H24問9

(1)
$$\sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i - cx_i^2)^2 \triangleq f(a,b,c)$$

$$\frac{\partial f}{\partial a} = \cancel{\sum 2(-1)}(y_i - a - bx_i - cx_i^2) = 0$$

$$n\bar{y} - an - bn\bar{x} - cnS_{x^2} = 0$$

$$-an - cn = 0 \quad \therefore \cancel{a=c} \; a = -c$$

$$\frac{\partial f}{\partial b} = \sum \cancel{2}(+x_i)(y_i - a - bx_i - cx_i^2) = 0$$

$$= \sum x_i y_i - a\sum x_i - b\sum x_i^2 - c\sum x_i^3 = 0$$

$$nS_{xy} - an\bar{x} - bnS_{x^2} - cnS_{x^3} = 0$$

$$S_{xy} - b - cS_{x^3} = 0 \qquad b = S_{xy} - cS_{x^3}$$

$$\frac{\partial f}{\partial c} = \sum x_i^2 (y_i - a - bx_i - cx_i^2) = 0$$

$$\sum x_i^2 y_i - a\sum x_i^2 - b\sum x_i^3 - c\sum x_i^4 = 0$$

$$nS_{x^2y} - an - bnS_{x^3} - cnS_{x^4} = 0$$

$$S_{x^2y} - \underset{+c}{\cancel{a}} - bS_{x^3} - cS_{x^4} = 0$$

$$S_{x^2y} - bS_{x^3} + c(1 \pm S_{x^4}) = 0$$

$$S_{x^2y} + S_{x^3}(cS_{x^3} - S_{xy}) + c(1 \pm S_{x^4}) = 0$$

$$c\{(S_{x^3})^2 + 1 - S_{x^4}\} = -S_{x^2y} + S_{x^3}S_{xy}$$

$$\therefore c = \frac{S_{x^3}S_{xy} - S_{x^2y}}{(S_{x^3})^2 + 1 - S_{x^4}} \;\text{〃} \quad \rightarrow \quad \underline{a = -c} = \frac{S_{x^2y} - S_{x^3}S_{xy}}{(S_{x^3})^2 + 1 - S_{x^4}}$$

$$b = S_{xy} - cS_{x^3} \;\text{〃} \quad = \frac{1}{(S_{x^3})^2 + 1 - S_{x^4}} \left\{ \cancel{S_{xy}(S_{x^3})^2} + S_{xy} - S_4 S_{xy} - \cancel{(S_{x^3})^2 S_{xy}} + S_{x^2y}S_{x^3} \right\}$$

$$= \frac{S_{xy}(1 - S_{x^4} + S_{x^3})}{(S_{x^3})^2 + 1 - S_{x^4}}$$