



Vehicle Dynamics FE8 Final Design Review

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Takeaways:

Comprehensive Status of VD



Vehicle Dynamics' Purpose

How does the Vehicle Dynamics subteam support FRUCD's mission of developing Formula SAE vehicles for competition?

• VD's Roles within FRUCD

- Teach – Ensure team understands fundamental vehicle principles
- Simulate – Predict complex design implications on performance
- Target – Direct a cohesive early system level vehicle concept
- Adjust – Accommodate arising low level constraints by refining concept
- Verify & Validate – Support predictions through experimentation

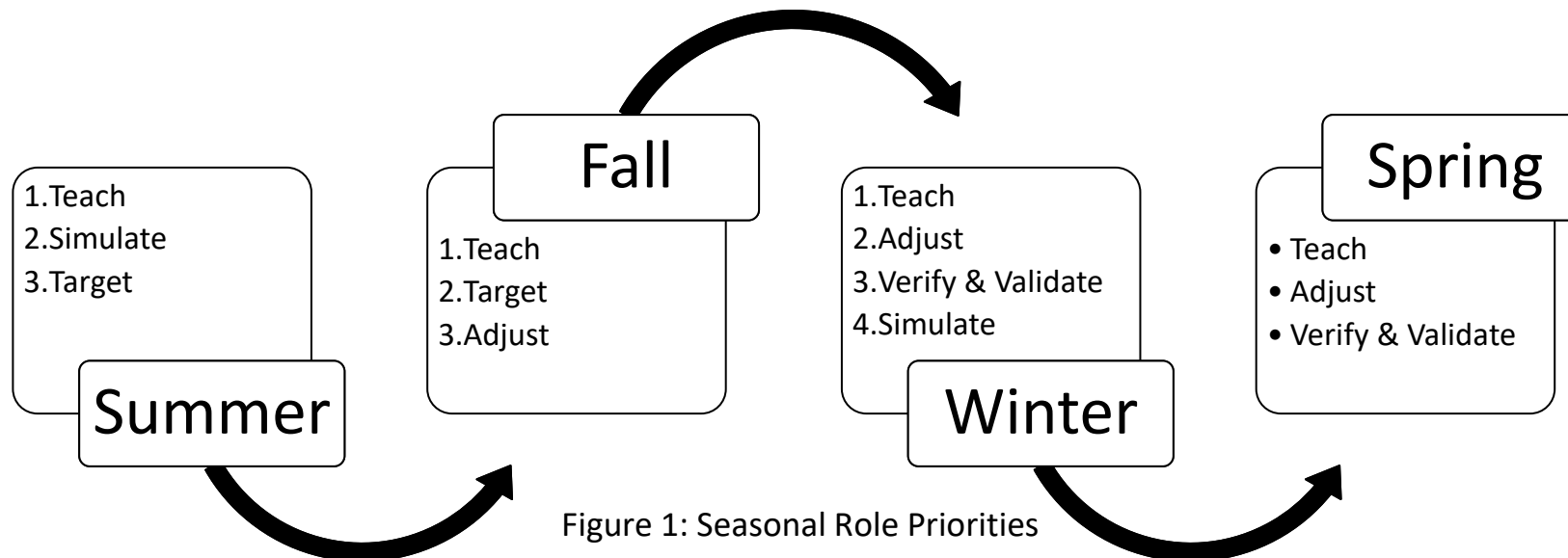


Figure 1: Seasonal Role Priorities

• Promoting Systems Engineering Approach

- Cohesive design:
 - Propagates down
 - Integrates up
- Points sensitivities drive decisions
 - One overarching goal: **Competition**
- System oriented designs are often more apparent and effective
 - Leverage constraints and priorities established at higher levels

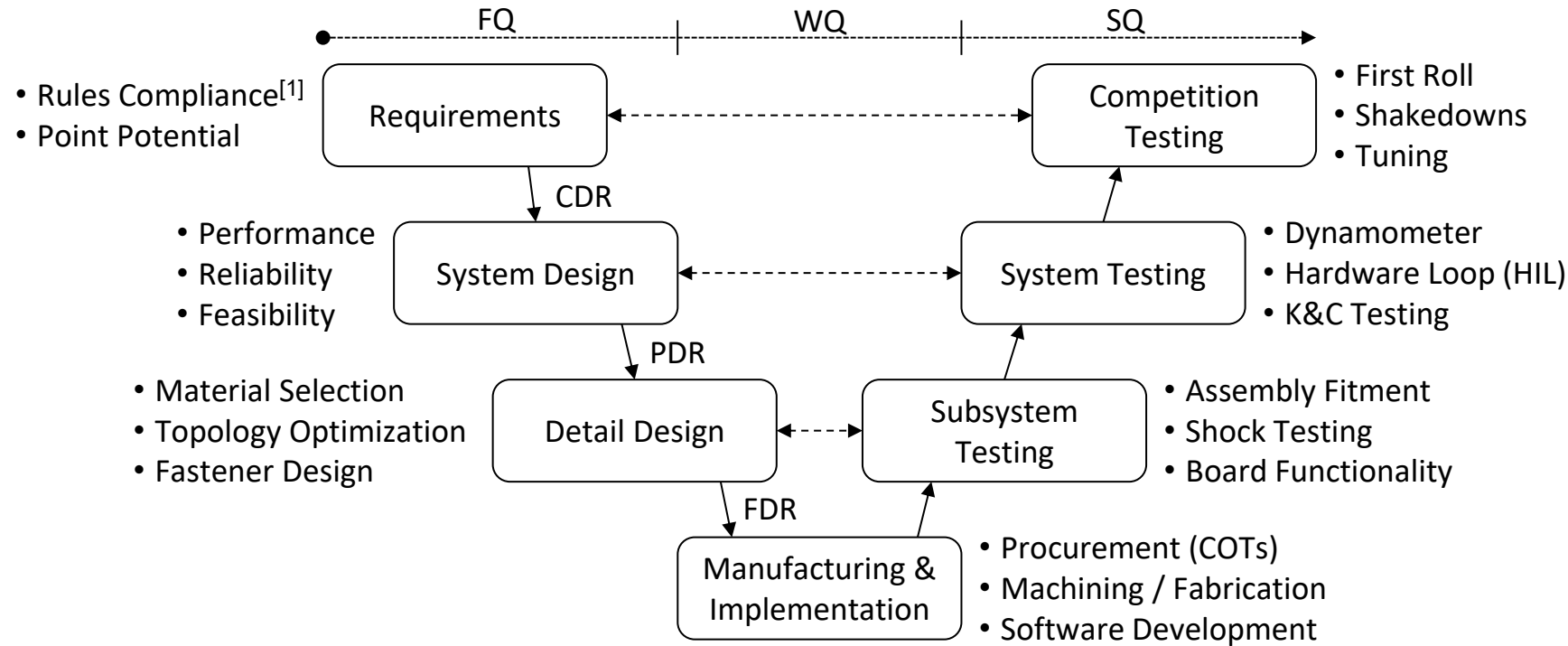


Figure 2: Systems Engineering "V" Diagram

Guiding Vehicle Concept Tradeoffs

- Often subsystem objectives conflict
 - Creates engineering tradeoffs (“Pareto Fronts”)
- Sensitivity analysis provides priorities
 - Mechanical grip is king with FSAE
 - Mass reduction is a distant second

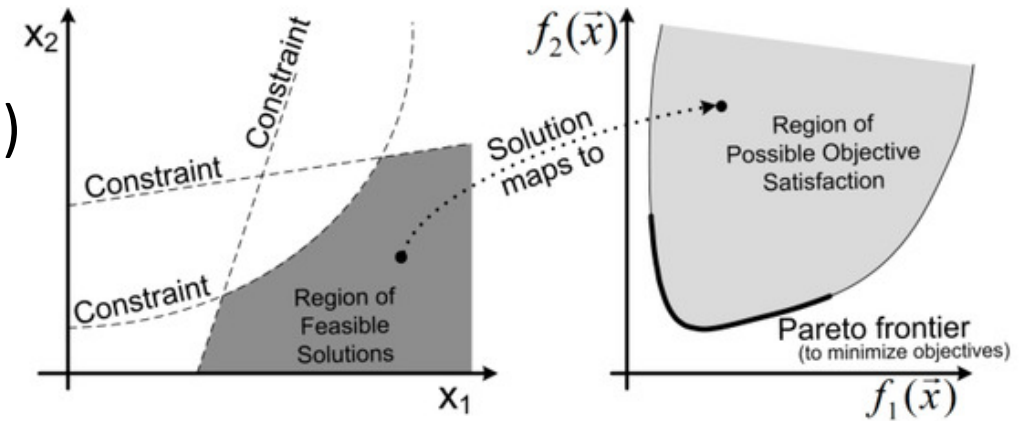


Figure 3: Multi-Objective Pareto Fronts^[2]

Parameter	Sensitivities [%/ χ]					
	Sapienza ^[3]	GFR ^[4]	Eindhoven ^[5]	Belfast ^[6]	Nominal [χ]	5% Delta ⁽¹⁾ [%]
Mass, m [kg]	0.025	0.029	0.021	0.023	275	0.337
Drag, C_D []	0.157		1.097	0.478	0.6 ⁽²⁾	0.017
Downforce, $-C_L$ []	-1.890			-1.433	1.9 ⁽²⁾	-0.158
Lat. Grip, μ_y []	-23.387			-28.634 ⁽³⁾	1.5	-1.742
Long. Grip, μ_x []	-5.646				1.5	-0.421
Power, P [kW]	-0.058	-0.140	-0.076		60	-0.274

Table 1: Endurance Lap Time Parameter Sensitivities

(1) 5% Delta $\equiv \frac{\partial \bar{T}}{\partial x} \cdot x_0 \cdot 0.05$

(2) Nominal C_D, C_L chosen based on estimates from aerodynamics.

(3) Belfast study lumped lateral & longitudinal grip.



Vehicle Simulation Overview

What are the major requirements of vehicle simulation and how have these formed FRUCD's simulation approach?

• Provide Flexible Simulation for Design

- Fidelity: Representational accuracy
 - How close is the model to reality?

