H20-9

(1)

$$L(\theta)=\prod_{i=1}^{n}P(X_i=x_i)P(Y_i=y_i)$$

$$=\prod_{i=1}^{n}\binom{m}{x_i}\theta^{x_i}(1-\theta)^{m-x_i}\binom{m}{y_i}\theta^{2y_i}(1-\theta^2)^{m-y_i}$$

$$=\left\{\prod_{i=1}^{n}\binom{m}{x_i}\binom{m}{y_i}\right\}\theta^{n\bar{x}}(1-\theta)^{nm-n\bar{x}}\ \theta^{2n\bar{y}}(1-\theta^2)^{nm-n\bar{y}}$$

$$=\left\{\prod_{i=1}^{n}\binom{m}{x_i}\binom{m}{y_i}\right\}\theta^{n(\bar{x}+2\bar{y})}(1-\theta)^{n(m-\bar{x})}(1-\theta^2)^{n(m-\bar{y})}$$

(2)

$\dfrac{\partial \log L}{\partial \theta}=0$ より、

$$n(\bar{x}+2\bar{y})\frac{1}{\theta}+n(m-\bar{x})\frac{-1}{1-\theta}+n(m-\bar{y})\frac{-2\theta}{1-\theta^2}=0$$ より、

$$3m\theta^2+(m-\bar{x})\theta-(\bar{x}+2\bar{y})=0$$

$0\leq\theta\leq 1$ より、

$$\hat{\theta}=\frac{1}{6m}\left(\bar{x}-m+\sqrt{m^2+10m\bar{x}+\bar{x}^2+24m\bar{y}}\right)$$