H24問6.

(1)

$$E(t) \triangleq \int_0^\pi \left(u_t^2 + k u_x^2\right) dx$$

$$E'(t) = \int_0^\pi \left(2u_t \underbrace{u_{tt}}_{\substack{u_{xx} \\ (u_x)'}} + 2k u_{xt} u_x\right) dx$$

$$= \underbrace{\left[2u_t u_x\right]_0^\pi}_{0} - \int_0^\pi \left(2u_{tx} u_x\right) dx + \int_0^\pi \left(2k \underbrace{u_{xt}}_{u_{tx}} u_x\right) dx$$

$$= \int_0^\pi 2u_{tx} u_x (k-1)\, dx = 0 \quad (\forall t)$$

$$\therefore k = 1$$

(2) $f \equiv 0$.

$$E(0) = \int_0^\pi \left(u_t(x,0)^2 + u_x(x,0)^2\right) dx = 0 = E(t) \quad (\forall t)$$

$$= \int_0^\pi \left(u_t(x,t)^2 + u_x(x,t)^2\right) dx$$

$$\therefore u_t(x,t) = u_x(x,t) = 0 \quad (\forall x, t)$$

$$\therefore u = A(x) \text{ かつ } u = B(t) \implies u = \text{Const} = 0$$

$$u(x,0) = 0$$

(3)

$$u(x,t) = \frac{1}{2}(0) + \frac{1}{2}\int_{x-t}^{x+t} f(x)\, dx = \frac{1}{2}\int_{x-t}^{x+t} (\sin 2x + \sin 5x)\, dx$$

$$= \frac{1}{2}\left\{\left[-\frac{1}{2}\cos 2x\right]_{x-t}^{x+t} + \left[-\frac{1}{5}\cos 5x\right]_{x-t}^{x+t}\right\}$$

$$= -\frac{1}{4}\left(\cos 2(x+t) - \cos 2(x-t)\right) - \frac{1}{10}\left(\cos 5(x+t) - \cos 5(x-t)\right)$$

$$= \frac{1}{2}\left(\sin 2x \sin 2t\right) + \frac{1}{5}\sin 5x \sin 5t$$