Contingency tables creation examples

Anton Antonov MathematicaForPrediction at GitHub MathematicaVsR project at GitHub October, 2016

Introduction

In statistics contingency tables are matrices used to show the co-occurrence of variable values of multidimensional data. They are fundamental in many types of research. This document shows how to use several functions implemented in *Mathematica* for the construction of contingency tables.

Code

In this document we are going to use the implementations in the package MathematicaForPredictionUtilities.m from MathematicaForPrediction at GitHub, [1].

```
In[1]:= Import[
```

"https://raw.githubusercontent.com/antononcube/MathematicaForPrediction/master/ MathematicaForPredictionUtilities.m"]

The implementation of CrossTabulate in [1] is based on Tally, GatherBy, and SparseArray. The implementation of xtabsViaRLink in [1] is based on R's function xtabs called via RLink.

Other package used in this document are [2] and [4] imported with these commands:

```
Import[
    "https://raw.githubusercontent.com/antononcube/MathematicaForPrediction/master/
    MosaicPlot.m"]
Import[
    "https://raw.githubusercontent.com/antononcube/MathematicaForPrediction/master/
    Misc/RSparseMatrix.m"]
```

For a different approach to implementing cross-tabulation than those taken in [1] see the Stack Over-flow answer http://stackoverflow.com/a/8101951 by Mr.Wizard.

Using Titanic data

Getting data

```
in[4]:= titanicData = Flatten@*List@@@ ExampleData[{"MachineLearning", "Titanic"}, "Data"];
titanicData = DeleteCases[titanicData, {___, _Missing, ___}];
```

```
In[6]:= titanicColumnNames = Flatten@*List@@
       ExampleData[{"MachineLearning", "Titanic"}, "VariableDescriptions"];
   aTitanicColumnNames = AssociationThread[
       titanicColumnNames → Range[Length[titanicColumnNames]]];
```

Note that we have removed the records with missing data (for simpler exposition).

Data summary

```
In[8]:= Dimensions[titanicData]
Out[8]= \{1046, 4\}
In[9]:= RecordsSummary[titanicData, titanicColumnNames]
                     2 passenger age
                     Min
                             0.1667
      1 passenger class
                     1st Qu 21.
                                        3 passenger sex
                                                      4 passenger survival
      3rd 501
                     Median 28.
                                      , male
                                                658 , died
                                                                 619
      1st 284
                     Mean
                             29.8811 female 388
                                                      survived 427
      2nd 261
                     3rd Qu 39.
                     Max
                             80.
```

Using CrossTabulate

Assume that we want to group the people according to their passenger class and survival and we want to find the average age for each group.

We can do that by

- 1. finding the counts contingency table C for the variables "passenger class" and "passenger survival",
- **2.** finding the age contingency table *A* for the same variables, and
- **3.** do the element-wise division A/C.

```
In[10]:= ctCounts = CrossTabulate[titanicData[All,
         aTitanicColumnNames /@ {"passenger class", "passenger survival"}]];
    MatrixForm[#1, TableHeadings → {#2, #3}] &@@ctCounts
```

Out[11]//MatrixForm=

```
|died survived
1st 103
            181
2nd 146
            115
3rd | 370
```

```
In[12]:= ctTotalAge = CrossTabulate[titanicData[All, aTitanicColumnNames /@
          {"passenger class", "passenger survival", "passenger age"}]];
    MatrixForm[#1, TableHeadings → {#2, #3}] &@@ctTotalAge
```

Out[13]//MatrixForm=

```
survived
     died
1st 4454.5 6666.92
2nd 4842.5 2858.75
3rd 9610.58 2822.42
```

In[14]:= MatrixForm[ctTotalAge[[1]] / Normal[ctCounts[[1]]], TableHeadings → Values[Rest[ctTotalAge]]]

Out[14]//MatrixForm=

(We have to make the sparse array ctCounts a regular array because otherwise we get warnings for division by zero because of the sparse array's default value.)

