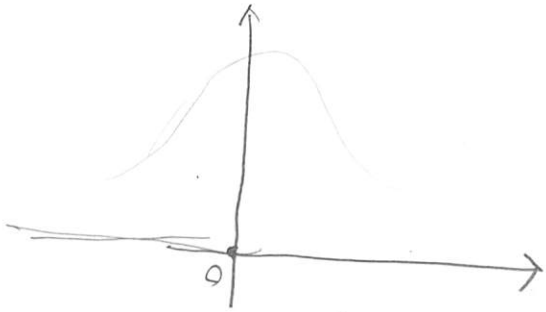
問8 H23

(1)

$$\begin{cases} W = X \\ Z = X - Y \end{cases}$$ で変数変換. $X(W,Z)$, $Y(W,Z)$



$$\begin{cases} X = W \\ Y = W - Z \end{cases}$$

さらにAbs

$$f_{W,Z}(w,z) = f_{X,Y}(w, w-z)\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = +f(w)f(w-z)$$

$$g(z) = f_Z(z) = \int_{-\infty}^{\infty} f_{W,Z}(w,z)\,dw = +\int_{-\infty}^{\infty} f(w)f(w-z)\,dw = +\int_{-\infty}^{\infty} f(y+z)f(y)\,dy \quad //$$

$$g(-z) = \int_{-\infty}^{\infty} f(t-z)f(t)\,dt \underset{\uparrow}{=} \int_{-\infty}^{\infty} f(y)f(y+z)\,dy = g(z) \quad //$$

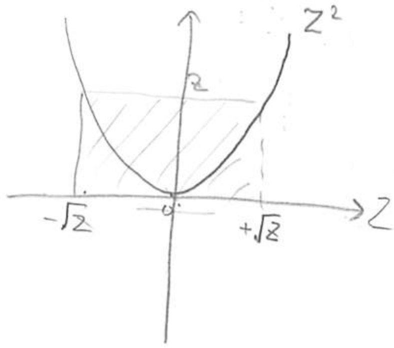
$t-z=y$

(2)

$$\mathrm{Cov}(Z,Z^2) = E(Z^3) - E(Z)E(Z^2)$$

$$= \underbrace{\int_{-\infty}^{\infty} z^3 g(z)\,dz}_{奇\ =\ 0} - \underbrace{\int_{-\infty}^{\infty} z g(z)\,dz}_{奇\ =\ 0}\int_{-\infty}^{\infty} z^2 g(z)\,dz = 0.$$

(3) Z と Z^2 は独立でない $\Longleftrightarrow f_{Z,Z^2} \neq f_Z \cdot f_{Z^2}$



$$F_{Z^2}(z) = P(Z^2 \le z) = \begin{cases} z<0 \Rightarrow P(\phi) = 0 \\ z \ge 0 \Rightarrow P(-\sqrt{z} \le Z \le \sqrt{z}) \quad (\because 連続) \end{cases}$$

$$F_{h(z)} = \begin{cases} 0 & z<0 \\ F_Z(\sqrt{z}) - F_Z(-\sqrt{z}) & z \ge 0. \end{cases}$$

$$f_{Z^2}(z) = F'_{Z^2}(z) = \begin{cases} 0 & z<0 \\ \frac{1}{2\sqrt{z}} g(\sqrt{z}) + \frac{1}{2\sqrt{z}} \underbrace{g(-\sqrt{z})}_{=\,g(\sqrt{z})} \end{cases} = \begin{cases} 0 & z \le 0 \\ \frac{1}{\sqrt{z}} g(\sqrt{z}) \end{cases}$$

($z=0$の点はP-零集合)

$\begin{cases} A = Z \\ B = Z^2 \end{cases}$ で → Z 一対一じゃないからムリ

$$E(e^{sZ+tZ^2}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{sz+ty^2} P(Z \le z,\ Z^2 \le y)\,dz\,dy$$

(3) ZとZ^2が独立と仮定すると.

$F_Z(x)F_{Z^2}(y) = F_{Z,Z^2}(x,y) \quad (\forall x, y)$

$P(Z \leq x)P(Z^2 \leq y) = P(Z \leq x,\ Z^2 \leq y)$

~~$y \geq 0$ とし~~ $y > 0$ として

$P(Z \leq x)\underbrace{P(-\sqrt{y} \leq Z \leq \sqrt{y})}_{= 2P(0 \leq Z \leq \sqrt{y})} = P(Z \leq \underset{= -\sqrt{y}}{x},\ -\sqrt{y} \leq Z \leq \sqrt{y})$

$2P(Z \leq x)P(0 \leq Z \leq 5) = P(Z \leq x, -5 \leq Z \leq 5)$

$x = -5$

$P(Z \leq -\sqrt{y})P(0 \leq Z \leq \sqrt{y}) = 0 \quad (\forall y > 0)$

$P(Z \leq +\infty) = 1$より、$\exists k > 0,\ P(Z \leq k) > 0$ $P(Z$

$\sqrt{y} = k-1$などとれば、

$P(Z \leq -\sqrt{y}) = P(Z \leq -k+1) \geq P(Z \leq -k) > 0$

$P(0 \leq Z \leq \sqrt{y}) = P(0 \leq Z \leq k-1) > 0$

よりムジュン.