Multifidelity simulation uses a range of models to predict outcomes

- Systematic design evolution
 - Simple models inform complex models
- Knowledge transfer
 - Model progression supports learning
- Resource optimization
 - Cheaply eliminates large sets of designs

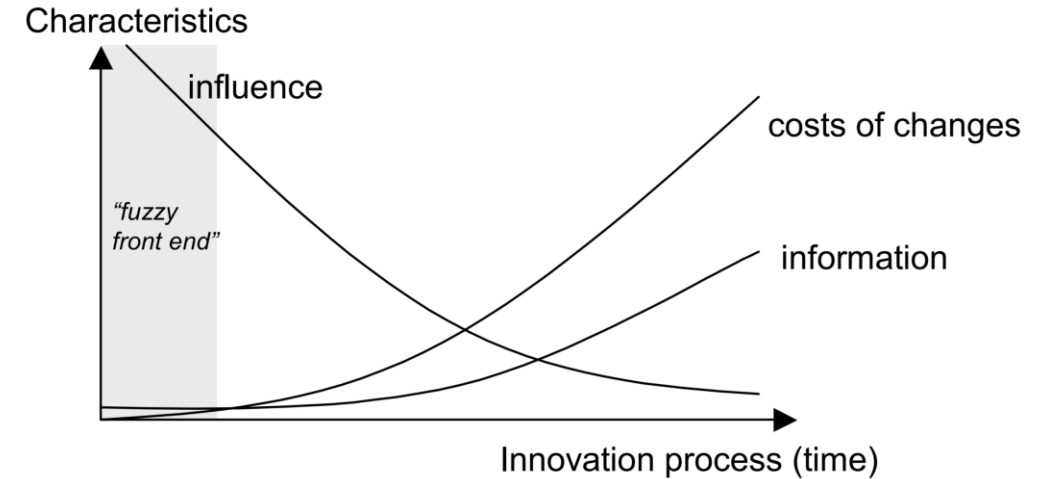


Figure 4: Project Evolution^[7]

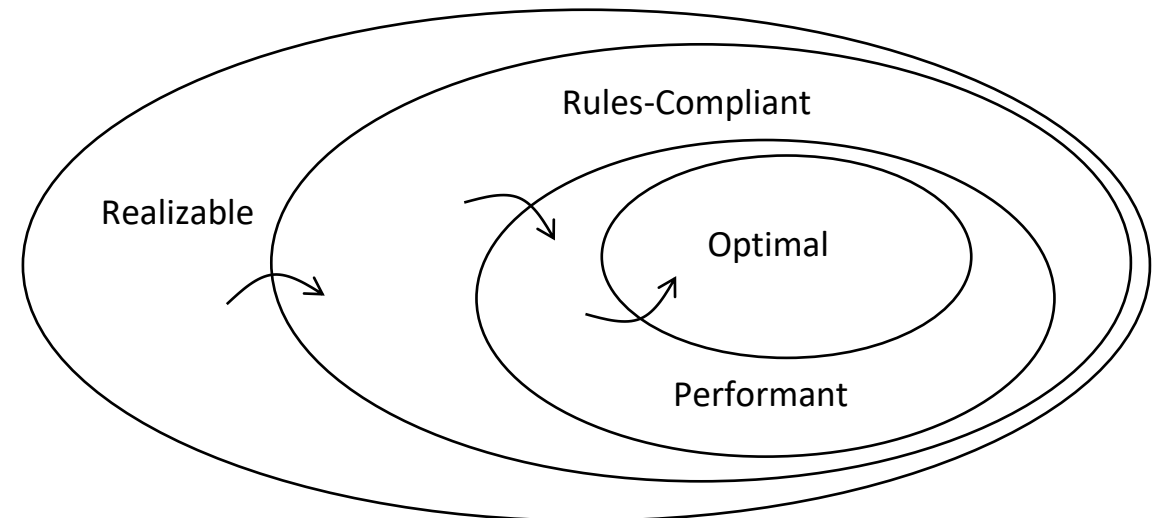


Figure 5: Vehicle Design Space Concentration

Simulation Workflows & Platforms

- Fidelity is manipulated by both:
 - Vehicle Model: Structural Error
 - Simulation: Computational Error
- Most design simulations value efficiency
 - Mostly addressed by performance envelopes and QSS lap sims
 - Transient simulation is only needed in ISO7401 for stability
- Focus is currently addressing design needs and not tuning
 - Expanding vehicle model resource function library
 - Developing performance envelopes and lap time simulation
- Transient simulations will be much more useful for tuning
 - Much higher fidelity required for purposeful tuning information

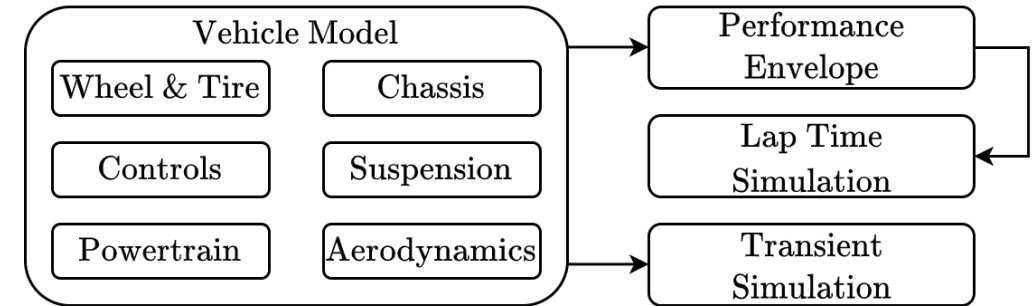


Figure 6: Planned Vehicle & Platform Interactions

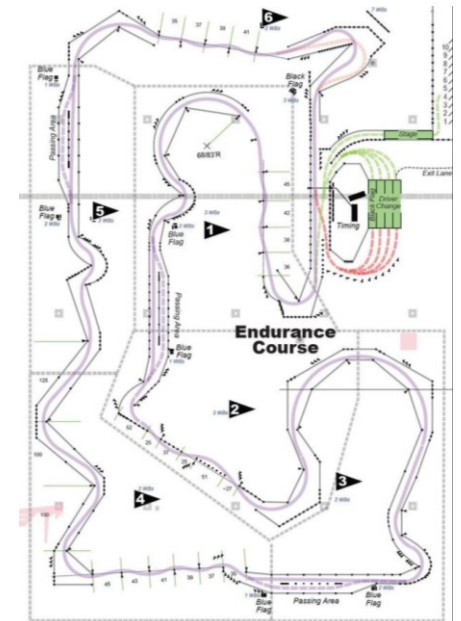


Figure 7: Lincoln 2019 Endurance Layout^[8]



Established Conventions

How can the complexities of automotive engineering be communicated efficiently and systematically?

• Nomenclature, Coordinate & Unit Systems

- Adopted SAE-J670-2008^[9] Z-Up
 - Descriptive nomenclature
 - Coordinate systems (sign conventions)
 - World, Intermediate, Body, Wheel, Tire
- Model handbook^[10] documents general symbolic nomenclature
- All vehicle simulation is performed in metric
 - Imperial inputs can be scripted but should be immediately converted
 - Angular unit is not standardized (beware of using degrees in differential equations)

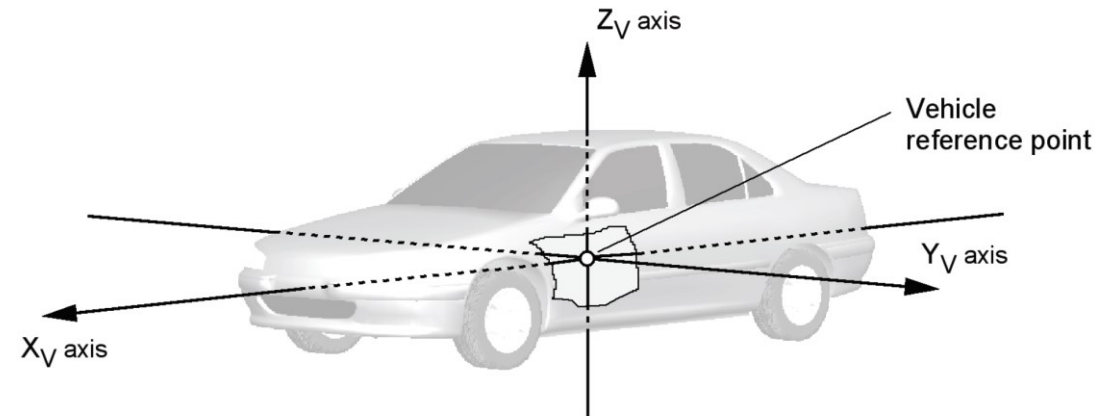
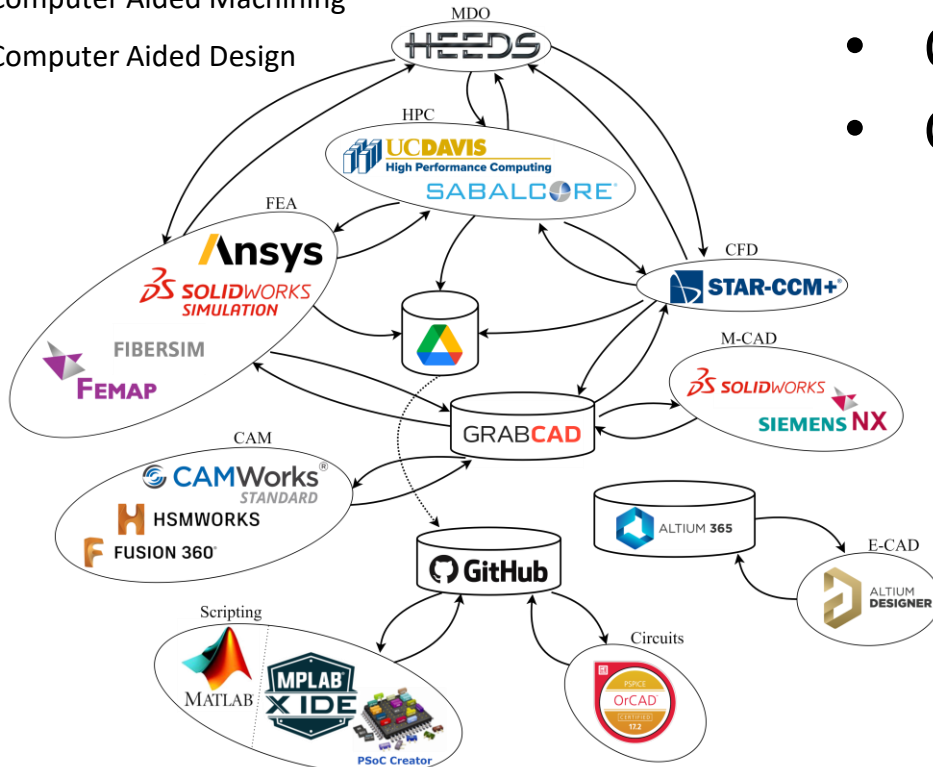


Figure 8: SAE Z-Up Vehicle (Body/Chassis) Coordinate^[9]

