

H22-9

(1)

$$P(A \cap B) = P(A|B)P(B) = (1 - P(A^c|B))P(B) = (1 - q)p$$

$$P(A \cap B^c) = P(A|B^c)P(B^c) = r(1-p)$$

$$P(A^c \cap B) = P(A^c|B)P(B) = qp$$

$$P(A^c \cap B^c) = P(A^c|B^c)P(B^c) = (1-r)(1-p)$$

(2)

全確率の公式より、

$$P(A) = P(B)P(A|B) + P(B^c)P(A|B^c)$$

$$= p(1-q) + (1-p)r$$

より、

$$\hat{p} = \frac{1}{1-q-r} \left( \frac{M}{N} - r \right)$$

(3) 尤度関数は、 $L(p) \triangleq \binom{N}{M} p^M (1-p)^{N-M}$

$$\frac{\partial \log L}{\partial p} = 0 \text{ より、}$$

$$\hat{p}_{MLE} = \frac{1}{1-q-r} \left( \frac{M}{N} - r \right)$$