H26問7

(1) $\min\ (1+\theta)x_1 - 4x_2 + x_3$

(P_θ) s.t. $3x_1 + x_2 \le 3$

$5x_1 + 2x_2 - 4 \le x_3$

$7x_1 + 3x_2 - 8 \le x_3$

$0 \le x_3$

$x_1, x_2 \ge 0$

$\longrightarrow (P_0)$

$$\begin{array}{llll}\min & 1x_1 - 4x_2 + x_3 & & \\ \text{s.t.} & 3x_1 + x_2 & \le 3 & y_1 \\ & 5x_1 + 2x_2 - x_3 & \le 4 & y_2 \\ & 7x_1 + 3x_2 - x_3 & \le 8 & y_3 \\ & x_1, x_2, x_3 \ge 0 & & \end{array}$$

(2) (P_0)で $\min \to \max$ として最後に-1倍する。

minのままやっている。

max $Z = 1x_1 - 4x_2 + x_3$

$x_4 = 3 - 3x_1 - x_2$ 　1

$x_5 = 4 - 5x_1 - 2x_2 + x_3$ 　$\frac{4}{5}$

$x_6 = 8 - 7x_1 - 3x_2 + x_3$ 　$\frac{8}{7}$

$x_1 \leftrightarrow x_5$

$x_1 = \frac{4}{5} - \frac{1}{5}x_5 - \frac{2}{5}x_2 + \frac{1}{5}x_3$

$x_3 \ge -4$ ×

$\frac{4}{5} + \frac{1}{5}x_3 \ge 0$ ○

$Z = \frac{4}{5} - \frac{1}{5}x_5 - \frac{22}{5}x_2 + \frac{6}{5}x_3$

$x_4 = 3 + \left(-\frac{12}{5} + \frac{3}{5}x_5 + \frac{6}{5}x_2 - \frac{3}{5}x_3\right)$

$= \frac{3}{5} + \frac{3}{5}x_5 + \frac{6}{5}x_2 - \frac{3}{5}x_3$ 　1

$x_6 = 8 + \left(-\frac{28}{5} + \frac{7}{5}x_5 + \frac{14}{5}x_2 - \frac{7}{5}x_3\right)$

$= \frac{12}{5} + \frac{7}{5}x_5 + \frac{14}{5}x_2 - \frac{7}{5}x_3$ 　$\frac{12}{7}$

$x_3 \longleftrightarrow x_1$

$x_3 = -4 + x_5 + 2x_2 - 5x_1$

$Z = \frac{4}{5} - \frac{1}{5}x_5 - \frac{22}{5}x_2 + \left(-\frac{24}{5} + \frac{6}{5}x_5 + \frac{12}{5}x_2 - 6x_1\right)$ ×

$\cancel{x_3} \leftrightarrow x_4$

$x_3 = 1 + x_5 + 2x_2 - \frac{5}{3}x_4$

$Z = \frac{4}{5} - \frac{1}{5}x_5 - \frac{22}{5}x_2$

$+ \left(\frac{6}{5} + \frac{6}{5}x_5 + \frac{12}{5}x_2 - 2x_4\right) =$

$= 2 + x_5 - 2x_2 - 2x_4$

$x_1 = \frac{4}{5} - \frac{1}{5}x_5 - \frac{2}{5}x_2$

$+ \left(\frac{1}{5} + \frac{1}{5}x_5 + \frac{2}{5}x_2 - \frac{1}{3}x_4\right)$

$= 1 - \frac{1}{3}x_4$

$x_6 = \frac{12}{5} + \frac{7}{5}x_5 + \frac{14}{5}x_2$

$+ \left(-\frac{7}{5} - \frac{7}{5}x_5 - \frac{14}{5}x_2 + \frac{7}{3}x_3\right)$

$= 1 + \frac{7}{3}x_3$

$-1-\frac{5}{2}\cdot 4$

(2)

$\max\ Z = -x_1+4x_2-x_3$

$\text{s.t.}\ x_4 = 3-3x_1-x_2$ 3

$x_5 = 4-5x_1-2x_2+x_3$ ②

$x_6 = 8-7x_1-3x_2+x_3$ $\frac{8}{3}$

$x_2 \longleftrightarrow x_5$

$x_2 = 2-\frac{5}{2}x_1-\frac{1}{2}x_5+\frac{1}{2}x_3$

$Z = 8-11x_1-2x_5+x_3$

$x_4 = 1-\frac{1}{2}x_1+\frac{1}{2}x_5-\frac{1}{2}x_3$

$x_6 = 2+\frac{1}{2}x_1+\frac{3}{2}x_5-\frac{1}{2}x_3$

$x_3 \longleftrightarrow x_4$

$x_3 = 2-x_1+x_5-2x_4$

$Z = 10-12x_1-x_5-2x_4$

$x_2 = 3-3x_1-x_4$

$x_6 = 1+x_1+x_5+x_4$

(minでは−10)

$Z^* = 10,\ (x_1^*, x_2^*) = \underline{(0,3)}$ $x_3^* = 2$.

(3) $\max\ 3y_1+4y_2+8y_3$

(D_θ) s.t. $3y_1+5y_2+7y_3 \geq 1+\theta$ x_1

$y_1+2y_2+3y_3 \geq -4$ x_2

$-y_2-y_3 \geq 1$ x_3

$y_i \geq 0$

(D_0)と(D_θ)でも最適解は同じになるので、相補性より

$x_2^* \neq \cancel{3}0 \rightarrow y_1+2y_2+3y_3 = -4$ $y_1^* = -2$.

$y_3^* = 0$

$x_3^* \neq \cancel{3}0 \rightarrow -y_2-y_3 = 1$ $y_2^* = -1$

$\therefore\ -6-5 \geq 1+\theta,\quad -12 \geq \theta \iff$ 最適解のまま. //