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Moment Method - A Comprehensive Tool for Race Car Development

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ABSTRACT

The MRA Moment Method computer program automates an original approach to the analysis of race car handling. For a given speed and power/braking condition, the computer solution covers the full maneuvering envelope and presents the results graphically in one figure. The model is based on a comprehensive nonlinear vehicle representation utilizing tire, chassis, and aerodynamic input data as available to race car designers and developers. The paper will first explain and illustrate the Moment Method in general terms, with some graphical examples.

The ability of the technique to model the behavior of an F.1 car traversing a small bump in mid-corner on a Grand Prix circuit is then reviewed. The results correlate well with the driver comments relative to changes in directional stability (plow/spin) as the car pitches over the bump. It also correlates well with the lateral acceleration/speed that the driver is willing to use in this corner.

INTRODUCTION

The MRA Moment Method is a technique for analyzing and graphically portraying the stability and control of an automobile. In principle, it is analogous to the way aeronautical engineers determine the stability, control and damping of an airplane from wind tunnel tests on an aircraft model. The embodiment discussed in this paper is based on computer calculations with a nonlinear ride/handling model consisting of fourteen (14) degrees of freedom, (six chassis, four vertical unsprung, four wheel rotations) and numerous "add-ons" for other compliance and kinematic effects. Constraints of the d'Alembert type are introduced for certain degrees of freedom thus bringing them into static equilibrium. Unlike the complete constraint system of the airplane wind tunnel model, the proper wheel loads will only occur if the body is free to heave, pitch, and roll along with the corresponding four vertical wheel motions. Thus these seven degrees of freedom are unconstrained. Constraints exist in longitudinal and lateral force, and yaw moment, and also on the four wheel torques to a total of seven. As well as simulating true steady-state, it possible to simulate steady longitudinally accelerated conditions (traction/braking) if the tire forces at the ground due to the wheel torques are placed in d'Alembert equilibrium with the longitudinal force constraint, noted above. Since it is convenient to perform analyses at a constant speed, the wheels do not angularly accelerate and the condition studied is a momentary one. The torques at the wheels are constrained in the sense that they must be adjusted for the specific operating mode. Steer angle is a pre-set variable.

In short, the Moment Method for the automobile is analogous to aircraft "statics". It is concerned with the balanced and unbalanced forces/moments that act on the automobile as opposed to time history analysis. By working at the force/

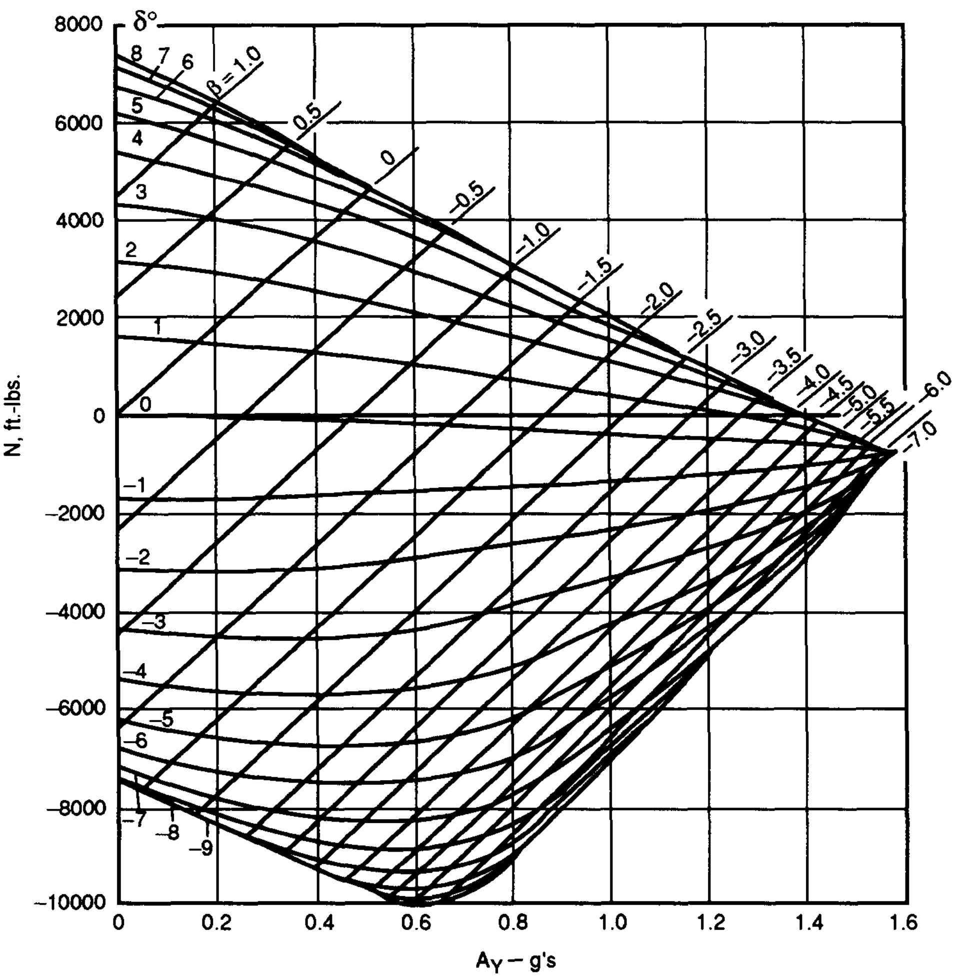


Figure 1: N-A_Y Moment Diagram; Road Load, 140 mph

moment level instead of at the motion level, one avoids the filtering effects of the inertias and is able to isolate results of small changes in the vehicle configuration not discernible in a transient response. However, transient responses can be developed by integrating the unbalanced forces/moments on the Moment Method diagram. Incidentally, MRA also offers a full transient model, Vehicle Dynamics Simulation (VDS) for use in special cases where a complete transient model is required to study a particular problem. The Moment Method is more fully described in References 1 — 5. The aircraft stability and control concepts that Moment Method technology is based upon are described in References 6 and 7.