Data Management

MDO	Multidisciplinary Design Optimization
HPC	High Performance Computing
FEA	Finite Element Analysis
CFD	Computational Fluid Dynamics
CAM	Computer Aided Machining
CAD	Computer Aided Design



- Simulation infrastructure has grown rapidly
 - Accommodating expanding analyses
- Simulations and data are now stored in Github
 - Operating processes still maturing
 - Currently separating public and private data
 - Store simulation scripts in public “ “-Modeling repos
 - Store simulation inputs/outputs in private “ “-Data repos
- Still hurdles in managing long workflows
 - FEA derived compliance lookup tables
 - CFD derived aerodynamic load map

Figure 9: FRUCD Data Management Network^[11]

Software Development Conventions

- Previous simulation development efforts have failed due to the following:
 - Documentation: Lack of documentation prevents transfer
 - Implemented standardized comment requirements to ensure readability
 - Interdependency: Software inflexible to necessary architecture changes
 - New parsing – resource function development architecture standardizes sustainable functions
 - Variance: Code base written in many different styles and symbolic naming
 - Adopted uniform syntax^[12] and descriptive naming (e.g. *Beta* → *BodySlip*)

```
function [Output, ...] = FunctionName(Input, ...)
%% FunctionName - Short Description
% Detailed Description ...
%
% Inputs:
%   Input - (dataType) Description {symbol} [unit]
%   ...
%
% Outputs:
%   Output - (dataType) Description {symbol} [unit]
%   ...
%
% Notes:
%   Recommendations, References, ...
%
% Author(s):
%   First Last (email@ucdavis.edu) [MMM YYYY - MMM YYYY]
%
% Last Updated: DD-MMM-YYYY
```

Figure 10: Standardized Comment Header

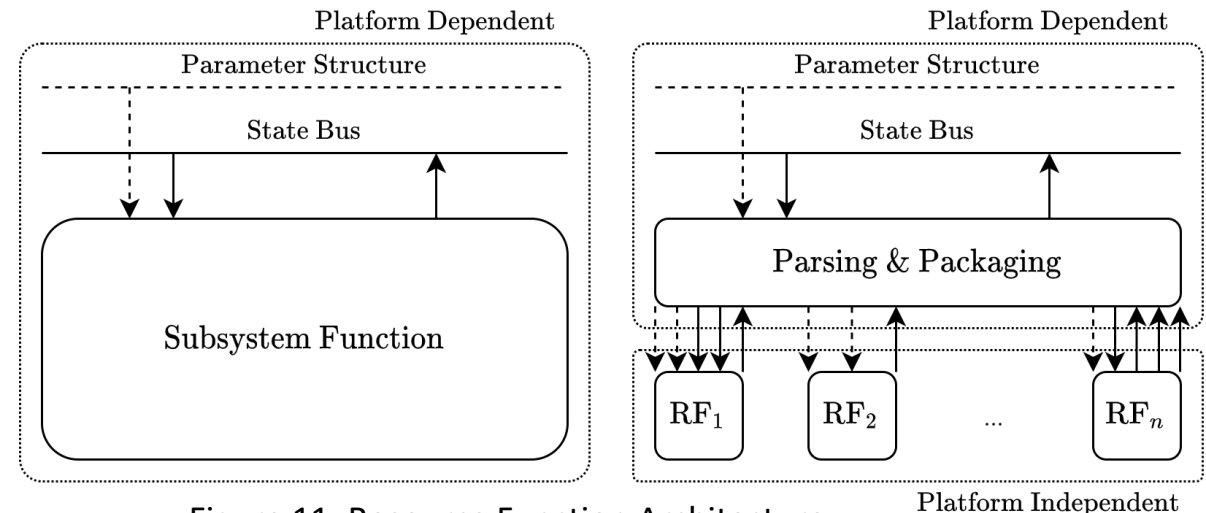


Figure 11: Resource Function Architecture

Vehicle Systems Decomposition

- Vehicles are complex nonlinear systems
 - 1000's of parameters, 100's of constitutive relationships, & 10's of dynamic states
- Decomposition promotes accessibility
 - Members can directly develop in their area of expertise
- Subsystems broken down in resource functions
 - Highly modular across platforms & fidelities

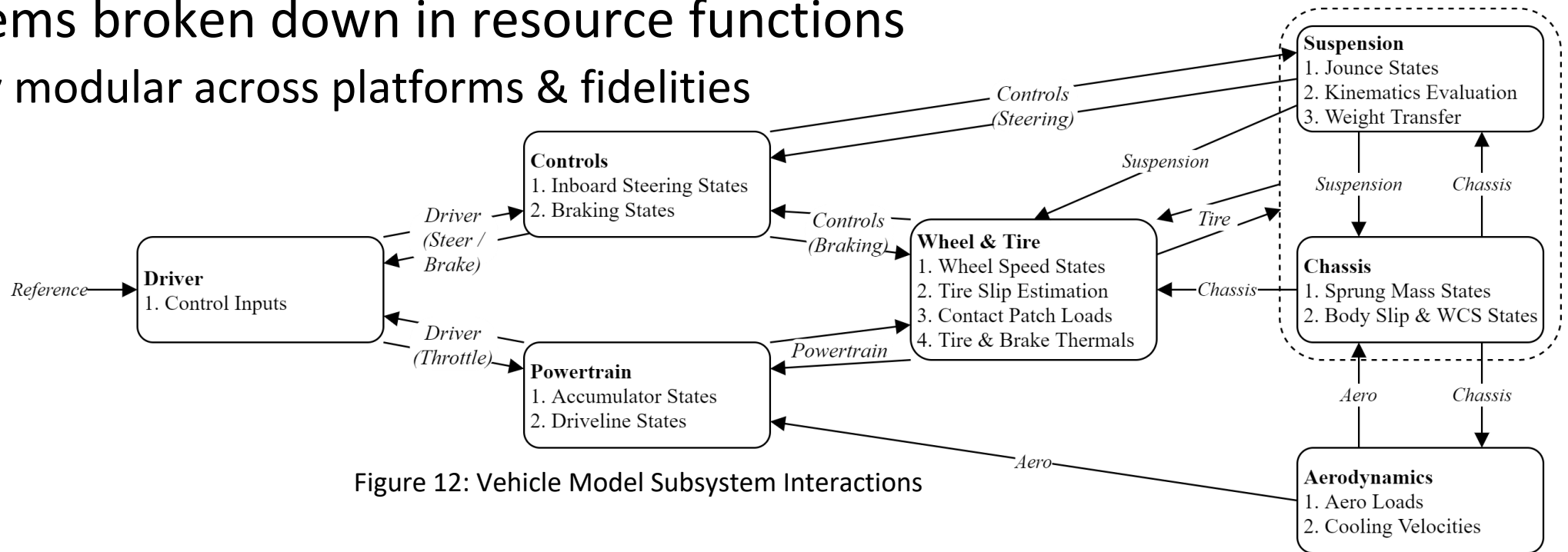


Figure 12: Vehicle Model Subsystem Interactions



Tire Mechanics

How do we represent the most important interaction between the vehicle and its environment?

Snapshotting Tire Selection

- Tire selection is very constrained
 - Limited market for slicks of correct size
 - Continentals only available through sponsorship
- Quick Notes^[13]:
 - Avon 13”s have no redeeming qualities
 - Avon 10”s have lateral peaks (hard to drive)
 - Hoosier is the main supplier
 - 13”: Heavy and do not provide more grip than 10”
 - 6.0: Reduced tread widths destroy available grip
 - LCO: Compound requires low temps and wears quickly
- This leaves two contenders:
 - Hoosier R25B 16.0x7.5-10x7
 - Hoosier R25B 18.0x7.5-10x7

Tire			Cornering	Braking
Hoosier	R25B	16.0x7.5-10x7	8-[1,2,4]	X
Hoosier	R25B	16.0x6.0-10x7	8-[11-13]	X
Hoosier	LCO	16.0x7.5-10x7	8-[17-19]	X
Hoosier	LCO	16.0x6.0-10x7	8-[23-25]	X
	Avon	7.0/16.0-10x7	7-[1-3]	X
Hoosier	R25B	18.0x7.5-10x7	6-[20-22]	6-[34-36]
Hoosier	R25B	18.0x6.0-10x7	X	X
Hoosier	R25B	20.5x7.0-13x7	6-[8-10]	6-[54-56]
Hoosier	R25B	20.5x6.0-13x7	8-[11-13]	6-[51-53]
	Continental	205/510R13x7	6-[14-16]	6-[60-62]
	Continental	205/470R13x7	8-[35-37]	8-[47-49]
	Avon	7.2/20.0-13x7	7-[14-16]	7-[32-34]
	Avon	6.2/20.0-13x7	7-[23-25]	7-[29-31]

Table 2: Tire Data Availability

(Supplier) (Compound) (Tire Diameter)x(Tire Width)–(Rim Diameter)x(Rim Width)

Figure 13: Tire Naming Convention

Tire Modeling Conventions

- Tires are the most critical and complex aspect of the vehicle model
 - Describe the main interaction between the vehicle and its environment
- Operating conditions determine a complex load and radial deflections

$$[F_x, F_y, M_z, M_x, M_y, r_e, r_l] = f(\alpha, \kappa, F_z, P_i, \gamma, T_C, v_T)$$

Equation 1: Tire Functional Relationship

Inputs	Symbol	Unit	Outputs	Symbol	Unit
Slip Angle	α	[deg]	Longitudinal Force	F_x	[N]
Slip Ratio	κ	[]	Lateral Force	F_y	[N]
Normal Load	F_z	[N]	Aligning Moment	M_z	[Nm]
Inflation Pressure	P_i	[kPa]	Overtuning Moment	M_x	[Nm]
Inclination Angle	γ	[deg]	Rolling Resistance	M_y	[Nm]
Carcass Temperature	T_C	[°C]	Effective Radius	r_e	[m]
Contact Patch Velocity	v_T	[m/s]	Loaded Radius	r_l	[m]

