



THE GRAINGER COLLEGE OF ENGINEERING

CS 521

Technological Foundations of Blockchain and Cryptocurrency

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Topic 4 – Ethereum

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Bitcoin's Limitations – Why Something More?

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- Lack of Turing-completeness
 - Script language supports no loops; simulating them requires duplicating code
- Value-blindness
 - UTXO is all-or-nothing; no fine-grained control over withdrawal amounts
- Lack of state
 - UTXO are either spent or unspent; no multi-stage contracts or persistent variables
- Blockchain-blindness
 - Scripts cannot access block data (nonce, timestamp, previous hash)



Ethereum

A next-generation smart contract
and decentralized application platform

Ethereum: A Programmable Blockchain

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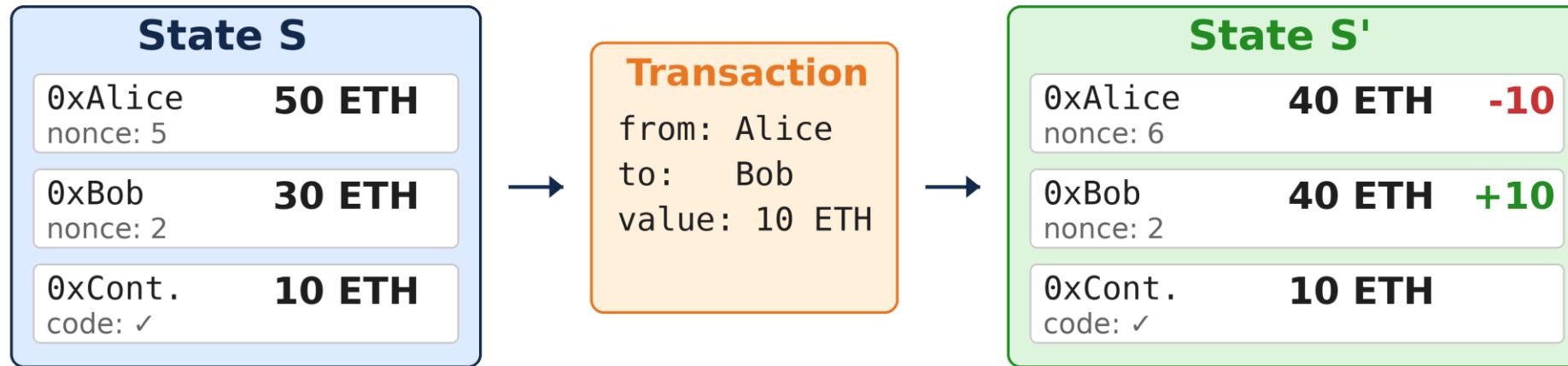
- Proposed by Vitalik Buterin in late 2013; whitepaper published 2014
- Goal: a blockchain with a built-in Turing-complete programming language
- Anyone can write smart contracts and decentralized applications (dApps)
 - Users define their own logic in a few lines of code
- Create arbitrary rules for ownership, transaction formats, and state transitions
- Not just digital cash – a general-purpose decentralized world computer
 - Tokens, DeFi, NFTs, DAOs, identity, governance, and more
- Launched July 30, 2015 (Frontier release)

Ethereum as a State Transition System

- State: mapping from addresses (160-bit) to account states
 - Every account has: nonce, balance, storage root, code hash
- State transition function: $\text{APPLY}(S, \text{TX}) \rightarrow S'$ or ERROR
 - Validates signature, deducts gas fees, runs code, refunds unused gas
- Transactions morph the state incrementally, block by block
- Key difference from Bitcoin:
 - Bitcoin = UTXO model (stateless); Ethereum = account model (stateful)
- Ethereum blocks contain both the transaction list AND the most recent state (root hash)
- State stored in a Modified Merkle Patricia Trie
 - Allows efficient state lookups and proofs without storing full history

