H22問8

$$f(x,y)=\begin{cases}\frac{1}{3}(1+2x+2y) & 0\le x,y\le 1\\ 0 & \text{o.w.}\end{cases}$$

E

(1)

$$f_X(x)=\int_0^1 f(x,y)\,dy=\int_0^1 \frac{1}{3}(1+2x+2y)\,dy=\frac{1}{3}\Big[(1+2x)y+y^2\Big]_0^1=\frac{1}{3}\{1+2x+1\}=\frac{2}{3}(x+1)$$

$(0\le x\le 1)$

(2)

$$E(X)=\int_0^1 x f_X(x)\,dx=\int_0^1\frac{2}{3}(x^2+x)\,dx=\frac{2}{3}\Big[\frac{x^3}{3}+\frac{x^2}{2}\Big]_0^1=\frac{2}{3}\Big(\frac{1}{3}+\frac{1}{2}\Big)=\frac{2}{3}\cdot\frac{5}{6}=\frac{5}{9}$$ //

$$V(X)=E(X^2)-E(X)^2=\int_0^1\frac{2}{3}(x^3+x^2)\,dx-\Big(\frac{5}{9}\Big)^2=\frac{\cancel{2}}{3}\cdot\underbrace{\Big(\frac{1}{4}+\frac{1}{3}\Big)}_{\frac{7}{\cancel{12}\,6}}-\frac{25}{81}=\frac{63-50}{162}$$

$$=\frac{13}{162}$$ //

(3)

$$\mathrm{Cov}(X,Y)=E(XY)-E(X)E(Y)=\int_0^1\int_0^1 xyf(x,y)\,dx\,dy-\frac{5}{9}\cdot\frac{5}{9}$$

$$=\int_0^1 x\frac{1}{3}\underbrace{\int_0^1 (1+2x)y+2y^2\,dy}_{\frac{1+2x}{2}+\frac{2}{3}}-\frac{25}{81}=\frac{1}{3}\underbrace{\int_0^1\Big(x^2+\frac{7}{6}x\Big)dx}_{\frac{1}{3}+\frac{7}{12}=\frac{11}{12}}-\frac{25}{81}=\frac{11}{36}-\frac{25}{81}=\frac{}{\cancel{162}}$$

$\frac{3+4}{6}$ $\frac{\cancel{12\times3}}{\cancel{4}}$ 9×9 9×4 $36.$

$$=\frac{99-100}{324}=-\frac{1}{324}$$ //

$$\rho(X,Y)=\frac{\mathrm{Cov}(X,Y)}{\sqrt{V(X)V(Y)}}=\frac{''}{\frac{13}{162}}=\frac{\cancel{162}}{13}\cdot-\frac{1}{\cancel{324}\,2}=-\frac{1}{26}$$ //

(4)

$$V(X+Y)=V(X)+2\mathrm{Cov}(X,Y)+V(Y)=\frac{13}{\cancel{81}\,162}+\cancel{2}\cdot\Big(-\frac{1}{\cancel{324}}\Big)=\frac{26-1}{162}=\frac{25}{162}$$ //

(5).

$$P(X+Y\le x)=\cancel{\int f_X(y-x)f_Y(x)\,d}=\int_0^x f_X(x-y)f_Y(y)\,dy$$ ✗ 独立でない。

$0\le x-y\le 1$ $y\le x\le 1+y$
$0\le y\le 1$
$0\le y\le x$

$$\cancel{=\frac{1}{9}\int_0^x (1+2(x-y)+}\quad=\frac{4}{9}\int_0^x \underbrace{(x-y+1)(y+1)}_{\cancel{(x+1)y}\ xy+x-y^2-\cancel{y}+\cancel{y}+1}\,dy=\frac{4}{9}\Big[\frac{x}{2}y^2+xy-\frac{y^3}{3}+y\Big]_0^x=\frac{4}{9}\Big(\frac{x^3}{2}+x^2-\frac{x^3}{3}+x\Big)$$

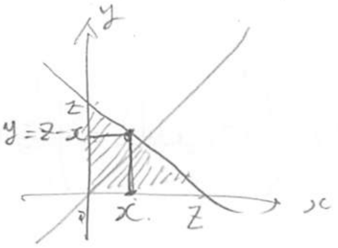
$$\therefore\ \cancel{P(X+Y\le x)}=P(X+Y\le 1)=\frac{\cancel{4}\,2}{9}\Big(\frac{1}{2}+1-\frac{1}{3}+1\Big)\qquad \frac{3-2+12}{6}\quad\frac{13}{\cancel{6}\,3}$$

$$=\frac{26}{27}$$ //

$$P(X+Y \le \cancel{x}\, y) = \int_0 \int_0^y f_{X,Y}(x, y-x)\, \cancel{dy}\, dx$$

$$= \frac{1}{3}\int_0^y \underbrace{1+2x+2(y-x)}_{1+2y}\, dx$$

$$= \frac{1}{3} \int_0 \left[y+y^2\right]_0^y = \frac{1}{3}(y+y^2) \rightarrow \frac{2}{3}\ ?$$



$$P(X+Y \le Z) = \int_0^Z \int_0^{Z-x} f(x,y)\, dy dx$$ 되.

$$P(X+Y \le 1) = \int_0^1 \int_0^{1-x} \frac{1}{3}(1+2x+2y)\, dy\, dx$$

$$= \frac{1}{3}\int_0^1 \left[(1+2x)y + y^2\right]_0^{1-x} dx$$

$$= \frac{1}{3}\int_0^1 (1+2x)(1-x) + (1-x)^2\, dx$$

$1-x = y$

$1-y = x$

$2-2y$

$$= \frac{1}{3}\int_1^0 (3-2y)\,y + y^2\,(-dy)$$

$$= \frac{1}{3}\int_0^1 (3y - y^2)\, dy$$

$$= \frac{1}{3}\cdot\left(\frac{3}{2} - \frac{1}{3}\right) = \frac{1}{3}\cdot\frac{9-2}{6} = \frac{7}{18},$$