H21問4.

(1) $W(y,t) \triangleq u(y+ct, t)$ $\begin{cases} x = y + ct \\ \cancel{z} = t \end{cases}$ 妻 $\dot{y} = -c$

$W_y = \frac{\partial u}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial t} = u_x + u_t$

~~$W_{yy} = \partial$~~

$u_t = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial t} = W_y \cdot (-c) + W_t$

chain rule あやしい

$u_x = \frac{\partial w}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial w}{\partial t}\frac{\partial t}{\partial x} = W_y + W_t \cdot 0 = W_y$

$u_{xx} = \frac{\partial W_y}{\partial y}\frac{\partial y}{\partial x} + \partial\, 0 = W_{yy}$

$\therefore \cancel{-cW_y} + W_t = W_{yy} - \cancel{cW_y} \longrightarrow W_{yy} = W_t$ //」

$u(y,0) \equiv W(y,0) = \sin y + \cos 2y$ //」

(2) 無限区間の熱方程式.

形式解 ~~$u(t,x)$~~ $W(y,t) = \int_{-\infty}^{\infty} G(t, x-y)a(y)\,dy$ , $G(t,x) = \frac{1}{\sqrt{4\pi t}} \exp\left(-\frac{x^2}{4t}\right)$

$a(y) = \sin y + \cos 2y$ .

$W(y,t) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi t}} \exp\left(-\frac{(y-s)^2}{4t}\right)(\sin s + \cos 2s)\,ds$

WL 一般に $\int_{-\infty}^{\infty} e^{-a(x-b)^2} \sin cx\,dx = \sqrt{\frac{\pi}{a}} e^{-\frac{c^2}{4a}} \sin bc$ $(a, b > 0)$

(cos) (cos)

より、

$W(y,t) = \frac{1}{\sqrt{4\pi t}} \cdot \left(\sqrt{4\pi t}\, e^{-t} \sin y + \sqrt{4\pi t}\, e^{-4t} \sin 2y\right)$

cos.

$= e^{-t}\sin y + e^{-4t}\sin 2y$ //

cos

$\therefore u(x,t) = W(x-ct, t) = e^{-t}\sin(x-ct) + e^{-4t}\sin 2(x-ct)$ //」

cos.