H21問1

(1) $Sa_i = \lambda_i a_i$ $a_i^T a_j = \left(\frac{Sa_i}{\lambda_i}\right)^T \left(\frac{Sa_j}{\lambda_j}\right) = \frac{1}{\lambda_i \lambda_j} a_i \underset{S}{\underline{S^T}} S a_j$

$Sa_j = \lambda_j a_j$. $(\underline{Sa_j})^T a_i = \lambda_j a_j^T a_i$

$PD\cancel{P^T}\cancel{P}DP^T$

$S = PDP^T$, $\cancel{PDT}$ $a_j^T \underline{Sa_i} = \lambda_j a_j^T a_i$

PD^2P^T

$PDP^T a_i = \lambda_i a_i$ $\lambda_i a_i$

$= \frac{1}{\lambda_i \lambda_j} PD^2P^T a_j$

$DP^T a_i = \lambda_i P^{-1} a_i$ $\lambda_i a_j^T a_i = \lambda_j a_j^T a_i$

$(\underset{0}{\underline{\lambda_i - \lambda_j}}) a_i^T a_j = 0$

$\therefore a_i^T a_j = 0$ //

(2) $\|Sx\|^2 = \|PDP^T x\|^2$

$= x^T S^T S x$

$= x^T (PDP^T)^T (PDP^T) x$ $= (DP^T x)^T (DP^T x)$

$= x^T PDP^T \cancel{P^T}PDP^T x$

$= x^T PD^2 P^T x$ $= \|DP^T x\|^2$

$P^T x = y$ で変換すると、P^Tは全単射より $\forall x \to \forall y$
$\|x\| = 1$ $\|y\| = 1$

$= \|Py\|^2 = \lambda_1^2 y_1^2 + \cdots + \lambda_n^2 y_n^2$,

$\begin{pmatrix} \lambda_1 y_1 \\ \vdots \\ \lambda_n y_n \end{pmatrix}$

より、 $\lambda_1 > \cdots > 0$, $\underline{y_1 = 1}$ のとき

$\max \lambda_1^2 \to$

$\max \|Sx\| = \lambda_1$

(3) $2 \le i \le n$, $\|x\|=1$ かつ $\forall j<i$ で $x^T a_j = 0$

$\cancel{\|Sx}$ $\max \|Sx\|$

$\forall j<i$,

$x^T a_j = 0$

$\|x\|=1$

λ_i

$Sa_j = \lambda_j a_j$

$x^T(Sa_j) = x^T(\lambda_j a_j)$

$= \lambda_j x^T a_j = 0$

$x^T S a_j$

$x^T (PDP^T) a_j = 0$

$\|Sx\|^2 = \cdots = \|D \underbrace{P^T x}_{y}\|^2$

$P^T x = y$

~~$P = (a_1\ a_2 \cdots a_n)\ x =$~~ $(a_1 x\ \ a_2 x\ \cdots\ a_n x)$

$P^T = \begin{pmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_n^T \end{pmatrix}$ $P^T x = \begin{pmatrix} a_1^T x \\ a_2^T x \\ \vdots \\ a_n^T x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ a_i^T x \\ \vdots \\ a_n^T x \end{pmatrix} = P^T y = \begin{pmatrix} 0 \\ 0 \\ y_i \\ \vdots \\ y_n \end{pmatrix}$

$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = a$

$\downarrow$

$\|Dy\|^2 = \lambda_i^2 y_i^2 + \cdots + \lambda_n^2 y_n^2$

$\le \max \lambda_i^2$