H20 問9

(1) $L(\theta) = \prod_{i=1}^{n} P(X_i = x_i) \prod_{j=1}^{n} P(Y_i = y_i)$

$= \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i} \prod_{j=1}^{n} \binom{m}{y_i} \theta^{2y_i} (1-\theta^2)^{m-y_i}$

$= \left\{ \prod_{i=1}^{n} \binom{m}{x_i} \right\} \theta^{\sum x_i} (1-\theta)^{nm - \sum x_i} \left\{ \prod_{j=1}^{n} \binom{m}{y_j} \right\} \theta^{2\sum y_j} (1-\theta^2)^{mn - \sum y_j}$

$= \left\{ \prod_{i=1}^{n} \binom{m}{x_i} \binom{m}{y_i} \right\} \theta^{n\bar{x} + 2n\bar{y}} (1-\theta)^{nm - n\bar{x}} (1-\theta^2)^{nm - n\bar{y}}$ //

(2)

$l(\theta) \triangleq \log L(\theta) = \log\{ \cdots \} + \cdots\cdots$

$l'(\theta) = \cancel{n}(\bar{x} + 2\bar{y}) \cdot \frac{1}{\theta} + \cancel{n}(m - \bar{x}) \frac{-1}{1-\theta} + \cancel{n}(m - \bar{y}) \cdot \frac{-2\theta}{1-\theta^2} = 0$ より

$\times \theta(1-\theta)(1+\theta)$ $\quad (1-\theta)(1+\theta)$

$(\bar{x} + 2\bar{y})(1-\theta^2) + (m - \bar{x})(-1)(\theta + \theta^2) + (m - \bar{y})(-2)\theta^2 = 0$

$\theta^2 \{ -\cancel{\bar{x}} - \cancel{2\bar{y}} - m + \cancel{\bar{x}} - 2m + \cancel{2\bar{y}} \} + \theta \{ -m + \bar{x} \} + \{ \bar{x} + 2\bar{y} \} = 0$

$-3m\theta^2 - (m - \bar{x})\theta + (\bar{x} + 2\bar{y}) = 0$

$3m\theta^2 + (m - \bar{x})\theta - (\bar{x} + 2\bar{y}) = 0$

$\theta = \frac{-(m-\bar{x}) \pm \sqrt{(m-\bar{x})^2 + 12m(\bar{x} + 2\bar{y})}}{6m}$ $\quad \frac{-b \pm \sqrt{\ }}{2a}$

$1 \geq \theta \geq 0$ より、

$\hat{\theta} = \frac{\bar{x} - m + \sqrt{m^2 + 10m\bar{x} + \bar{x}^2 + 24m\bar{y}}}{6m}$ //