Part I — FORCE—MOMENT DIAGRAM

A general description and interpretation of a Moment Diagram is given in this section. Figure 1 shows a diagram of yawing moment about the CG (N), vs. lateral acceleration, A_Y, for a race car traveling at 140 mph under road load conditions, with considerable aero down force, based on the rather complete data available in Reference 8. Comparable diagrams can be developed for other operating conditions such as other speeds or under acceleration/braking.

This diagram portrays all right hand turn maneuvers, with lateral acceleration plotted along the horizontal axis. Since the diagram is plotted for constant speed, this axis covers all turn radii from infinite (straight ahead) to the tightest possible. The vertical axis is the actual yawing moment (in lb.-ft.) about a vertical axis through the vehicle CG; it can also be expressed as yawing moment coefficient by dividing by Wl, the weight times the wheelbase.

A vehicle's control is most fundamentally expressed as a yawing moment. Similarly, the directional stability can be quantified in yawing moment terms. Thus a vehicle is statically directionally stable if upon disturbance in slip angle (β) the yawing moment (N) tends to reduce the slip angle back to its original trim value. The stable vehicle acts like a weathervane in trying to reduce its slip angle (conversely it will increase β if unstable). A vehicle has no knowledge of compass heading, hence has no path stability, only slip angle stability.

Two sets of lines are shown on the diagram. These result from connecting solution points generated by the MMM solver, in this case generated by varying average steer angles, δ, over a ± 8 degree range and vehicle slip angle, β, from +1 degree to −7 degrees. The sign conventions are per SAE, thus in a right hand turn a normally stable vehicle operates at positive δ and negative β.

Some of the characteristic features of the diagram are:

1. The upper boundary represents saturation of the front tires, i.e., the front tires are on the peak of their cornering force curves. The lower boundary is saturation of the rear tires.
2. The area of the diagram is the vehicle maneuvering area in a right-hand turn at 140 mph, at road load power (variable power as required to maintain $A_X = 0$. In practical operation this area cannot be exceeded.
3. Trimmed, equilibrium, maneuvers such as steady turns take place along the horizontal axis. Trim is defined as yawing moment, $N = 0$.
4. Turn entry or recovery occur off the horizontal axis, i.e., $N \neq 0$. For a constant yawing moment of inertia (so-called polar moment), the yawing moment, N, can be replaced by the angular acceleration in yaw. The diagram can than be interpreted in acceleration terms as a plot of angular acceleration in yaw versus lateral acceleration, which are related to the attitude of the vehicle on the path and the curvature of the path.
5. Most racing occurs in a small band about the horizontal axis since the best drivers are very smooth and tend to go from one steady-state to another. They recognize that it is important to use the lateral tire forces for creating high steady-state lateral accelerations, minimizing their use in angularly accelerating the vehicle.
6. The slope of a steer angle, δ line as it crosses the horizontal axis is a numerical measure of the directional stability. This definition is directly analogous to aircraft practice. As described in References 1, 2, and 3, this slope on the N-A_Y diagram has two components, that due to under/oversteer and that due to yaw damping. Yaw damping is the yawing moment due to yawing velocity—a large and important moment normally overlooked in traditional automobile stability analysis.
7. Control is numerically measured in yawing moment terms as one moves along a constant β line.
8. Conventional skid pad tests for determining under/oversteer are performed under steady-state conditions. Thus lateral acceleration along the horizontal axis at the crossing points of the δ lines, enable the construction of a typical skid pad result, a plot of δ vs. A_Y.
9. Consider now the behavior at limit cornering. Moving out along the horizontal axis under trimmed conditions, the limit of the front tires is reached at 1.4 g. At this point no further yawing moment can be obtained from the front end but some 3000 lb.-ft. of negative yawing moment is still available from the rear pair of tires. This vehicle has become very stable (moment from the rear tires is stabilizing) and the front end pushes (or plows). If the RH endpoint of the diagram is below the A_Y axis, the vehicle pushes at the limit. If the

