

問8H19

(1)

① $y < 0$ のとき

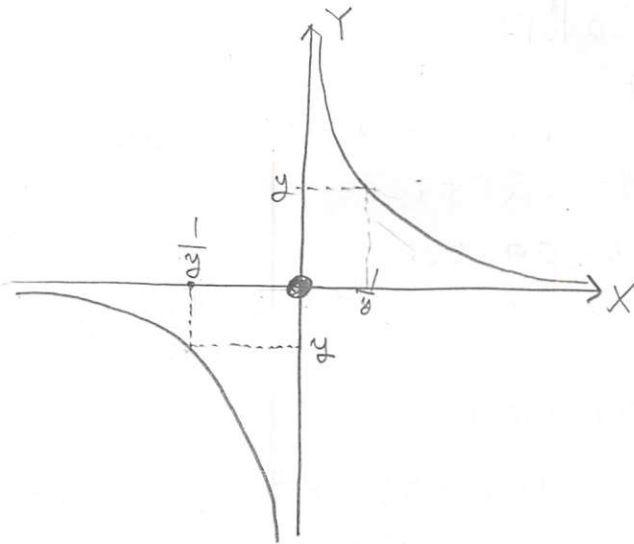
$$F_Y(y) = P(Y \leq y) = P\left(\frac{1}{y} \leq X < 0\right) \\ = F_X(0-0) - F_X\left(\frac{1}{y}-0\right)$$

② $y = 0$ のとき

$$F_Y(y) = P(Y \leq y) = P(X \leq 0) = F_X(0)$$

③ $y > 0$ のとき

$$F_Y(y) = P_X\left(\frac{1}{y} \leq X\right) + P(X \leq 0) = 1 - F_X\left(\frac{1}{y}-0\right) + F_X(0)$$



(2)

① $y < 0$ のとき

$$F_Y(y) = F_X(0-0) - F_X\left(\frac{1}{y}-0\right) \\ = \lim_{x \rightarrow 0-0} p e^{\alpha x} - \lim_{x \rightarrow \frac{1}{y}-0} (p e^{\alpha x}) \\ = p - p e^{\frac{\alpha}{y}}$$

② $y = 0$ のとき

$$F_Y(y) = F_X(0) = 1 - p$$

③ $y > 0$ のとき

$$F_Y(y) = 1 - \left(1 - p e^{-\frac{\alpha}{y}}\right) + (1 - p) \\ = p e^{-\frac{\alpha}{y}} + 1 - p$$

(3)

$$F_X(x) = \int_{-\infty}^x \frac{1}{\pi} \frac{a}{a^2 + x^2} dx = \frac{1}{\pi} \int_{-\infty}^x \frac{a}{a^2 + x^2} dx$$

(1) ①,

$$f_Y(y) = F_Y'(y) = \begin{cases} +\frac{1}{y^2} f_X\left(\frac{1}{y}\right) & y < 0 \\ 0 & y = 0 \\ +\frac{1}{y^2} f_X\left(\frac{1}{y}\right) & y > 0 \end{cases}$$

f_X は C^∞ 可

$$= \begin{cases} \frac{1}{y^2} \cdot \frac{1}{\pi} \cdot \frac{a}{a^2 + \left(\frac{1}{y}\right)^2} & y \neq 0 \\ 0 & y = 0 \end{cases} = \begin{cases} \frac{a}{\pi \cdot a^2 y^2 + 1} & y \neq 0 \\ 0 & y = 0 \end{cases}$$

一点は零集合

$\sim \text{Cauchy}\left(\frac{1}{a}\right)$ と見做す