H22問9

(1) $P(A\cap B) = P(A|B)P(B) = (1-P(A^c|B))P(B) = (1-q)p$ //

$P(A\cap B^c) = P(A|B^c)P(B^c) = r(1-p)$

$P(A^c\cap B) = P(A^c|B)P(B) = qp$

$P(A^c\cap B^c) = P(A^c|B^c)P(B^c) = (1-r)(1-p)$

(2)

× $P(A) = P(A|B)+P(A|B^c) = (1-q)+r = \frac{M}{N}$ より.

$\cancel{P(A) = \frac{P(A)}{1} = \frac{P(}{P(B)+P(B^c)}}$

$$P(A) = \sum_i P(D_i)P(A|D_i) = P(B)P(A|B) + P(B^c)P(A|B^c)$$

$$= p\cdot(1-q) + (1-p)\cdot r = \frac{M}{N} \text{より.}$$

$$p(1-q-r) = \frac{M}{N} - r \quad \therefore \hat{p} = \frac{1}{1-q-r}\left(\frac{M}{N}-r\right)$$

(3)

$$\binom{N}{M} P(A)^M (1-P(A))^{N-M} = \binom{N}{M}(p-pq+r-pr)^M(1-p+pq-r+pr)^{N-M} = L(p)$$

$$l(p) = \log L(p) = \log\binom{N}{M} + M\log(\quad) + (N-M)\log(\quad)$$

$$l'(p) = M\frac{1-q-r}{p-pq+r-pr} + (N-M)\frac{-1+q+r}{1-p+pq-r+pr} = 0$$

$$\frac{M}{p(1-q-r)+r} = \frac{N-M}{p(-1+q+r)+1-r}$$

$$pM(q+r-1) + M(1-r) = p(N-M)(1-q-r) + (N-M)r$$

$$p\{\cancel{M(q+r-1)} + \underline{M(1-q-r)} - N(1-q-r)\} = \cancel{Mr} - M + Nr - \cancel{Mr}$$

$$-M(q+r-1)$$

$$\therefore p = \frac{-M+Nr}{N(q+r-1)} = \frac{M-rN}{N(1-q-r)} \quad //$$

" $\frac{1}{(1-q-r)}\left(\frac{M}{N}-r\right)$ で(2),