問9H18　　$r \triangleq 1-p-q.$

(1)

$$L(p,q) = \binom{N}{\underbrace{N_{11}\ N_{12}\ \cdots\ N_{33}}_{9通り}} (pp)^{N_{11}} (pq)^{N_{12}} (pr)^{N_{13}} (qp)^{N_{21}} (qq)^{N_{22}} (qr)^{N_{23}} (rp)^{N_{31}} (rq)^{N_{32}} (rr)^{N_{33}}$$

$$= \binom{N}{N_{11}\ \cdots\ N_{33}} p^{2N_{11}} (pq)^{N_{12}+N_{21}} (pr)^{N_{13}+N_{31}} q^{2N_{22}} (qr)^{N_{23}+N_{32}} r^{2N_{33}}$$

$$= \binom{N}{N_{11}\ \cdots\ N_{33}} p^{2N_{11}} q^{2N_{22}} r^{2N_{33}} (pq)^{N_{12}+N_{21}} (qr)^{N_{23}+N_{32}} (rp)^{N_{13}+N_{31}} \quad //$$

$$= \binom{N}{N_{11}\ \cdots\ N_{33}} p^{2N_{11}+N_{12}+N_{21}+N_{13}+N_{31}}$$
$$\times q^{2N_{22}+N_{21}+N_{21}+N_{23}+N_{32}}$$
$$\times r^{2N_{33}+N_{23}+N_{32}+N_{13}+N_{31}} \quad //$$

p^{M_p}, q^{M_q}, r^{M_r} とおく.

$$l(p,q) \triangleq \log L(p,q) = \log\binom{N}{N_{11}\cdots N_{33}} + M_p \log p + M_q \log q + M_r \log(1-p-q)$$

$$\frac{\partial l}{\partial p} = \frac{M_p}{p} + \frac{-M_r}{1-p-q} = 0$$

$$\frac{\partial l}{\partial q} = \frac{M_q}{q} + \frac{-M_r}{1-p-q} = 0$$

$$\frac{M_p}{p} = \frac{M_q}{q} \quad — ①$$

$$p = \frac{M_p}{M_q} q$$

$$(1-p-q)M_q - qM_r = 0 \rightarrow M_q - \underbrace{pM_q}_{M_p q} - qM_q - qM_r = 0 \rightarrow M_q = q(M_p+M_q+M_r)$$

$$\hat{q} = \frac{M_q}{M_p+M_q+M_r}, \quad \hat{p} = \frac{M_p}{M_p+M_q+M_r} \quad //$$

$$\cancel{\left(= \frac{M_q}{\sum_{i,j} N_{ij} + \sum_i N_{ii}} \right)} \quad //$$

よりきれいに

$$\hat{p} = \frac{2N_{11}+N_{12}+N_{21}+N_{13}+N_{31}}{2N}$$

$$\hat{q} = \frac{2N_{22}+N_{21}+N_{21}+N_{23}+N_{32}}{2N} \quad //$$

(2) $L(P,Q) \triangleq \begin{pmatrix} & N & \\ K & L & N-K-L \end{pmatrix} \cdot P^K Q^L R^{N-K-L}$, $P+Q+R=1$?

Aが勝つかそうでないかの2通りしか考えなければ、

$$L(P) \triangleq \binom{N}{K} P^K (1-P)^{N-K}$$

$$l(P) \triangleq \log L(P) = \log \binom{N}{K} + K \log P + (N-K) \log (1-P)$$

$$l'(P) = \frac{K}{P} + \frac{-(N-K)}{1-P} = 0$$

$$K - KP - (N-K)P = 0$$

$$K = P\{K+N-K\}$$

$$\therefore \hat{P} = \frac{K}{N}$$

X 題意的にX.

$$P = pq + qr + rp = pq + q(1-p-q) + p(1-p-q)$$

$$= \cancel{pq} + q - \cancel{pq} - q^2 + p - p^2 - pq = q - q^2 + p - p^2 - pq = q(1-q) + p(1-p) - pq$$

$Q = P$

$R = 1 - P - Q = 1 - 2P$

$$\therefore L(P) = \begin{pmatrix} & N & \\ K & L & N-K-L \end{pmatrix} P^{K+L} (1-2P)^{N-K-L}$$

$$l(P) = \log(\,〃\,) + (K+L)\log P + (N-K-L)\log(1-2P)$$

$$l'(P) = \frac{K+L}{P} - 2\frac{N-K-L}{1-2P} = 0$$

$$(K+L) - 2(K+L)P - 2(N-K-L)P = 0$$

$$\hat{P} = \frac{K+L}{2K+2L+2N-2K-2L} = \frac{K+L}{2N}$$