H17問2

(1)

$$\lim_{(x,y)\to 0} \frac{x^2}{x^2+y^2} = \lim_{y\to 0} \frac{0}{y^2} = \lim_{y\to 0} 0 = 0.$$

(2)

$$|f(x,y)| = \left|\frac{x}{x^2+y^2}\right| = \left|\frac{x}{x^2+k^2x^2}\right| = \left|\frac{1}{(1+k^2)x}\right| = \frac{1}{1+k^2}\cdot\frac{1}{|x|} \longrightarrow +\infty \quad (|x|\to 0)$$

(3) $\forall a \in \mathbb{R}$,

$$(x_n, y_n) \triangleq \left(\frac{a}{n}, \frac{1}{\sqrt{n}}\right) \quad (n\in\mathbb{N})$$

$$(x_n, y_n) \longrightarrow (0,0) \quad (n\to\infty)$$

$$\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} \frac{\frac{a}{n}}{\left(\frac{a}{n}\right)^2 + \left(\frac{1}{\sqrt{n}}\right)^2} = \lim_{n\to\infty} \frac{a}{\frac{a^2}{n} + 1} = \frac{a}{0+1} = a$$