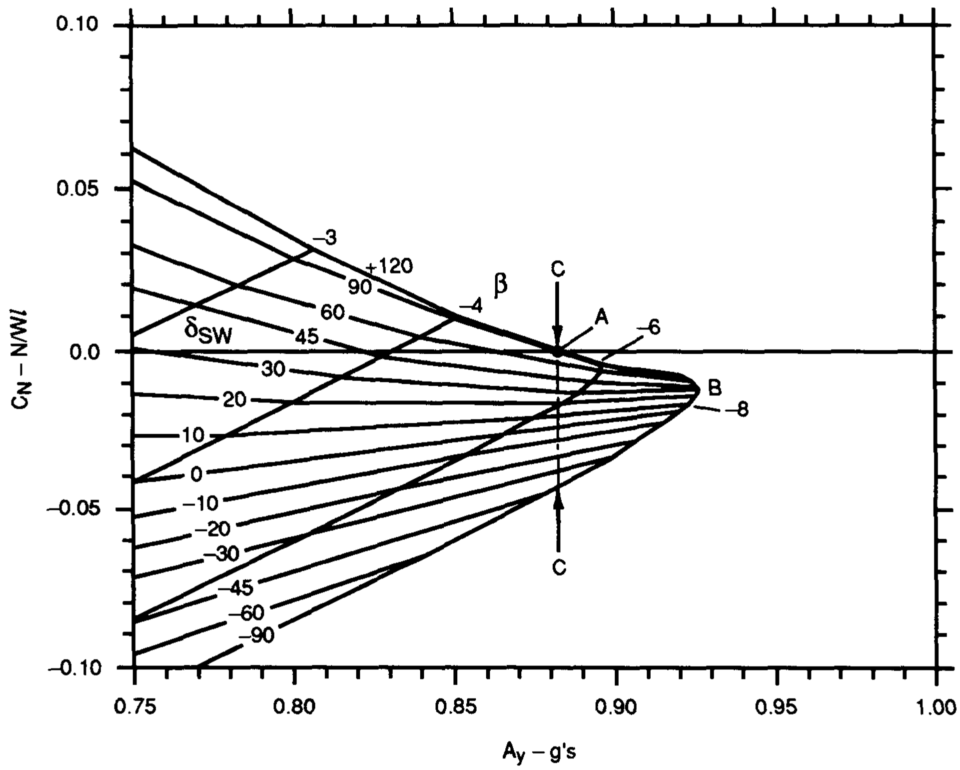


Figure 2: C_N-A_Y Moment Diagram, high lateral acceleration region, road load, 100 mph

endpoint is above the axis, the vehicle is "loose" with a tendency to spin. A numerical measure of "push" or "loose" is the vertical distance between the endpoint of the diagram and the axis.

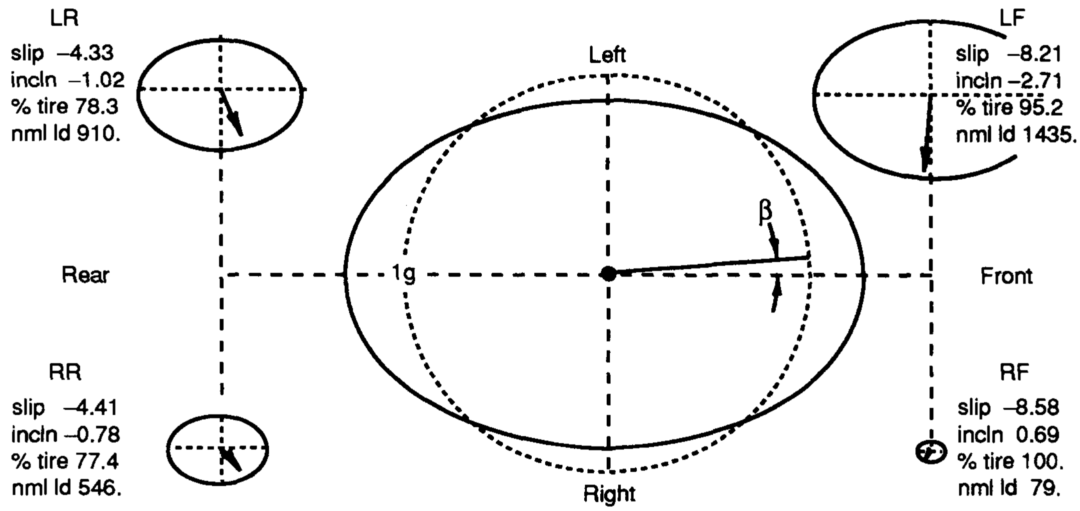
If the endpoint of the diagram falls on the axis, the vehicle is neutral at the limit and the highest trimmed lateral acceleration is obtained because all of the front and rear tire capabilities are being used. In this example, the maximum lateral acceleration is 1.57 g. This is the condition normally sought in balancing out the race car, however, it is usually not possible to achieve this for all operating conditions of speed, traction and braking, nor is it always desirable to have the neutral stability that this implies. Thus the skill of balancing out the vehicle is one of achieving the optimum compromise in relation to driver feel, track conditions, etc. The Moment Diagram is an analytical device which enables an understanding of the process and quantifies it.

LIMIT BEHAVIOR

The use of the Moment Method for race car analysis centers around the endpoint of the diagram. Figure 2 is a typical example, enlarged for clarity. The characteristics at the limit can be defined by several parameters, thus,

1. Maximum trimmed lateral acceleration (Point A) = 0.881 g.
2. Maximum untrimmed lateral acceleration (Point B) = 0.926 g.
3. Residual moment coefficient at maximum trimmed lateral acceleration, distance C—C = –0.042 C_N (at rear, thus push).
4. Stability at maximum trimmed lateral acceleration, the slope of the δ line through Point A = –0.083 C_N/g (stable).

