H27問6

~~(1) $\xi = x + t$, $\eta = x - t$ に対して.~~

(2) $u(t, x)$

$$V_{\eta\xi} = 0 \rightarrow V_\eta = A(\eta) \rightarrow V = \int A(\eta)d\eta + B(\xi)$$

不定積分として、$\int_0^\eta$ を採用すると、 $V = \underbrace{\int_0^\eta A(w)dw}_{\varphi(\eta)\text{とおく。}} + B(\xi)$

$V(\xi, \eta)$

$V = \varphi(\eta) + B(\xi)$, $\underset{u(x,t)}{\underset{\parallel}{V(x+t, x-t)}} = \underline{B(x+t) + \varphi(x-t)}$

$$u_t = \frac{\partial u}{\partial t} = \frac{\partial V}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial V}{\partial \eta}\frac{\partial \eta}{\partial t} = B'(\xi) + \varphi'(\eta)(-1)$$

$$u_t(0, x) = V(x, x) = B'(x) - \varphi'(x) = g(x) \rightarrow \int g(x)dx = B(x) - \varphi(x) + C_1$$

$$u(0, x) = \cancel{u} V(x, x) = B(x) + \varphi(x) = f(x)$$

$B(x) = \frac{1}{2}\left(\int_0^x g(x) + f(x) - \frac{1}{2}C_1\right)$ $\qquad u(t, x) = V(x+t, x-t)$

$$\varphi(x) = \frac{1}{2}\left(f(x) - \int_0^x g(x) + \frac{1}{2}C_1\right)$$

$$\therefore u(t, x) = \frac{1}{2}\left(f(x+t) + f(x-t) + \int_{x-t}^{x+t} g(w)dw\right) _{//}$$

(3) $t = 1$, $|x| \geq 2$, $u(1, x) = 0$.

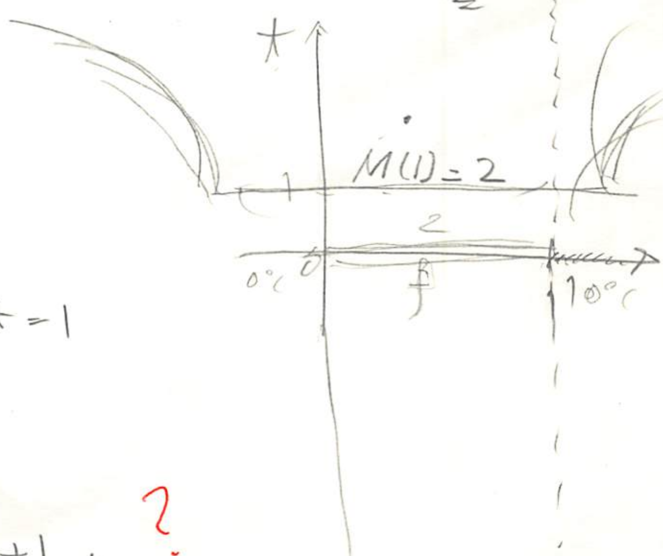
$$u(1, x) = \frac{1}{2}\left(\underset{0}{\underline{f(x+1)}} + \underset{0}{\underline{f(x-1)}} + \int_{x-1}^{x+1} g(w)dw\right)$$

$x+1$ $|x| \geq 2$ ならば、

$x + 1 \geq 3$

$1 \leq 2 - 1 \leq |x| - 1 = ||x| - 1| = ||x| - |1|| \leq |x - 1| \leq |x| + |1|$

$$= \int_{x-1}^{x+1} \cancel{g(w)}dw = 0$$



(4) $|x| \geq t + 1$, $\geq t$ $|x| \geq t$

$$u(t, x) = \frac{1}{2}(\cdots\cdots)$$

$$|x + t| \geq ||x| - |t|| = ||x| - t| = |x| - t \geq (t+1) - t = 1$$

$|x - t| \geq ||x| - |t|| = 1$ より、

$$u(t, x) = 0 _{//}$$

$$\therefore M(t) = t + 1 _{//}$$

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