

# CMOS Transistor Small signal model

## What do you need to know to understand this topic?

- The MOSFET model
- Channel length modulation
- The body effect

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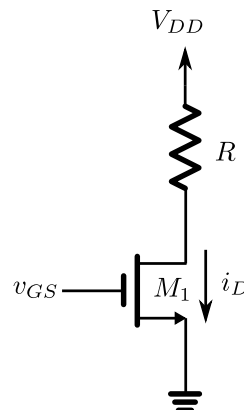
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## What is the small-signal model?

The CMOS transistor is normally used as an amplifier when it is working in the saturation region (it has a low dependence with  $V_{DS}$  and a high sensitivity to  $V_{GS}$ ). To characterize the transistor for that purpose, it is used a **model that considers that small signals are injected in the terminals of the transistor**. That model linearizes the  $i_D$  and  $v_{DS}$  curves around a point of operation, called the **bias or operating point**.

### The bias or operating point

Consider the following circuit:



A gate voltage  $v_{GS}$  is applied to the transistor. A current will be generated that will match the current that flows through the resistance. This will in turn generate a voltage drop across the resistance and set the drain-source voltage of the transistor. This will lead to a stable condition for the transistor, for which we call bias or operating point.

The slider represents  $R_S$ . As you decrease  $R_S$  (go left), the current sunk by the transistor has a small voltage drop at the resistance and the operating point goes to higher  $I_{DS}$ . If you increase  $R_S$  (go right), the voltage drop is higher and  $I_{DS}$  decreases. If the resistance was too large, it would put the transistor in triode region for this  $V_{GS}$ .



### Linearizing the curves

If a small signal  $v_{gs}$  is superimposed on a DC signal  $V_{GS}$  to produce  $v_{gs} + V_{GS}$  at the gate of the transistor, the DC signal sets the bias and the small-signal will swing around that bias point. The curves around the bias point are not linear and to get a simple model for small signals that can be analyzed, we approximate the curves to lines to get a linear model.

The linear model is based on taking the derivatives of the drain current with respect to the voltages at the terminals of the

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First let us find what components should make up this model. First we see that the transistor is a current source controlled by the gate voltage, so a voltage-controlled current source should be there. Next, we see that the current is also influenced by the drain-source voltage from the **channel length modulation**, which translates into an output resistance. There is no input resistance because there is no current flowing through the gate.

Now it is time to determine the values of the **transconductance** and the **output resistance**.

### Transconductance

is the change in current caused by a change in  $V_{GS}$ , so it is:

The equation for  $g_m$  depends whether the transistor is in triode or in saturation.

in triode	in saturation
$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{\partial}{\partial V_{GS}} \left( \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_{DS} \right)$	$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{\partial}{\partial V_{GS}} \left( \frac{\mu C_{ox} W}{2L} (V_{GS} - V_{TH})^2 \right)$

When in triode,  $g_m$  does not really depend on  $V_{DS}$  because  $I_{DS}$  is linear with  $V_{DS}$  in this region. However,  $V_{GS}$  affects the transconductance linearly. When in saturation,  $g_m$  depends on  $V_{GS}$  and it is a parameter that can be tweaked to control the gain of the transistor.

### Output resistance

is the inverse of a change in current caused by a change in the drain-source voltage, so it is:

$g_o$  is the output conductance and is commonly used in exchange of  $1/r_o$  when it simplifies the equations. The equation for  $g_o$  depends whether the transistor is in triode or in saturation.

in triode	in saturation
$g_o = \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}} \left( \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_{DS} \right)$	$g_o = \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}} \left( \frac{\mu C_{ox} W}{2L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \right)$

The output resistance is influenced both by  $V_{GS}$  and  $V_{DS}$  when the transistor is in triode. When in saturation, the output resistance is only defined by the bias current.

From here we see that the transistor is not a very good amplifier in the triode region, because its gain is changed with the output voltage and we can only play with the transistor's size to control the gain. It is not a very good current source either, because its gain (and therefore its current) and its output resistance changes with the output voltage. **The transistor is usually used in the saturation region for amplification.** Also, to reduce the distortion caused by the amplification, the bias point should be somewhere in the middle of the saturation region, so that large swings of the drain voltage will not bring the transistor into the triode region or saturate to the power supply voltage.

Include body effect

The **body effect** changes the threshold voltage, which in turn affects the drain current. How can this be incorporated in the model? The body is like another gate (it is sometimes called a back-gate) and it is modelled as just that:

Since  $V_{BS}$  is always negative, the current direction of the  $I_{BS}$  current source is actually the opposite of the  $I_{DS}$  current source. To calculate

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is expressed as:

$$\frac{\partial I_D}{\partial V_{BS}}$$

As in the other cases, the expression depends if the transistor is in triode or in saturation

in triode	in saturation
$\frac{\partial I_D}{\partial V_{BS}} = \frac{\partial}{\partial V_{BS}} \left( \frac{\mu C_{ox} W}{2L} (V_{GS} - V_{TH})^2 (V_{GS} - V_{DS}) \right)$	$\frac{\partial I_D}{\partial V_{BS}} = \frac{\partial}{\partial V_{BS}} \left( \frac{\mu C_{ox} W}{2L} (V_{GS} - V_{TH})^2 \right)$

Summary

The model has three parameters: the gate-source transconductance  $g_m$ , the bulk-source transconductance  $g_{bs}$  and the output resistance  $r_o$ . The expressions for these parameters are summarized in the following table:

Parameter	Triode	Saturation
$g_m$	$\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$	$\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$
$g_{bs}$	$-\mu C_{ox} \frac{W}{L} \frac{V_{GS} - V_{TH}}{2}$	$-\mu C_{ox} \frac{W}{L} \frac{V_{GS} - V_{TH}}{2}$
$r_o$	$\frac{1}{\lambda I_D}$	$\frac{1}{\lambda I_D}$

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