H28-6

(1)

$$u_t = \left(-\frac{1}{2}t^{-\frac{3}{2}}\right)v\left(\frac{x}{\sqrt{t}}\right) + t^{-\frac{1}{2}}\cdot\left(-\frac{x}{2}t^{-\frac{3}{2}}\right)v'\left(\frac{x}{\sqrt{t}}\right) = t^{-\frac{3}{2}}\left(-\frac{1}{2}v\left(\frac{x}{\sqrt{t}}\right) - \frac{x}{2}t^{-\frac{1}{2}}v'\left(\frac{x}{\sqrt{t}}\right)\right)$$

$$u_x = t^{-1}v'\left(\frac{x}{\sqrt{t}}\right)$$

$$u_{xx} = t^{-\frac{3}{2}}v''\left(\frac{x}{\sqrt{t}}\right)$$

$u_{xx} = u_t$ を $t^{-\frac{3}{2}} > 0$ で割って、$y = \frac{x}{\sqrt{t}}$ で置き換えると、

$$v''(y) = -\frac{1}{2}v(y) - \frac{1}{2}yv'(y)$$

$$= -\frac{1}{2}\left(yv(y)\right)'$$

従って、

$$\frac{1}{2}\left(yv(y)\right)' + v''(y) = 0$$

(2) (1)より、

$$\frac{1}{2}yv(y) + v'(y) = C \qquad (C：積分定数)$$

両辺 $|y| \to \infty$ とすると、

$$C = 0 \qquad \left(\because \lim_{|x|\to\infty} x u(x,t) = 0 \text{ より、} \lim_{|y|\to\infty} yv(y) = 0 \quad \lim_{|x|\to\infty} u_x(x,t) = 0 \text{ より、} \lim_{|y|\to\infty} v'(y) = 0\right)$$

従って、

$$-\frac{1}{2}y = \frac{v'(y)}{v(y)}$$

$$v(y) = De^{-\frac{y^2}{4}} \qquad (D：積分定数)$$

$$D = 1 \qquad (\because u(0,1) = 1)$$

従って、

$$u(x,t) = \frac{1}{\sqrt{t}}e^{-\frac{x^2}{4t}}$$