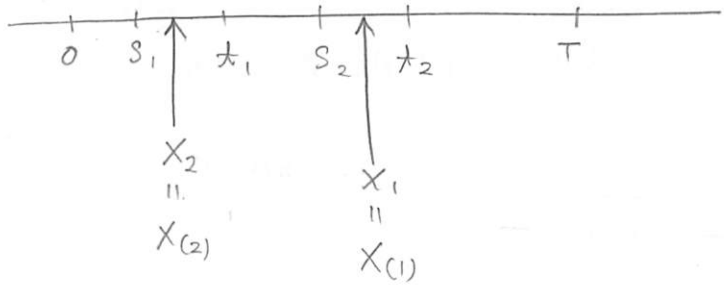
H25問8

(1) 右の場合,



$P(s_1 < X_{(1)} \leq t_1, s_2 < X_{(2)} \leq t_2)$

$= P(s_1 < X_1 \leq t_1, s_2 < X_2 \leq t_2)$

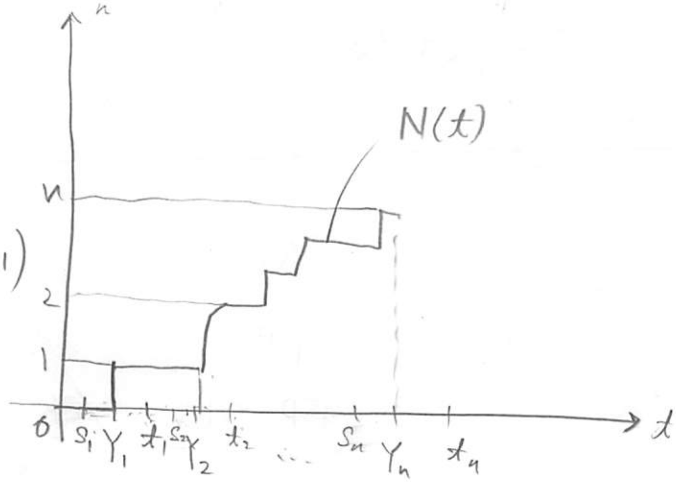
$= \dfrac{t_1 - s_1}{T} \cdot \dfrac{t_2 - s_2}{T}$

$P(s_1 \leq X_{(1)} \leq t_1, \cdots, s_n \leq X_{(n)} \leq t_n) = P(s_1 \leq X_{i_1} \leq t_1, \cdots, s_n \leq X_{i_n} \leq t_n)\ (\exists i_1, \cdots, i_n)$

$(X_{(1)}, \cdots, X_{(n)})$ の組で場合分けすれば、各場合の生起確率は等しく, $n!$通りある.

$= \prod_{i=1}^{n} \left(\dfrac{t_i - s_i}{T}\right) n!$ //

(2)



$P(s_1 < Y_1 \leq t) = P(s_1 < \inf\{t > 0 ; N(t) \geq 1\} \leq t_1)$

$= P(\ N(s_1) = 0, N(t_1) = 1)$ をもとに,

$P(s_1 < Y_1 \leq t_1, \cdots s_n < Y_n \leq t_n) = P(N(s_1) = 0, N(t_1) = 1 = N(s_2), N(t_2) = 2 = N(s_3)$

$t_i \sim s_{i+1}$ 間の情報がない. $, N(s_n) = n-1, N(t_n) = n)$

~~$= P(N(t_1) - N(s_1) = 1, N(t_2) - N(s_2) = 1, \cdots, N(t_n) - N(s_n) = 1)$~~

~~$= \prod_{i=1}^{n} P(N(t_i) - N(s_i) = 1) = \prod (t_i - s_i) e^{-(t_i - s_i)} =$~~

$P(s_1 < Y_1 \leq t_1, \cdots, s_n \leq Y_n \leq t_n) = P(N(s_1) - N(0) = 0, N(t_1) - N(s_1) = 1, N(s_2) - N(t_1) = 0,$
$\cdots, N(s_n) - N(t_{n-1}) = 0, N(t_n) - N(s_n) = 1)$

$= e^{-(s_1 - 0)} \cdot \prod_{i=1}^{n} P(N(t_i) - N(s_i) = 1) \cdot \prod_{i=1}^{n-1} \underbrace{P(N(s_{i+1}) - N(t_i) \neq 0)}_{e^{-(s_{i+1} - t_i)}}$

$= e^{-s_1} \cdot \prod_{i=1}^{n} (t_i - s_i) e^{-(t_i - s_i)} \cdot \prod_{i=1}^{n-1} e^{-(s_{i+1} - t_i)}$

$= e^{-s_1} \cdot (t_1 - s_1) \cdots (t_n - s_n) \cdot e^{-(t_1 - s_1)} \cdot e^{-(t_2 - s_2)} \cdot \cdots \cdot e^{-(t_n - s_n)}$
$\cdot e^{-(s_2 - t_1)} \cdot e^{-(s_3 - t_2)} \cdot \cdots \cdot e^{-(s_n - t_{n-1})}$

$= \prod_{i=1}^{n} (t_i - s_i) \cdot e^{-t_n}$ //

$0 \longrightarrow T$ へ整理したほうがいいかも.

(3)

$$P(s_1 < Y_1 \le t_1, \cdots, s_n < Y_n \le t_n \mid N(T) = n)$$

$t_n \sim T$ の情報

$$\cancel{=} P(\quad 〃 \quad , N(T) - N(t_n) = 0)$$

$$= \frac{P(s_1 < Y_1 \le t_1, \cdots, s_n < Y_n \le t_n, N(T) = n)}{P(N(T) = n)}$$

$$= \frac{P(\quad (2) \quad \cdots \quad , N(t_n) - N(s_n) = 1, N(T) - N(t_n) = 0)}{P(N(0) = 0, N(T) = n)}$$

$N(T) - N(0) = n$

$$= \frac{\prod_{i=1}^{n} (t_i - s_i) \cdot e^{-t_n} \cdot e^{-(T - t_n)}}{\frac{(T-0)^n}{n!} e^{-T}}$$

$$= \prod_{i=1}^{n} (t_i - s_i) \cdot \exp\{\cancel{-t_n} \cancel{-T} + \cancel{t_n} + \cancel{T}\} \cdot T^{-n} \cdot n!$$

$$= \prod_{i=1}^{n} \left(\frac{t_i - s_i}{T}\right) \cdot n!$$

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