H21 問9　$y \geq 1$としているので

$$P(Y=y) = \binom{m}{y}\theta^y(1-\theta)^{m-y}$$

(1)
$$P(Y=y \mid Y \geq 1) = \frac{P(Y=y, Y\geq 1)}{P(Y\geq 1)} = \frac{}{\sum_{y=1}^{m} P(Y=y)}$$

$(1-\theta)^m$

$$= \frac{}{1-P(Y=0)}$$

$$= \frac{P(Y=y)}{1-P(Y=0)} = \frac{\binom{m}{y}\theta^y(1-\theta)^{m-y}}{1-(1-\theta)^m} \quad //$$

こういう$Y\geq 1$データの処理はよくあるの！

(2)
$$L(\theta) = P(Y=y) \to l(\theta) = \log\binom{m}{y} + y\log\theta + (m-y)\log(1-\theta) - \log\{1-(1-\theta)^m\}$$

$$l'(\theta) = y\frac{1}{\theta} - \frac{m-y}{1-\theta} - \frac{m(1-\theta)^{m-1}}{1-(1-\theta)^m} = 0$$

$$\cancel{-m(1-\theta)^{m-1}\cdot(-1)}$$

$$y(1-\theta)\{1-(1-\theta)^m\} - (m-y)\theta\{1-(1-\theta)^m\} - m(1-\theta)^{m-1}\theta(1-\theta) = 0$$

$$y - y(1-\theta)^m \cancel{- y\theta} + \cancel{y\theta(1-\theta)^m}$$
$$-m\theta + \cancel{m\theta(1-\theta)^m} \cancel{+y\theta} - \cancel{y\theta(1-\theta)^m}$$
$$\cancel{-m\theta(1-\theta)^m}$$
$$= y - y(1-\theta)^m - m\theta = 0$$

×

$\prod P(Y=$

$$L(\theta) = \prod_{i=1}^{n} P(X=x_i) = \frac{\left(\prod_{i=1}^{n}\binom{m}{x_i}\right)\theta^{\sum x_i}(1-\theta)^{nm-\sum x_i}}{\{1-\underset{A}{(1-\theta)^m}\}^n}$$

$$l(\theta) = \log\left(\prod\binom{m}{x_i}\right) + n\bar{x}\log\theta + (nm-n\bar{x})\log(1-\theta) - n\log\{1-(1-\theta)^m\}$$

$$l'(\theta) = n\bar{x}\frac{1}{\theta} + (nm-n\bar{x})\frac{-1}{1-\theta} - n\frac{m(1-\theta)^{m-1}}{1-(1-\theta)^m} = 0$$

$$n\bar{x}(1-\theta)\{1-(1-\theta)^m\} - (nm-n\bar{x})\theta\{1-(1-\theta)^m\} - nm\theta(1-\theta)^m = 0$$

$$n\bar{x} - n\bar{x}(1-\theta)^m - \cancel{n\bar{x}\theta} + n\bar{x}\cancel{(1-\theta)^m}$$
$$-nm\theta + nm\theta(1-\theta)^m + \cancel{n\bar{x}\theta} - n\bar{x}\theta\cancel{(1-\theta)^m}$$
$$-nm\theta(1-\theta)^m = 0$$

$$= \frac{\bar{x}}{m}n - \frac{\bar{x}}{m}n(1-\theta)^m - n\theta + \cancel{n\theta(1-\theta)^m} - \cancel{n\theta(1-\theta)^m} = 0$$

$$\frac{\bar{x}}{m} - \frac{\bar{x}}{m}(1-\theta)^m - \theta = 0$$

$$\hat{\theta} = \underbrace{\frac{\bar{x}}{m}\{1-(1-\hat{\theta})^m\}}_{= g(\hat{\theta})}$$

76 100.

2

(3)

$\hat{\theta}_{\cancel{k+1}}$ $\hat{\theta}_1 = \frac{\bar{x}}{m}g(\hat{\theta}_1) = \frac{1}{4}\cdot 2\cdot\{1-(\underbrace{1-\hat{\theta}_0}_{=1})^4\}$

$$\bar{x} = \frac{1\times 28 + 2\times 48 + 3\times 20 + 4\times 4}{\cancel{100}\ 100}$$

$$= \frac{1}{100}\{28+96+60+16\}$$

124 184 200

$$= \frac{1}{100}\cdot 200 = 2_{\text{//}}$$

$$= \frac{1}{2}\{1-0\} = \frac{1}{2}$$

$$\hat{\theta}_2 = \frac{1}{4}\cdot 2\cdot\{1-\underbrace{(1-\hat{\theta}_1)^4}_{=\frac{1}{16}}\}$$

$$= \frac{1}{2}\cdot\frac{15}{16} = \frac{15}{32} \text{ //}$$