With the Moment Method computer program it is possible to rapidly create these limit diagrams (and the above parameters) for numerous changes in the vehicle configuration and operating conditions.



delsw (δ)	beta (β)	Ax	Ay	Cn	N	V	r	R	phi (φ)
deg	deg	g	g	–	ft - lb	mph	deg / sec.	ft	deg
74.31	−4.189	0.0	0.848	0.0	0.0	100.0	10.66	788.0	−1.102

Figure 3: Friction Circle Diagram, A_Y = 0.848; road load, 100 mph

To examine in still more detail what is occurring on the vehicle, one of the post-processing routines generates the operating conditions at each wheel for any vehicle lateral acceleration and yaw moment. With this Friction Circle program, a cursor is moved to the desired operating condition on the Moment Diagram and a Friction Circle plot such as Figure 3 appears. This plot is for cornering at 0.848g, under road load at 100 mph. The four ellipses show the horizontal force capability at each wheel for the load associated with this speed and cornering condition. The vector at each wheel is the actual horizontal force being developed. Because a very large anti-roll bar was installed on the front, a large part of the lateral load transfer is taken on the front in this configuration. The inside front wheel is experiencing a load of only 79 lbs., compared to 1435 for the outside wheel. The inside wheel has reached its lateral force limit and the outside wheel is also close to its limit. The rear drive wheels are only utilizing about 78% of their lateral force capability, thus the vehicle is pushing (plowing). Considerable tractive effort is required for this road load condition at 100 mph as indicated by the forward sloping vectors on the rear tires. The front wheels are producing some slip angle drag, although most of the drag is aerodynamic.

The large ellipse in the center of the figure is a theoretical "g" diagram for this operating condition, with a line indicating β, and a 1g reference circle (this reference circle scales up for cars with higher acceleration capability). Detailed operating conditions are tabulated at the bottom of the diagram and further detail is saved in a text file for later inspection.

A great virtue of the Friction Circle presentation is that it shows immediately which wheel is critical and limiting performance. Attention may then be directed toward improving the performance of that wheel and hence the lateral acceleration and the limiting behavior of the vehicle.

Part II — VALIDATION

For the Moment Method to be an effective tool for the development of racing cars it must be validated under conditions representative of those experienced by a racing car on a race track. Team Lotus set out to do this in 1991 using a Lotus T102B—Judd EV Formula 1 car.

Of the three parts that make up any statics or dynamics model, namely:

1. Force generator characteristics — tires, aerodynamic devices
2. Kineo-elastic load paths characteristics — suspension, structure

3. Inertias — masses, inertias

the last two are reasonably straightforward to validate by measurement and rig testing, the first is not so easy. Flat belt tire testing generates good linear range characteristics, but it is the non-linear range characteristics that determine the ultimate performance of a racing car and these are highly dependent on a number of factors, including:

1. Road surface smoothness
2. Texture and temperature
3. Amount and type of rubber ingrained into the road surface
4. Compound and temperature of the tire tread

None of these effects can be accurately reproduced on the test machine. Aerodynamic characteristics suffer a similar problem in that it is not feasible to reproduce in the wind tunnel all the factors, such as Reynold's Number, surface boundary layer, turbulence and dynamic effects caused by vehicle movement that significantly affect the aerodynamic loads on the car.

TEST TRACK REQUIREMENTS

To run believable model validation tests required a perfectly flat, clean but rubbered, windless track with sufficient space to carry out maneuvers at up to 200kph and sufficient run in track to bring the tires up to operating temperature is needed. Initial tests were run on the Dynamics Pad (Black Lake) at General Motors' Milford Proving Ground, but it proved impossible to bring the tires up to temperature due to insufficient space and a lack of grip (dusty surface and no rubber). A new (in 1991) Grand Prix track—Catalunya, near Barcelona in Spain—proved more suitable. Newly laid, it was the smoothest GP track at that time and included 180 degree corners taken at 220, 120, and 80kph. Once other cars had run on it the surface grip was reasonable and as the car could run continuously it was possible to bring the tires up to operating temperature. Data from runs at competitive speeds at this track are used to compare with output from Moment Method analysis of a car configured to represent the Lotus T102B.

VEHICLE MEASUREMENT SYSTEM

The car was fitted with an 18-channel measurement and data acquisition system. The system measured:

1. Driver inputs
2. Suspension vertical forces and displacements
3. CG accelerations and yaw rate

at 50 samples per channel per second. Data was transferred to a PC for analysis using Team Lotus proprietary software.

MOMENT METHOD INPUT FILES

TIRE - Tire data was supplied by Goodyear for 1991 F1 race tires, derived from measurements made on the TIRF facility at Calspan. The rig test tires were not the same compound as the tires used for the tests at Catalunya, and the flat belt surface characteristics are different from the track surface. The data was factored to give the same peak coefficient of friction as the track test tires, based on peak braking and acceleration figures. Tire dynamics were ignored because at a typical relaxation length (about one meter) the tire response time is about 0.018 sec. at 200kph—much faster than the suspension natural frequencies in bump and pitch.

VEHICLE - The vehicle file was assembled from design and rig test measurements made on a Lotus T102B. Aerodynamic coefficients (very sensitive to ride heights and the Moment Method does not currently accommodate aerodynamic mapping) and roll stiffnesses (very sensitive to tire pressures which change significantly due to tire carcass temperature) were adjusted on each run to match the four wheel load distribution to the data. Changes required were small.