Let us repeat the steps above by dividing the passengers before-hand according to their sex.

```
In[15]:= Association@
      Map[
        (mCount = CrossTabulate[
            #[[All, aTitanicColumnNames/@{"passenger class", "passenger survival"}]];
          mAge = CrossTabulate[#[All, aTitanicColumnNames /@
               {"passenger class", "passenger survival", "passenger age"}]];
          #[1, aTitanicColumnNames["passenger sex"]] -> MatrixForm[
            mAge[1] / Normal[mCount[1]], TableHeadings → Values[Rest[mAge]]]) &,
       GatherBy[titanicData, #[aTitanicColumnNames["passenger sex"]] &]]
                  1st
                                                    1st 43.6582 36.1682
                                          \mid , male \rightarrow
Out[15]= \langle | female \rightarrow \rangle
                  2nd 34.0909 26.7111
                                                     2nd 33.0926 17.4493
```

Using R's xtabs (via RLink)

2nd 146.

3rd 370.

115.

131.

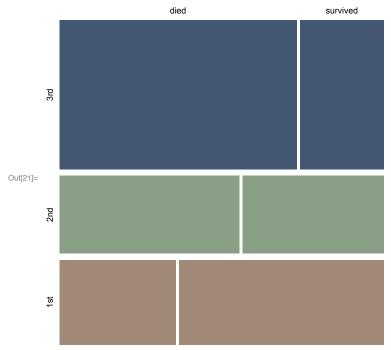
The alternative of CrossTabulate is xtabsViaRLink that is uses R's function xtabs via RLink.

```
In[16]:= Needs["RLink`"]
      RLinkResourcesInstall[]
      InstallR[]
 Out[17]= {Paclet[RLinkRuntime, 9.0.0.0, <> ]}
 In[19]:= ctCounts = FromRXTabsForm@xtabsViaRLink[titanicData[All,
            aTitanicColumnNames /@ {"passenger class", "passenger survival"}],
           {"passenger.sex", "passenger.survival"},
           " ~ passenger.sex + passenger.survival"];
      MatrixForm[#1, TableHeadings → {#2, #3}] &@@ctCounts
Out[20]//MatrixForm=
            died survived
        1st 103.
                    181.
```

Relation to mosaic plots

The graphical visualization of a dataset with mosaic plots, [2,3], is similar in spirit to contingency tables. Compare the following mosaic plot with the contingency table in the last section.

In[21]:= MosaicPlot[titanicData[All, aTitanicColumnNames /@ {"passenger class", "passenger survival"}]]



Straightforward calling of MatrixForm

At this point we might want to be able to call MatrixForm more directly for the output of CrossTabulate and FromRXTabsForm. One way to do this is to define an up-value for Association.

```
In[22]:= Unprotect[Association];
    MatrixForm[
        x_Association /; (KeyExistsQ[x, "XTABMatrix"] || KeyExistsQ[x, "XTABTensor"])] ^:=
       (MatrixForm[#1, TableHeadings → Rest[{##}]] &@@ x);
    Protect[Association];
    Now we can do this:
```

```
In[25]:= MatrixForm@CrossTabulate[titanicData[All,
        aTitanicColumnNames /@ {"passenger class", "passenger survival"}]]
```

Out[25]//MatrixForm=

(died	survived	
1st	103	181	
2nd	146	115	
3rd	370	131	

Remark: Because of this up-value definition for Association with MatrixForm we have the associations returned by CrossTabulate and FromRXTabsForm to have the key "XTABMatrix" instead of "Matrix", the former assumed to be much more rarely to be used than the latter.

Using larger data

Let us consider an example with larger data that has larger number of unique values in its columns.