Table 3: Tire Mechanics Nomenclature

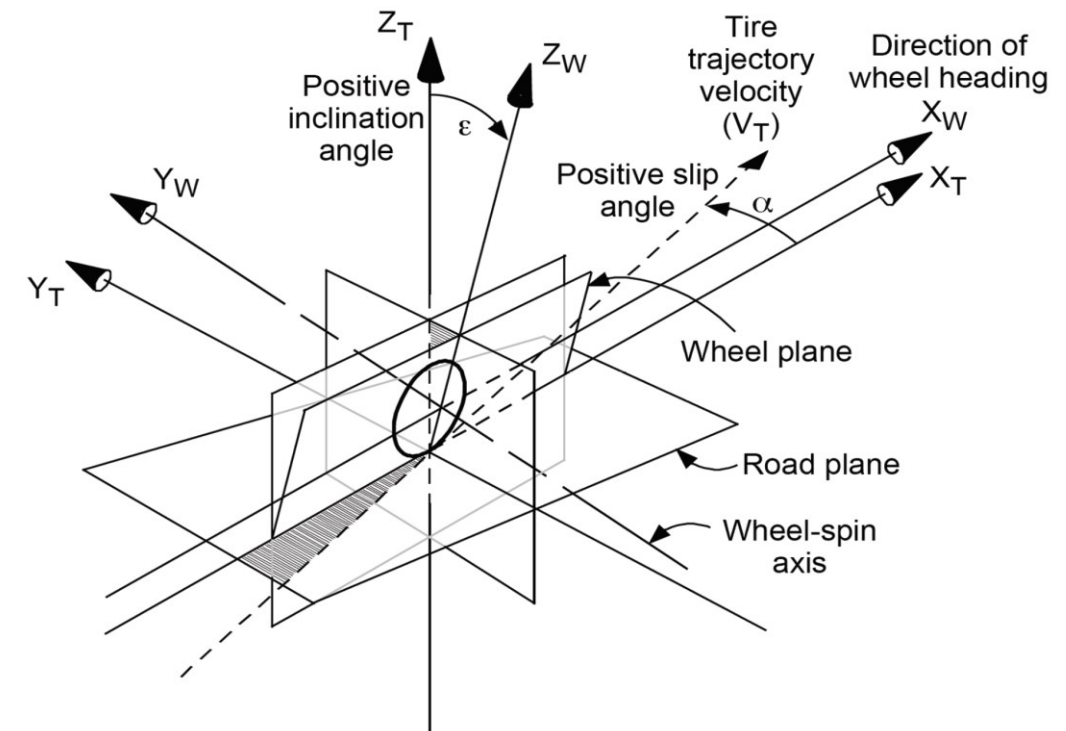


Figure 14: SAE Z-Up Wheel & Tire Coordinates^[9]

Tire Model Decomposition

- Evaluation involves a cascading set of models which improve predictions
 - Pure: No longitudinal-lateral interaction effects
 - Combined: “Friction Ellipse” behavior
- Recent Additions:
 - Overturning Moment
 - Relaxation Length
- Current Priorities:
 - Radial Deflection
 - Combined Slip Aligning Moment
 - Rolling Resistance
 - Thermal Effects

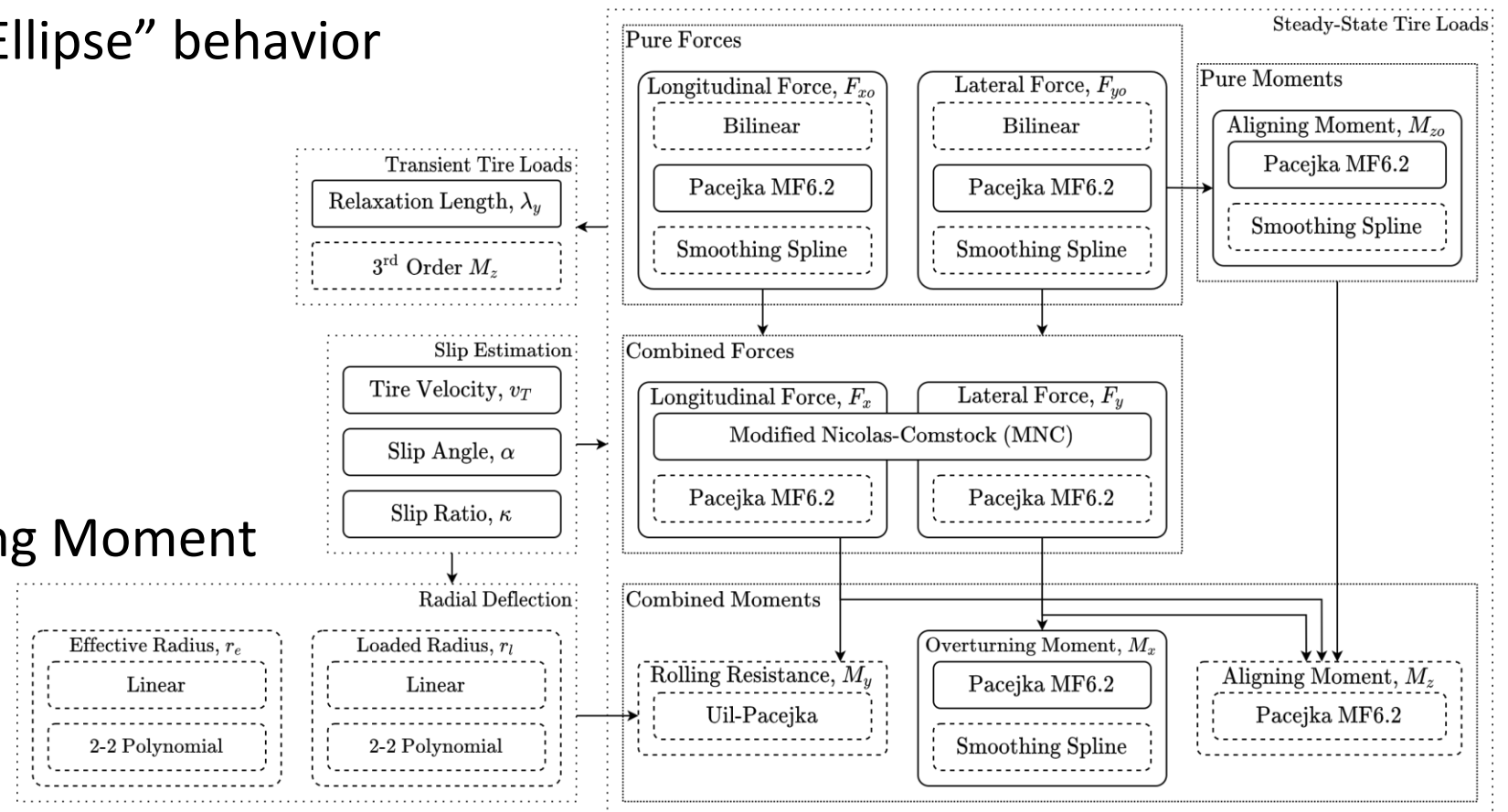


Figure 15: Tire Computation Flowchart

Tire Contact Patch Load Methodology



Figure 16: FSAE Tire Testing Consortium^[17]

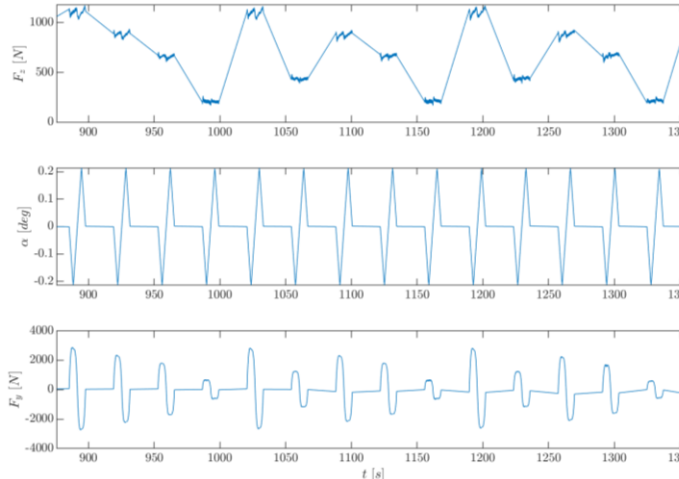


Figure 17: Tire Data^[18]

- Data-driven Pacejka^[14] model
 - Quick to evaluate and scales nicely
 - Smoothing spline models will be more locally accurate
- Modified-Nicolas-Comstock^[15] (MNC) model is used to overcome longitudinal data limitations of 16"s
- Wong-Pacejka^[16,14] rolling resistance model is used again due to data limitations for all tires

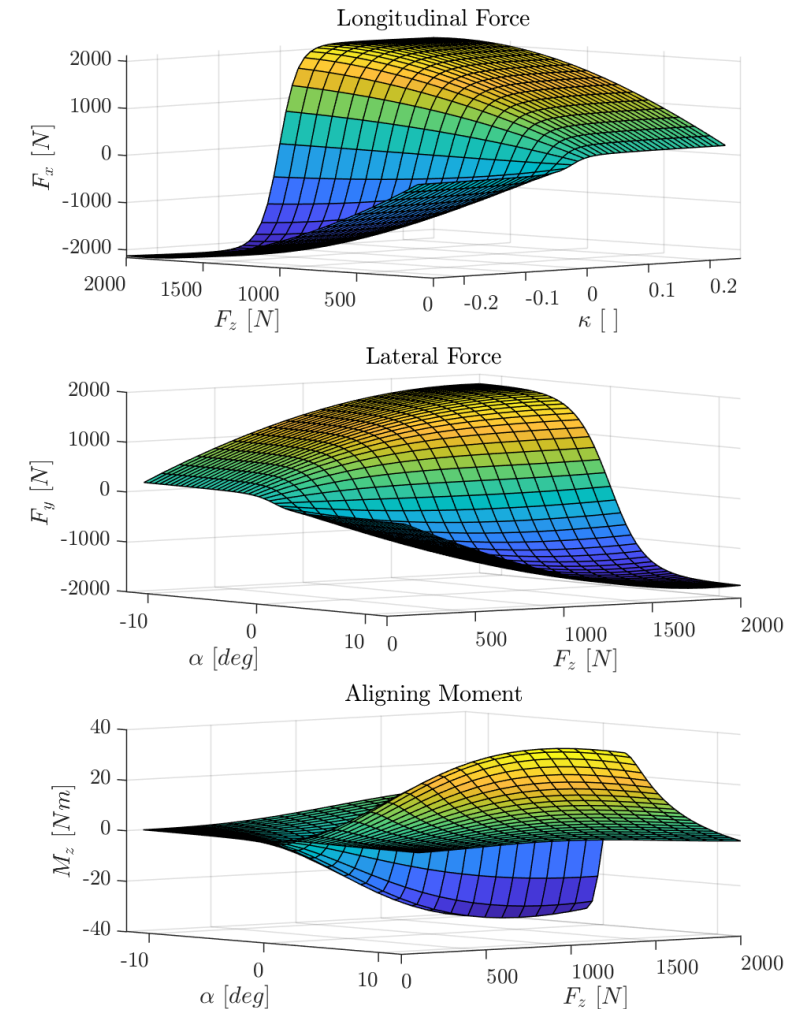


Figure 18: Pacejka Pure Slip Surfaces

• Overturning Moment, M_x

- Heavily influences load distribution between A-Arms
 - Manipulates roll dynamics
 - Complex behavior at large inclination
- Fitted with single regression
 - No need for bootstrapping since $p = 12$
- Fits are functional, but not tuned
 - Diverging behavior at high load

