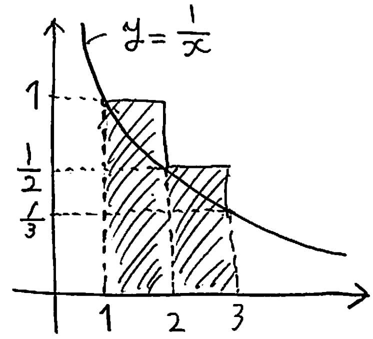
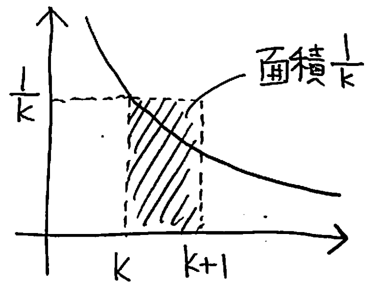
H24-2

(1) 右図より

$$\frac{1}{k} > \int_k^{k+1} \frac{1}{x}\,dx \qquad (k=1,2,\cdots)$$





より

$$\sum_{k=1}^{\infty} \frac{1}{k} > \sum_{k=1}^{\infty} \int_k^{k+1} \frac{1}{x}\,dx = \int_1^{\infty} \frac{1}{x}\,dx = \Big[\log x\Big]_1^{\infty} = \infty$$

従って、

$\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ は発散する。

(2)
$$\sum_{n=1}^{l} (-1)^{n+1} \frac{2n+1}{n(n+1)} = \sum_{n=1}^{l} (-1)^{n+1} \left(\frac{1}{n} + \frac{1}{n+1}\right)$$

$$= \left(\frac{1}{1} + \cancel{\frac{1}{2}}\right) + \left(-\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \cdots + (-1)^{l+1}\left(\cancel{\frac{1}{l}} + \frac{1}{l+1}\right)$$

$$= 1 + (-1)^{l+1} \frac{1}{l+1}$$

従って、

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+1}{n(n+1)} = \lim_{l \to \infty} \left(1 + \frac{(-1)^{l+1}}{l+1}\right) = 1$$

$$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{2n+1}{n(n+1)} \right| = \sum_{n=1}^{\infty} \left(\frac{1}{n} + \frac{1}{n+1}\right) \geq \sum_{n=1}^{\infty} \frac{1}{n}$$

従って、(1)より左辺も収束しない。