Ethereum State Transition Function

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APPLY(S , TX) → S' or ERROR

Yellow Paper: $\sigma(t+1) \equiv Y(\sigma(t), T)$

APPLY({ Alice: 50, Bob: 30 }, "send 70")

→ **ERROR (insufficient balance: 50 < 70)**

Ethereum Accounts

- Two types of accounts sharing the same 20-byte address space:
 - **Externally Owned Accounts (EOAs)**
 - Controlled by private keys (humans/wallets)
 - Can send transactions by creating and signing them; has no code
 - **Contract Accounts**
 - Controlled by their contract code; activates on every message received
 - Can read/write to internal storage, send messages, create contracts
- Every account has four fields:
 - nonce – transaction counter (prevents replay)
 - balance – amount of Wei ($1 \text{ ETH} = 10^{18} \text{ Wei}$)
 - storageRoot – hash of persistent key/value storage
 - codeHash – hash of EVM bytecode (empty for EOAs)

Ethereum Account Types

Externally Owned Account

(controlled by private key)

Private Key

Signs transactions
Derives address

nonce	7	TX count
--------------	---	----------

balance	2.5 ETH	Wei amount
----------------	---------	------------

storageRoot	EMPTY	—
--------------------	-------	---

codeHash	Keccak256("")	—
-----------------	---------------	---

Contract Account

(controlled by its own code)

EVM Bytecode

Executes on every
message/TX received
PUSH MSTORE LT

nonce	1	Created
--------------	---	---------

balance	100 ETH	Hold/send
----------------	---------	-----------

storageRoot	0xa3f2e8...	Merkle root
--------------------	-------------	-------------

codeHash	0x7b9ef1...	Keccak256
-----------------	-------------	-----------

Persistent Storage

slot 0	→ 0x42f...
slot 1	→ 0xab3...
slot 2	→ 0x000...

TX / Message Call

The diagram illustrates the interaction between Ethereum Account Types. It shows two main components: an 'Externally Owned Account' (blue box) and a 'Contract Account' (orange box). The 'Externally Owned Account' contains fields for nonce (7), balance (2.5 ETH), storageRoot (EMPTY), and codeHash (Keccak256("")). The 'Contract Account' contains fields for nonce (1), balance (100 ETH), storageRoot (0xa3f2e8...), and codeHash (0x7b9ef1...). A red box labeled 'TX / Message Call' is positioned below both accounts. A red arrow points from the 'TX / Message Call' box to the 'Contract Account' codeHash field, indicating that a transaction or message call can interact with the contract's code hash.

Both account types share 20-byte address space (160 bits)

EOA = Keccak-256(publicKey) [rightmost 20 bytes]

Contract = Keccak-256(RLP(sender, nonce)) [rightmost 20 bytes]

EOAs are controlled by private keys; Contract Accounts are controlled by code

Transactions and Messages

- Transaction: a signed data package from an EOA containing:
 - recipient – target address
 - signature – identifies the sender
 - value – amount of Ether to transfer
 - data – optional payload (e.g., function call + arguments)
 - gasLimit – max computational steps allowed
 - gasPrice – fee per computational step
- Messages: virtual objects sent between contracts (internal calls)
 - Like transactions but produced by contracts via CALL opcode
 - Never serialized; exist only in the EVM execution environment
- Gas allocation applies to the entire call chain:
 - If A sends TX with 1000 gas, B uses 600, calls C which uses 300, B has 100 left

The Gas Mechanism

- Problem: Turing-complete code can loop forever – how to prevent DoS?
- Solution: every computational step costs gas
 - An attacker must pay proportionally for computation, bandwidth, and storage
- How it works:
 - Sender pre-pays $\text{gasLimit} \times \text{gasPrice}$ in Ether
 - Each opcode consumes a defined amount of gas
 - Unused gas is refunded after execution
- Gas costs reflect resource consumption:
 - ADD = 3 gas, MUL = 5 gas, SSTORE = 20,000 gas (new slot)
 - Transaction data: 4 gas/zero byte, 16 gas/nonzero byte
- If execution runs out of gas:
 - All state changes revert, but gas is NOT refunded (prevents spam)

