 $1 < p < 2$ 时收敛; (21) 收敛.
 3. (1) 条件收敛; (2) 条件收敛; (3) 绝对收敛; (4) $p > -2, q > p+1$ 时绝对收敛, $p > -2$,

$p < q \leq p+1$ 时条件收敛.

$$4. (1) 7!; (2) \frac{3}{4}\sqrt{\pi}; (3) \frac{\sqrt{\pi}}{4}; (4) \frac{\sqrt{\pi}}{2\sqrt{3}\ln 4}; (5) \frac{6}{(\alpha-1)^4}; (6) \frac{(2n-1)!!}{2^{n+1}}\sqrt{\pi}; (7) \frac{\pi}{8};$$

$$(8) \frac{\pi a^4}{16}; (9) \frac{2\pi}{3\sqrt{3}}; (10) \frac{\pi}{n \sin \frac{\pi}{n}}.$$

习题 9.2

$$1. (1) \frac{\operatorname{sh}(2\pi)}{2\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \operatorname{sh}(2\pi)}{4+n^2} \cos nx + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n \operatorname{sh}(2\pi)}{4+n^2} \sin nx$$

$$= \begin{cases} e^{2x}, & -\pi < x < \pi, \\ \operatorname{ch}(2\pi), & x = \pm\pi; \end{cases}$$

$$(2) \frac{\pi(b-a)}{4} + \frac{b-a}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} \cos nx + (b+a) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$$

$$= \begin{cases} ax, & -\pi < x < 0, \\ bx, & 0 \leq x < \pi, \\ \frac{\pi(b-a)}{2}, & x = \pm\pi; \end{cases}$$

$$(3) \frac{1+\pi-e^{-\pi}}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1+(-1)^{n+1}e^{-\pi}}{1+n^2} \cos nx$$

$$+ \frac{1}{\pi} \sum_{n=1}^{\infty} \left[\frac{1}{1+n^2} (-n + n(-1)^n e^{-\pi}) + \frac{1}{n} (1 - (-1)^n) \right] \sin nx = \begin{cases} e^x, & -\pi < x < 0, \\ 1, & 0 \leq x < \pi, \\ \frac{e^{-\pi}+1}{2}, & x = \pm\pi. \end{cases}$$

$$(4) \frac{3}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} (1 - \cos n\pi) \sin nx = \begin{cases} 1, & -\pi < x < 0, \\ \frac{3}{2}, & x = 0, \pm\pi, \\ 2, & 0 < x < \pi. \end{cases}$$

$$(5) \sum_{n=1}^{\infty} \frac{1}{n} \sin nx = \begin{cases} -\frac{\pi+x}{2}, & -\pi \leq x < 0, \\ 0, & x = 0, \\ \frac{\pi-x}{2}, & 0 < x \leq \pi. \end{cases}$$

$$2. (1) \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx \quad (|x| \leq \pi);$$

$$(2) \sum_{n=1}^{\infty} \left((-1)^{n+1} \frac{2\pi}{n} + \frac{4((-1)^n - 1)}{n^3\pi} \right) \sin nx \quad (0 \leq x < \pi);$$

$$(3) \frac{4\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} - 4\pi \sum_{n=1}^{\infty} \frac{\sin nx}{n} \quad (0 < x < 2\pi); \quad (4) \frac{\pi^2}{6}, \frac{\pi^2}{12}, \frac{\pi^2}{8}.$$

$$3. \sum_{n=1}^{\infty} \left(\frac{2}{n^2\pi} \sin \frac{n\pi}{2} + \frac{(-1)^{n+1}}{n} \right) \sin nx.$$

$$4. 3 = \frac{12}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1} \quad (0 < x < \pi). \text{ 令 } x = \frac{\pi}{2} \text{ 就可得到 } \frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}.$$

