Міністерство освіти і науки України

Національний технічний університет України

«Київський політехнічний інститут імені Ігоря Сікорського»

Факультет інформатики та обчислювальної техніки

Кафедра обчислювальної техніки

Лабораторна робота №5

«Проведення трьохфакторного експерименту при використанні рівняння регресії з урахуванням квадратичних членів (центральний ортогональний композиційний план)»

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Мета роботи: Провести трьохфакторний експеримент з урахуванням квадратичних членів ,використовуючи центральний ортогональний композиційний план. Знайти рівняння регресії, яке буде адекватним для опису об'єкту.

Завдання на лабораторну роботу:

1. Взяти рівняння з урахуванням квадратичних членів.

2. Скласти матрицю планування для ОЦКП

3. Провести експеримент у всіх точках факторного простору (знайти значення функції відгуку Y). Значення функції відгуку знайти у відповідності з варіантом діапазону, зазначеного далі.

$$y_{max} = 200 + x_{cp\,max};$$

$$y_{min} = 200 + x_{cp\,min}$$

де $x_{cp\,max} = \frac{x_{1max} + x_{2max} + x_{3max}}{3}$, $x_{cp\,min} = \frac{x_{1min} + x_{2min} + x_{3min}}{3}$

4. Розрахувати коефіцієнти рівняння регресії і записати його.

5. Провести 3 статистичні перевірки.

Варіант завдання:

