

H22-8

$$(1) f_X(x) = \int_0^1 f(x, y) dy = \frac{2}{3}(x+1) \mathbb{1}_{\{0 \leq x \leq 1\}}$$

$$(2) E(X) = \int_0^1 x f_X(x) dx = \frac{5}{9}$$

$$E(X^2) = \frac{7}{18}$$

$$V(X) = E(X^2) - E(X)^2 = \frac{13}{162}$$

$$(3) E(XY) = \int_0^1 \int_0^1 xy f(x, y) dx dy = \frac{11}{36}$$

X と Y の対称性より、

$$\text{Cov}(X, Y) = E(XY) - E(X)^2 = -\frac{1}{324}$$

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{V(X)} = -\frac{1}{26}$$

$$(4) V(X+Y) = V(X) + 2\text{Cov}(X, Y) + V(Y) = \frac{25}{162}$$

$$(5) P(X+Y \leq 1) = \int_0^1 \int_0^{1-x} f(x, y) dy dx = \frac{7}{18}$$