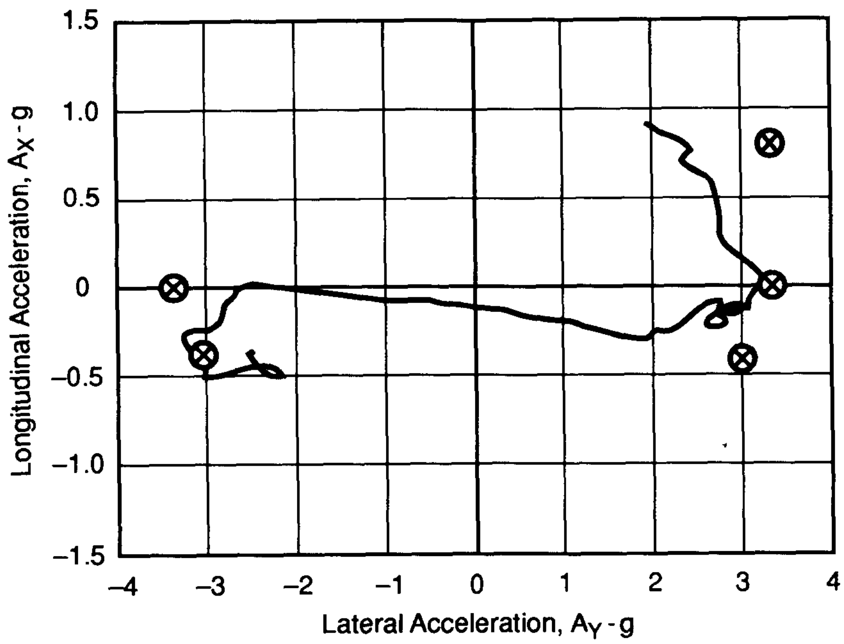


Figure 4: g-g Diagram for 220 kph Chicane

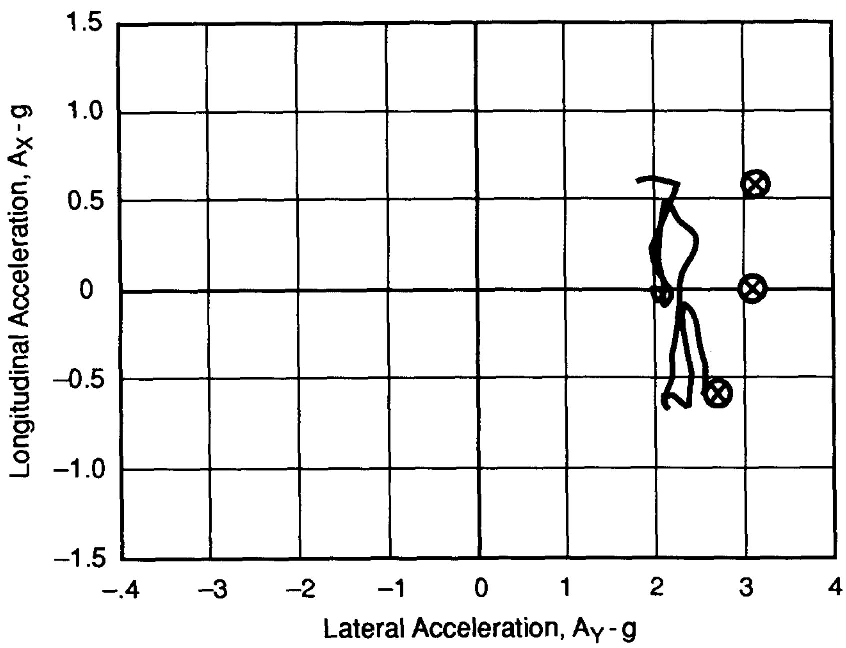


Figure 5: g-g Diagram for 200 kph Corner

OUTPUT AND ANALYSIS

Certain corners, with a range of speeds from 220kph to 80 kph, were selected for analysis and the segment of the g-g diagram appropriate to that corner plotted using the data from the car (Figures 4-8). Note that $+A_X$ is braking and $-A_X$ is acceleration. By inspecting the data through the corner a few points in each corner were selected at which the car was "steady state". This was not easy as both the driver and the track surface disturb the car most of the time.

The speed, longitudinal acceleration and wheel loads at these points were used to set up a vehicle file, as described above, and the C_N-A_Y Diagram calculated and plotted. Figure 9 shows an example for the 200kph corner.