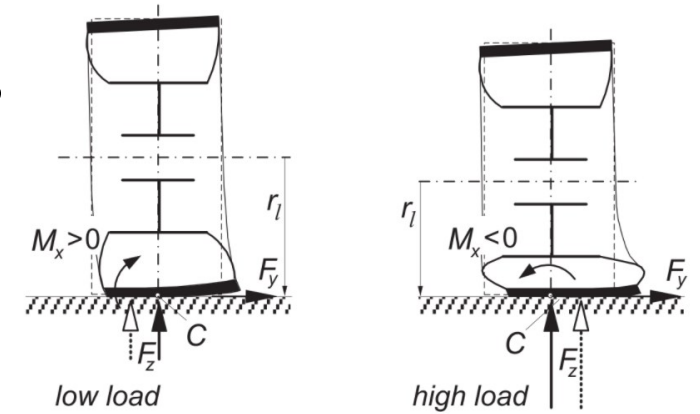


Figure 19: Overturning Moment Diagram [14]

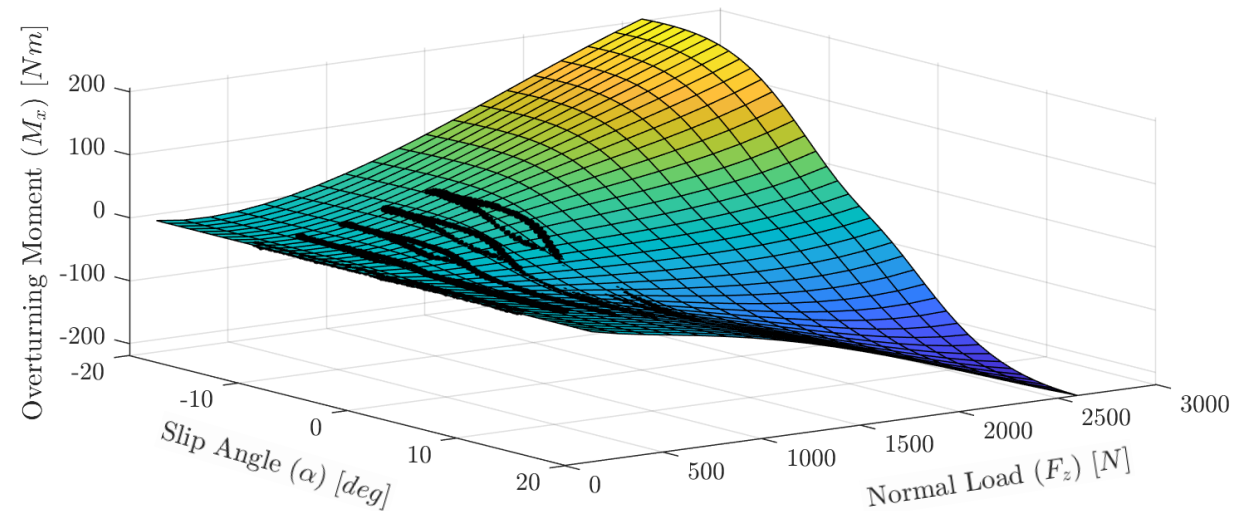


Figure 20: Overturning Moment Fit Example

• Transient Tire Modeling - Relaxation Length, λ

- FSAE style racing is highly transient due to tight corners
- A first order step response is fitted to TTC data:

$$\text{Equation 2: } \dot{F}_y = F_{y_{t \rightarrow \infty}} \left[1 - \exp\left(\frac{-t}{\tau}\right) \right] \quad \lambda = \tau \dot{x}_T$$

- The relaxation length was dependent on conditions:
 - Tire responded slower at high normal loads
 - Tire responded faster at high pressures
 - Unexpectedly tire responded faster on larger slip steps
- Transient response differentiates the 18"s and 16"s
 - Thinner sidewall 16"s respond faster in theory
 - λ is 45.6% shorter for the 16" tire

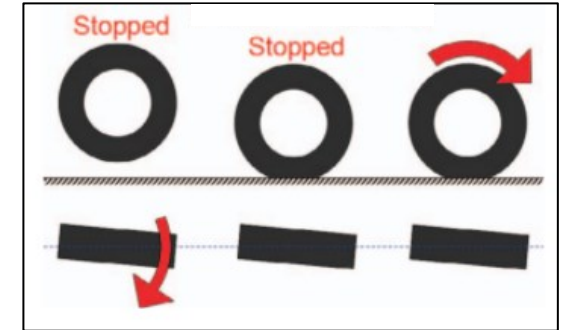


Figure 21: Lateral RL Test Procedure^[19]

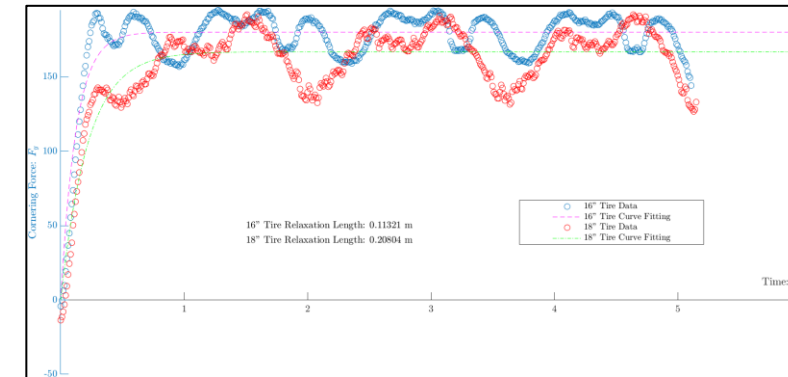


Figure 22: Lateral Force Response Comparison

Combined Slip Aligning Moment

- Gives feedback from longitudinal forces to driver
 - Gives scrub constraints on front wheel package
- Current method involves fitting larger tires
 - Will overpredict and over constrain targets
 - Data is limited compared to pure slip
- Fitting process involves scrub parameters
 - Only 4 parameters to fit total
 - Other parameters have already been fit

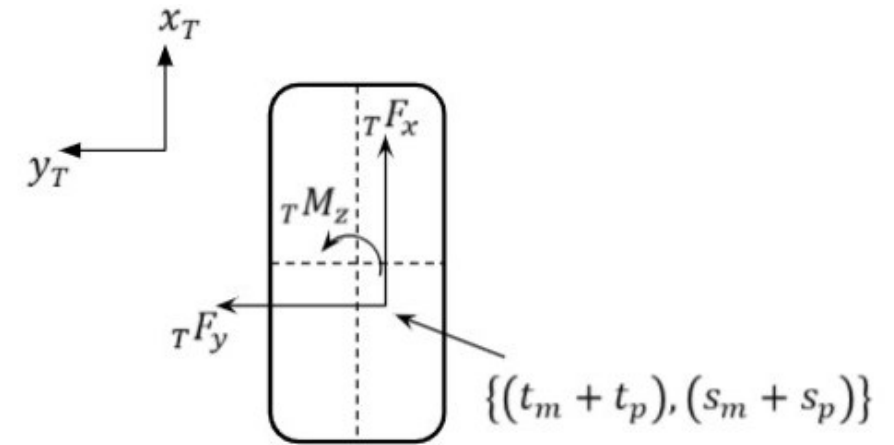


Figure 23: Combined aligning moment diagram

Thermal Tire Behavior

- Influential in force/moment generation
 - Changes how tire behavior generates
 - Creates pressure differences (impacts above)
- Tire Thermals are hard to compute
 - Very limited data (if any available)
 - Data to fit equations requires heat transfer properties
 - Many parameters to fit using these complex data sets
- Modelling will be hard to justify
 - Direction to be completed by end of sq

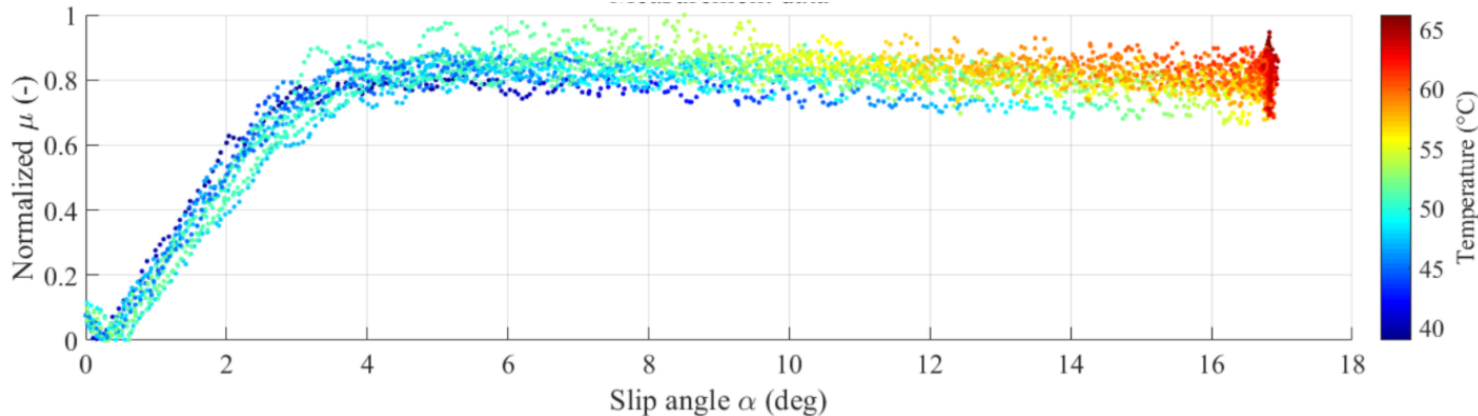


Figure 25: Coefficient of Friction vs Slip Angle

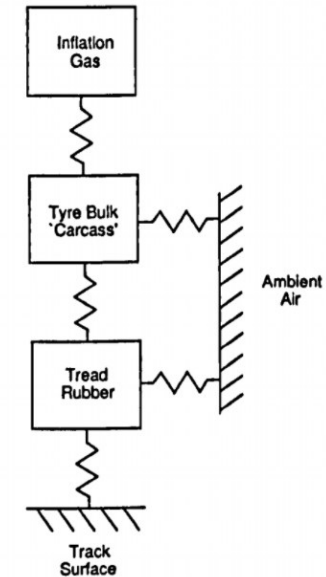


Figure 24: Thermal Schematic of Tire

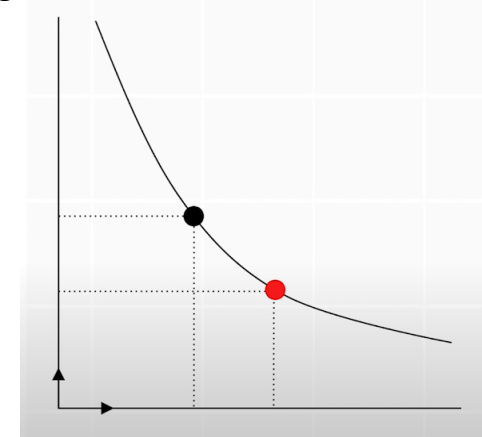


Figure 26: Grip vs Pressure Relationship

• Tire Model Validation Needs

- Friction scaling between data and reality
 - Discrepancies in testing and racing surfaces
 - Test data overpredicts by x1.25 – x1.6
- Tires cannot be verified in isolation
 - Requires expensive testing infrastructure
- Other systems must be tightly controlled
 - Allows for system error to be clearly attributed to tire uncertainty