Варіант	X_1		X_2		X_3	
	min	max	min	max	min	max
127	-4	4	-5	4	-5	4

Роздруківка тексту програми

```
from random import randint
from numpy.linalg import det
from copy import deepcopy
from scipy.stats import t


def naturalize(matrix_of_plan, min_max_arr, flag):
    result = []
    for i in range(len(matrix_of_plan)):
        if i < 8:
```

```
            result.append(min_max_arr[1]) if matrix_of_plan[i] == 1 else
result.append(min_max_arr[0])
        else:
            x0 = (max(min_max_arr) + min(min_max_arr)) / 2
            dx = x0 - min(min_max_arr)
            value = None
            if flag == 1:
                value = matrix_of_plan[i] * dx + x0 if i == 8 or 9 else x0
            elif flag == 2:
                value = matrix_of_plan[i] * dx + x0 if i == 10 or 11 else x0
            elif flag == 3:
                value = matrix_of_plan[i] * dx + x0 if i == 12 or 13 else x0
            result.append(value)
    return result


def cocharans_test(y_arr, y_avg, m, N):
    # Перевірка однорідності дисперсії за критерієм Кохрена
    dispersion = []
    for i in range(len(y_arr[0])):
        current_sum = 0
        for j in range(len(y_arr)):
            current_sum += (y_arr[j][i] - y_avg[j]) ** 2
        dispersion.append(current_sum / len(y_arr))

    print('dispersion:', dispersion)

    gp = max(dispersion) / sum(dispersion)
    print('Gp =', gp)

    # Рівень значимості q = 0.05
    # f1 = m - 1
    # f2 = N

    # За таблицею Gт = 0.3346
    if gp < 0.3346:
        print('Дисперсія однорідна')
        return dispersion
    else:
        print('Дисперсія неоднорідна')
        return None


def students_test(x0_plan1, x1_plan1, x2_plan1, x3_plan1, y_avg_arr,
dispersion, m):
    # Оцінка значимості коефіцієнтів регресії згідно критерію Стьюдента
    s2b = sum(dispersion) / 15
    s2bs_avg = s2b / 15 * m
    sb = s2bs_avg ** (1 / 2)

    beta_arr = [
        sum([y_avg_arr[i] * x0_plan1[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * x1_plan1[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * x2_plan1[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * x3_plan1[i] for i in range(15)]) / 15,
        sum([y avg arr[i] * x1 plan1[i] * x2 plan1[i] for i in range(15)]) /
15,
        sum([y_avg_arr[i] * x1_plan1[i] * x3_plan1[i] for i in range(15)]) /
15,
        sum([y_avg_arr[i] * x2_plan1[i] * x3_plan1[i] for i in range(15)]) /
15,
        sum([y_avg_arr[i] * x1_plan1[i] * x2_plan1[i] * x3_plan1[i] for i in
range(15)]) / 15,
        sum([y_avg_arr[i] * x1_plan1[i] ** 2 for i in range(15)]) / 15,
```

```
        sum([y_avg_arr[i] * x2_plan1[i] ** 2 for i in range(15)]) / 15,
        sum([y_avg_arr[i] * x3_plan1[i] ** 2 for i in range(15)]) / 15
