H22-4

(1) $\begin{cases} \xi = t + x \\ \eta = t - x \end{cases}$ $\begin{cases} t = \dfrac{\xi+\eta}{2} \\ x = \dfrac{\xi-\eta}{2} \end{cases}$

連鎖公式より、

$$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial t}\frac{\partial t}{\partial \xi} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi} = \frac{1}{2}(u_t + u_x)$$

$$\frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial t}\frac{\partial t}{\partial \eta} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial \eta} = \frac{1}{2}(u_t - u_x)$$

従って、

$$u_t = u_\xi + u_\eta$$
$$u_x = u_\xi - u_\eta$$

より、

$$2u_\xi = \cos\xi$$

(2)

(1)をξで積分すると、ηの関数φを用いて

$$2u = \sin\xi + \varphi(\eta)$$

初期条件より、

$$\sin x + \varphi(-x) = 0 \quad (x \in \mathbb{R})$$ より、

$$\varphi(x) = \sin x$$

従って、

$$2u(\xi, \eta) = \sin\xi + \sin\eta$$ より、

$$u(t, x) = \frac{1}{2}\left(\sin(t+x) + \sin(t-x)\right) \qquad (= \sin t \cos x)$$