The Gas Mechanism

1. User Submits TX

gasLimit = 100,000

gasPrice = 20 Gwei

Pre-pay:

$100,000 \times 20 \text{ Gwei}$

= 0.002 ETH upfront

2. EVM Executes

ADD → 3 gas

MUL → 5 gas

SLOAD → 2,100

SSTORE → 20,000

CALL → varies

Total: 65,000 gas

3a. SUCCESS ✓

State changes COMMIT

Refund unused gas:

$35,000 \times 20 \text{ Gwei}$

= 0.0007 ETH back

3b. OUT OF GAS ✗

ALL state changes

REVERT to original

Gas NOT refunded

(prevents spam)

Why Gas Matters

- Turing-complete code loops forever
- Gas ensures every computation has cost
- Attacker pays for CPU, BW, storage
- Halting problem solved economically

Selected Gas Costs (EIP-2929+)

TX base: 21,000 gas

ADD: 3 gas

SSTORE: 22,100 gas

MUL: 5 gas

CREATE: 32,000 gas

SLOAD: 2,100 gas

CALL: 2,600 gas

BALANCE: 2,600 gas

Gas prevents infinite loops and DoS attacks by metering every computation

The Ethereum Virtual Machine

A Turing-complete, stack-based execution environment
running identically on every node

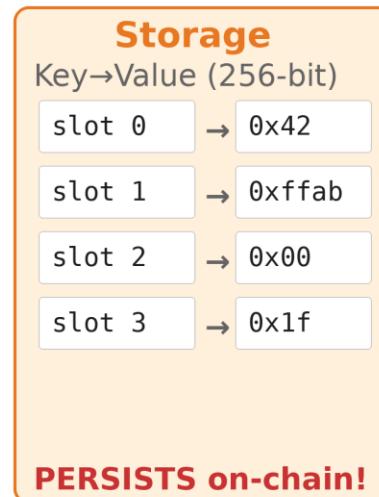
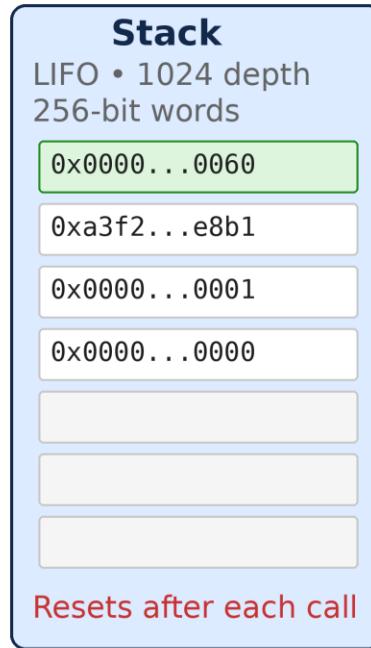
EVM Architecture



- Stack-based virtual machine with 1024-item depth, 256-bit words
- Three data storage areas:
 - Stack – LIFO container; push/pop 256-bit values; resets after execution
 - Memory – expandable byte array; resets after execution
 - Storage – persistent key/value store (256-bit → 256-bit); survives between calls
- Execution model:
 - tuple (block_state, transaction, message, code, memory, stack, pc, gas)
- Code = series of bytes; each byte is an opcode (ADD, SUB, SSTORE, CALL, ...)
 - Program counter increments through bytecode; STOP or RETURN halts
- Deterministic: given the same input, every node produces the same output
 - All nodes execute the same code – this IS the world computer