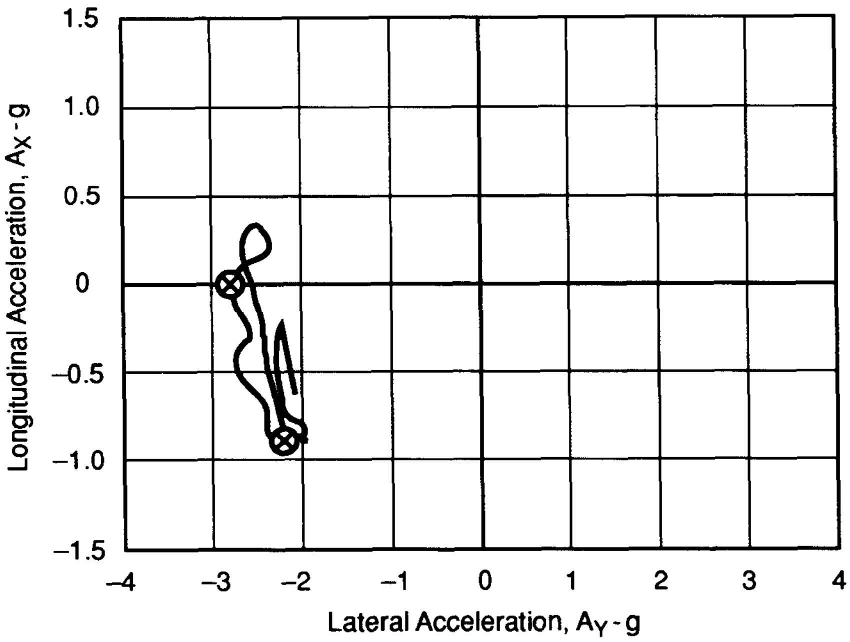


Figure 6: g-g Diagram for 165 kph Corner

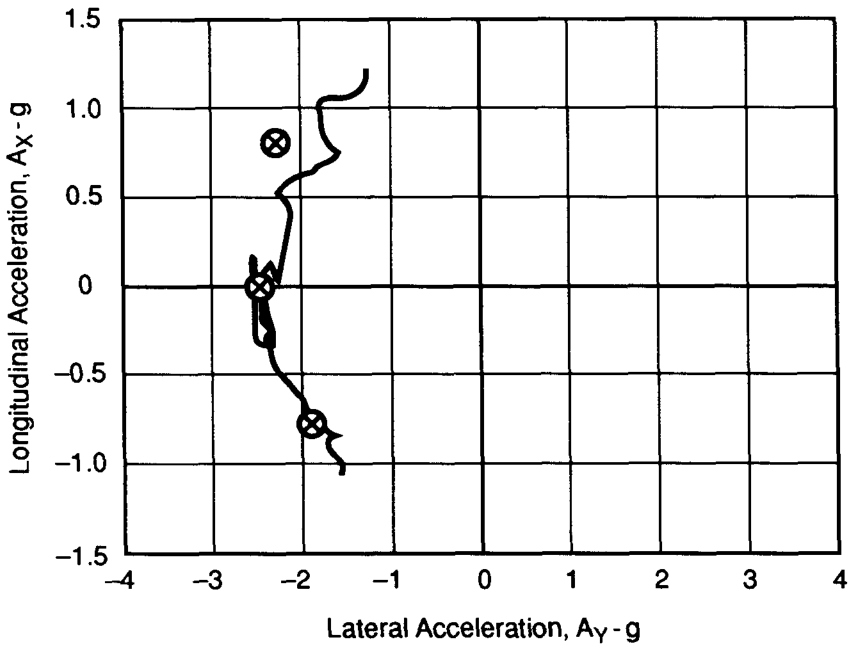


Figure 7: g-g Diagram for 125 kph Corner

Stirling Moss wrote that driving a racing car at the limit is akin to a bowler (pitcher) bowling a ball. All the spin, speed, swing and pitch are set before the ball leaves his hand; after that he cannot influence the trajectory. A driver, driving at the limit, sets up the car on entry to the corner, and if it is truly at the limit he cannot modify the trajectory subsequently—he is a passenger on the "ball". Inspecting Figure 9 it can be seen that the car is neutral at the limit and that at the limit the driver has zero C_N left with which to control the car. Experience shows that while a driver may drive the car like this in qualifying or while racing hard, he does not do so while testing—he reserves a small amount of C_N to permit corrections to be made. The magnitude of this reserve is an interesting measure of a racing driver's combined confidence, judgment, skill and bravery.