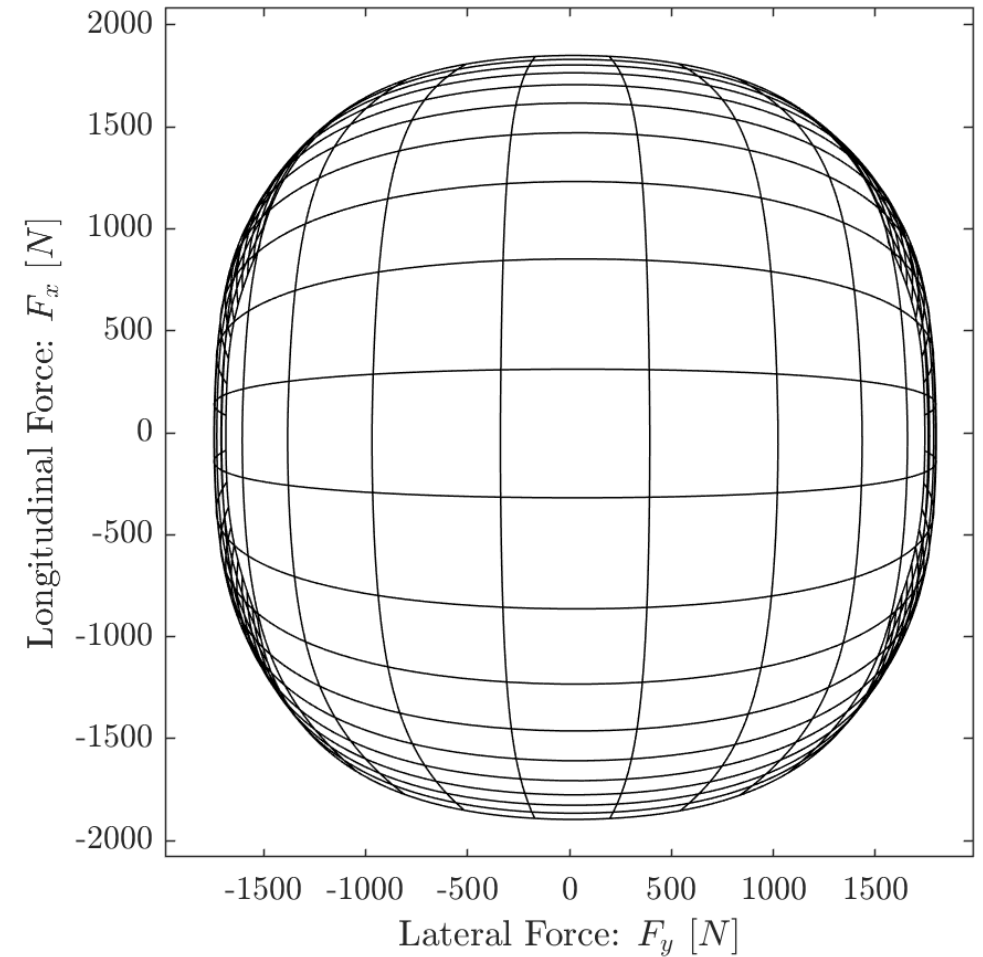


Figure 27: Friction Ellipse Example



• Suspension and Chassis Dynamics

How do we represent the relationship of tire loads, suspension, and the body frame of the car?

Suspension and Chassis Overview

- Suspension and tire loads
 - Reaction forces affect tire loads
 - Tire load manipulated by suspension
- Chassis
 - Reacts to the loads from suspension
- Body coordinate frame
 - x axis parallel to car heading
 - y axis is normal to driver's side of the car
 - z axis is positive upwards

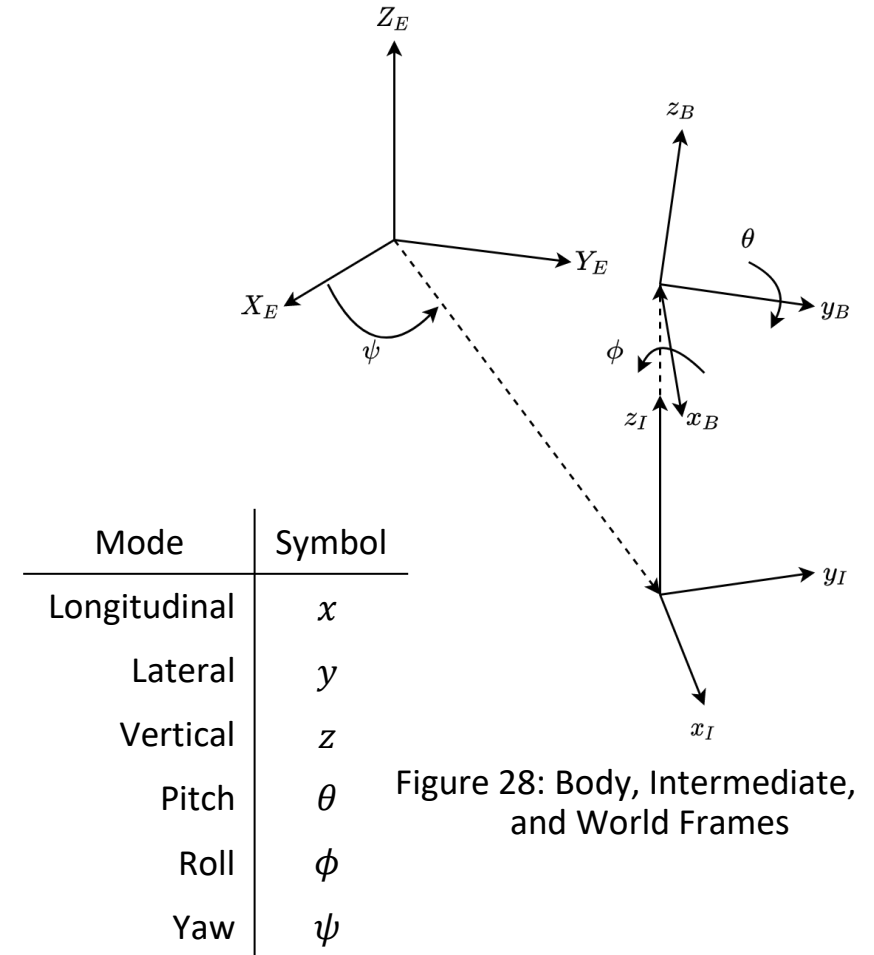


Figure 28: Body, Intermediate, and World Frames

Table 4: Chassis Mode Nomenclature



Underlying Physics

- Newton's laws govern reaction in the chassis
 - Tire loads from acceleration, braking, bumps
 - Aero loads
- Grip, balance, stability, & control
 - Grip and yaw moment
 - Quantifying stability and control
- Key decisions influenced by modeling
 - Mass distribution, center of gravity height, spring stiffness, wheelbase
- Suspension dynamics include jounce
 - Which are vertical DOFs which are coupled to chassis dynamics

Chassis & Suspension Modeling

3DOF Bicycle Model

- Basic yaw response
- Straight line performance
- Initial pitch and roll rates

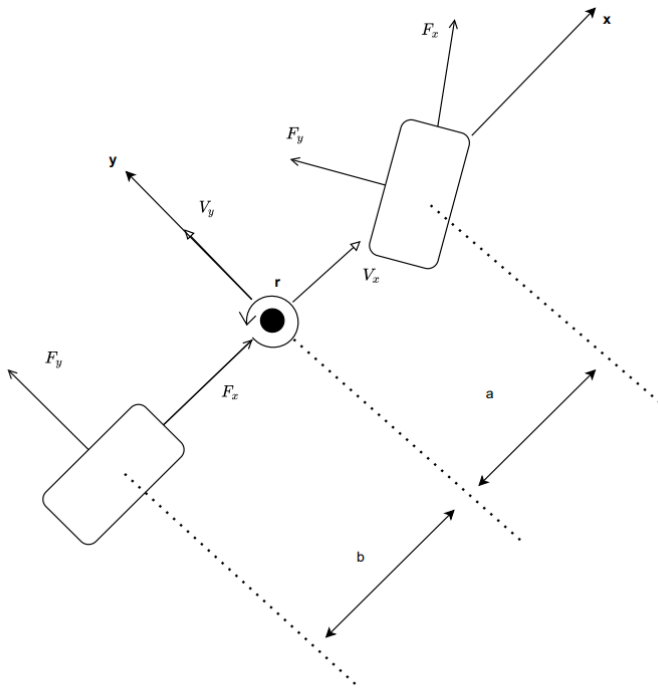


Figure 29: 3DOF Bicycle Model

Table 5: Chassis & Suspension Model Configurations

Model Name	Dynamics			Weight Transfer	Chassis Compliance
	Corner	Attitude	Planar		
10DOF FBRC	X	X	X	TD	
6DOF FBRC		X	X	RB	
5DOF Sampo		X	X	RB	X
4DOF Half Car	X	X		TD	
3DOF Table			X	RB	
3DOF Bicycle			X		

TD = Tire Deflection, RB = (Spring) Rate Based

Chassis & Suspension Modeling (2)

3DOF Table Model

- Lateral load transfer studies
- Initial Ackerman steering design

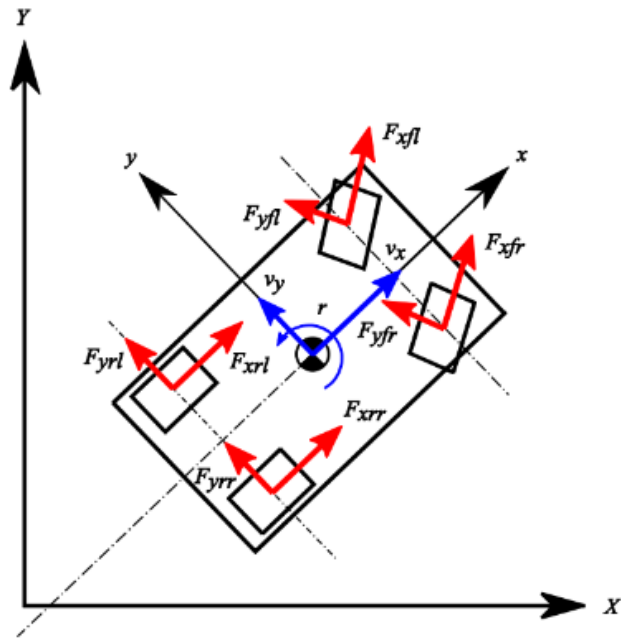


Figure 30: 3DOF Table Model^[?]

