

HW-8

$$(1) \quad M(i) = \sum_{j=0}^{\infty} j Q(j|i) = \sum_{j=0}^{\infty} j \frac{P(i,j)}{P_i(i)} \underset{\substack{\uparrow \\ \text{独立性}}}{=} \sum_{j=0}^{\infty} j P(j) = E(Y)$$

$$\begin{aligned} (2) \quad E(E(Y|X)) &= \sum_{i=0}^{\infty} E(Y|X=i) P(X=i) \\ &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} j P(Y=j|X=i) P(X=i) \\ &= \sum_i \sum_j j P(Y=j, X=i) \\ &= \sum_j j \sum_i P(Y=j, X=i) \\ &= \sum_j j P(Y=j) \\ &= E(Y) \end{aligned}$$

$$\begin{aligned} (3) \quad E(Y|X=i) &= \sum_{k=1}^i E(N_k|X=i) \\ &= \sum_{k=1}^i E(N_k) \\ &= i E(N) \end{aligned}$$

$$\begin{aligned} E(Y) &= E(E(Y|X)) \\ &= \sum_{i=0}^{\infty} E(Y|X=i) P(X=i) \\ &= \sum_{i=0}^{\infty} i E(N) P(X=i) \\ &= E(N) \sum_{i=0}^{\infty} i P(X=i) \\ &= E(N) E(X) \end{aligned}$$