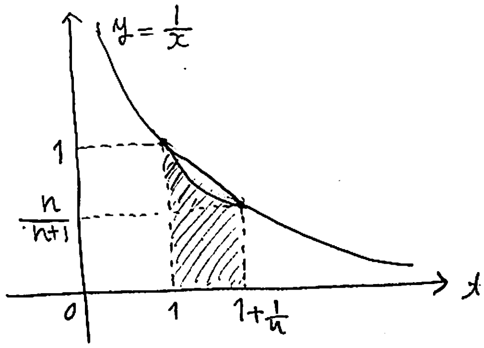
H27問2.

(1)

$$a_n - a_{n+1} = -\frac{1}{n+1} - \log n + \log(n+1) = \log\frac{n+1}{n} - \frac{1}{n+1} = \int_1^{1+\frac{1}{n}} \frac{dt}{t} - \frac{1}{n+1}$$

$$\underset{\text{長方形}}{\frac{1}{n}\cdot\frac{n}{n+1}} < \int_1^{1+\frac{1}{n}} \frac{dt}{t} < \underset{\text{台形近似}}{\frac{1}{2}\cdot\left(1+\frac{n}{n+1}\right)\cdot\frac{1}{n}}$$



より、

$$0 < a_n - a_{n+1} < \frac{2n+1}{2(n+1)n} - \frac{1}{n+1} = \frac{1}{2(n+1)n}$$ ■

(2)

$$S_n \triangleq \sum_{i=1}^{n}(a_i - a_{i+1}) = (a_1 - a_2) + (a_2 - a_3) + \cdots + (a_n - a_{n+1}) = a_1 - a_{n+1}$$

$$S_n < \sum_{i=1}^{n}\frac{1}{2i(i+1)} = \frac{1}{2}\sum_{i=1}^{n}\left(\frac{1}{i} - \frac{1}{i+1}\right) = \frac{1}{2}\left\{\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \cdots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)\right\} = \frac{1}{2}\left(1 - \frac{1}{n+1}\right)$$

より、 $$\lim_{n\to\infty} S_n = \lim(a_1 - a_{n+1}) = \underset{\substack{\shortparallel \\ 1}}{a_1} - \lim a_{n+1} < \lim_{n\to\infty}\frac{1}{2}\left(1 - \frac{1}{n+1}\right) = \frac{1}{2} \quad \text{— ①}$$

・ $\lim_{n\to\infty} a_n = -\infty$ とすると、 $+\infty < \frac{1}{2}$ でムジュンする。 $\therefore \lim a_n \in \mathbb{R}$ ■

(3)

① と (2) で収束性を示したので移項すると、 $\lim_{n\to\infty} a_n > \frac{1}{2}$ ■