习题 9.3

$$1. (1) \frac{2}{3} - \sum_{n=1}^{\infty} \frac{6}{n^2 \pi^2} \sin^2 \frac{n\pi}{3} \cos \frac{2n\pi x}{3};$$

$$(2) f(x) = \frac{11}{12} + \frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos 2n\pi x, x \in (-\infty, +\infty);$$

$$(3) f(x) = -\frac{1}{2} + \sum_{n=1}^{\infty} \left\{ \frac{6}{n^2 \pi^2} [1 - (-1)^n] \cos \frac{n\pi x}{3} + \frac{6}{n\pi} (-1)^{n+1} \sin \frac{n\pi x}{3} \right\}, x \neq 3(2k+1),$$

$$k = 0, \pm 1, \pm 2, \dots$$

$$2. \frac{1}{2} \sin \frac{\pi x}{l} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n n}{4n^2 - 1} \sin \frac{2n\pi x}{l} = \begin{cases} \sin \frac{\pi x}{l}, & 0 \leq x < \frac{l}{2}, \\ 1/2, & x = l/2, \\ 0, & \frac{l}{2} < x \leq l. \end{cases}$$

$$3. (1) f(x) = \frac{5}{2} + \sum_{n=0}^{\infty} \frac{-4}{(2n+1)^2 \pi^2} \cos(2n+1)\pi x, x \in [-1, 1].$$

$$(2) \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}. \quad (3) \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

$$4. a_{2n} = b_{2n} = 0 \quad (n = 0, 1, 2, \dots). \quad 5. a_{2n-1} = b_{2n-1} = 0 \quad (n = 1, 2, 3, \dots).$$

$$6. \alpha_n = a_n, \beta_n = -b_n.$$

$$7. \text{ 正弦级数 } \sum_{n=1}^{\infty} \frac{2}{n\pi} (1 - \cos nh) \sin nx; \text{ 余弦级数 } \frac{h}{\pi} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin nh \cos nx.$$

习题 10.2

$$2. (1) y = (x + y')y'; (2) x^2 y' - xy + y^2 = 0; (3) y'' - y = 0; (4) y' + xy'' = 0.$$

$$3. (1) y = e^{Ce^x}; (2) \sqrt{1-y^2} = \arcsin x + C, y = \pm 1, x = \pm 1;$$

$$(3) 3y^2 = 2 \ln |1+x^3| + C; (4) xy = Ce^{y-x}.$$

$$4. (1) y = e^{Cx}; (2) \ln(1+x^2) + 2 \ln |y| = C, y = 0; (3) \ln^2 x + \ln^2 y = C; (4) e^x + e^{-y} = C;$$

$$(5) (e^x + 1)(e^y - 1) = C; (6) y^2 + 2y + \ln(y-1)^2 + \frac{2}{x} = C, y = 1.$$

$$5. (1) y^2 = x^2 - Cx; (2) y = Cx(y-x); (3) \sin \frac{y}{x} = Cx; (4) \ln \left(1 + \frac{y}{x}\right) = Cx;$$

$$(5) \arcsin \frac{y}{x} = \ln |Cx|; (6) y = Ce^{\frac{y}{x}}.$$

$$6. (1) y + 2 = Ce^{2 \arctan \frac{x-3}{y+2}}; (2) (y-2x)^3 = C(y-x-1)^2, y = x+1;$$

$$(3) 9 \ln \left| 3y + 2x + \frac{22}{7} \right| = 21(2y-x+C), 21y + 14x + 22 = 0;$$

$$(4) \ln[(x+2)^2 + (y+3)^2] + 2 \arctan \frac{y+3}{x+2} = C.$$

$$7. (1) y(1 + \ln |x^2 - 1|) = 1; (2) \sqrt{y} = x \ln x - x + 1; (3) \tan 2y = x + \sin x \cos x; (4) y = xe^x.$$