    ]

    print('beta:', beta_arr)
    t_arr = [abs(beta_arr[i]) / sb for i in range(11)]
    print('t:', t_arr)

    # f3 = f1*f2 = 2*15 = 30
    f1 = m - 1
    f2 = 15
    f3 = f1 * f2

    b_arr = []
    for i in range(len(t_arr)):
        if t_arr[i] > t.ppf(q=0.975, df=f3):
            b_arr.append(t_arr[i])
        else:
            print(f'Коефіцієнт b{i} приймаємо не значним')
            b_arr.append(0)

    return b_arr, s2b


def fishers_test(b_arr, s2b, y_avg, y_res, m):
    # Критерій Фішера
    d = len([i for i in b_arr if i != 0])  # кількість значимих коефіцієнтів
    print(f'd = {d}')
    s2_ad = m * sum([(y_res[i] - y_avg[i]) ** 2 for i in range(15)]) / 15 - d
    fp = s2_ad / s2b
    print(f'Fp = {fp}')


    # Ft = 2.1
    if fp > 2.1:
        print('Рівняння регресії неадекватно оригіналу при рівні значимості
0.05')
    else:
        print('Рівняння регресії адекватно оригіналу при рівні значимості
0.05')


def main(m=3):
    N = 15

    x1 = [-4, 4]
    x2 = [-5, 4]
    x3 = [-5, 4]

    # Величина зоряного плеча
    l = 1.215

    # Матриця планування з нормованих значень
    x0_plan1 = [1 for _ in range(N)]
    x1_plan1 = [-1, -1, 1, 1, -1, -1, 1, 1, -l, l, 0, 0, 0, 0, 0]
    x2_plan1 = [-1, 1, -1, 1, -1, 1, -1, 1, 0, 0, -l, l, 0, 0, 0]
    x3_plan1 = [1, -1, -1, 1, -1, 1, 1, -1, 0, 0, 0, 0, -l, l, 0]
    print('x1:', x1_plan1)
    print('x2:', x2_plan1)
    print('x3:', x3_plan1)
    print('-' * 100)

    # Матриця планування з натуралізованих значень
    x1_plan2 = naturalize(x1_plan1, x1, 1)
```

```
    x2_plan2 = naturalize(x2_plan1, x2, 2)
    x3_plan2 = naturalize(x3_plan1, x3, 3)

    # Мультиплікативні значення факторів
    x4_plan2 = [x1_plan2[i] * x2_plan2[i] for i in range(len(x1_plan2))]
    x5_plan2 = [x1_plan2[i] * x3_plan2[i] for i in range(len(x1_plan2))]
    x6_plan2 = [x2_plan2[i] * x3_plan2[i] for i in range(len(x1_plan2))]
    x7_plan2 = [x1_plan2[i] * x2_plan2[i] * x3_plan2[i] for i in
range(len(x1_plan2))]

    # Квадратичні значення факторів
    x8_plan2 = [x1_plan2[i] ** 2 for i in range(len(x1_plan2))]
    x9_plan2 = [x2_plan2[i] ** 2 for i in range(len(x1_plan2))]
    x10_plan2 = [x3_plan2[i] ** 2 for i in range(len(x1_plan2))]

    print(f'x1: {x1_plan2}')
    print(f'x2: {x2_plan2}')
    print(f'x3: {x3_plan2}')
    print(f'x4: {x4_plan2}')
    print(f'x5: {x5_plan2}')
    print(f'x6: {x6_plan2}')
    print(f'x7: {x7_plan2}')
    print(f'x8: {x8_plan2}')
    print(f'x9: {x9_plan2}')
    print(f'x10: {x10_plan2}')

    x_avg_max = (max(x1_plan2) + max(x2_plan2) + max(x3_plan2)) / 3
    x_avg_min = (min(x1_plan2) + min(x2_plan2) + min(x3_plan2)) / 3
    print()
    print(f'x_avg_max = {x_avg_max}')
