H26-9

(1)

$$P(k+Y=l)=\binom{l-1}{k-1}p^{k-1}(1-p)^{l-k}\times p$$ より、

$$P_k(i)=P(Y=l-k)=\binom{i+k-1}{k-1}p^{k-1}(1-p)^{i}p=\binom{i+k-1}{i}p^{k}(1-p)^{i} \quad (i=0,1,2,\cdots)$$

(2)

$$E(Y)=\sum_{i=0}^{\infty} iP_k(i)=\sum_{i=0}^{\infty} i\binom{i+k-1}{i}p^{k}(1-p)^{i}$$

$$=\frac{k(1-p)}{p}\sum_{i=1}^{\infty}\frac{(k+i-1)!}{k!(i-1)!}p^{k+1}(1-p)^{i-1}$$

$$=\frac{k(1-p)}{p}\sum_{j=0}^{\infty}\binom{(k+1)+j-1}{j}p^{k+1}(1-p)^{j}$$

$$=\frac{k(1-p)}{p}$$ ($\because$ $NB(k+1,p)$の全確率)

(3)

$$\bar{y}=\frac{k(1-\hat{p})}{\hat{p}}$$ より、 $$\hat{p}_{ME}=\frac{k}{k+\bar{y}}$$

(4)

尤度関数 $$L(p)=\prod_{i=1}^{n}P(Y_i=y_i)=\left\{\prod_{i=1}^{n}\binom{y_i+k-1}{y_i}\right\}p^{nk}(1-p)^{n\bar{y}}$$

$$\frac{\partial \log L}{\partial p}=0$$ より、

$$nk\frac{1}{p}+n\bar{y}\frac{-1}{1-p}=0$$

$$\hat{p}_{MLE}=\frac{k}{k+\bar{y}}$$