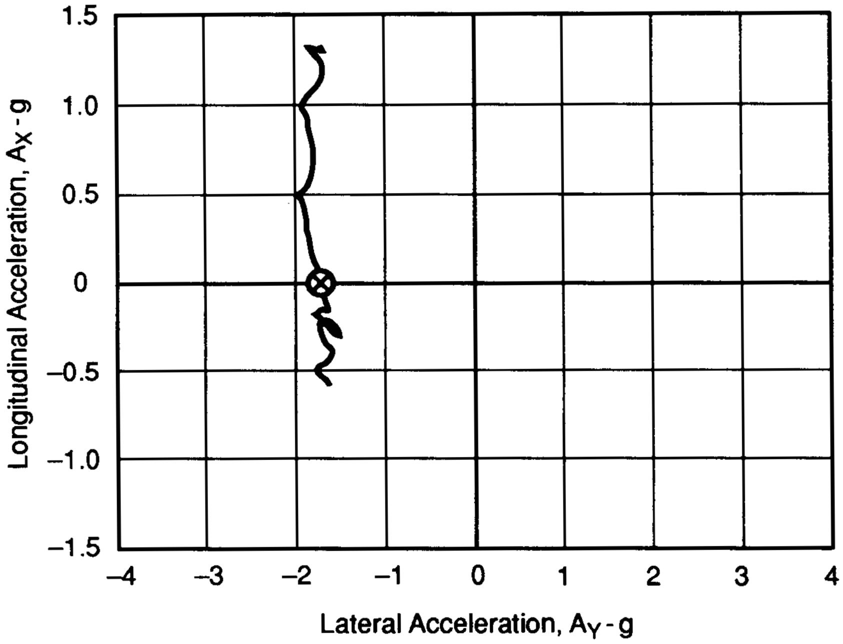


Figure 8: g-g Diagram for 80 kph Corner

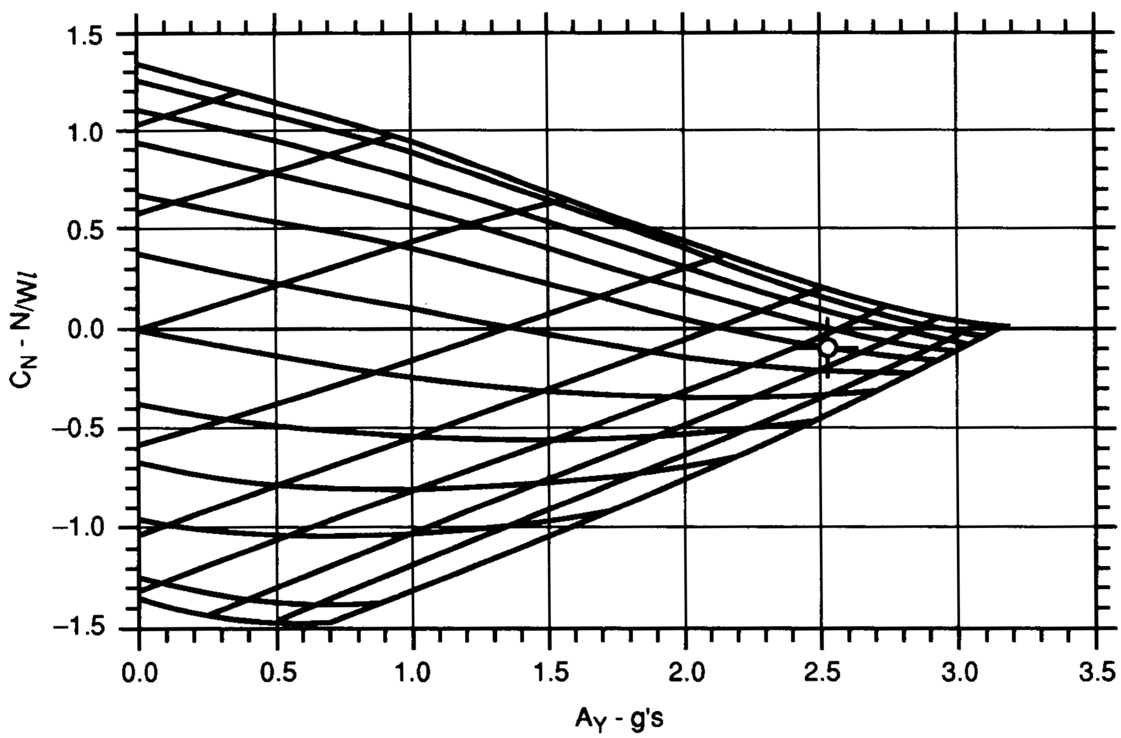


Figure 9: C_N-A_Y Diagram for 200 kph Corner, 0 Longitudinal Deceleration

Analysis of car data and discussion with the driver involved in the tests showed that he required a value of C_N in reserve that would enable him to induce a yaw acceleration of approximately 400 degrees sec^{-2}. For the test car this equated to a C_N = 0.02. When more was available he did not use it, and when there was less he complained of "understeer"—a confusion of control and stability.

This value of C_N = 0.02 was used to determine the limit value of A_Y which was then plotted for each "steady state" case onto the segment of the g-g diagrams of Figures 4-8, shown by ⊗.

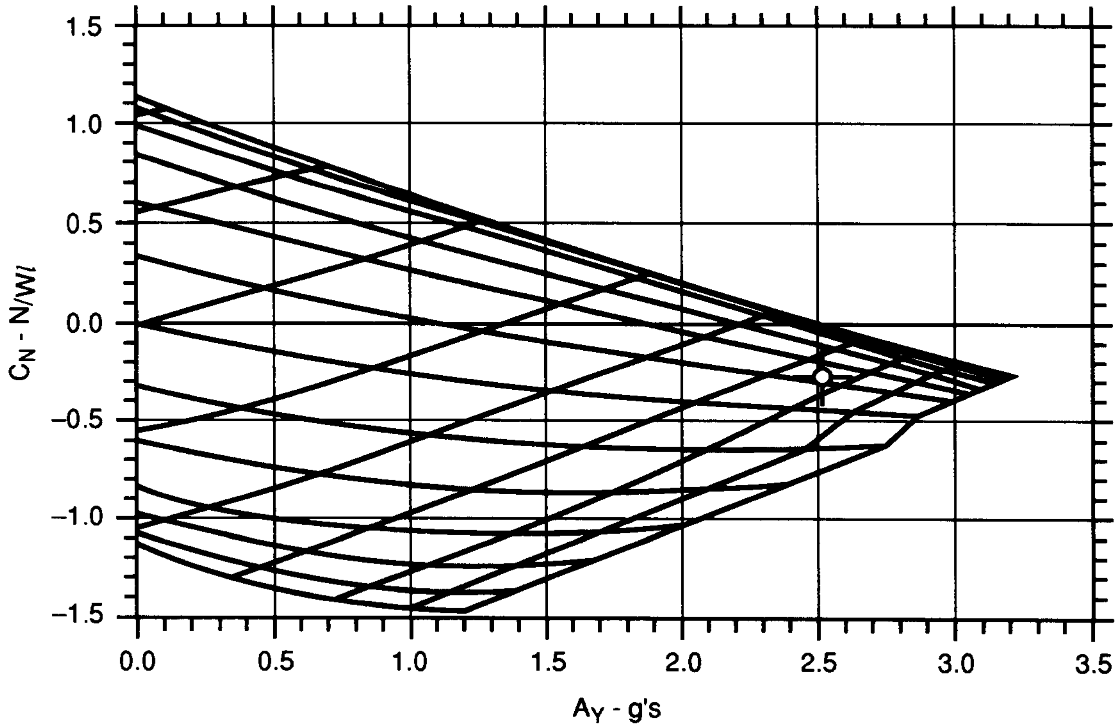


Figure 10: C_N-A_Y Diagram for "Nose-Up" Condition in Bumpy kph Corner

RESULTS

Figures 4-8 for 80, 125, 165 and 200 kph corners and 220 kph chicane illustrate the match achieved between the maximum lateral predicted by the Moment Method and that achieved by the driver. It is noticeable that the slower the corner the better the match, and that on power the match is good but under braking (very difficult to pick a steady state condition) the driver uses less lateral acceleration than predicted. The reasons for these features were evident upon discussion with the driver and analysis of the C_N-A_Y diagrams:

1. The stability of the car reduced with increase in speed.
2. The least stable condition is under braking (the braking—lateral acceleration quadrant of the g-g diagram is always the most difficult for the driver to fill).
3. The driver is able to easily control the car at the limit, with power, when accelerating out of a corner.