    print(f'x_avg_min = {x_avg_min}')
    print('-' * 100)

    # Діапазон y
    y_max = int(200 + x_avg_max)
    y_min = int(200 + x_avg_min)
    print(f'y_max = {y_max}')
    print(f'y_min = {y_min}')
    print()

    y_arr = [[randint(y_min, y_max) for _ in range(N)] for _ in range(m)]
    for i in range(len(y_arr)):
        print(f'y{i + 1}: {y_arr[i]}')

    y_avg = []
    for i in range(len(y_arr[0])):
        current_sum = 0
        for j in range(len(y_arr)):
            current_sum += y_arr[j][i]
        y_avg.append(current_sum / len(y_arr))
    print('y average:', y_avg)
    print('-' * 100)

    dispersion = cocharans_test(y_arr, y_avg, m, N)
    if dispersion:
        mx1 = sum(x1_plan2) / len(x1_plan2)
        mx2 = sum(x2_plan2) / len(x2_plan2)
        mx3 = sum(x3_plan2) / len(x3_plan2)
        mx4 = sum(x4_plan2) / len(x4_plan2)
        mx5 = sum(x5_plan2) / len(x5_plan2)
        mx6 = sum(x6_plan2) / len(x6_plan2)
        mx7 = sum(x7_plan2) / len(x7_plan2)
        mx8 = sum(x8_plan2) / len(x8_plan2)
        mx9 = sum(x9_plan2) / len(x9_plan2)
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        mx10 = sum(x10_plan2) / len(x10_plan2)
        my = sum(y_avg) / len(y_avg)

        a1 = sum([y_avg[i] * x1_plan2[i] for i in range(len(x1_plan2))]) /
len(x1_plan2)
        a11 = mx8
        a12 = mx4
        a13 = mx5
        a14 = sum([x1_plan2[i] * x4_plan2[i] for i in range(len(x1_plan2))])
/ len(x1_plan2)
        a15 = sum([x1_plan2[i] * x5_plan2[i] for i in range(len(x1_plan2))])
/ len(x1_plan2)
        a16 = sum([x1_plan2[i] * x6_plan2[i] for i in range(len(x1_plan2))])
/ len(x1_plan2)
        a17 = sum([x1_plan2[i] * x7_plan2[i] for i in range(len(x1_plan2))])
/ len(x1_plan2)
        a18 = sum([x1_plan2[i] * x8_plan2[i] for i in range(len(x1_plan2))])
/ len(x1_plan2)
        a19 = sum([x1_plan2[i] * x9_plan2[i] for i in range(len(x1_plan2))])
/ len(x1_plan2)

        a2 = sum([y_avg[i] * x2_plan2[i] for i in range(len(x1_plan2))]) /
len(x2_plan2)
        a21 = a12
        a22 = mx9
        a23 = mx6
        a24 = sum([x2_plan2[i] * x4_plan2[i] for i in range(len(x2_plan2))])
/ len(x2_plan2)
        a25 = sum([x2_plan2[i] * x5_plan2[i] for i in range(len(x2_plan2))])
/ len(x2_plan2)
        a26 = sum([x2_plan2[i] * x6_plan2[i] for i in range(len(x2_plan2))])
/ len(x2_plan2)
        a27 = sum([x2_plan2[i] * x7_plan2[i] for i in range(len(x2_plan2))])
/ len(x2_plan2)
        a28 = sum([x2_plan2[i] * x8_plan2[i] for i in range(len(x2_plan2))])
/ len(x2_plan2)
        a29 = sum([x2_plan2[i] * x9_plan2[i] for i in range(len(x2_plan2))])
/ len(x2_plan2)

        a3 = sum([y_avg[i] * x3_plan2[i] for i in range(len(x3_plan2))]) /
len(x3_plan2)
        a31 = a13
