問6 H20

・同次形

同次形では $u(0) \neq b$.

$u' + au = 0$, $\frac{u'}{u} = -a$, $\log u = -ax + C_1$, $\cancel{\log b = C_1}$

$u = e^{-ax+C_1} = \cancel{be^{-ax}}$

・非同次：定数変形法，C_1を改めて $C_1(x)$として，

$u = C_2(x)e^{-ax}$

$u' = C_2'(x)e^{-ax} - aC_2(x)e^{-ax}$

$u' + au = e^{-ax}\{C_2' - aC_2 + aC_2\} = C_2'e^{-ax} = \frac{f(x)}{n}$ $\cancel{=0 \to C_2'(x) = e^{ax}}$

$\cancel{C_2(x) = \frac{1}{a}e^{ax} + C_3}$

$\cancel{u(0) =}$

$\cancel{u(x) = \left(\frac{1}{a}e^{ax} + C_3\right)e^{-ax}}$

$\cancel{u(0) = \left(\frac{1}{a} + C_3\right) = b \quad \therefore C_3 = b - \frac{1}{a}}$

$\therefore C_2'(x) = \frac{f(x)}{n}e^{ax}$ $\cancel{= \frac{1}{n}g(x) \text{ とおくと}}$, $C_2(x) = \int \frac{f(x)}{n}e^{ax}dx + C_3 =$

$u(0) =$

$= \frac{1}{n}\underbrace{\int f(x)e^{ax}dx}_{= g(x)} + C_3$

不定積分として $\int_0^x$ をえらんで．

$u(0) = \left(\frac{1}{n}\underbrace{g(0)}_{=0} + C_3\right) = b \quad \therefore C_3 = b.$

$u_n(x) = \left(\frac{1}{n}g(x) + b\right)e^{-ax}$, $g(x) = \int_0^x f(w)e^{aw}dw < +\infty.$

$\forall x \in [0, \infty)$ fixed,

$u_n \to be^{-ax}$

$\therefore \lim u_n = be^{-ax}$ //