

# H A R D

It's too hard.

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## 1 Hard Problems

Many problems have polynomial  $O(n^k)$  time solutions. Some problems haven't had a polynomial time solution.

## 2 Complexity Classes

**P** Set of problems we can solve in polynomial time. Sorting, Searching...

**NP** Verifiable in polynomial time. If you can solve in polynomial time, you can verify in polynomial time, but it's not always true in the other direction.

**NP-Complete** Set of problems that ANY problem in NP can be transformed into in polynomial time. Decision.

**NP-Hard** At least as hard as hard as hard as hard as any problem in NP. Optimization.

### 3 P vs NP

$P = NP$ ? If you can find a polynomial solution in NP-Complete, then you can transform any NP problem into NP-Complete in polynomial time and then solve it. So if you can prove  $P = NP$  you're a genius millionaire.

## 4 NP-Complete Problems

### 4.1 Boolean Satisfiability

kCNF (k Conjunctive Normal Form) - A way to express a logical statement. Each or has exactly k variables.

- "AND" of many "OR" expressions, each of which contains K variables or their negations.
- $(P \vee !Q) \wedge (R \vee Q) \wedge (!P \vee !R)$  - 2 CNF because each or has 2 variables.

### 4.2 kSAT

Given an expression in KCNF is there an assignment of vars that makes it evaluate to true.

$k = 2$  (2SAT)  $\in P$

$k > 2 \in \text{NP-Complete}$