        a32 = a23
        a33 = mx10
        a34 = sum([x3_plan2[i] * x4_plan2[i] for i in range(len(x3_plan2))])
/ len(x3_plan2)
        a35 = sum([x3_plan2[i] * x5_plan2[i] for i in range(len(x3_plan2))])
/ len(x3_plan2)
        a36 = sum([x3_plan2[i] * x6_plan2[i] for i in range(len(x3_plan2))])
/ len(x3_plan2)
        a37 = sum([x3_plan2[i] * x7_plan2[i] for i in range(len(x3_plan2))])
/ len(x3_plan2)
        a38 = sum([x3_plan2[i] * x8_plan2[i] for i in range(len(x3_plan2))])
/ len(x3_plan2)
        a39 = sum([x3_plan2[i] * x9_plan2[i] for i in range(len(x3_plan2))])
/ len(x3 plan2)

        a4 = sum([y_avg[i] * x4_plan2[i] for i in range(len(x4_plan2))]) /
len(x4_plan2)
        a41 = a14
        a42 = a24
        a43 = a34
        a44 = sum([x4_plan2[i] ** 2 for i in range(len(x4_plan2))]) /
len(x4_plan2)
```

```
        a45 = sum([x4_plan2[i] * x5_plan2[i] for i in range(len(x4_plan2))])
/ len(x4_plan2)
        a46 = sum([x4_plan2[i] * x6_plan2[i] for i in range(len(x4_plan2))])
/ len(x4_plan2)
        a47 = sum([x4_plan2[i] * x7_plan2[i] for i in range(len(x4_plan2))])
/ len(x4_plan2)
        a48 = sum([x4_plan2[i] * x8_plan2[i] for i in range(len(x4_plan2))])
/ len(x4_plan2)
        a49 = sum([x4_plan2[i] * x9_plan2[i] for i in range(len(x4_plan2))])
/ len(x4_plan2)

        a5 = sum([y_avg[i] * x5_plan2[i] for i in range(len(x5_plan2))]) /
len(x5_plan2)
        a51 = a15
        a52 = a25
        a53 = a35
        a54 = a45
        a55 = sum([x5_plan2[i] ** 2 for i in range(len(x5_plan2))]) /
len(x5_plan2)
        a56 = sum([x5_plan2[i] * x6_plan2[i] for i in range(len(x5_plan2))])
/ len(x5_plan2)
        a57 = sum([x5_plan2[i] * x7_plan2[i] for i in range(len(x5_plan2))])
/ len(x5_plan2)
        a58 = sum([x5_plan2[i] * x8_plan2[i] for i in range(len(x5_plan2))])
/ len(x5_plan2)
        a59 = sum([x5_plan2[i] * x9_plan2[i] for i in range(len(x5_plan2))])
/ len(x5_plan2)

        a6 = sum([y_avg[i] * x6_plan2[i] for i in range(len(x6_plan2))]) /
len(x6_plan2)
        a61 = a16
        a62 = a26
        a63 = a36
        a64 = a46
        a65 = a56
        a66 = sum([x6_plan2[i] ** 2 for i in range(len(x6_plan2))]) /
len(x6_plan2)
        a67 = sum([x6_plan2[i] * x7_plan2[i] for i in range(len(x6_plan2))])
/ len(x6_plan2)
        a68 = sum([x6_plan2[i] * x8_plan2[i] for i in range(len(x6_plan2))])
/ len(x6_plan2)
        a69 = sum([x6_plan2[i] * x9_plan2[i] for i in range(len(x6_plan2))])
/ len(x6_plan2)

        a7 = sum([y_avg[i] * x7_plan2[i] for i in range(len(x7_plan2))]) /
len(x7_plan2)
        a71 = a17
        a72 = a27
        a73 = a37
        a74 = a47
        a75 = a57
        a76 = a67
        a77 = sum([x7_plan2[i] ** 2 for i in range(len(x7_plan2))]) /
