問9 H23. $P_{11} = P(A_t=1, B_t=1) = P_{11}$

(1) ~~$P_{00} = P(A_t=0, B_t=0) =$~~ $P_{00}+P_{11}+P_{01}+P_{10}=1$

周辺分布.

$P(A_t=a) = P(A_t=a, B_t=0) + P(A_t=a, B_t=1)$ $(\forall a)$ — ①

$P(B_t=b) = P(A_t=0, B_t=b) + P(A_t=1, B_t=b)$ $(\forall b)$ — ②

① で $a=1$ より.

$\frac{1}{2} = P_{10} + \theta$ $\quad P_{10} = \frac{1}{2} - \theta$

② で $b=1$ より. $\longrightarrow$ $P_{01} = \frac{1}{2} - \theta$

$\frac{1}{2} = P_{01} + \theta$ $\quad P_{00} = 1 - (P_{01}+P_{10}) - \theta = \theta$

③ $\theta + P_{00} + P_{01} + P_{10} = 1$

(2) $P(A_t=B_t) = P_{00}+P_{11} = 2\theta$

$L(\theta) = \binom{n}{m}(2\theta)^m(1-2\theta)^{n-m}$

$l'(\theta) = m\cdot\frac{2^1}{2\theta} + (n-m)\frac{-2}{1-2\theta} = 0$, $m - 2\theta m - 2\theta(n-m) = 0$,

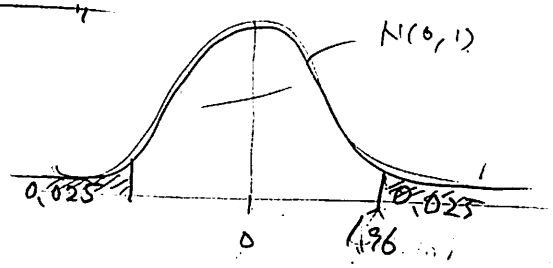
$-2\theta(m+n-m) = -m$, $\hat{\theta} = \frac{-m}{-2n} = \frac{m}{2n}$,

(3) $\theta = \frac{1}{4}$ のとき.

$\therefore B(n,\theta) \sim (A_t, B_t) \cong Z$

m

n が十分大きいので



$$P\left(\left|\frac{Z - n\cdot 2\theta}{\sqrt{n\cdot 2\theta(1-2\theta)}}\right| \le 1.96\right) \simeq \Phi(1.96) = 0.975$$

$\frac{1.96}{2} = 98$

m $\quad$ m

$\leftarrow \frac{Z - n\cdot\frac{1}{2}}{\sqrt{n\cdot\frac{1}{2}\cdot\frac{1}{2}}} = \frac{Z - 10^3\cdot 5}{10^2\cdot\frac{1}{2}} \rightarrow$ $|Z - 10^3\cdot 5| \le 98$

$|Z| \le 98$

$\left|m - \frac{n}{2}\right| \le 1.96 \times 10^2 \cdot \frac{1}{2} = 98$