H23 問6

$$(\sqrt{x^2+y^2})' = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{x}{r}$$

$$\left(\frac{x}{r}\right)' =$$

(1)

$$u_x = (\sqrt{x^2+y^2})' f'(r) = \frac{x}{r} f'(r)$$

$$u_{xx} = \left(\frac{x}{r}\right)' f'(r) + \left(\frac{x}{r}\right) f''(r) \cdot \text{(中のビブン)}$$

〃 $\frac{x}{r}$

$$\left(\frac{x}{r}\right)' = \frac{x'r - xr'}{r^2} = \frac{1}{r^2}\left\{r - x\cdot\frac{x}{r}\right\} = \frac{1}{r} - \frac{x^2}{r^3}$$ より、

$$u_{xx} = \left(\frac{1}{r} - \frac{x^2}{r^3}\right) f'(r) + \frac{x^2}{r^2} f''(r)$$

同様に、

$$u_{yy} = \left(\frac{1}{r} - \frac{y^2}{r^3}\right) f'(r) + \frac{y^2}{r^2} f''(r)$$

$$u_{xx} + u_{yy} = \frac{2}{r}f\cancel{(r} \ \frac{1}{r^3} f'(r) \underbrace{(x^2+y^2)}_{r^2} + \frac{1}{r^2} f''(r) \underbrace{(x^2+y^2)}_{r^2}$$

$$= f''(r) + \frac{2}{r} f'(r) - \frac{1}{r} f'(r) = f''(r) + \frac{1}{r} f'(r) \ //$$

(2) $h(r) = g'(r)$, $-h' - \frac{1}{r}h = 1 \rightarrow -rh' - h = r$, $rh' + h = -r$

$(rh)' = -r$

$rh = \frac{1}{2}r^2 + \underset{0}{C_1}$ r>0だから×

× $\begin{cases} h = \frac{1}{2}r \\ g = \frac{1}{4}r^2 + C_1 \end{cases}$

× $\begin{cases} g(1 \quad r=0 \Rightarrow 0 = C_1 \\ g(1) = \frac{1}{4} + C_1 \quad \therefore C_1 = -\frac{1}{4} \\ \therefore g(r) = \frac{1}{4}r^2 - \frac{1}{4} \end{cases}$ //

$rh = \frac{1}{2}r^2 + C$, $h(r) = g'(r) = -\frac{1}{2}r + \frac{C}{r}$

$$g(r) = -\frac{1}{4}r^2 + C\log r + D$$

$$g(1) = -\frac{1}{4} + D = 0$$

$$\therefore g(r) = \frac{1}{4}r^2 + C\log r + \frac{1}{4} \ //$$

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