問9H17

(1) $Pr(\mu \in (0, aX)) = P(\mu \leq aX) = P\left(\frac{\mu}{a} \leq X\right) = e^{-\frac{1}{a}} = \beta$ より、$-\frac{1}{a} = \log\beta$, $a = -\frac{1}{\log\beta}$ //

(2)

$Pr\{(\mu_1, \mu_2) \in (0, a_1X_1) \times (0, a_2X_2)\} = Pr\left(\frac{\mu_1}{a_1} \leq X_1, \frac{\mu_2}{a_2} \leq X_2\right) = Pr\left(\frac{\mu_1}{a_1} \leq X_1\right) Pr\left(\frac{\mu_2}{a_2} \leq X_2\right)$

$= e^{-\frac{1}{a_1} - \frac{1}{a_2}} = \beta$ より、

$$\underline{-\frac{1}{a_1} - \frac{1}{a_2} = \log\beta} \ //$$

(3)

$a_1 a_2 X_1 X_2$ の最小化 s.t. $\frac{1}{a_1} + \frac{1}{a_2} = -\log\beta = \log\frac{1}{\beta} > 0$

相加相乗より、

$$-\log\beta \geq 2\sqrt{\frac{1}{a_1}\cdot\frac{1}{a_2}} = \frac{2}{\sqrt{a_1 a_2}}$$

$$\sqrt{a_1 a_2} \geq \frac{2}{\log\frac{1}{\beta}}$$

よって、min 達成は、$\frac{1}{a_1} = \frac{1}{a_2}$, $a_1 = a_2$ のときで、$a_1 a_2 = a_1^2 = \left(\frac{2}{\log\frac{1}{\beta}}\right)^2$

$$\therefore a_1 = a_2 = \frac{2}{\log\frac{1}{\beta}} \ //$$