习题 10.3

1. (1) $y = -x + C\sqrt{1+x^2}$; (2) $y \sin x = C + x \sin x + \cos x$; (3) $2y = -\csc x + C \sin x$;
 (4) $2x = Cy + y^3, y = 0$; (5) $2xy = y^2 + C$; (6) $y = (C+x)e^{-\sin x}$; (7) $y = \frac{1}{x^2+1} \left(C + \frac{4}{3}x^3 \right)$;
 (8) $x = y^2(1 + Ce^{\frac{1}{y}}), y = 0$; (9) $y = \left(\frac{x^2}{2} + C \right) e^{-x^2}$; (10) $\frac{x}{y^3} + \frac{3}{4}x^2(2\ln x - 1) = C, y = 0$.
2. (1) $y^2 = Cx + x^2$; (2) $\frac{1}{y} = Ce^{-x} + \frac{1}{2}(\cos x - \sin x)$; (3) $y = \left(Cx^5 + \frac{5}{2}x^3 \right)^{-1/5}$;
 (4) $x(Ce^{-\frac{y^2}{2}} - y^2 + 2) = 1$; (5) $\frac{1}{y} = -\sin x + Ce^x$; (6) $\frac{1}{y^4} = -x + \frac{1}{4} + Ce^{-4x}$.
3. (1) $x = 2y - \ln y - 1$; (2) $y = x^2(e^x - e)$; (3) $y = \frac{2}{3}(x+1)^2[(x+1)^{\frac{3}{2}} - 1]$;
 (4) $y = \frac{1}{3}(8 - 5e^{-3x})$; (5) $y = e^{1-\sin \frac{x}{y}}$.

习题 10.4

1. (1) 是; (2) 是; (3) 是.
2. (1) $x^3 + 2x^2y + y^2 = C$; (2) $6xy^2 - \cos(2xy) = C$; (3) $\frac{x^3}{3} + \frac{x}{y} + \ln|y| = C$;
 (4) $x^3y - xy^4 + e^{xy} = C$; (5) $\frac{x^2}{2} + x - xy - \frac{y^3}{3} - 3y = C$; (6) $e^x \sin y + 2y \cos x = C$;
 (7) $\sin \frac{y}{x} - \cos \frac{x}{y} + x - \frac{1}{y} = C$; (8) $ye^x - xe^{-y} = C$.
3. (1) $A = \frac{3}{2}, 2x^3 + 9x^2y + 12y^2 = C$; (2) $A = -2, y(1-x) = Cx^2$; (3) $A = 3, x^3y + 2xy^2 = C$;
 (4) $A = -2, 2x^2 - 2y^2 - x = Cxy^2$.
4. (1) $P(x, y) = y^2e^x + y^3e^{3x} + \varphi(x), 3y^2e^x + y^3e^{3x} + 3 \int \varphi(x)dx = C$;
 (2) $Q(x, y) = 2x^{-1}y^{-3} - \frac{3}{2}x^2y^{-4} + \psi(y), x^2y^{-3} - 2x^{-1}y^{-2} + 2 \int \psi(y)dy = C$;
 (3) $P(x, y) = xy^4 + 2x^3y^2 + \varphi(x), x^2y^4 + x^4y^2 + 2 \int \varphi(x)dx = C$;
 (4) $Q(x, y) = x^2y + \psi(y), x^4 + 2x^2y^2 + 4 \int \psi(y)dy = C$.
5. (1) $xe^y(x^2 + 3y^2) = C$; (2) $x^3 - x^2y + x^2y^2 = C$; (3) $xy + x^2 = Cy, y = 0$;
 (4) $e^x(xy + \sin y) = C$; (5) $x + 3x^2y^2 - 4xy^3 = Cy^2, y = 0$; (6) $(x^3 + y^3)\ln^2 y = C$;
 (7) $x = Ce^{\frac{y^4}{4x^4}}$; (8) $\frac{e^y}{x} + y^2 = C, x = 0$; (9) $y = Ce^{\frac{x^3}{3y^3}}$; (10) $x^2y + \frac{1}{y} = C, y = 0$.
6. (1) $1 + 3x^2y^3 = Cx^3y^3$; (2) $\ln|x| - \frac{x}{y} = C$; (3) $2y = x \tan(2x + C)$; (4) $\frac{1}{xy} + \frac{1}{2}\ln^2 x = C$.

