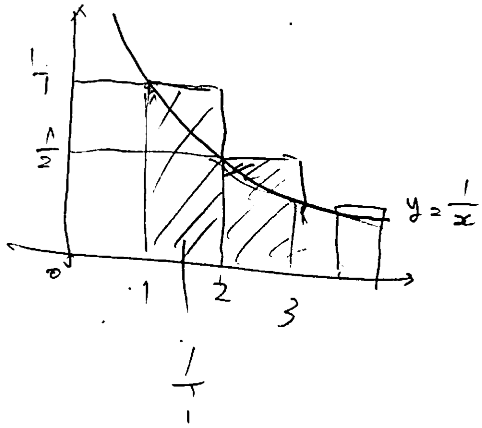
H19問2.

(1) $\sum_{i=1}^{\infty}\frac{1}{i} > \int_1^{\infty}\frac{1}{x}dx = [\log x] = \infty$



(2)

$a = \frac{S_1}{1} + \frac{S_2}{2} + \cdots + \frac{S_4}{n} + \cdots + \cdots$

$S_n \triangleq ?? \; S_{n-1} \; S_n$

$a=1 = \frac{1}{1} + \frac{0}{2} + \frac{0}{3} + \cdots$

$a=2 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} - \frac{1}{8} \cdots$

$a=\sqrt{2}$. 1, 0.5, 0.33, 0.25, 0.2, 0.16

1.83, 2.02, 1.82, 1.94

$a=3=$

$S_{n+1} \triangleq \left(\sum_{i=1}^{n}\frac{S_n}{i}\text{が、}a \leq \cdot \Rightarrow S_{n+1}=+1,\ \text{o.w} \Rightarrow -1\right)$ //

$a>0$としても一般性

これで、$\sum_{i=1}^{\infty}\frac{S_i}{i} = \lim_{n\to\infty}\sum_{i=1}^{n} = a$を示す。

$\sum_{i=1}^{\infty}\frac{1}{i} = +\infty$より、$\sum_{i=1}^{\infty}\frac{S_i}{i} = -\infty, +\infty$をとれる $\Rightarrow \exists k_0, \sum_{i=1}^{k_0}\frac{1}{i} > a$ となるmin $N_{\ldots}$

$\sum_{i=1}^{k_0}\frac{S_i}{i} + \frac{S_{k_0+1}}{k_0+1}$ $\exists k_1, \sum_{k=k_0+1}^{k_1}\frac{S_i}{i} < a$.

$\forall \varepsilon > 0$,

$\exists k_0, \sum_{i=1}^{k_0}\frac{1}{i} > a$

$\exists k_1, \sum_{i=1}^{k_0}\frac{1}{i} - \sum_{i=k_0}^{k_1}\frac{1}{i} < a$ (∵ どこから初めても発散するから).

$\exists k_2, \sum_{i=1}^{k_0}\frac{1}{i} - \sum_{i=k_0}^{k_1}\frac{1}{i} + \sum_{i=k_1}^{k_2}\frac{1}{i} > a$.

しかも、$\left|\sum_{i=1}^{k_0}\frac{1}{i} - a\right| > \left|\left(\sum_{i=1}^{k_0}\frac{1}{i} - \sum_{i=k_0}^{k_1}\right) - a\right|$ (∵ $\left|\sum^{k_0}\right| > \left|\sum_{k_0}^{k_1}\right|$)

$> \left|\sum^{k_0} - \sum^{k_1} + \sum^{k_2} - a\right|$ //