(3) $\hat{Y}$ が μ の不偏推定量となる条件は変わらない。

$$V[\hat{Y}] = \sum_{i=1}^{2n} \alpha_i^2 V[X_i] = \sum_{i=1}^{n} \alpha_i^2 \cdot \sigma^2 + \sum_{i=n+1}^{2n} \alpha_i^2 \cdot \frac{\sigma^2}{2} = \sigma^2 \sum_{i=1}^{n} \alpha_i^2 + \frac{\sigma^2}{2} \sum_{i=n+1}^{2n} \alpha_i^2$$

$$g(\alpha_1, \cdots, \alpha_{2n}, \lambda) = V[\hat{Y}] - \lambda\left(\sum_{i=1}^{2n} \alpha_i - 1\right)$$

$k=1, \cdots, n$ のとき、

$$\frac{\partial g}{\partial \alpha_k} = 2\sigma^2 \alpha_k - \lambda$$

$k=n+1, \cdots, 2n$ のとき、

$$\frac{\partial g}{\partial \alpha_k} = \sigma^2 \alpha_k - \lambda$$

$\frac{\partial g}{\partial \alpha_k} = 0$ ($\forall k$) とすると、

$$\alpha_k = \frac{\lambda}{2\sigma^2} \quad (k=1, \cdots, n) \qquad \alpha_k = \frac{\lambda}{\sigma^2} \quad (k=n+1, \cdots, 2n)$$

$$\sum_{i=1}^{2n} \alpha_i = \frac{n\lambda}{2\sigma^2} + \frac{n\lambda}{\sigma^2} = \frac{3n\lambda}{2\sigma^2} = 1$$ より、$\lambda = \frac{2\sigma^2}{3n}$.

$$\therefore \alpha_k = \begin{cases} \frac{1}{3n} & (k=1, \cdots, n) \\ \frac{2}{3n} & (k=n+1, \cdots, 2n) \end{cases}$$

このとき，

$$V[\hat{Y}] = \sigma^2 \cdot n \cdot \frac{1}{9n^2} + \frac{\sigma^2}{2} \cdot n \cdot \frac{4}{9n^2} = \frac{\sigma^2}{9n} + \frac{2\sigma^2}{9n} = \frac{\sigma^2}{3n} \quad //$$