4DOF Half Car Model

- Brake & acceleration design
- Initial roll and pitch rate design

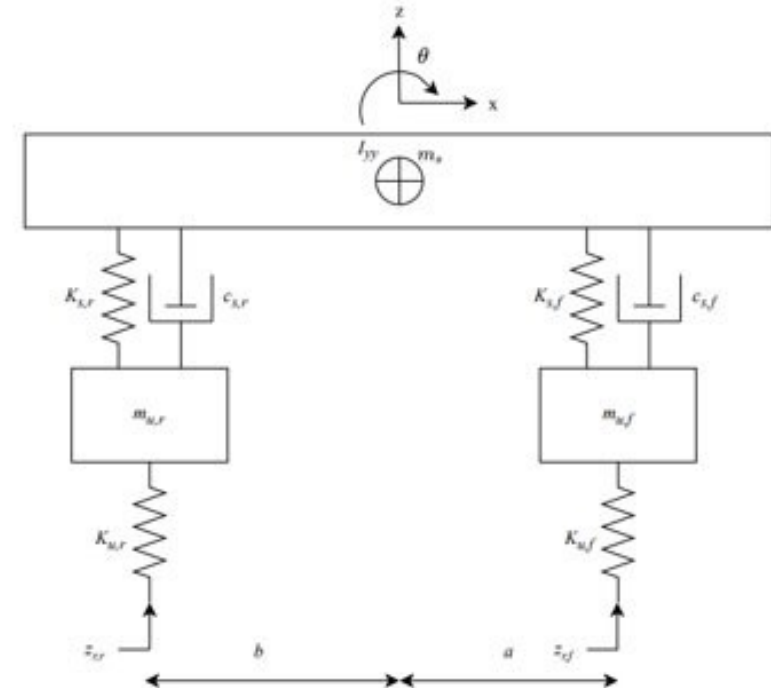


Figure 31: 4DOF Half Car Model

Chassis and Suspension Modeling (3)

5DOF Sampo Model

- Validate chassis torsional stiffness target

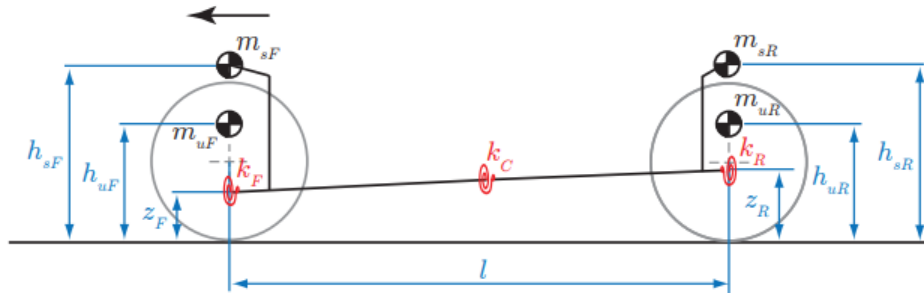


Figure 32: 5DOF Sampo Flexible Lateral Load Transfer Model^[?]

6DOF Force Based Roll Center

- Lap Time Simulation

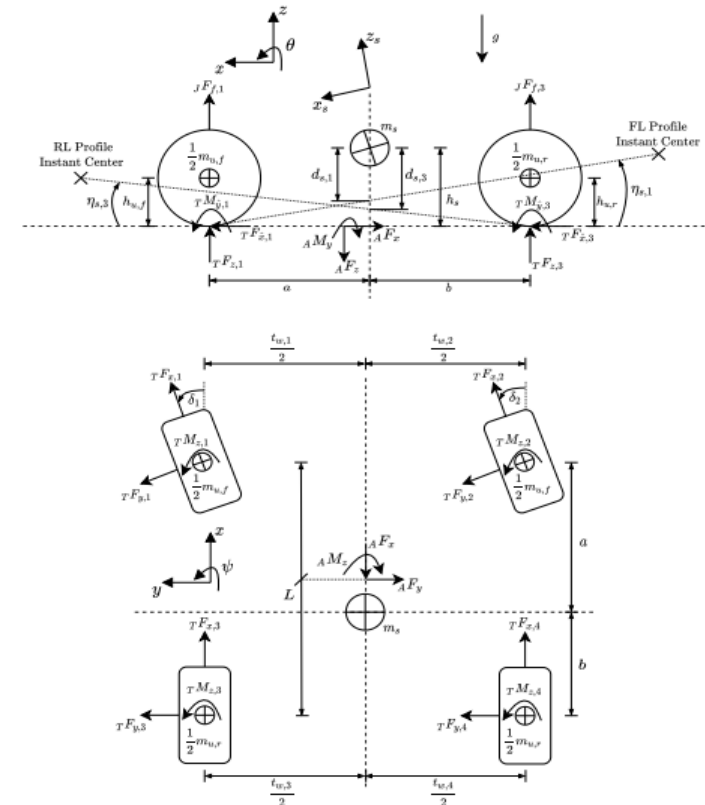


Figure 33: 6 DOF Force Based Roll Center Model

● Chassis and Suspension Modeling (4)

10DOF Force Based Roll Center

- Suspension instant center design
- Final spring rates and damping design

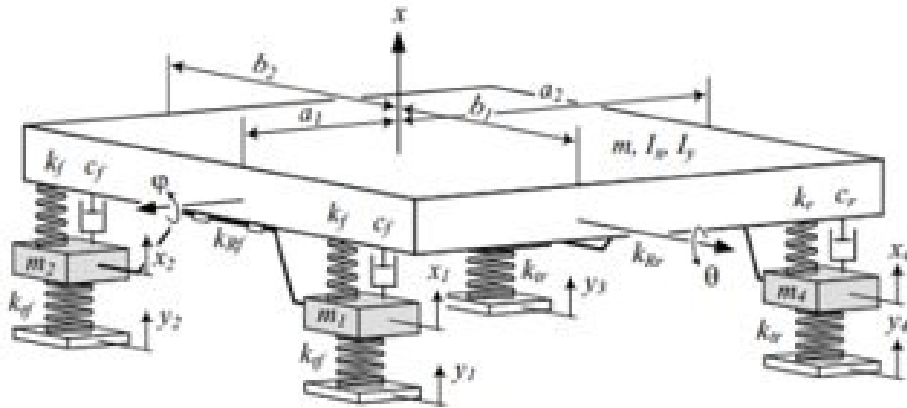


Figure 34: 10DOF Force Based Roll Center (FBRC) Model^[6]

Resource Functions

- Advantages
 - Reusability
 - Flexibility
 - Standardized conventions
- Weight Transfer
- Kinematics Evaluation
- Jounce States
- Sprung Mass States
- WCS States



Controls Modeling

How is the driver integrated with the rest of the vehicle?



Controls Overview

Inboard Steering

Description	Symbol	Unit
Steering Torque	τ_{sw}	$[Nm]$
Pedal Displacement	δ_{sw}	$[deg]$
Rack Displacement	δ_r	$[m]$
Rack Force	F_r	$[N]$

Table 6: Steering State Nomenclature

- Models are very simplistic
- Incorporates nonlinear U-Joints

Brake System

Description	Symbol	Unit
Pedal Force	F_p	$[N]$
Pedal Displacement	θ_p	$[m]$
Line Pressure	$P_{b,i}$	$[MPa]$
Brake Torque	τ_B	$[Nm]$

Table 7: Brake State Nomenclature

- Models are very simplistic
- Use algebraic relationships to convert force into brake torque
- Displacements are currently unavailable (see next)



• Open Ended Controls Modeling Questions

- Controls models are simplistic, but estimating parameters is not
 - Physics heavily depend on finicky friction parameters
- Steering nonlinearities have a large impact on feedback
 - Deadzones created by joint lash
 - Hysteresis from rack and column collar friction
- Braking pressure and pedal displacement are dependent on compliances
 - Hardlines and fluid compliance are reliably estimated
 - Softline compliance must be tested due to the complexity of softline construction



Powertrain Modeling

How is the power generated and distributed to the tire?

Powertrain Overview

- Differential modeling efforts have stalled
- Powertrain models will be oversimplified
- Will require major future efforts to provide justification for design parameters

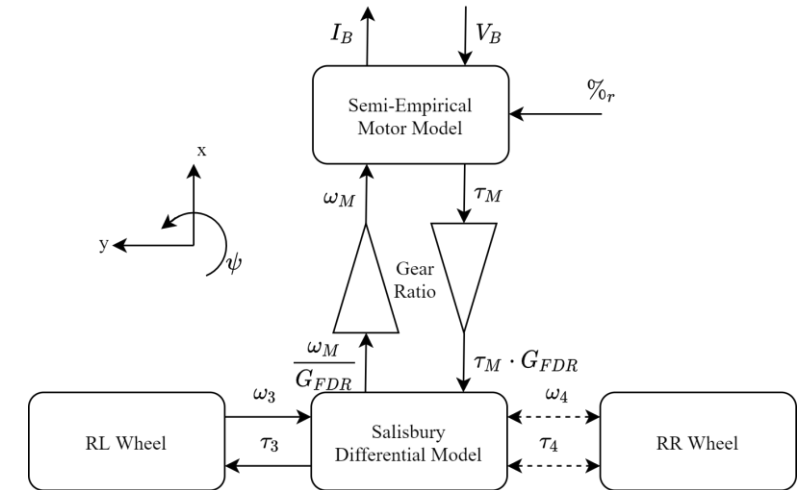


Figure 35: RWD Powertrain Block Diagram

Symbol	Description
V_B	Voltage Supplied to Motor
$\%r$	Torque Request
I_B	Current Drawn by Motor
τ_A	Axle Torque
$\omega_{3,4}$	Left & Right Wheel Speeds
$\tau_{3,4}$	Left & Right Wheel Torque
ω_D	Rotation Speed of Differential
η_D	Differential Efficiency

Table 8: Powertrain Nomenclature Table



Integrated Simulation Platforms

How are vehicle models evaluated to characterize performance?

