H14問8

$\overline{H}(x,y) = \min\{F(x), G(y)\}$, $\underline{H}(x,y) = \max\{0, F(x)+G(y)-1\}$

(1)

$\cancel{P(\quad)} = P(F^{-1}(U) \le x, G^{-1}(U) \le y)$

$= P(U \le F(x),\ U \le G(y))$

$= P(U \le \min(F(x), G(y)))$

$= \min(F(x), G(y))$

$= \overline{H}(x,y)$

$P(F^{-1}(U) \le x, G^{-1}(1-U) \le y) = P(U \le F(x),\ 1-U \le G(y))$

$1-G(y) \le U)$

$\times = P(\{1-G(y) \le U \le F(x)\} \cup \underbrace{\{F(x) \le 1-G(y)\}}_{\phi})$

$\cancel{= P(U \le F(x),\ 1-G(y) \le U) =}$

$\times \begin{cases} = P(1-G(y) \le U \le F(x)) \\ = F(x) - (1-G(y)) = F(x)+G(y)-1 \end{cases}$

$\cancel{= P(1-G(y) \le U \le F(x)) + P(F(x) \le 1-G(y))}$

$= \begin{cases} \cancel{P(1-G(y)} \quad F(x)+G(y)-1 & \text{if } 1-G(y) \le F(x) \\ 0 & \text{o.w.} \end{cases}$

$= \max(F(x)+G(y)-1,\ 0)$

(2)

$\underline{H}(x,y) \le H(x,y) \le \overline{H}(x,y)$

$\max\{0, F(x)+G(y)-1\} \le$

$\cancel{P(X\ P(F^{-1}(U) \le x, G^{-1}(1-U)) \le P(}$

• $F(x) = \cancel{\lim}\ H(x,\infty)$ $\quad$ $H(x,\infty),\ H(y,\infty) \ge H(x,y)$

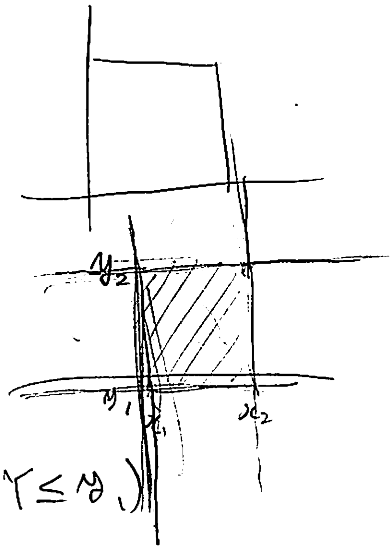
~~また~~ $G(y) = H(\infty, y)$ より.

$\max\{0, H(x,\infty) + H(\infty,y) - 1\} \le H(x,y)$

$0 \quad H(x,\infty)+$

$H(x,y) - H(x,\infty) - H(\infty,y) + 1$

$\cancel{= 1 - H(\infty,y) \ge 0, \quad H(x,y) - H} \quad \cancel{P(}$



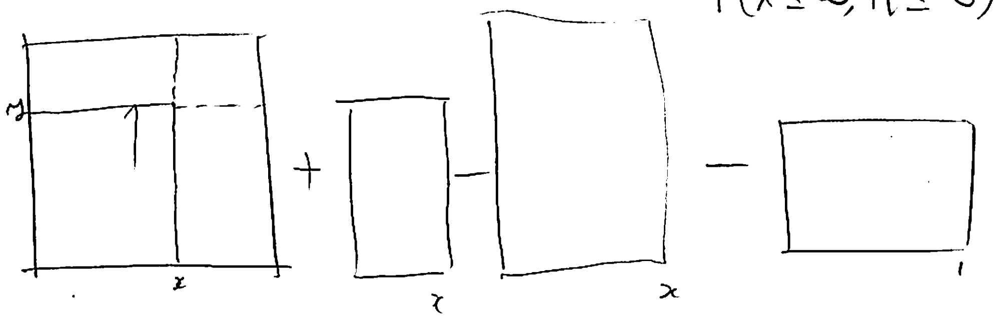
· $P(x_1 \le X \le x_2, y_1 \le Y \le y_2)$

$\parallel$

$P(X \le x_2) + P(Y \le y_2) - P(X \le x_1) - P(Y \le y_1)$

$= H(x,y) - P$

· $P_{XY}(X \le x, Y \le y) - P(X \le x) - P(Y \le y) + \underbrace{1}_{P(X \le \infty,\ Y \le \infty)}$

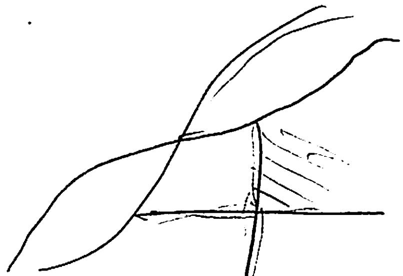


$\overline{H}(x,y) - H(x,y) = \min\{P(X \le x), P(Y \le y)\} - P(X \le x, Y \le y)$

$$P(X \le x),\ P(Y \le y)$$

$$\overset{\shortparallel}{P(X \le x, Y \le \infty)},\ P(\cancel{X \le \infty}), P(Y \le y) \ge P(X \le x, Y \le y)$$

(3) $\mathrm{Cov}_{\underline{H}}(X,Y) = E_{\underline{H}}(X,Y) - E_{\underline{H}}(X)E_{\underline{H}}(Y)$



$$E_H(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{P_H(\cancel{X} > x, Y > y)}_{\shortparallel}\, dx\,dy$$

$$1 + \underbrace{P(X \le x, \cancel{Y} \le y)}_{\underline{H}(x,y)} - \underline{P(X \le x)} - P(Y \le y)$$

$$E_{\underline{H}}(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$

$$E_{\underline{H}}(XY) = \int_{\cancel{-\infty}\ 0}^{\infty}\int_0^{\infty} \underbrace{P(F^{-1}(U) > x, G^{-1}(U) > y)}_{\shortparallel}\, dx\,dy \quad \cancel{= E_{U,U}}\ E$$

$$P(U > F(x), U > G(y))$$

~~E~~

$\mathrm{Cov}_H(X,Y) = E_H(XY) - E_H(X)E_H(Y)$ 同じ $\quad E_H(X) = \int_0^\infty \underline{P(\cancel{x > x}\ X > x)}\, dx$

$$= \int_0^\infty\int_0^\infty 1 + H(x,y) - F(x) - G(y)\, dx\,dy \ - \ ''$$

$$\cancel{\text{for}}\ {}^{?}E_{\underline{H}}(X,Y) = \int_0^\infty\int_0^\infty 1 + \underline{H}(x,y) - F(x) - G(y)\, dx\,dy$$

~~##~~ $\underline{F}(x) = \underline{H}(x,\infty) = \min\{F(x), \underset{1}{\underline{G(\infty)}}\} = F(x)$

$\underline{G}(y) = G(y)$

$\overline{F}(x) = \overline{H}(x,\infty) = \max(0, F(x) + \underset{0}{\underline{G(\infty) - 1}}) = F(x)$

$\overline{G}(y) = G(y)$

$\therefore$ $\underline{\mathrm{Cov}_H}$ $\quad \underline{\mathrm{Cov}} \le \mathrm{Cov} \le \overline{\mathrm{Cov}} \iff \underline{H} \le H \le \overline{H}$