H15-8　PrをPとかく。

(1)

$$F_n(x) = P(U_1 \leq x, U_2 \leq x, \cdots, U_n \leq x)$$
$$= x^n \quad (\because \text{i.i.d.}) \qquad (x \in [0,1])$$

従って、$F_n(x) = \begin{cases} 0 & (x \leq 0) \\ x^n & (0 \leq x \leq 1) \\ 1 & (1 \leq x) \end{cases}$

(2)

$$G_n(y) = 1 - P\left(X_n < 1 - \frac{y}{n}\right)$$
$$= 1 - F_n\left(1 - \frac{y}{n}\right)$$
$$= \begin{cases} 1 & \left(1 - \frac{y}{n} \leq 0\right) \\ 1 - \left(1 - \frac{y}{n}\right)^n & \left(0 \leq 1 - \frac{y}{n} \leq 1\right) \\ 0 & \left(1 \leq 1 - \frac{y}{n}\right) \end{cases}$$
$$= \begin{cases} 0 & (y \leq 0) \\ 1 - \left(1 - \frac{y}{n}\right)^n & (0 \leq y \leq n) \\ 1 & (n \leq y) \end{cases}$$

従って、

$$G(y) = \lim_{n \to \infty} G_n(y) = \begin{cases} 0 & (y \leq 0) \\ 1 - e^{-y} & (0 \leq y) \end{cases}$$

(3)

$$P(Z_n \leq z) = 1 - P(\max\{V_1, V_2, \cdots, V_n\} < \log n - z)$$
$$= 1 - \left(1 - e^{-(\log n - z)}\right)^n \quad (\because \text{i.i.d.}) \qquad (0 \leq \log n - z)$$
$$= 1 - \left(1 - \frac{e^z}{n}\right)^n$$

従って、

$$H(z) = 1 - \exp(-e^z) \qquad (z \in \mathbb{R})$$