

H18問1

(a) $x = -1$ のとき

$$\left(\begin{array}{ccc|c} -5 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & -1 & -2 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 0 & 1 \\ 0 & -1 & -2 & \frac{6}{5} \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & \frac{11}{5} \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -\frac{11}{10} \end{array} \right)$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{5} \\ 1 \\ -\frac{11}{10} \end{pmatrix}$$

(b) $x = 0$

$$\left(\begin{array}{ccc|c} 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 1 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\therefore \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \rightarrow \begin{cases} x_1 + \frac{1}{2}x_3 = \frac{1}{2} \\ x_2 + \frac{1}{2}x_3 = \frac{1}{2} \end{cases}$$

$$\begin{aligned} x_1 &= \frac{1}{2}(1-x_3) \\ x_2 &= \frac{1}{2}(1-x_3) \end{aligned} \rightarrow \begin{pmatrix} \frac{1}{2}(1-c) \\ \frac{1}{2}(1-c) \\ c \end{pmatrix} \quad c \in \mathbb{R} = c \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

(c)

$$f(x) \triangleq \det A = x(-x+1)(8x+3)$$

$$f(x) = 0 \Leftrightarrow x = 0, 1, -\frac{3}{8}$$

$x = 1$ のとき

$$A = \begin{pmatrix} 5 & 4 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 3 & 1 & 4 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 5 & 4 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 4 & -8 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$x = -\frac{3}{8}$ のとき

$$\begin{pmatrix} -\frac{15}{8} & \frac{10}{8} & \frac{5}{8} & 1 \\ \frac{5}{8} & 1 & \frac{5}{8} & 1 \\ \frac{13}{8} & -\frac{3}{8} & -\frac{1}{8} & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -15 & 10 & 5 & 8 \\ 5 & 8 & 5 & 8 \\ 13 & -3 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} -20 & 12 & 0 & 0 \\ 5 & 8 & 5 & 8 \\ 13 & -3 & -1 & 1 \end{pmatrix}$$

$0 = -3$ 矛盾なし。

$$\rightarrow \begin{pmatrix} -5 & 3 & 0 & 0 \\ 0 & 11 & 5 & 8 \\ 13 & -3 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -\frac{3}{5} & 0 & 0 \\ 0 & 11 & 5 & 8 \\ 8 & 0 & -1 & 1 \end{pmatrix} \rightarrow \begin{cases} x_1 = \frac{3}{5}x_2 \\ 11x_2 + 5x_3 = 8 \\ 8x_1 - x_3 = \frac{24}{5}x_2 - x_3 = 1 \end{cases}$$

(余)