問4H22.

(1) $\xi = t + x$, $\eta = t - x$ $\quad t = \frac{\xi+\eta}{2}$, $x = \frac{\xi-\eta}{2}$.

~~$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial u}{\partial}$~~

$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial t}\frac{\partial t}{\partial \xi} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi}$, $\frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial t}\frac{\partial t}{\partial \eta} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial \eta}$

$= \frac{1}{2}u_t + \frac{1}{2}u_x$ $\quad = \frac{1}{2}u_t - \frac{1}{2}u_x$

$\frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta} = u_t$, $\quad \frac{\partial u}{\partial \xi} - \frac{\partial u}{\partial \eta} = u_x$

~~$u(0,x) = \cdots$~~ より、

$2\frac{\partial u}{\partial \xi} = \cos\xi$ //

(2) (1)を両辺ξで積分して、

$2u = \sin\xi + C(\eta)$

$u(0,x) = 0$より、

$\xi = x$, $\eta = -x$のとき、 $0 = \sin\xi + C(\eta)$ $\quad (\forall x \in \mathbb{R})$

$C(\eta) = -\sin x = \sin(-x) = \sin\eta$

$\therefore u(t,x) = \frac{1}{2}(\sin\xi + \sin\eta)$

$= \frac{1}{2}(\sin(t+x) + \sin(t-x))$ //