

H20問7.

3行=2列へ導く清め.

(1)

$$\min 2x_1 + 3x_2 + 8x_3$$

$$\text{s.t. } \begin{aligned} 2x_1 + 2x_2 + 6x_3 - x_4 &= 6 \\ x_1 + 2x_2 + 4x_3 - x_5 &= 4 \end{aligned}$$

$$x_i \geq 0$$

★

$$\max -2x_1 - 3x_2 - 8x_3$$

$$\text{s.t. } \begin{aligned} 2x_1 + 2x_2 + 6x_3 - x_4 &\leq 6 \\ -2x_1 - 2x_2 - 6x_3 + x_4 &\leq -6 \end{aligned} \quad \text{対偶問題}$$

$$\min 2x_1 + 3x_2 + 8x_3$$

$$\text{s.t. } \begin{aligned} 2x_1 + 2x_2 + 6x_3 - x_4 &\leq 6 \\ -2x_1 - 2x_2 - 6x_3 + x_4 &\leq -6 \\ x_1 + 2x_2 + 4x_3 - x_5 &\leq 4 \\ -x_1 - 2x_2 - 4x_3 + x_5 &\leq -4 \end{aligned}$$

$$x_i \geq 0$$

(P) $\max 6y_1 + 4y_2$

$$\text{s.t. } 2y_1 + y_2 \geq 2$$

$$2y_1 + 2y_2 \geq 3$$

$$6y_1 + 4y_2 \geq 8$$

(D) $\max 6y_1 - 6y_2 + 4y_3 - 4y_4$

$$\text{s.t. } 2y_1 - 2y_2 + y_3 - y_4 \geq 2$$

$$2y_1 - 2y_2 + 2y_3 - 2y_4 \geq 3$$

$$6y_1 - 6y_2 + 4y_3 - 4y_4 \geq 8$$

$$-y_1 + y_2 \geq 0$$

$$-y_3 + y_4 \geq 0$$

$$y_i \geq 0$$

X

$$\min 2x_1 + 3x_2 + 8x_3$$

s.t.

$$x_B = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad x_N = \begin{pmatrix} x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} \quad N = \begin{pmatrix} 6 & -1 & 0 \\ 4 & 0 & -1 \end{pmatrix}$$

$$b = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \quad C_B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad C_N = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix}$$

$$\min C_B^T x_B + C_N^T x_N$$

$$s.t. \quad Bx_B + Nx_N = b$$

x_1, x_2, x_3

$$x_B + B^{-1}Nx_N = B^{-1}b, \quad x_B = B^{-1}b - B^{-1}Nx_N$$

$$C_B^T (B^{-1}b - B^{-1}Nx_N) + C_N^T x_N$$

$$= C_B^T B^{-1}b - C_B^T B^{-1}Nx_N + C_N^T x_N$$

$$= \underbrace{C_B^T B^{-1}b}_{\text{const}} + \underbrace{(C_N^T - C_B^T B^{-1}N)}_{-4} x_N$$

$$B^{-1}b = \frac{1}{4-2} \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \geq 0 \text{ 可行}$$

$$(8 \ 0 \ 0) - \left| (2 \ 3) \frac{1}{2} \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \right| \begin{pmatrix} 6 & -1 & 0 \\ 4 & 0 & -1 \end{pmatrix}$$

$$= \left| \frac{1}{2} (1 \ 2) \right|$$

$$= (8 \ 0 \ 0) - \frac{1}{2} \begin{pmatrix} 14 & -1 & -2 \end{pmatrix}$$

$$= (1 \ \frac{1}{2} \ 1) \geq 0 \text{ 可行, 最小}$$

(2)

\Leftrightarrow ~~Base~~ Non Base ~~feasible~~ \Leftrightarrow Opt D.

$$B^{-1}(b+\Delta b) = \frac{1}{2} \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 4+2s \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2+s \end{pmatrix} = \begin{pmatrix} 6-4-2s \\ -3+4+2s \end{pmatrix}$$

$$= \begin{pmatrix} 2-2s \\ 1+2s \end{pmatrix} \geq 0$$

$$1 \geq s$$

$$s \geq -\frac{1}{2}$$

$$\underline{-\frac{1}{2} \leq s \leq 1}$$