Getting online retail invoices data

The following dataset is taken from [6].

```
In[26]:= data = Import[ "/Volumes/WhiteSlimSeagate/Datasets/UCI
          Online Retail Data Set/Online Retail.csv"];
     columnNames = First[data];
    data = Rest[data];
In[29]:= aColumnNames = AssociationThread[columnNames → Range[Length[columnNames]]];
```

Data summary

We have ≈ 66 000 rows and 8 columns:

```
In[30]:= Dimensions[data]
Out[30]= \{65499, 8\}
```

Here is a summary of the columns:

IN[31]:= Magnify[#, 0.75] &@RecordsSummary[data, columnNames]

```
2 StockCode
              1 InvoiceNo
                                                                       3 Description
              537 434 675 85123A 353 WHITE HANGING HEART T-LIGHT HOLDER 358
              538 071 652 22 423 279 REGENCY CAKESTAND 3 TIER 278
224
                                                                                                                                    213
207
                                                                      SCOTTIE DOG HOT WATER BOTTLE
JUMBO BAG RED RETROSPOT
              536 876 593
                                                                                                                                    205
               (Other) 61761 (Other) 64018 (Other)
                                                                                                                                     64 014

    4 Quantity
    5 InvoiceDate
    6 UnitPrice
    7 CustomerID
    8 Country

    Min
    -74 215
    12/6/10 16:57 675
    Min
    0
    25 281
    United Kingdom 61 186

    1st Qu
    1
    12/9/10 14:09 652
    1st Qu
    1.25
    12 748 695
    Germany
    982

    Median
    2
    12/10/10 14:59 621
    Median
    2.51
    17 841 481
    France
    967

    Median 2
    12/10/10 14:59 621
    Median 2.51
    17.841 481
    France

    3rd Qu 8
    12/7/10 15:28 601
    Median 2.51
    14606 421
    EIRE

    Mean 182 660 21833
    12/3/10 11:36 593
    Mean 5.85759
    15311 418
    Spain 14911 377

    Max 74 215
    (Other)
    61 760

                                                                                                                                                                      504
                                                                                                                                                                       355
                                                                                                                                                                     1293
```

Contingency tables

xtabsViaRLink:

Country vs. StockCode

There is no one-to-one correspondence between the values of the column "Description" and the column "StockCode" which can be seen with this command:

```
In[32]:= MinMax@Map[Length@*Union,
        GatherBy[data[All, aColumnNames /@ {"Description", "StockCode"}]], First]]
Out[32]= \{1, 144\}
     The way in which the column "StockCode" was ingested made it have multiple types for its values:
In[33]:= Tally[NumberQ/@data[All, aColumnNames["StockCode"]]]]
Out[33]= { {False, 9009}, {True, 56490}}
     So let us convert it to all strings:
In[34]:= data[All, aColumnNames["StockCode"]] =
        ToString /@ data[All, aColumnNames["StockCode"]];
     Here we find the contingency table for "Country" and "StockCode" over "Quantity" using
     CrossTabulate:
In[35]:= AbsoluteTiming[
       ctRes =
         CrossTabulate[data[All, aColumnNames /@ {"Country", "StockCode", "Quantity"}]];
     1
Out[35]= { 0.272178, Null}
```

