H18問4

(1)

$$\|Tu-Tv\| = \left\| \frac{1}{a}\int_0^t (u(\tau)-v(\tau))\sin(t-\tau)d\tau \right\|$$

$$= \frac{1}{a}\sup_{0\le t\le a}\left| \int_0^t (u(\tau)-v(\tau))\sin(t-\tau)d\tau \right|$$

$$\le \frac{1}{a}\sup_{0\le t\le a}\int_0^t |(u(\tau)-v(\tau))\sin(t-\tau)|d\tau$$

$$\le \frac{1}{a}\sup_{0\le t\le a}\int_0^t |(u(\tau)-v(\tau)|d\tau$$

$$= \frac{1}{a}\int_0^a |u(\tau)-v(\tau)|d\tau$$

$$\le \frac{1}{a}\int_0^a \|u-v\| d\tau$$

$$= \frac{1}{a}\cdot\|u-v\|\cdot a$$

$$= \|u-v\|$$

$k<1$ とならない。

(2) Tは縮小写像で、$(C[0,a], \|\cdot\|)$ はバナッハ space より、縮小写像の原理より

$\exists^1 x\in C[0,a]$ $T(x)=x$ "

$F(t,\tau)$ 　$F(t,t)-F(t,0)$

$$1+\frac{1}{a}\int_0^t u(\tau)\sin(t-\tau)d\tau = u(\tau)$$

で

$$\frac{1}{a}u(t)\sin(t-t) = 0 = u'(\tau) \rightarrow u(\tau)=\mathrm{Const}$$

?

$t=0$ として

$$1=u(0) \quad \therefore u(t)=1$$

一致しない。

実際、

$$1+\frac{1}{a}\int_0^t 1\cdot\sin(t-\tau)d\tau = 1+\frac{1}{a}\Big[\underbrace{\cos(t-\tau)}_{1-\cos t}\Big]_0^t = 1+\frac{1}{a}(1-\cos t) \ ?$$