习题 10.6

1. (1) $y = \int \arctan x dx + \int C_1 dx = x \arctan x - \frac{1}{2} \ln(1+x^2) + C_1 x + C_2$;
 (2) $y = \frac{1}{2} x^2 \ln|x| + C_1 x^3 + C_2 x^2 + C_3 x + C_4$;
 (3) $y = \frac{1}{2} \ln|\csc x - \cot x| + C_1 \cos x + C_2$;
 (4) $y = -x \ln x + C_1 x^2 + C_2$;
 (5) $y = -\frac{x}{C_1} + \frac{1}{C_1} \ln|e^x + C_1| + C_2$, 奇解为 $y = C, y = e^{-x} + C$;
 (6) $x + C_1 \ln|y + C_1| - y + C_2 = 0, y = C$;
 (7) $y = -\ln|\cos(x + C_1)| + C_2$;
 (8) $C_1 y = C_3 e^{C_1 x} + 1, y = C, y = C - x$.
2. (1) $y = \left(1 + \frac{3}{4}x\right)^{\frac{4}{3}}$; (2) $y = \sqrt{2x - x^2}$; (3) $y = -x^5 + \frac{1}{6}(x^6 - 1)$; (4) $y = \frac{1}{2}(x^2 - 4x + 5)$.
3. (1) $y^3 = C_1 x + C_2$; (2) $C_1 y^2 - 1 = (C_1 x + C_2)^2$; (3) $y = -(\ln|\cos(\sqrt{2}x + C_1)|)/2 + C_2$;
 (4) $\sin(y + C_1) = C_2 e^x$; (5) $y = \operatorname{ch}(x + C_1) + C_2$; (6) $y = C_2 e^{C_1 x}$.
4. (1) $y = (C_1 \cos \sqrt{2}x + C_2 \sin \sqrt{2}x)e^x$; (2) $y = C_1 e^{\frac{1}{2}x} + C_2 e^{-x}$; (3) $y = (C_1 + C_2 x)e^{-4x}$;
 (4) $y = C_1 \cos 2x + C_2 \sin 2x$; (5) $y = C_1 + C_2 e^{-\frac{2}{3}x}$; (6) $y = C_1 e^x + C_2 e^{3x}$;
 (7) $y = C_1 e^x + C_2 x e^x$; (8) $y = e^{3x}(C_1 \cos \sqrt{2}x + C_2 \sin \sqrt{2}x)$;
 (9) $y = C_1 e^{2x} + e^{-x}(C_2 \cos \sqrt{3}x + C_3 \sin \sqrt{3}x)$; (10) $y = (C_1 + C_2 x)e^x + C_3 e^{2x} + C_4 e^{3x}$.
5. (1) $y^* = x e^{2x}(Ax + B)$; (2) $y^* = A \cos 2x + B \sin 2x$; (3) $y^* = e^x(A \cos x + B \sin x)$;
 (4) $y^* = x^2(Ax + B)e^x$; (5) $y^* = x e^x(A \cos x + B \sin x)$; (6) $y^* = x(Ax^2 + Bx + C)$;
 (7) $y^* = e^x(Ax^2 + Bx + C) + x e^{2x}(Dx + E)$;
 (8) $y^* = x e^x[(Ax^2 + Bx + C) \cos 2x + (Dx^2 + Ex + F) \sin 2x]$.
6. (1) $y = C_1 e^{2x} + C_2 e^{-2x} + \frac{1}{4} x e^{2x}$; (2) $y = C_1 + C_2 e^{-\frac{5}{2}x} + \frac{1}{3} x^3 - \frac{3}{5} x^2 + \frac{7}{25} x$;
 (3) $y = C_1 e^{-x} + C_2 e^{-2x} + \frac{1}{2} e^{-x}(\sin x - \cos x)$; (4) $y = (C_1 + C_2 x)e^{2x} + \frac{1}{8} \cos 2x + \frac{1}{2} x^2 e^{2x}$;
 (5) $y = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x} + \frac{2}{9}[(2 - 3x) \cos x + (3x + 2) \sin x]$;
 (6) $y = C_1 \cos x + C_2 \sin x - x \cos x + \sin x \ln|\sin x|$;
 (7) $y = C_1 e^x + C_2 x e^x - \frac{e^x}{2} \ln(1+x^2) + x e^x \arctan x$; (8) $y = e^{3x}(C_1 \cos x + C_2 \sin x) + \frac{1}{2}$;
 (9) $y = C_1 + C_2 e^{-x} + 3x - x^2 + \frac{x^3}{3}$; (10) $y = C_1 e^{2x} + C_2 e^{-x} + \frac{1}{3} x e^{2x}$;
 (11) $y = (C_1 + C_2 x)e^{4x} + \frac{x}{16} + \frac{1}{32} + \frac{x^3}{6} e^{4x}$; (12) $y = C_1 e^x + C_2 e^{-x} + (x^2 - x)e^x$;

