H26 問9

(1)

$Z \triangleq k+Y$

$P(Z=l) = P($ l回目に1が出て、$l-1$回までに1が$\overset{k-1}{\cancel{k}}$回, 0が$l-k$ ~~k~~回出る$)$

$$= p \cdot \binom{l-1}{k-1} p^{k-1}(1-p)^{l-k}$$

$$\underset{\shortparallel}{P(Y=l-k)} = \binom{l-1}{k-1} p^{k}(1-p)^{l-k}$$

$$P_k(Y=i) = \binom{k+i-1}{k-1} p^{k}(1-p)^{i} = \binom{k+i-1}{i} p^{k}(1-p)^{i}, \quad l-k \geq 1 \text{ より}, \ i=0,1,\cdots$$

(2)

$$E(Y) = \sum_{i=0}^{\infty} i P_k(Y=i) = \sum_{i=0}^{\infty} i \binom{k+i-1}{i} p^{k}(1-p)^{i+1}$$

$$= \frac{k(1-p)}{p} \cdot \sum_{i=0}^{\infty} \frac{i}{k} \frac{(k+i-1)!}{i!\,(k-1)!} \cdot p^{k+1} \cdot (1-p)^{i}$$

$$= \frac{k(1-p)}{p} \underbrace{\sum_{i=1}^{\infty} \frac{(k+i-1)!}{(i-1)!\,k!} p^{k+1}(1-p)^{i}}$$ $(k+i-1$

$$= \frac{k(1-p)}{p} \underbrace{\sum_{j=0}^{\infty} \frac{((k+1)+j-1)!}{j!\,k!} p^{k+1}(1-p)^{j}}_{\shortparallel\ 1}$$ NB$(k+1, p)$の全かくりつ.

$$= \frac{k(\cancel{1+}-p)}{p} \quad /\!/$$

(3) 母集団平均と標本平均が等しいとする。

$\underset{\overline{y}}{\underset{\shortparallel}{E(\overline{Y})}} = \frac{k(1-p)}{p}$ より $\quad \overline{y} = \frac{k(1-p)}{p}, \quad p\overline{y} = k - kp$

$p(\overline{y}+k) = k, \quad \hat{p}_{ME} = \frac{k}{k+\overline{y}}$

(4) $l(p) = \log\left\{\prod_{i=1}^{n} P(Y_i = y_i)\right\} = \log\left\{\prod \binom{i+k-1}{i}\right\} + \log\left\{(pk)^n\right\} + \log\left\{(1-p)^{\sum y_i}\right\}$

$l'(p) = nk \cdot \frac{1}{p} + \sum y_i \cdot \frac{-1}{1-p} = \frac{nk}{p} - \overline{y} \cdot \frac{n}{1-p} = 0$

$\cancel{n}k - \cancel{n}kp - \overline{y}\cancel{n}p = 0$

~~nk~~ $k = p(k+\overline{y})$

$$\therefore \hat{p}_{MLE} = \frac{k}{k+\overline{y}} \quad /\!/$$