len(x7_plan2)
        a78 = sum([x7_plan2[i] * x8_plan2[i] for i in range(len(x7_plan2))])
/ len(x7 plan2)
        a79 = sum([x7_plan2[i] * x9_plan2[i] for i in range(len(x7_plan2))])
/ len(x7_plan2)

        a8 = sum([y_avg[i] * x8_plan2[i] for i in range(len(x8_plan2))]) /
len(x8_plan2)
        a81 = a18
        a82 = a28
        a83 = a38
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        a84 = a48
        a85 = a58
        a86 = a68
        a87 = a78
        a88 = sum([x8_plan2[i] ** 2 for i in range(len(x8_plan2))]) /
len(x8_plan2)
        a89 = sum([x8_plan2[i] * x9_plan2[i] for i in range(len(x8_plan2))])
/ len(x8_plan2)

        a9 = sum([y_avg[i] * x9_plan2[i] for i in range(len(x9_plan2))]) /
len(x9_plan2)
        a91 = a19
        a92 = a29
        a93 = a39
        a94 = a49
        a95 = a59
        a96 = a69
        a97 = a79
        a98 = a89
        a99 = sum([x9_plan2[i] ** 2 for i in range(len(x9_plan2))]) /
len(x9_plan2)

        a10 = sum([y_avg[i] * x10_plan2[i] for i in range(len(x10_plan2))]) /
len(x10_plan2)
        a101 = sum([x10_plan2[i] * x1_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a102 = sum([x10_plan2[i] * x2_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a103 = sum([x10_plan2[i] * x3_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a104 = sum([x10_plan2[i] * x4_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a105 = sum([x10_plan2[i] * x5_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a106 = sum([x10_plan2[i] * x6_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a107 = sum([x10_plan2[i] * x7_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a108 = sum([x10_plan2[i] * x8_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a109 = sum([x10_plan2[i] * x9_plan2[i] for i in
range(len(x10_plan2))]) / len(x10_plan2)
        a1010 = sum([x10_plan2[i] ** 2 for i in range(len(x10_plan2))]) /
len(x10_plan2)

        main_determinant = [[1, mx1, mx2, mx3, mx4, mx5, mx6, mx7, mx8, mx9,
mx10],
                            [mx1, a11, a21, a31, a41, a51, a61, a71, a81,
a91, a101],
                            [mx2, a12, a22, a32, a42, a52, a62, a72, a82,
a92, a102],
                            [mx3, a13, a23, a33, a43, a53, a63, a73, a83,
a93, a103],
                            [mx4, a14, a24, a34, a44, a54, a64, a74, a84,
a94, a104],
                            [mx5, a15, a25, a35, a45, a55, a65, a75, a85,
a95, a105],
                            [mx6, a16, a26, a36, a46, a56, a66, a76, a86,
a96, a106],
                            [mx7, a17, a27, a37, a47, a57, a67, a77, a87,
a97, a107],
                            [mx8, a18, a28, a38, a48, a58, a68, a78, a88,
a98, a108],
