

H22-2

(1)

$$\int \frac{\log x}{(1+x)^3} dx = \int \left(-\frac{1}{2}(1+x)^{-2}\right) \log x dx = -\frac{\log x}{2(1+x)^2} + \int \frac{1}{2x(1+x)^2} dx$$

$$\frac{1}{x(1+x)^2} = \frac{a}{x} + \frac{b}{1+x} + \frac{c}{(1+x)^2} \quad (\forall x) \text{ 在 } \mathbb{R}^+ \text{ 上, } a=1, b=-1, c=-1 \text{ 也})$$

$$\int \frac{\log x}{(1+x)^3} dx = -\frac{\log x}{2(1+x)^2} + \frac{1}{2} \left(\log x - \log(1+x) + \frac{1}{1+x} \right) + \text{Const.}$$

(2)

$$F(x) = \int \frac{\log x}{(1+x)^3} dx \quad x \in \mathbb{R}^+.$$

$$\lim_{x \rightarrow \infty} F(x) = \lim_{x \rightarrow \infty} \left(-\frac{\log x}{2(1+x)^2} + \frac{1}{2} \log \frac{1}{\frac{1}{x}+1} + \frac{1}{2(1+x)} \right) = 0$$

$$\lim_{x \rightarrow 0} \frac{\log x}{(1+x)^2} = \lim_{y \rightarrow -\infty} \frac{y}{(1+e^y)^2} = 0 \text{ 也})$$

$$\lim_{x \rightarrow +0} F(x) = \lim_{x \rightarrow +0} \left(-\frac{\log x}{2(1+x)^2} + \frac{1}{2} \log \frac{x}{x+1} + \frac{1}{2(1+x)} \right) = \frac{1}{2}$$

$$\therefore \textcircled{I} = F(\infty) - F(0) = -\frac{1}{2}$$