Homogeneous Coordinates

a) 直线: $\vec{l}=(a,b,c)^T$ 即 $ax+by+c=0$.

若x在$\vec{l}_1$与$\vec{l}_2$上,则 $\begin{cases} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{cases}$

$$x=\frac{1}{a_1}(-b_1y-c_1)$$

$$b_2y+c_2=\frac{a_2}{a_1}(b_1y+c_1) \longrightarrow y=\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1} \qquad x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$$

$$\vec{l}_1\times\vec{l}_2=\vec{x}=\begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{bmatrix} b_1c_2-b_2c_1 \\ c_1a_2-c_2a_1 \\ a_1b_2-a_2b_1 \end{bmatrix}=\begin{bmatrix} \frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \\ \frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1} \\ 1 \end{bmatrix}=\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

b) $\vec{l}$上有$\vec{x}_1$, $\vec{x}_2$,则$\vec{l}$的方程可写作 $y=\frac{y_2-y_1}{x_2-x_1}x+\frac{x_2y_1-x_1y_2}{x_2-x_1} \Rightarrow \vec{l}=\begin{bmatrix} y_2-y_1 \\ x_1-x_2 \\ x_2y_1-x_1y_2 \end{bmatrix}$

$$\vec{x}_1\times\vec{x}_2=\begin{vmatrix} i & j & k \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=\begin{bmatrix} y_1-y_2 \\ x_2-x_1 \\ x_2y_1-x_1y_2 \end{bmatrix}=\vec{l}.$$

c) $\begin{cases} x+y+3=0 \\ -x-2y+7=0 \end{cases} \longrightarrow \begin{cases} x=-13 \\ y=10 \end{cases}$

$\vec{l}_1=(1\ 1\ 3)^T \quad \vec{l}_2=(-1\ -2\ 7)^T$

$$\vec{l}_1\times\vec{l}_2=\begin{vmatrix} i & j & k \\ 1 & 1 & 3 \\ -1 & -2 & 7 \end{vmatrix}=\vec{i}\begin{vmatrix} 1 & 3 \\ -2 & 7 \end{vmatrix}-\vec{j}\begin{vmatrix} 1 & 3 \\ -1 & 7 \end{vmatrix}+\vec{k}\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix}$$

$$\vec{x}=13\vec{i}-10\vec{j}-\vec{k}.=(13\ -10\ -1)^T \rightarrow (-13\ 10\ 1)^T$$

所得交点相同.

a) 法向量为$(3,4)^T$,则该直线可写作 $3x+4y+c=0$

离原点距离 $d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}=\frac{|c|}{5}=3 \qquad |c|=15$

$\therefore$ $\vec{l}$为 $3x+4y\pm15=0$

e) 法向量为$(2,5)^T$ $\quad d=\frac{\frac{\sqrt{29}}{5}}{\sqrt{29}}=\frac{1}{5}$

Transformations

a) $T=(0,3)^T-(1,2)^T=(-1,1)^T$ $\therefore$ 平移矩阵为 $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

验证: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

b) $T=\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \qquad T\vec{x}_i=\begin{bmatrix} x_1^i+t_x \\ x_2^i+t_y \end{bmatrix}$

$$E(T)=\sum_{i=1}^{N}((x_1^i+t_x-y_1^i)^2+(x_2^i+t_y-y_2^i)^2)$$

$$\frac{\partial E}{\partial t_x}=\sum_{i=1}^{N}2(x_1^i+t_x-y_1^i)=0 \qquad \frac{\partial E}{\partial t_y}=\sum_{i=1}^{N}2(x_2^i+t_y-y_2^i)=0$$

解得 $t_x=\frac{1}{N}\sum_{i=1}^{N}(y_1^i-x_1^i) \qquad t_y=\frac{1}{N}\sum_{i=1}^{N}(y_2^i-x_2^i)$

c) $t_x=\frac{1}{3}[(3-0)+(7-5)+(5-4)]=2 \qquad t_y=\frac{1}{3}[(-5-1)+(6-7)+(-4-1)]=-4$

$T=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{bmatrix}$

Camera Projections

a) $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ $t = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ $K = \begin{bmatrix} 100 & 0 & 25 \\ 0 & 100 & 25 \\ 0 & 0 & 1 \end{bmatrix}$

$$\tilde{P} = \begin{bmatrix} K & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 100 & 0 & 25 & 0 \\ 0 & 100 & 25 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 100 & 25 & 0 & 150 \\ 0 & 25 & -100 & 50 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $$\tilde{x}_w = \tilde{P}^{-1}\tilde{x}_s = \begin{bmatrix} 0.01 & 0 & -0.25 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & -0.01 & 0.25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 25 \\ 50 \\ 1 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -0.25 \\ 0.5 \\ -0.25 \\ 0.25 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

