H28 問8

(1)

$$E(S^{N(C)}) = \sum_{k=0}^{\infty} S^k P(N(C)=k) = \sum_{k=0}^{\infty} S^k \cdot e^{-|C|}\frac{|C|^k}{k!} = e^{-|C|} \cdot \sum_{k=0}^{\infty}\frac{(S|C|)^k}{k!} \underset{\underset{|C|<+\infty}{\uparrow}}{=} e^{-|C|}\cdot e^{S|C|}$$

$$= e^{-(1-S)|C|} \quad //$$

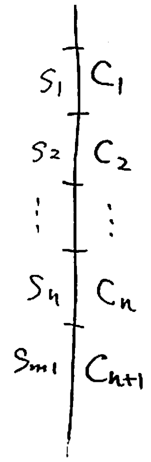
(2)

$S_{n+1} := 1$, $C_{n+1} := \left(\bigcup_{i=1}^{n} C_i\right)^C$ とすると、

$f(x) = S_i \quad x \in C_i \quad (i=1,2,\cdots,n+1)$ とかけ、

$C_1, C_2, \cdots, C_n, C_{n+1}$ は互いに排反な区間．（C_{n+1} は有界でない）

$Y_i := f(X_i) \quad (i=1,2,\cdots)$ とおく．$\Lambda := \{S_1, S_2, \cdots, S_{n+1}\}$



$$E\left(\prod_{X_j\in\Phi} f(X_j)\right) = E\left(\prod_{i=1}^{\infty} Y_i\right) = \sum_{\substack{y_1\in\Lambda\\ y_2\in\Lambda\\ \vdots}} \prod_{i=1}^{\infty} y_i P(Y_1=y_1, Y_2=y_2, \cdots)$$

（あやしい？）

$$= \sum_{k_1=0}^{\infty}\cdots\sum_{k_{n+1}=0}^{\infty} S_1^{k_1}\cdots S_{n+1}^{k_{n+1}} P(N(C_1)=k_1, \cdots, N(C_{n+1})=k_{n+1})$$

$(\because \text{(ii)})$

$$= \sum_{k_1=0}^{\infty} S_1^{k_1} P(N(C_1)=k_1) \sum_{k_2=0}^{\infty} S_2^{k_2} P(N(C_2)=k_2) \cdots \underbrace{\sum_{k_{n+1}=0}^{\infty} S_{n+1} P(N(C_{n+1})=k_{n+1})}_{\sum_{k_{n+1}=0}^{\infty} P(N(C_{n+1})=k_{n+1}) = 1}$$

$$= E(S_1^{N(C_1)}) \cdots E(S_n^{N(C_n)}) \cdot 1$$

$$= \prod_{i=1}^{n} e^{-(1-S_i)|C_i|} = \exp\left(-\sum_{i=1}^{n}(1-S_i)|C_i|\right)$$

$\cancel{(3)}$

$$E\left(\prod_{X_j\in\Phi} g_n(X_j)\right) = \exp\left\{-\sum_{i=0}^{n-1}\left(1-\frac{i}{n}\right)\cdot\frac{1}{n}\right\}$$

$$E\left(\prod_{X_j\in\Phi} g(X_j)\right) = E\left(\lim_{n\to\infty}\prod g_n(X_j)\right) = \lim E\left(\prod g_n(X_j)\right) = \exp\left\{-\lim_{n\to\infty}\sum_{i=0}^{n-1}\left(1-\frac{i}{n}\right)\frac{1}{n}\right\}$$

ここで、

$$\int_{\frac{i}{n}\le g(x)\le\frac{i+1}{n}} g_n(x)\,dx = \left[\frac{i}{n}x\right].$$