$$(13) y = C_1 e^x + C_2 e^{3x} - \frac{3}{8} e^x (\cos 2x + \sin 2x);$$

$$(14) y = C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2 - 1} \sin x, (a \neq 1),$$

$$y = C_1 \cos x + C_2 \sin x - \frac{1}{2} x \cos x, (a = 1);$$

$$(15) y = C_1 e^{-3x} + C_2 e^x - x - 1 - \frac{1}{5} \cos x + \frac{1}{10} \sin x;$$

$$(16) y = C_1 e^{-x} + C_2 e^{2x} + C_3 x e^{2x} + 3x^2 + \frac{9}{2} + 7 \cos x + \sin x.$$

$$7. (1) y = C_1 \sqrt{x} + C_2 x^{-2}; (2) y = C_1 x + C_2 x \ln |x| + x \ln^2 |x|;$$

$$(3) y = C_1 x + C_2 x^2 + \frac{1}{2} (\ln^2 |x| + \ln |x|) + \frac{1}{4}; (4) y = C_1 x^2 + C_2 x^{-2} + \frac{1}{5} x^3;$$

$$(5) y = x [C_1 \cos(\sqrt{3} \ln x) + C_2 \sin(\sqrt{3} \ln x)] + \frac{1}{2} x \sin(\ln x);$$

$$(6) y = C_1 x^2 + C_2 x^2 \ln x + x + \frac{1}{6} x^2 \ln^3 x.$$

习题 10.7

$$1. xy = 6, (x > 0). \quad 2. y = Cx^3 \text{ 或 } x = Cy^3. \quad 3. \text{需要 60 分钟此物体温度降至 } 30^\circ \text{C}.$$

$$4. \frac{df(t)}{dt} + \frac{2}{t} f(t) = \frac{3}{t^2} f^2(t), f(x) = \frac{x}{1+x^3}. \quad 5. \text{和函数 } f(x) = (1+x)^\alpha.$$

$$6. k(t) = \frac{1000e^{\frac{t}{2}}}{49 + e^{\frac{t}{2}}}. \quad 7. x^2 + y^2 = C. \quad 8. y^2 = \frac{1}{3}x.$$

$$9. V = \left(V_0^{\frac{1}{3}} - \frac{1}{3} (36\pi)^{\frac{1}{3}} kt \right)^3. \quad 10. S = \begin{cases} \frac{Ft^2}{2m}, & 0 < t \leq 1, \\ \frac{F - F_1}{2m} t^2 + \frac{F_1}{m} t - \frac{F_1}{2m}, & t > 1. \end{cases}$$