H30-4

(1)

初期辞書

$$Z = 6x_1 - 3x_2 + 4x_3$$
$$x_4 = 5 + 3x_1 - 2x_2 + 4x_3$$
$$x_5 = 2 - 2x_1 - 2x_2 - x_3$$
$$x_6 = 5 - 4x_1 - 2x_2 - 3x_3$$

最適辞書

$$Z = 7 - 7x_2 - x_5 - x_6$$
$$x_1 = \frac{1}{2} - 2x_2 - \frac{3}{2}x_5 + \frac{1}{2}x_6$$
$$x_3 = 1 + 2x_2 + 2x_5 - x_6$$
$$x_4 = \frac{21}{2} \qquad + \frac{7}{2}x_5 - \frac{5}{2}x_6$$

$$Z^* = 7, \quad \left(\frac{1}{2}, 0, 1\right)$$

(2)

$$b = \begin{pmatrix} 5 \\ 2 \\ 5 \end{pmatrix} \quad x_B = \begin{pmatrix} x_1 \\ x_3 \\ x_4 \end{pmatrix}, \quad x_N = \begin{pmatrix} x_2 \\ x_5 \\ x_6 \end{pmatrix}, \quad C_B = \begin{pmatrix} 6 \\ 4+\theta \\ 0 \end{pmatrix} \quad C_N = \begin{pmatrix} -3+\theta \\ 0 \\ 0 \end{pmatrix}$$

$$B = \begin{pmatrix} -3 & -4 & 1 \\ 2 & 1 & 0 \\ 4 & 3 & 0 \end{pmatrix}, \quad N = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

とすると、最適辞書

$$Z = C_B^T B^{-1} b + (C_N^T - C_B^T B^{-1} N) x_N$$
$$x_B = B^{-1} b - B^{-1} N x_N$$

題意 $\iff B^{-1} b \geq 0$ かつ $C_N^T - C_B^T B^{-1} N \leq 0$

$$\iff -1 \leq \theta \leq \frac{1}{2}$$

(3)

$$Z^*(\theta) = 6x_1^*(0) + (-3+\theta)x_2^*(0) + (4+\theta)x_3^*(0)$$
$$= 7 + \theta$$