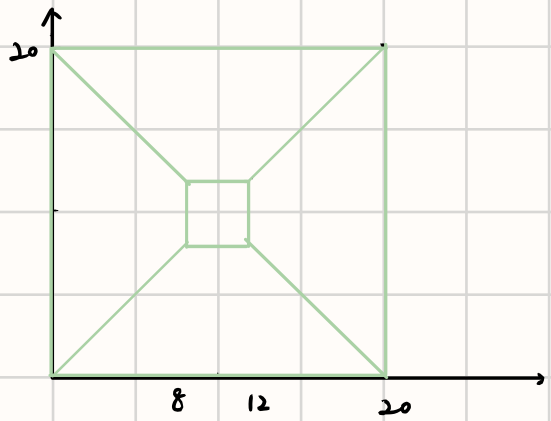
$$x_w = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

c)

i $K = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$ 原顶点为 $\begin{bmatrix} -10 \\ -10 \\ 5 \end{bmatrix}\begin{bmatrix} -10 \\ -10 \\ 25 \end{bmatrix}\begin{bmatrix} -10 \\ 10 \\ 5 \end{bmatrix}\begin{bmatrix} -10 \\ 10 \\ 25 \end{bmatrix}\cdots$

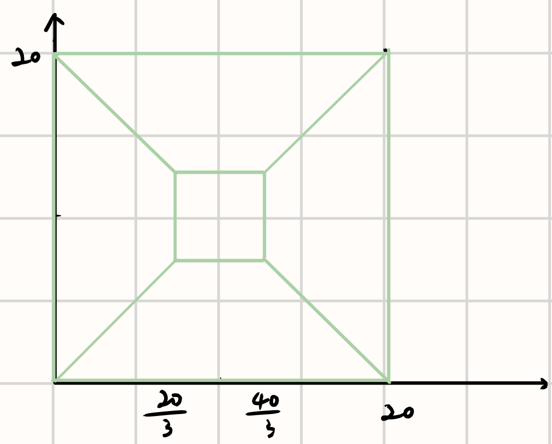
$$\tilde{x}_s = K\tilde{x}_c = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{bmatrix}\tilde{x}_c$$

则转换后 x_s 有 (0,0) (8,8) (0,20) (8,12) (20,0) (12,8) (20,20) (12,12)



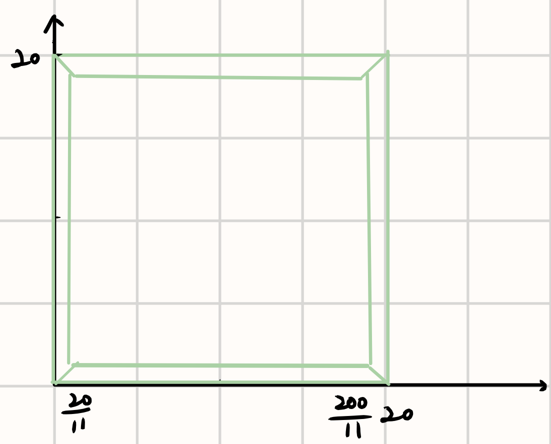
ii $K = \begin{bmatrix} 10 & 0 & 10 \\ 0 & 10 & 10 \\ 0 & 0 & 1 \end{bmatrix}$ 原顶点为 $\begin{bmatrix} -10 \\ -10 \\ 10 \end{bmatrix}\begin{bmatrix} -10 \\ -10 \\ 30 \end{bmatrix}\cdots$

转换后 x_s 有 (0,0) $(\frac{20}{3}, \frac{20}{3})$ $(\frac{20}{3}, \frac{40}{3})$ $(\frac{40}{3}, \frac{20}{3})$ $(\frac{40}{3}, \frac{40}{3})$ (0,20) (20,20) (20,0)



iii $K = \begin{bmatrix} 90 & 0 & 10 \\ 0 & 90 & 10 \\ 0 & 0 & 1 \end{bmatrix}$ 顶点为 $\begin{bmatrix} -10 \\ -10 \\ 90 \end{bmatrix}\begin{bmatrix} -10 \\ -10 \\ 110 \end{bmatrix}\cdots$

转换后 x_s 有 (0,0) $(\frac{20}{11}, \frac{20}{11})$ $(\frac{200}{11}, \frac{200}{11})\cdots$



iv ∞时没有透视，直接投影

v. 正方体离镜头越远，镜头焦距越大时.

Photometric Image Formation

a) $\frac{1}{f} = \frac{1}{z_s} + \frac{1}{z_c}$

$\frac{1}{0.1} = \frac{1}{0.102} + \frac{1}{z_c} \Rightarrow z_c = 5.1\,m$

b) $\frac{c}{d} = \frac{\Delta z_s}{z_s} \Rightarrow c = \frac{f}{N} \cdot \frac{\Delta z_s}{z_s}$

c) 当 $\Delta z_s = 0.1\,mm$, $c_1 = \frac{35}{1.4 \cdot 40} \cdot 0.1 = 0.0625\,mm$

当 $\Delta z_s = 0.03\,mm$, $c_2 = \frac{35}{1.4 \cdot 40} \cdot 0.03 = 0.01875\,mm$

每个像素点的面积为 $S = \frac{64}{1600} = 0.0004\,mm^2$，边长为 $\sqrt{0.0004} = 0.02\,mm$

而像素点边长 $> c$ 才会不模糊. 故 $\Delta z_s = 0.1\,mm$ 时是模糊的. $\Delta z_s = 0.03\,mm$ 时是清晰的