EVM: Stack, Memory, Storage

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Data Areas

PC→	0x00 STOP	0x01 ADD	0x02 MUL	0x10 LT
0x35	CALLDATALOAD	0x52 MSTORE	0x54 SLOAD	0x55 SSTORE
0x56	JUMP	0x57 JUMPI	0x60 PUSH1	0xF1 CALL
0xF3	RETURN	0xFD REVERT	0xF0 CREATE	0xFF SELFDESTRUCT

Gas Counter

Remaining: 21,347 gas

Each opcode costs gas
Out-of-gas = REVERT

Execution Context

msg.sender – who called
msg.value – ETH sent
msg.data – calldata
block.number, timestamp
tx.origin – original EOA
address(this) – contract

Execution Engine

The EVM is a stack-based machine with three distinct data areas

Smart Contracts and Solidity

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- Smart contracts:
 - Reactive agents, executing code when “poked” by a transaction or message
 - Have direct control over their own ETH balance and storage
- Solidity:
 - Primary high-level language for smart contracts; compiles to EVM
 - High-level, statically typed, object-oriented; influenced by C++, JavaScript, Python
- Development workflow:
 - Write Solidity → Compile to bytecode → Deploy → Interact
 - Tools: Remix (web IDE), Hardhat, Foundry
- Key properties:
 - Immutable once deployed (code cannot be changed)
 - Composable – contracts can call other contracts
- Other smart contract languages: Vyper (Python-like), Yul (low-level)

ERC-20: The Token Standard

- A token is just a mapping: address → balance
 - Subtract X from A, give X to B (if A has at least X)
- ERC-20 (2015) standardizes this into 6 functions + 2 events:
 - balanceOf(owner) → uint256
 - transfer(to, amount) — direct send
 - approve(spender, amount) — grant spending rights
 - transferFrom(from, to, amount) — spend on behalf
 - totalSupply() and allowance(owner, spender)
 - Events: Transfer(from, to, value), Approval(owner, spender, value)
- Why it matters:
 - Any ERC-20 token works with every wallet, DEX, and dApp
 - USDC, USDT, UNI, LINK, DAI — all ERC-20
 - Composability: one standard → entire DeFi ecosystem

SimpleToken.sol — Minimal ERC-20 Implementation

```
// SPDX-License-Identifier: MIT
pragma solidity ^0.8.20;

contract SimpleToken {
    string public name = "MyToken";
    string public symbol = "MTK";
    uint8 public decimals = 18;
    uint256 public totalSupply;

    mapping(address => uint256) public balanceOf;
    mapping(address => mapping(address => uint256))
        public allowance;

    event Transfer(address indexed from,
                  address indexed to, uint256 val);
    event Approval(address indexed owner,
                  address indexed spender, uint256);

    constructor(uint256 _supply) {
        totalSupply = _supply * 10**decimals;
        balanceOf[msg.sender] = totalSupply;
    }

    function transfer(address to, uint256 amt)
        public returns (bool) {
        require(balanceOf[msg.sender] >= amt);
        balanceOf[msg.sender] -= amt;
        balanceOf[to] += amt;
        emit Transfer(msg.sender, to, amt);
        return true;
    }
}
```

State variables:

name, symbol, decimals
are token metadata

The core state:

mapping(addr → bal)

transfer() is the heart of ERC-20:

check → subtract → add
→ emit event

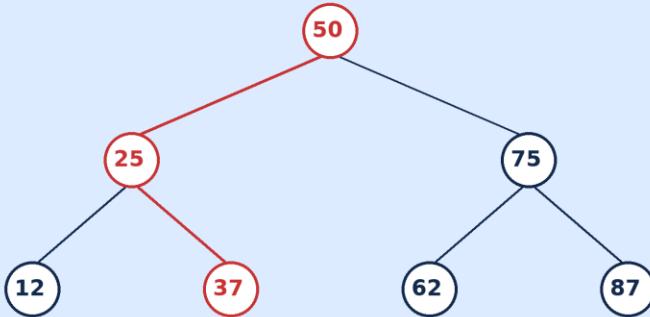
A minimal but complete ERC-20 token — the building block of DeFi

Tree vs. Trie

Binary Search Tree

Compare keys at each node

$O(\log n)$ lookup



Find 37:

50 → go left

25 → go right

37 → found!

