H30-3

(1)

$V_{tt} = V_{xx}$, $V(0,x) = \frac{1}{6}x^3 + x$ $\frac{\partial V}{\partial t}(0,x) = 0$

(2)

ダランベールの解法より、

$$V(t,x) = \frac{1}{2}\left(V(0,x+t) + V(0,x-t)\right) + \frac{1}{2}\int_{x-t}^{x+t} \frac{\partial V}{\partial t}(0,x)\,dx$$ より、

$$V(t,x) = \frac{1}{2}\left\{\frac{1}{6}(x+t)^3 + (x+t) + \frac{1}{6}(x-t)^3 + (x-t)\right\}$$

$$= \frac{1}{6}x^3 + \frac{1}{2}xt^2 + x$$

$$u(t,x) = V(t,x) - \frac{1}{6}x^3$$

$$= \frac{1}{2}xt^2 + x$$