Here we find the contingency table for "Country" and "StockCode" over "Quantity" using

```
In[36]:= AbsoluteTiming[rres =
         xtabsViaRLink[data[All, aColumnNames /@ {"Country", "StockCode", "Quantity"}],
           {"Country", "StockCode", "Quantity"}, "Quantity ~ Country + StockCode"];
        ctRRes = FromRXTabsForm[rres];
      1
Out[36] = \{0.912233, Null\}
      Both functions produce the same result:
In[37]:= ctRRes["XTABMatrix"] == N@ctRRes[[1]]
Out[37]= True
      Note that xtabsViaRLink is slower but still fairly quick.
      Here we plot the contingency table using MatrixPlot:
In[38]:= MatrixPlot[ctRRes["XTABMatrix"], AspectRatio → 1 / 1.5, FrameTicks →
         {{#, #} &@Table[{i, ctRRes["RowNames"][i]}}, {i, Length[ctRRes["RowNames"]]}],
           {Automatic, Automatic}}, ImageSize → 550]
                                         1000
           Australia
                                                                                          Australia
            Austria
                                                                                          Austria
            Bahrain
                                                                                          Bahrain
            Belgium
                                                                                          Belgium
      Channel Islands
                                                                                          Channel Islands
            Cyprus
                                                                                          Cyprus
           Denmark
                                                                                          Denmark
              EIRE
                                                                                          EIRE
            Finland
                                                                                          Finland
            France
                                                                                          France
           Germany
                                                                                          Germany
            Iceland
                                                                                          Iceland
Out[38]=
              Israel
                                                                                          Israel
              Italy
                                                                                          Italy
             Japan
                                                                                          Japan
           Lithuania
                                                                                          Lithuania
         Netherlands
                                                                                          Netherlands
            Norway
                                                                                          Norway
            Poland
                                                                                          Poland
            Portugal
                                                                                          Portugal
             Spain
                                                                                          Spain
            Sweden
                                                                                          Sweden
         Switzerland
                                                                                          Switzerland
      United Kingdom
                                                                                          United Kingdom
                                                                2000
                                         1000
```

Country vs. Quarter

Let us extend the data with columns that have months and quarters corresponding to the invoice dates.

The following commands computing date objects and extracting month and quarter values from them are too slow.

```
In[39]:= (*AbsoluteTiming[
     dobjs=DateObject[{#,{"Month","/","Day","/","Year"," ","Hour",":","Minute"}}]&/@
         data[All,aColumnNames["InvoiceDate"]];
    ]*)
```

```
In[40]:= (*AbsoluteTiming[
      dvals=DateValue[dobjs,{"MonthName","QuarterNameShort"}];
    ]*)
```

We can use the following ad hoc computation instead.

```
In[41]:= dvals = StringSplit[#, {"/", " ", ":"}] & /@ data[All, aColumnNames["InvoiceDate"]];
     This summary shows that the second value in the dates is day of month, and the first value is most
     likely month.
```

```
log(42):= Magnify[#, 0.75] &@RecordsSummary[dvals[All, 1;; 3]], "MaxTallies" \rightarrow 16]
```

```
2 column 2
                            5710
                    6
                   17
                            5672
                   7
                            4757
                   10
                            4734
                   5
                            4468
                            4008
        1 column 1
                   13
                            3728
                                   3 column 3
                            3597 , 10 42481
Out[42]= { 12 42481 , 14
        1 23018
                   12
                            3260
                                    11 23 018
                            3108
                   20
                            2851
                   8
                            2647
                            2436
                   3
                            2202
                    2
                            2109
                    (Other) 10 212
```

These commands extend the data and the corresponding column-name-to-index association.

```
In[43]:= ms = DateValue[Table[DateObject[{2016, i, 1}], {i, 12}], "MonthName"];
    dvals =
       Map[{ms[#], "Q" <> ToString[Quotient[#, 4] + 1]} &, ToExpression@dvals[All, 1]];
     dataM = MapThread[Join[#1, #2] &, {data, dvals}];
     aColumnNamesM = Join[aColumnNames, <|"MonthName" → (Length[aColumnNames] + 1),
         "QuarterNameShort" → (Length[aColumnNames] + 2) |>];
```

Here is the contingency table for "Country" vs "QuarterNameShort" over "Quantity".

```
In[47]:= ctRes = CrossTabulate[
        dataM[All, aColumnNamesM /@ {"Country", "QuarterNameShort", "Quantity"}]];
    Magnify[#, 0.75] &@MatrixForm[#1, TableHeadings → {#2, #3}] &@@ ctRes
```

