問2H23

(1)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \cdots$$

$$f'(x) = e^{\alpha x}(\alpha + \alpha\sin x + \cos x)$$

$f'(0) = \alpha + 1 = 0$ より、$\alpha = -1$

$$f''(x) = e^{-x}(1 - 2\cos x)$$

$$f''(0) = -1$$

$$a_2 = \frac{f''(0)}{2} = -\frac{1}{2}$$

(2)

$$I \triangleq \int_0^\infty f(x)dx = \int_0^\infty e^{-x}(1+\sin x)dx = \left[-e^{-x}(1+\sin x)\right]_0^\infty + \int_0^\infty e^{-x}\cos x\,dx$$

$$= (0+1) + \left[-e^{-x}\cos x\right]_0^\infty + \int_0^\infty e^{-x}(-\sin x)\,dx$$

$$= 1 + (0+1) - \int_0^\infty e^{-x}\sin x\,dx$$

$$= 2 - \int_0^\infty e^{-x}(1+\sin x)\,dx + \int_0^\infty e^{-x}dx$$

$$= 2 - I + \left[-e^{-x}\right]_0^\infty$$

$$= 2 - I + 1$$

$$\therefore I = \frac{3}{2}$$