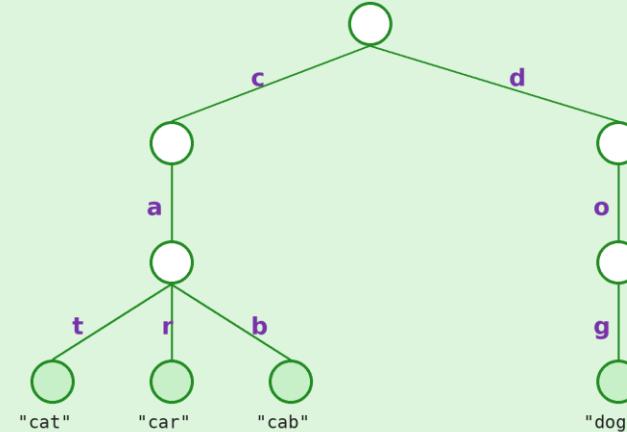
Properties:

- Full key stored in each node
- Compare & branch left/right
- Good for ranges, bad for strings
- $O(\log n)$ — depends on # entries
- Non-deterministic (insert order matters)

Trie (Prefix Tree)

Path from root encodes key

$O(\text{key length})$ lookup



Find "car":

root → c → a → r → found!

Follow path edges, no comparisons

3 steps = length of key

Properties:

- Key encoded in the path (edges)
- Shared prefixes = shared paths
- Perfect for address lookups
- $O(\text{key length})$ — independent of # entries
- **Deterministic (same keys → same trie)**

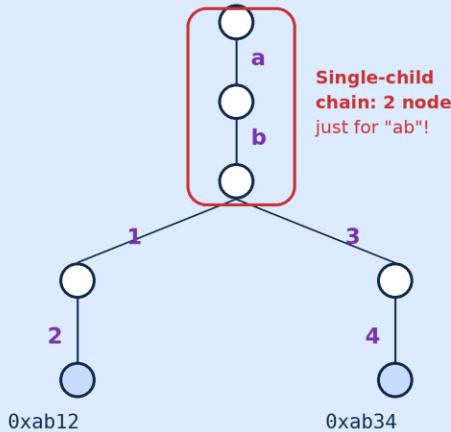
Ethereum needs deterministic lookups by address — tries provide exactly this

Patricia Trie: Path Compression

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Standard Trie (wasteful)

Single-child chains waste space



7 nodes for just 2 keys

Each nibble = one node, one edge

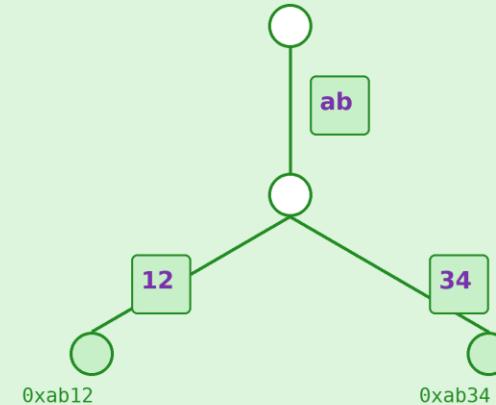
→ Wastes nodes where there is no branching

Why Ethereum needs this:

- Keys = Keccak-256 hashes, 40 nibbles (hex digits)
- Max path depth: 40 edges
- Patricia compresses non-branching segments
- Each internal node stores hash of its subtree
- Merkle proof: $O(\log n)$ hashes to verify
- **Result: Modified Merkle Patricia Trie**
- $= \text{trie} + \text{path compression} + \text{Merkle hashing}$

Patricia Trie (compressed)

Collapse single-child chains



4 nodes for 2 keys!