	(Q1	Q4	١
	Australia	5534	454	
	Austria	0	3	
	Bahrain	-54	54	
	Belgium	675	1755	
	Channel Islands	0	80	
	Cyprus	144	917	
	Denmark	0	454	
	EIRE	7796	5381	
	Finland	0	1254	
	France	6581	4978	
	Germany	4495	6723	
=	Iceland	0	319	
	Israel	100	-56	
	Italy	745	293	
	Japan	-45	4093	
	Lithuania	0	652	
	Netherlands	14 570	6811	
	Norway	0	3582	
	Poland	288	140	
	Portugal	1841	945	
	Spain	3091	867	
	Sweden	292	3714	
	Switzerland	2653	714	
	United Kingdom	157 046	298 101	,

Uniform tables

Often when making contingency tables over subsets of the data we obtain contingency tables with different rows and columns. For various reasons (programming, esthetics, comprehension) it is better to have the tables with the same rows and columns.

Here is an example of non-uniform contingency tables derived from the online retail data of the previous section. We split the data over the countries and find contingency tables of "MonthName" vs "QuarterNameShort" over "Quantity".

```
In[49]:= tbs = Association@Map[
          (xtab = CrossTabulate[
              #[All, aColumnNamesM /@ {"MonthName", "QuarterNameShort", "Quantity"}]];
            #[1, aColumnNamesM["Country"]] \rightarrow xtab) &,
         GatherBy[dataM, #[aColumnNamesM["Country"]] &]];
```

Out[48]=

$$| \text{United Kingdom} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad 0 \quad 298101 \\ \overline{\text{January}} \quad | 157046 \quad 0 \end{array} \right), \text{ France} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad 0 \quad 4978 \\ \overline{\text{January}} \quad | 6581 \quad 0 \end{array} \right),$$

$$| \text{Australia} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad 0 \quad 454 \\ \overline{\text{January}} \quad | 5534 \quad 0 \end{array} \right), \text{ Netherlands} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad 0 \quad 6811 \\ \overline{\text{January}} \quad | 14570 \quad 0 \end{array} \right),$$

$$| \text{Germany} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad 0 \quad 6723 \\ \overline{\text{January}} \quad | 4495 \quad 0 \end{array} \right), \text{ Norway} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 3582 \end{array} \right), \text{ EIRE} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 5381 \end{array} \right),$$

$$| \text{Switzerland} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{January}} \quad | 2653 \quad 0 \end{array} \right), \text{ Spain} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 867 \end{array} \right),$$

$$| \text{Poland} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{January}} \quad | 288 \quad 0 \end{array} \right), \text{ Portugal} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 945 \end{array} \right), \text{ Italy} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 293 \end{array} \right),$$

$$| \text{Belgium} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 1755 \end{array} \right), \text{ Lithuania} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 4093 \end{array} \right),$$

$$| \text{Locland} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 1755 \end{array} \right), \text{ Channel Islands} \rightarrow \left(\begin{array}{c} | \text{Q4} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 3714 \end{array} \right), \text{ Austria} \rightarrow \left(\begin{array}{c} | \text{Q4} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 454 \end{array} \right),$$

$$| \text{Cyprus} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 917 \end{array} \right), \text{ Sweden} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 3714 \end{array} \right), \text{ Austria} \rightarrow \left(\begin{array}{c} | \text{Q4} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 544 \end{array} \right),$$

$$| \text{Israel} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 556 \\ \overline{\text{January}} \quad | 144 \quad 0 \end{array} \right), \text{ Finland} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 1254 \end{array} \right), \text{ Bahrain} \rightarrow \left(\begin{array}{c} | \text{Q1} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 54 \end{array} \right), \text{ Austria} \rightarrow \left(\begin{array}{c} | \text{Q4} \quad \text{Q4} \\ \overline{\text{December}} \quad | 0 \quad 54 \end{array} \right),$$

Using the object RSparseMatrix, see [4,5], we can impose row and column names on each table. First we convert the contingency tables into RSparseMatrix objects:

And then we impose the desired row and column names:

In[52]:= tbs2 = Map[ImposeColumnNames[

ImposeRowNames[#, {"January", "December"}], {"Q1", "Q2", "Q3", "Q4"}] &, tbs2]; Magnify[#, 0.75] &@ (MatrixForm /@ tbs2)

Generalization: CrossTensorate

A generalization of CrossTabulate is the function CrossTensorate implemented in [1] that takes a "formula" argument similar to R's xtabs.