Performance Envelope Generation (1)

- Provides steady state performance envelope
 - Steady state is less expensive allowing for large studies
 - Enables vehicle level optimization ($p > 100$)
- Plots Yaw Acceleration vs Lateral Acceleration
 - Mesh of body slip and steer and combinations
- Final oversteer and understeer
 - Limit stability & trim acceleration are easily marked

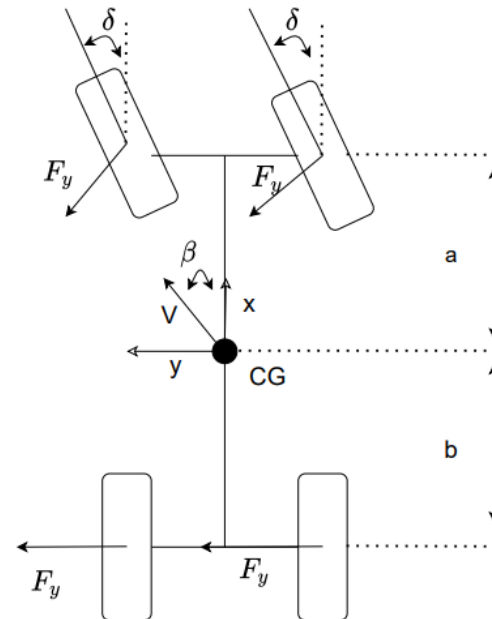


Figure 37: Body Slip Angle Diagram

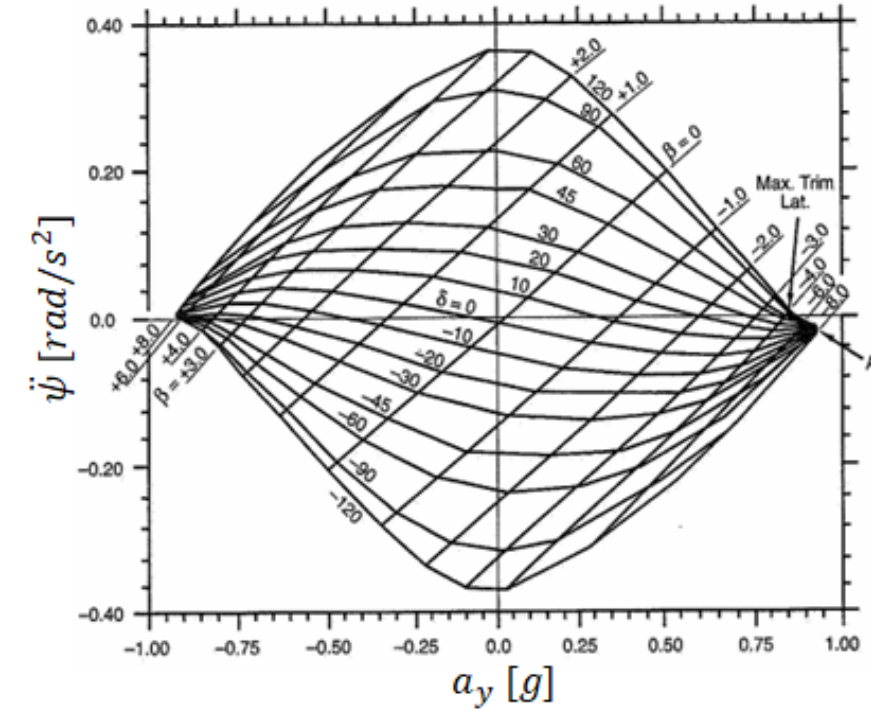


Figure 38: Typical MMM Diagram^[?]

Symbol	Description
δ	Steering Wheel Angle
β	Vehicle Sideslip
a_y	Lateral Acceleration
$\dot{\psi}$	Yaw Acceleration

Table 10: MMM Nomenclature Table

Performance Envelope Generation (2)

- Chassis
 - Chassis Dynamics, Sideslip
- Wheel and Tyre
 - Wheel Speeds, Tire Evaluation, Slip estimation
- Suspension
 - Kinematics and Compliances
- Controls
 - Driveline and Brakes
- Powertrain
 - Driveline and Brakes
- Aerodynamics

- Timeline
 - Debugging, adding more systems/models
 - Goal to have it running by end of quarter

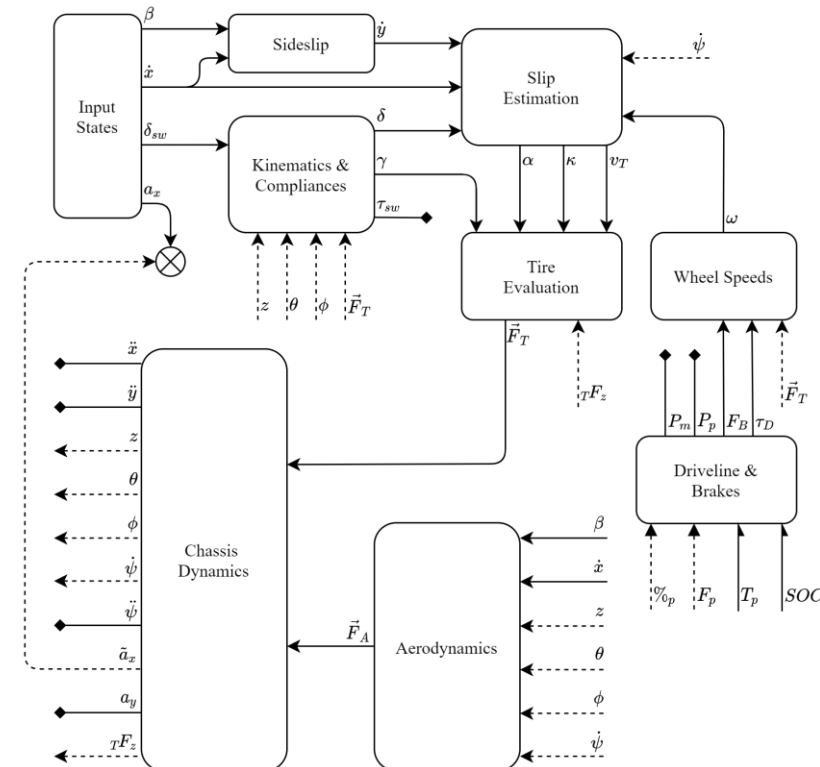


Figure 39: Vehicle Model Flowchart

Quasi-Transient Lap Time Simulation

- Leverages performance envelopes
- Solves planar curvilinear motion with a body slip
 - Fully addresses yaw dynamics
- Projected to simulate 5 sec / simulation
- More to come at the end of SQ

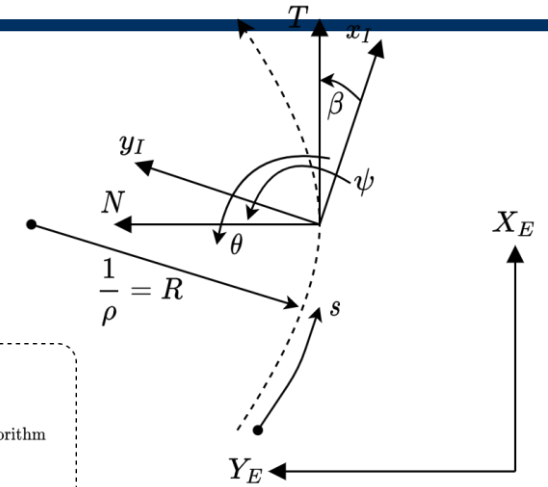


Figure 40: Curvilinear Motion

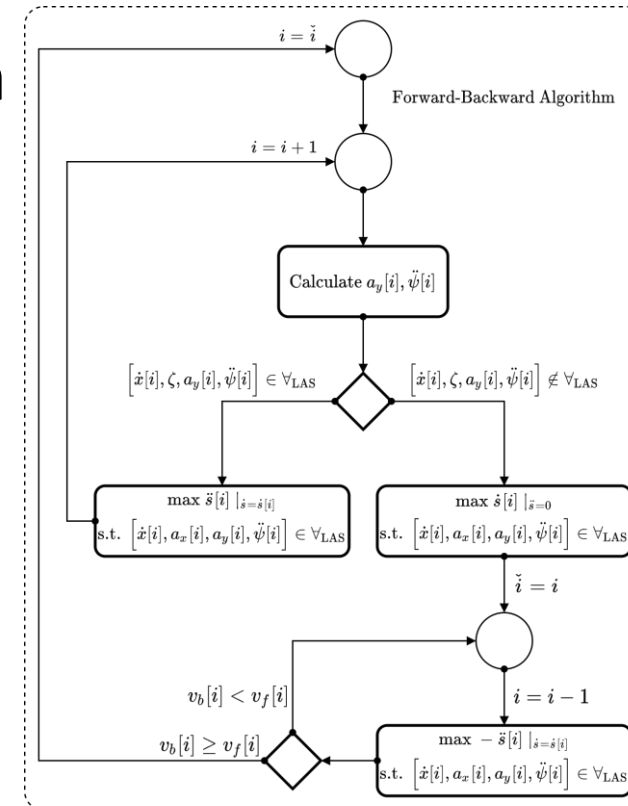


Figure 41: Stepping Solver



Summary



Recapping Actions on FE8 Goals

- Maximize knowledge transfer to ensure consistent VD growth
 - Season had the largest and most active VD subteam in FRUCD history
 - Next year will present challenges; however, VD is in a good place
- Expand VD knowledge across team for more integrated system design
 - Accomplished via All-Hands, and integration lectures
- Develop user-friendly vehicle simulation platforms to expediate design
 - Created more sustainable simulation software architecture
 - Did not finish implementation of a vehicle simulation platform
- Prepare future leads on how to correlate models with testing data
 - Not addressed completely, will be a focus in remaining time



Goals for VD

- Complete performance envelope generator
- Utilize senior design LTS tool to establish and justify vehicle concept
- Vehicle modeling platform will make dynamic simulation more accessible
 - Allows future efforts to be focused on specific thrusts instead of architecture



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