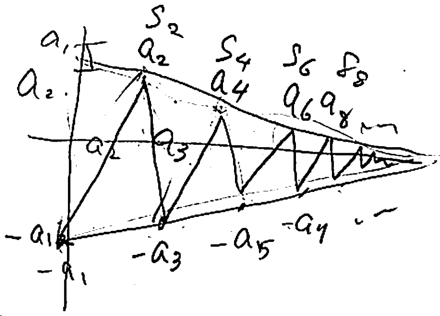
H20問2

(1) $a_n > a_{n+1}$, $\lim a_n = 0$ $\quad a_n \geq 0$

$S_n = \sum_{k=1}^{n} (-1)^k a_k$, $\lim_{n\to\infty} S_n$,



$S_n = -a_1 + a_2 - a_3 + a_4 - \cdots (-1)^{n-1} a_{n-1} + (-1)^n a_n$

$\limsup_{n\to\infty} S_n = \inf_{n\geq 1} \sup_{k\geq n} S_k =$

$|S_n| = \left| \sum \right| \leq \quad = \sum_{k=1}^{n} a_k$

$S_{2n} = \sum_{k=1}^{2n} (-1)^k a_k = -a_1 + a_2 - a_3 + a_4 - \cdots - a_{2n-1} + a_{2n}$

$S_{2n+2} = \sum_{k=1}^{2n+2} (-1)^k a_k = -a_1 + a_2 - a_3 + a_4 - \cdots - a_{2n-1} + a_{2n} - a_{2n+1} + a_{2n+2}$

$S_{2n+2} - S_{2n} = -a_{2n+1} + a_{2n+2} < 0 \therefore$ $\cancel{S_n = S_{2n-2}}$ $\quad S_{n+2} = S_{2n} + a_{2n+2} - a_{2n+1}$

$\lim_{n\to\infty}$ $\quad \{S_{2n}\} \downarrow$

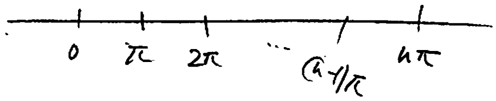
$$S_{2n+2} = -a_1 + \underset{0}{\underset{\vee}{(a_2 - a_3)}} + \cdots + \underset{0}{\underset{\vee}{(a_{2n} - a_{2n+1})}} + \underset{0}{\underset{\vee}{a_{2n+2}}} > -a_1$$

$\therefore \exists \lim_{n\to\infty} S_{2n}$

$\lim S_{2n+1} = \underset{\exists\alpha}{\underset{\downarrow}{S_{2n}}} + \underset{0}{\underset{\downarrow}{a_{2n+1}}} \quad \therefore \quad \lim S_{2n} = \lim S_{2n+1}$ //

(2)

$$\lim_{n\to\infty}\int_0^{n\pi}\frac{\sin x}{x+1}dx = \lim_{n\to\infty}\sum_{k=1}^{n}\underbrace{\int_{(k-1)\pi}^{k\pi}\frac{\sin x}{x+1}dx}_{=(-1)^{k}a_k}$$



$(k-1)\pi + y = x$

~~k=0~~ k=1, y

k=2

kπ − π + y

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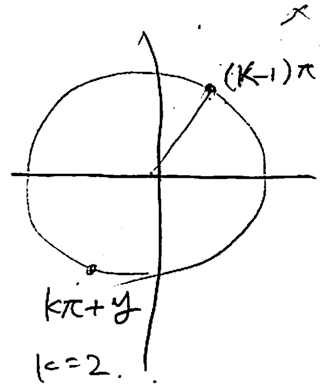
$$\int_0^{\pi}\frac{\sin\{(k-1)\pi+y\}}{y+(k-1)\pi+1}dy$$

$k\pi-\pi+y$

$\sin\{(k-1)\pi+y\} = \sin y\cdot(-1)^{k-1}$

$$\| $$

$$(-1)^{k-1}\underbrace{\int_0^{\pi}\frac{\sin y}{y+(k-1)\pi+1}dy}_{=a_k}$$



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6.

$\{a_k\}\downarrow$, $\lim a_k = 0$ より、Exist //