This finds number of people of different sub-groups of Titanic data:

```
In[54]:= ctRes =
       CrossTensorate[Count == "passenger survival" + "passenger sex" + "passenger class",
        titanicData, aTitanicColumnNames];
    MatrixForm[
      ctRes]
```

Out[55]//MatrixForm=

(female		male	
		/1st	5 \	/1st	98 \
	died	2nd	L1	2nd	135
		3rd 8	30	3rd	290
	survived	(1st 1	28 \	(1st	53 \
		2nd 9	92	2nd	23
		3rd	72	3rd	59

We can verify the result using Count:

```
In[56]:= Count[titanicData, {"1st", , "female", "died"}]
Out[56]= 5
In[57]:= Count[titanicData, {"2nd", _, "male", "survived"}]
Out[57]= 23
```

To split the cross-tensor across its first variable we can use this command:

```
In[60]:= sctRes =
       Association@MapThread[Rule[#1, Join[<|"XTABTensor" → #2|>, Rest@Rest@ctRes]] &,
         {ctRes[[2]], # & /@ctRes["XTABTensor"]}];
    MatrixForm /@
     sctRes
```

$$\mathsf{Out}[\mathsf{61}]= \left\langle \left| \, \mathsf{died} \rightarrow \left(\begin{array}{c|ccc} & \mathsf{1st} & \mathsf{2nd} & \mathsf{3rd} \\ \hline \mathsf{female} & \mathsf{5} & \mathsf{11} & \mathsf{80} \\ \mathsf{male} & \mathsf{98} & \mathsf{135} & \mathsf{290} \end{array} \right), \, \mathsf{survived} \rightarrow \left(\begin{array}{c|ccc} & \mathsf{1st} & \mathsf{2nd} & \mathsf{3rd} \\ \hline \mathsf{female} & \mathsf{128} & \mathsf{92} & \mathsf{72} \\ \mathsf{male} & \mathsf{53} & \mathsf{23} & \mathsf{59} \end{array} \right) \, \right| \, \rangle$$

References

- [1] Anton Antonov, MathematicaForPrediction utilities, (2014), source code MathematicaForPrediction at GitHub, package MathematicaForPredictionUtilities.m.
- [2] Anton Antonov, Mosaic plot for data visualization implementation in Mathematica, (2014), MathematicaForPrediction at GitHub, package MosaicPlot.m.
- [3] Anton Antonov, "Mosaic plots for data visualization", (2014), MathematicaForPrediction at Word-Press blog. URL: https://mathematicaforprediction.wordpress.com/2014/03/17/mosaic-plots-for-datavisualization/.
- [4] Anton Antonov, RSparseMatrix Mathematica package, (2015) MathematicaForPrediction at GitHub. URL: https://github.com/antononcube/MathematicaForPrediction/blob/master/Misc/RSparseMatrix.m .
- [5] Anton Antonov, "RSparseMatrix for sparse matrices with named rows and columns", (2015), MathematicaForPrediction at WordPress blog. URL: https://mathematicaforprediction.wordpress.-

 $com/2015/10/08/r sparse matrix-for-sparse-matrices-with-named-rows-and-columns/\ .$

[6] Daqing Chen, Online Retail Data Set, (2015), UCI Machine Learning Repository. URL: https://archive.ics.uci.edu/ml/datasets/Online+Retail .