H23-6

(1)

$$u_x = \frac{x}{\sqrt{x^2+y^2}} f'(\sqrt{x^2+y^2})$$

$$u_{xx} = \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}} f'(\sqrt{x^2+y^2}) + \frac{x^2}{x^2+y^2} f''(\sqrt{x^2+y^2})$$

対称性より、

$$u_{yy} = \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}} f'(\sqrt{x^2+y^2}) + \frac{y^2}{x^2+y^2} f''(\sqrt{x^2+y^2})$$

従って、

$$u_{xx}+u_{yy} = \frac{1}{r} f'(r) + f''(r)$$

(2)

与式'より、

$$g''r+g' = -r$$

$$(g'r)' = -r$$

$$g'(r) = -\frac{r}{2} + \frac{C}{r} \qquad (C\text{は定数})$$

$$g(r) = -\frac{r^2}{4} + C\log r + D \qquad (D\text{は定数})$$

初期条件より、

$$g(r) = -\frac{r^2}{4} + C\log r + \frac{1}{4}$$