                            [mx9, a19, a29, a39, a49, a59, a69, a79, a89,
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a99, a109],
                            [mx10, a101, a102, a103, a104, a105, a106, a107,
a108, a109, a1010]]

        column_to_change = [my, a1, a2, a3, a4, a5, a6, a7, a8, a9, a10]
        main_determinant_value = det(main_determinant)

        matrices = []
        for i in range(len(main_determinant[0])):
            new_matrix = deepcopy(main_determinant)
            for j in range(len(main_determinant)):
                new_matrix[j][i] = column_to_change[j]
            matrices.append(new_matrix)

        b_list = []
        for i in range(len(matrices)):
            b_list.append(det(matrices[i]) / main_determinant_value)
        print('-' * 100)
        print(f'b: {b_list}')

        y_list = []
        for i in range(len(x1_plan2)):
            y = b_list[0] + b_list[1] * x1_plan2[i] + b_list[2] * x2_plan2[i]
+ b_list[3] * x3_plan2[i] +\
                b_list[4] * x4_plan2[i] + b_list[5] * x5_plan2[i] + b_list[6]
* x6_plan2[i] + b_list[7] * x7_plan2[i] +\
                b_list[8] * x8_plan2[i] + b_list[9] * x9_plan2[i] +
b_list[10] * x10_plan2[i]
            y_list.append(y)
            print(f'y = {y}; y avg = {y_avg[i]}')
        print('-' * 100)

        t_arr, s2b = students_test(x0_plan1, x1_plan1, x2_plan1, x3_plan1,
y_avg, dispersion, m)

        b_arr = []
        for i in range(len(b_list)):
            b = b_list[i] if t_arr[i] != 0 else 0
            b_arr.append(b)
        print('-' * 100)

        y_res = []
        for i in range(N):
            y = b_arr[0] + b_arr[1] * x1_plan1[i] + b_arr[2] * x2_plan1[i] +
b_arr[3] * x3_plan1[i] +\
                b_arr[4] * x1_plan1[i] * x2_plan1[i] + b_arr[5] * x1_plan1[i]
* x3_plan1[i] +\
                b_arr[6] * x2_plan1[i] * x3_plan1[i] + b_arr[7] * x1_plan1[i]
* x2_plan1[i] * x3_plan1[i] +\
                b_arr[8] * x1_plan1[i] ** 2 + b_arr[9] * x2_plan1[i] ** 2 +
b_arr[10] * x3_plan1[i] ** 2
            print(f'ŷ = {y}')
            y_res.append(y)

        fishers_test(b_arr, s2b, y_avg, y_res, m)
    else:
        main(m+1)
        exit()


if __name__ == '__main__':
    main()
```
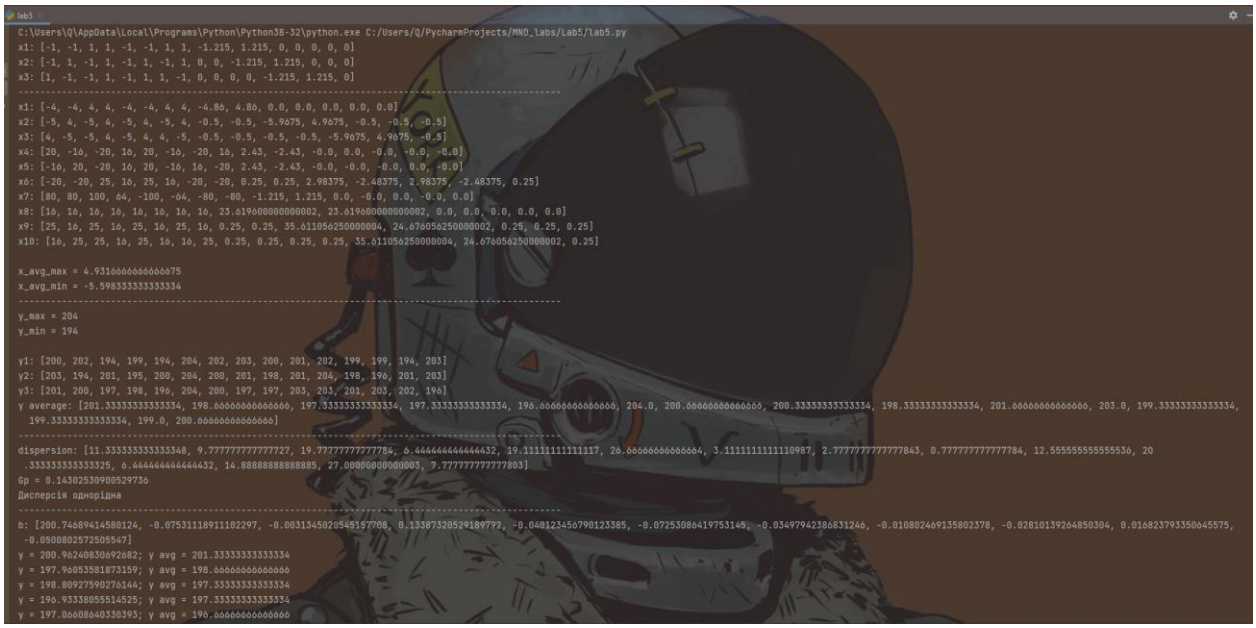
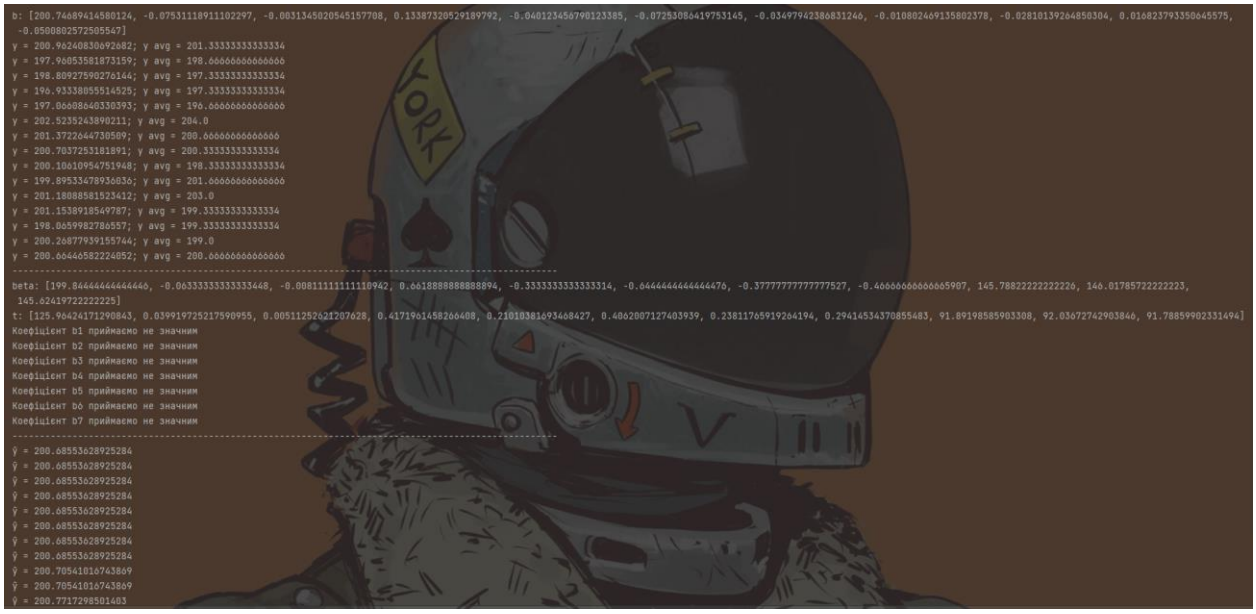
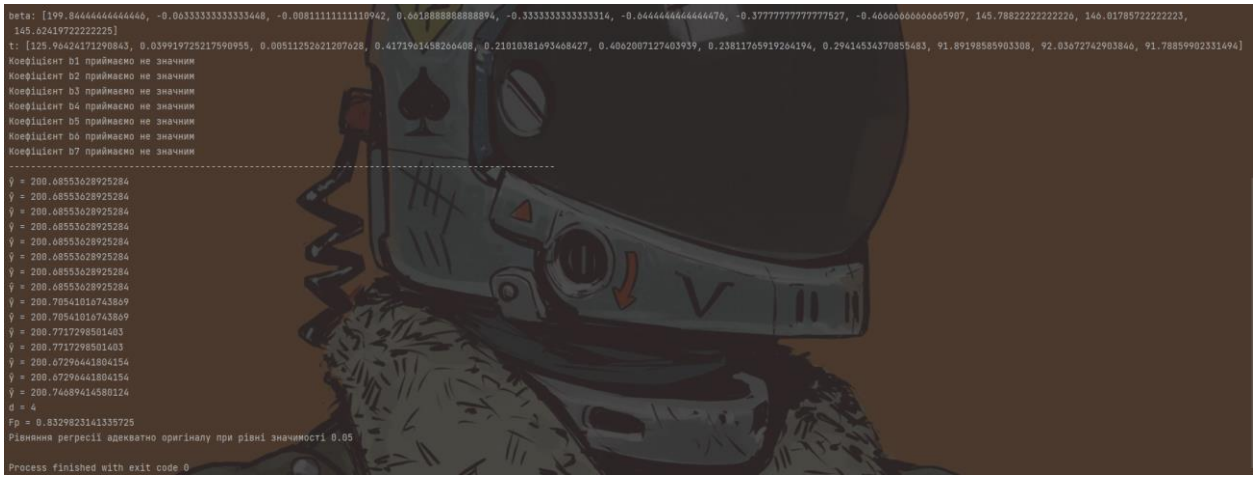

Результати роботи програми







Висновок:

У ході лабораторної роботи я змоделював трьохфакторний експеримент з урахуванням квадратичних членів ,використовуючи центральний ортогональний композиційний план, провів 3 статистичні перевірки, відкинувши незначні коефіцієнти рівняння регресії і отримав рівняння регресії, яке адекватне оригіналу при заданому рівні значимості для опису об'єкту.