"ab" collapsed into single edge

Branch only where keys diverge

Key insight:

Edges store multi-character strings, not single characters. This is what makes it "Patricia" (Practical Algorithm To Retrieve Information Coded In Alphanumeric)

Patricia = Practical Algorithm To Retrieve Information Coded In Alphanumeric

Modified Merkle Patricia Trie (MPT)

- Combines three ideas:
 - 1. Trie – deterministic key→value lookups
 - 2. Patricia – path compression, saves space
 - 3. Merkle – nodes store hash of children
- “Modified” = Ethereum’s variant:
 - 16-way branching (one per nibble)
 - Node types: branch, extension, leaf, empty
 - Keys/values serialized before hashing
 - Root hash = Keccak-256 of entire trie
- Why blockchains need this:
 - Root hash in header commits to all data
 - $O(\log n)$ proof verifies any key-value pair
 - Deterministic: same data → same root hash

Branch Node

16 children (one per nibble 0-f)
+ optional value field



↑ orange = nibbles with children

Extension Node

Shared prefix
(path compressed)
Points to next node

prefix:"a7b" → next

= **Patricia compression!**
Collapses single-child chains

Leaf Node

Remaining key suffix + value
End of a key’s path in the trie

suffix:"c9"→[nonce, bal]

Merkle Hashing

Every node is hashed: hash = Keccak-256(serialize(node))
Node hash commits to all children’s hashes at once
→ **Change any leaf → all hashes up to root change**

Root Hash

= fingerprint of entire trie
Stored in block header for consensus

Ethereum's Three Tries

- Each block header stores three trie root hashes:
 - Root hash = cryptographic commitment to data set
- State Trie – persistent across blocks, updated not rebuilt
 - Maps account address → account data (nonce, balance, ...)
 - Only rehash changed leaf-to-root path ($O(\log n)$)
 - Merkle proofs: verify any account without full state
- Transaction Trie – rebuilt fresh each block
 - Maps tx index → transaction data for that block
 - Proves a transaction was included in a block
- Receipt Trie – rebuilt fresh each block
 - Maps tx index → execution result (status, gas, logs)
 - Proves tx outcome (e.g., event was emitted)
 - Separate from tx trie: receipts depend on execution context
- All three roots in every block header

Block Structure and Validation

- Ethereum blocks store more than Bitcoin:
 - Parent hash, timestamp, block number
 - State root, transaction root, receipts root (three MPTs!)
 - Gas limit, gas used, beneficiary (validator address)
- Block validation algorithm (simplified):
 - 1. Verify previous block reference is valid
 - 2. Check timestamp and other header constraints
 - 3. Execute each TX: $S[i+1] = \text{APPLY}(S[i], \text{TX}[i])$
 - 4. Check total gas \leq gas limit
 - 5. Verify final state root matches header
- State root \rightarrow any node can independently verify

Ethereum Block Structure

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Block N	
parentHash	0x1a2b...c3d4
number	18,000,000
timestamp	1695801600
beneficiary	0x9c0d...ef12
stateRoot	0x3c..e5f6
txRoot	0x5e..a7b8
receiptsRoot	0x7a..c9d0
gasLimit	30,000,000
gasUsed	15,234,567
baseFee	12 Gwei
Transaction List	
TX ₀ , TX ₁ , TX ₂ ...	

State Trie

MPT of all account states.

Key = Keccak-256(address)

Value = [nonce, balance, storageRoot, codeHash]

Persistent; most subtrees shared,
only changed paths rehashed



Transaction Trie

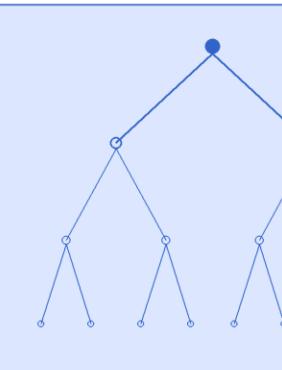
MPT of all TXs in this block.

