H29問6.

$X \sim Po(\lambda)$, $Y \sim Po(\nu)$

(1) $\sum_{k=0}^{\infty} P(X=k;\lambda) = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} \cdot e^{\lambda} = 1$

(2) $L(\lambda) = \prod_{i=1}^{n} P(X=x_i;\lambda) = \prod_{i=1}^{n} \left(\frac{\lambda^{x_i}}{x_i!} e^{-\lambda}\right) = \left(\prod \frac{1}{x_i!}\right) \cdot \lambda^{\sum x_i} \cdot e^{-n\lambda}$

$l(\lambda) = \log\left(\prod \frac{1}{x_i!}\right) + \sum x_i \log \lambda - n\lambda \underbrace{\log e}_{1}$ $\cancel{=0}$

$l'(\lambda) = \frac{\sum x_i}{\lambda} - n = 0$ $\therefore \hat{\lambda} = \frac{\sum x_i}{n} = \bar{x}$

$\hat{\nu} = \bar{y}$

(3) $N = X+Y \sim Po(\lambda+\nu)$ (再生?)

$P(X=x \mid N=X+Y) = \frac{P(X=x, N)}{P(N)} = \frac{P_{XN}(x,n)}{P_N(n)} =$

$P(x,n) = P(X=x, Y=n-x) = P(X=x)P(Y=n-x) =$

$\therefore P(X=x \mid N=X+Y) = \dfrac{\frac{\lambda^x}{\cancel{x!}} \cancel{e^{-\lambda}} \cdot \frac{\nu^{n-x}}{(n-x)!} \cancel{e^{-\nu}}}{\frac{(\lambda+\nu)^n}{\cancel{n!}} \cancel{e^{-(\lambda+\nu)}}}$

$= \binom{n}{x} \frac{\lambda^x \nu^{n-x}}{(\lambda+\nu)^n}$ //

$\prod f_a(x_i)^{y_i}$

(4) $L(a) = \prod_{i=1}^{n} P(Y=y_i \mid X=x_i; a) = \prod \frac{1}{y_i!} \cdot \cancel{f_a(x_i)}^{\sum y_i} \cdot \cancel{\prod} e^{-\sum f_a(x_i)}$

$l(a) = \log\left(\prod \frac{1}{y_i!}\right) + \cancel{n\bar{y} \log f_a(x_i)} - \sum f_a(x_i)$

$\sum y_i \log f_a(x_i)$

$f_a'(x) = 1+x$

$l'(a) = \sum \frac{y_i f_a'(x_i)}{f_a(x_i)} - \sum f_a'(x_i) = 0$

$\sum \frac{y_i (1+\cancel{x_i})}{a(1+\cancel{x_i})} - \underbrace{\sum (1+x_i)}_{n+n\bar{x}} = \frac{1}{a} \cancel{n} \cdot \bar{y} - \cancel{n} - n\bar{x} = 0.$

$\frac{1}{a}\bar{y} - 1 - \bar{x} = 0$

$\frac{1}{a}\bar{y} = 1+\bar{x}$

$a = \frac{\bar{y}}{1+\bar{x}}$ //