

H18問4

$$\begin{aligned}
 (1) \quad \|Tu - Tv\| &= \left\| \frac{1}{a} \int_0^t (u(z) - v(z)) \sin(t-z) dz \right\| \\
 &= \frac{1}{a} \sup_{0 \leq t \leq a} \left| \int_0^t (u(z) - v(z)) \sin(t-z) dz \right| \\
 &\leq \frac{1}{a} \sup_{0 \leq t \leq a} \int_0^t |u(z) - v(z)| \sin(t-z) dz \\
 &\leq \frac{1}{a} \sup_{0 \leq t \leq a} \int_0^t |u(z) - v(z)| dz \\
 &= \frac{1}{a} \int_0^a |u(z) - v(z)| dz \\
 &\leq \frac{1}{a} \int_0^a \|u - v\| dz \\
 &= \frac{1}{a} \cdot \|u - v\| \cdot a \\
 &= \|u - v\|
 \end{aligned}$$

$K < 1$ とはならない。

(2) T は縮小写像で、 $(C[0, a], \|\cdot\|)$ はバナッハ space より、縮小写像の原理より

$$\exists! x \in C[0, a] \text{ such that } T(x) = x$$

$$\begin{aligned}
 T(x) &= 1 + \frac{1}{a} \int_0^t u(z) \sin(t-z) dz = u(t) \\
 \text{c" } \left\{ \begin{aligned} &\frac{1}{a} u(t) \sin(t-t) = 0 = u'(t) \rightarrow u(t) = \text{const} \\ &t=0 \text{ or } a: \quad 1 = u(0) \quad \therefore u(t) = 1 \end{aligned} \right.
 \end{aligned}$$

実際、

$$1 + \frac{1}{a} \int_0^t 1 \cdot \sin(t-z) dz = 1 + \frac{1}{a} \underbrace{\left[\cos(t-z) \right]_0^t}_{1 - \cos t} = 1 + \frac{1}{a} (1 - \cos t) ?$$

一致しない。