Key = TX index (0, 1, 2...)

Value = serialized TX

Rebuilt fresh each block.

Proves inclusion via O(log n) Merkle proof



Receipts Trie

Execution results per TX:

status, gas used, logs, bloom filter

Rebuilt fresh each block.

Separate: receipts depend on execution context



Three MPTs provide cryptographic commitments to state, transactions, and receipts

The Merge: From Proof-of-Work to Proof-of-Stake

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- Ran Proof-of-Work from 2015 until The Merge (Sept 2022)
- Beacon Chain launched Dec 2020 – parallel PoS chain
 - Validators staked 32 ETH; tested PoS consensus independently
- The Merge: September 15, 2022
 - Execution layer merged with consensus layer (Beacon Chain)
 - Mining permanently ended; PoS validators now produce blocks
 - Triggered by Terminal Total Difficulty (difficulty bomb)
- Impact:
 - ~99.95% reduction in energy consumption
 - Block time stabilized at exactly 12 seconds
 - Issuance fell ~97%: PoS costs negligible → still profitable (~4-5% APY)
- No TX history lost – first PoS block attached to last PoW block

The Merge: PoW to PoS Timeline



Impact of The Merge

Energy:	~99.95%	$\sim 93.95 \rightarrow \sim 0.01 \text{ TWh/yr}$
Block time:	Exactly 12 sec	Steady, no PoW variance
Reward:	-97%	TX fee revenue increased
Validators:	500,000+	Lido, Coinbase, Rocket Pool
TX history:	Fully preserved	No data lost

How The Merge Worked

1. Beacon Chain launched Dec 2020
2. Ran parallel ~21 months testing
3. Validators staked 32 ETH
4. Terminal Total Difficulty reached
5. Execution layer swapped to PoS
6. Mining permanently discontinued

Proof-of-Stake Mechanics

- Validators replace miners; must stake 32 ETH to participate
 - Over 500,000 validators active post-merge
- Every 12 seconds (one slot), the protocol:
 - Randomly selects a block proposer from the validator set
 - Assigns a committee of attesters (~1/32 of all validators)
 - Attesters vote on the proposed block's validity
- Slashing: penalties for malicious behavior
 - Double-signing or contradictory attestations → lose portion of staked ETH
 - Inactivity leak: offline validators gradually lose stake
- Finality: Casper FFG (Friendly Finality Gadget)
 - Checkpoints every 32 slots (1 epoch ~6.4 min); finalized after 2 epochs
- Shanghai upgrade (April 2023) finally enabled staking withdrawals

Ethereum Applications: DeFi, NFTs, DAOs

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- DeFi (Decentralized Finance)
 - Automated Market Makers (Uniswap), lending (Aave, Compound)
 - Stablecoins (DAI, USDC) – hedging contracts from the whitepaper, realized
 - Over \$100B total value locked at peak
- NFTs (Non-Fungible Tokens – ERC-721)
 - Unique digital ownership: art, collectibles, gaming items, real-world assets
 - Each token has a unique ID; ownership tracked on-chain
- DAOs (Decentralized Autonomous Organizations)
 - On-chain governance with token-weighted voting
 - Treasury management without traditional legal structures
 - The original whitepaper vision of “decentralized autonomous organizations”
- Other applications:
 - Identity systems (ENS), prediction markets, decentralized storage, layer-2 rollups

