H24 問7

無関係な2つの部分問題を解く.

$$\min \max\{\underbrace{y_1 - y_2}_{=k_1}, \underbrace{3y_1 - 2y_2}_{=k_2}) = y_4$$

(P1) s.t. $\frac{3}{2}y_1 + y_2 \leq 3$

$\frac{1}{2}y_1 + y_2 \geq -1$

$-2y_1 + y_2 \leq \frac{19}{4}$

$y_2 \geq 0,\ y_1 \leq 0$

$\underbrace{(-y_1)}_{= y_5} \geq 0$

$\longrightarrow$ min y_4

$y_1 - y_2 \leq y_4$

$3y_1 - 2y_2 \leq y_4$

他は同じ

× $y_4 \geq 0$

$\xrightarrow{y_5 = -y_1}$ min y_4

s.t. $-y_5 - y_2 \leq y_4$

$-3y_5 - 2y_2 \leq y_4$

$-\frac{3}{2}y_5 + y_2 \leq 3$

$-\frac{1}{2}y_5 + y_2 \geq -1$

$2y_5 + y_2 \leq \frac{19}{4}$

$y_1, \cdots y_5 \geq 0$

max min $(-k_1, -k_2)$

min $y_4^+ - y_4^-$

s.t. $y_6 = 0 + y_2 + y_4^+ - y_4^- + y_5$

$y_7 = 0 + 2y_2 + y_4^+ - y_4^- + 3y_5$

$y_8 = 3 - y_2 + \frac{3}{2}y_5$

$y_9 = 1 + y_2 - \frac{1}{2}y_5$

$y_{10} = \frac{19}{4} - y_2 - 2y_5$

$y_i \geq 0 \quad (i = 2, \cdots, 10)$

シンプレックスきつい $\xleftarrow{y_4^- \leftrightarrow}$

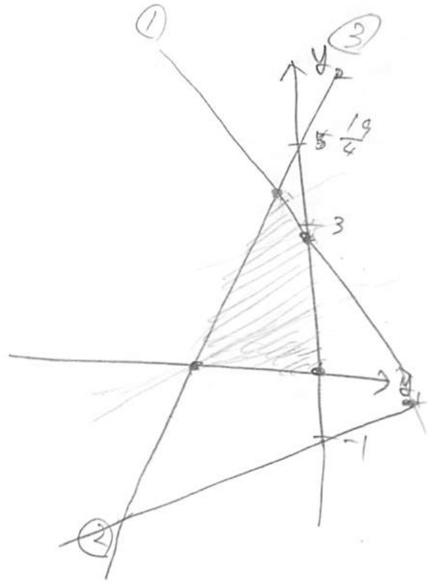
(P1)

s.t. $y_2 \leq -\frac{3}{2}y_1 + 3$ — ①

$y_2 \geq \frac{1}{2}y_1 - 1$ — ②

$y_2 \leq 2y_1 + \frac{19}{4}$ — ③

max (k_1, k_2)



①, ②

$0 = 2y_1 - 4$, $y_1 = 2$, $y_2 = -3 + 3 = 0$

① ③

$0 = \frac{7}{2}y_1 + \frac{7}{4}$, $y_1 = -\frac{7}{4} \cdot \frac{2}{7} = -\frac{1}{2}$, $y_2 = \frac{3}{4} + 3 = \frac{15}{4}$

② ③

$0 = \frac{3}{2}y_1 + \frac{23}{4}$, $y_1 = -\frac{23}{4} \times \frac{2}{3} = -\frac{23}{6}$, $y_2 = -\frac{23}{12} - 1 = -\frac{35}{12}$

$y_2 = y_1 - k_1$, $y_2 = \frac{3}{2}y_1 - \frac{k_2}{2}$

ニーほぼ端点 $(y_1, y_2) \to (k_1, k_2)$

$(0, 0) \to (0, 0)$

$-\frac{3}{2} - \frac{15}{2}$

$\underset{※}{\underline{(-\frac{1}{2}, \frac{15}{4})}} \to (-\frac{17}{4}, -9) \longrightarrow \underline{-9}_{/\!/}$

max は $-\frac{17}{4}$

$\cancel{4)}$ $\min \frac{1}{3}x_1 - x_2 - \frac{5}{12}x_3$

s.t. $x_1 + x_2 + \frac{1}{3}x_3 \le \frac{1}{5}$

$x_i \ge 0 \ (\forall i)$

$\rightarrow$ $\min \frac{1}{3}x_1 - x_2 - \frac{5}{12}x_3$

$x_4 = \frac{1}{5} - x_1 - x_2 - \frac{1}{3}x_3$

$x_2 \longleftrightarrow x_4$

$x_2 = \frac{1}{5} - x_1 - x_4 - \frac{1}{3}x_3$

$\min \ \frac{1}{3}x_1 - \frac{1}{5} + x_1 + x_4 + \underset{\frac{4}{12}}{\frac{1}{3}}x_3 - \frac{5}{12}x_3 = -\frac{1}{5} + \frac{4}{3}x_1 + x_4 - \frac{1}{12}x_3$

$x_3 \longleftrightarrow x_2$

$x_3 = \frac{3}{5} - 3x_1 - 3x_4 - 3x_2$

$\min \ \cancel{\frac{1}{5} + \frac{2}{3}} \ -\frac{1}{5} + \frac{4}{3}x_1 + x_4 + \left(-\frac{1}{20} + \frac{1}{4}x_1 + \frac{1}{4}x_4 + \frac{1}{4}x_2\right)$

$= -\frac{5}{20} + \frac{19}{12}x_1 + \frac{5}{4}x_4 + \frac{1}{4}x_2$

$\vee \quad \vee \quad \vee$

$0 \quad 0 \quad 0$

$-\frac{17}{4} = -\frac{18}{4} = -\frac{9}{2}$

$\therefore$ Opt $-\frac{\cancel{5}\,1}{\cancel{20}\,4} + (-9) = -\frac{37}{4}$

$(x_1, x_2, x_3, y_1, y_2) = (0, 0, \frac{3}{5}, -\frac{1}{2}, \frac{15}{4})$ //