Figure 9 for a 200 kph corner indicates that the driver drove through the corner (the marked point) at around 0.75g less than the potential predicted by the Moment Method analysis. The reasons for this were not obvious until inspection of the data showed that there was a bump in the middle of the corner which caused the car to pitch at around 7Hz for half a second exciting a yaw acceleration oscillation with an amplitude of ±250 degrees sec^{-2}. C_N-A_Y diagrams were derived for both the nose-up and nose-down cases at the extremes of the pitch cycle (by adjusting axle loadings to match the car data) to investigate the effect on the trimmed limit—Figures 10 and 11.

It will be seen that in the nose-up case the trimmed limit has been reduced from 3.2g neutral to 2.5g understeer; and in the nose-down case to 2.7g oversteer. The driver drove through the corner at 2.5g in the knowledge of the limitations imposed on the car by the bump.

The critical condition during the oscillation is shown in Figure 11. Here the car is basically unstable (loose) and the driver is operating (marked point) at $C_N = -0.1$ correcting an incipient spin (negative yawing moment, C_N, is counter-clockwise) by reducing the A_Y in this righthand turn. The distance from the operating point to the lower edge of the diagram (the rear tire limit) is

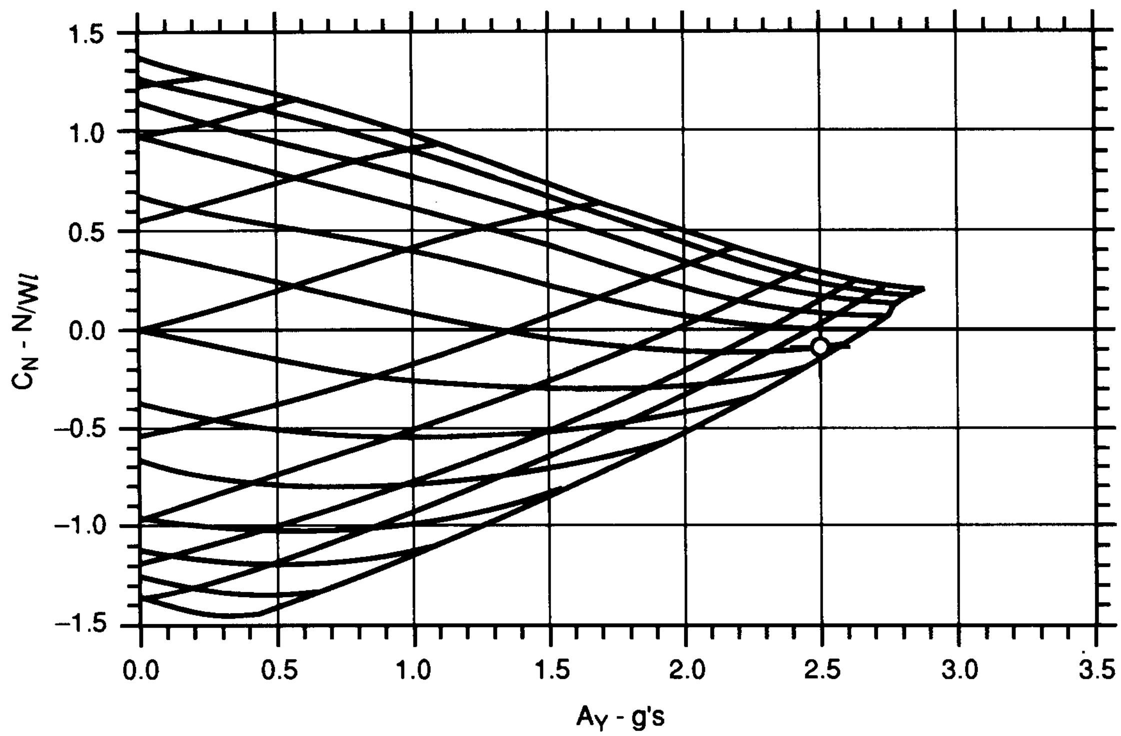


Figure 11: C_N-A_Y Diagram for "Nose-Down" Condition in Bumpy 200 kph Corner

$C_N = -0.02$. As in the earlier trimmed cases, this C_N represents the additional "margin-of-safety" that the driver retains to catch the incipient spin.

Figure 4 shows the plot for a 220kph chicane which was also bumpy. However it shows reasonable correlation in spite of the additional dynamic effects of a fast change of direction.

CONCLUSIONS

Using the Moment Method in this way enables the racing car engineer to do two things: first, the peak performance through any corner (i.e., the theoretical g-g diagram) can be plotted and the actual performance of the driver and car compared to help determine the reasons for a shortfall against potential; second, any of the car characteristics modeled can be varied to establish their sensitivity in affecting the potential performance and characteristics of the car using standard Moment Method interpretation techniques.

The limitation of this approach is the usual one in vehicle modeling: how representative is the model of the tires? As confidence of the vehicle model increases however, and with the addition of a slip angle sensor, which was not available in a robust, high bandwidth form in 1991, the car plus the C_N-A_Y diagram become an on-track tire test machine. Iteratively modifying the tire model until a good match is obtained with the C_N-A_Y diagram through a range of steer and vehicle slip angles allows tire characteristics to be established under real track conditions and temperatures.

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