Ethereum Milestones

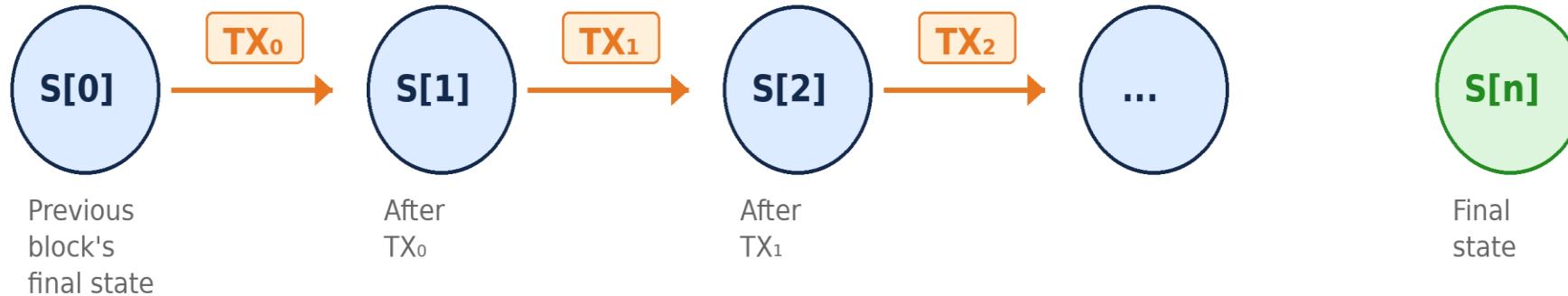


- Nov 2013 – Vitalik Buterin publishes the Ethereum Whitepaper
- Jul 2014 – Public crowdsale raises ~\$18M (31,500 BTC)
- Jul 30, 2015 – Frontier launch (genesis block)
- Jun 2016 – The DAO hack (\$60M); hard fork creates ETH/ETC split
- Dec 2017 – CryptoKitties congests the network; scaling debate intensifies
- Jan 2018 – ETH price peaks near \$1,400
- Aug 2021 – EIP-1559 (London): base fee burn makes ETH deflationary
- Sep 15, 2022 – The Merge: PoW → PoS
- Apr 2023 – Shanghai: staking withdrawals enabled
- 2024–2025 – Dencun (proto-danksharding) and Pectra upgrades

- The Yellow Paper defines the EVM – but in informal mathematics
 - Ambiguities and inconsistencies discovered over the years
- KEVM: a complete, executable formal semantics of the EVM in the K Framework
 - Developed at UIUC and Runtime Verification
 - The “Jello Paper” – jellopaper.org – a readable presentation of the semantics
- What makes KEVM special:
 - Complete – passes the entire official Ethereum test suite
 - Executable – can run EVM programs directly
 - Formal – enables mathematical proofs about smart contract correctness
- Practical impact:
 - Formal verification of smart contracts (e.g., ERC-20 implementations)
 - Found real bugs in the Yellow Paper specification

Block Processing: Sequential State Transitions

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$S[i+1] = \text{APPLY}(S[i] , TX[i])$

$S[0]$ = previous block's final state

If ANY returns ERROR → block INVALID

$S[n]$ = final state → Merkle root in header

Block Validation:

1. Previous block exists and is valid
2. Timestamp in valid range
3. Total gas \leq GASLIMIT
4. **MerkleRoot($S[n]$) == stateRoot ✓**

Each transaction is applied sequentially; the final state root must match the header

Ethereum vs. Bitcoin: How It All Fits Together

- State model: UTXO (Bitcoin) vs. Accounts (Ethereum)
 - Stateless vs. stateful; simple transfers vs. complex interactions
- Scripting: limited stack-based (Bitcoin) vs. Turing-complete EVM (Ethereum)
- Consensus: PoW (Bitcoin) vs. PoS (Ethereum, since 2022)
- Block time: ~10 min (Bitcoin) vs. ~12 sec (Ethereum)
- Data in blocks: transactions only (Bitcoin) vs. transactions + state root (Ethereum)
- Supply: capped at 21M BTC vs. no hard cap, but deflationary since EIP-1559
- Purpose: peer-to-peer electronic cash vs. general-purpose world computer
 - Digital gold vs. decentralized application platform