Turbulent Fluid Flows in Definite Geometries and Numerical Solutions

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Complex Fluid Flows and Its Environmental Applications

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The Abdus Salam International Centre for Theoretical Physics

IAEA.org International Atomic Energy Agency

Presidenza del Consiglio dei Ministri

UNESCO United Nations Educational, Scientific and Cultural Organization

The lectures aims to convey the following information/message to the students:

♦ Fundamentals of turbulent flows

→ Definitions

→ Statistical descriptions

→ Equations of turbulent flows

♦ Approaches to study turbulence

→ Numerical *vs* Analytical *vs* Experimental

→ Numerical modeling

♦ Channel flow

♦ Turbulent thermal convection in an enclosed cavity

“We all pass through life surrounded -and even sustained- by the flow of fluids. Blood moves through the vessels in our bodies, and air (a fluid-properly speaking) flows into our lungs”

“Our vehicles move through our planet’s blanket of air or across its lakes and sea, powered by still other fluids, such as fuel and oxidizer, that mix in the combustion chambers of engine.”

“Indeed many of the environmental or energy related issues we face today cannot possibly be confronted without detailed knowledge of mechanics of fluids”

Parviz Moin and John Kim

Scientific American

January 1997

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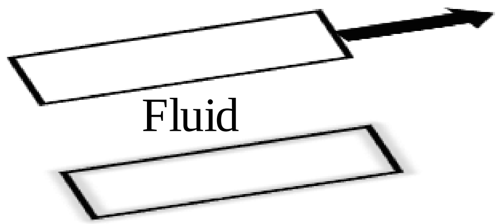
Some application areas of fluid mechanics.

What is a fluid?

"A substance that deforms continuously when acted on by a shearing stress of any size".



Important characteristics of fluid, from a fluid mechanics point of view, are density (ρ), pressure (P),viscosity (μ), surface tension (τ) and compressibility.

Internal/External flow

Viscous/Inviscid regions of flow

Laminar/Turbulent

Forced/Neutral flow

Steady/Unsteady

Compressible/Incompressible

Newtonian/non-Newtonian

Laminar & turbulent flows

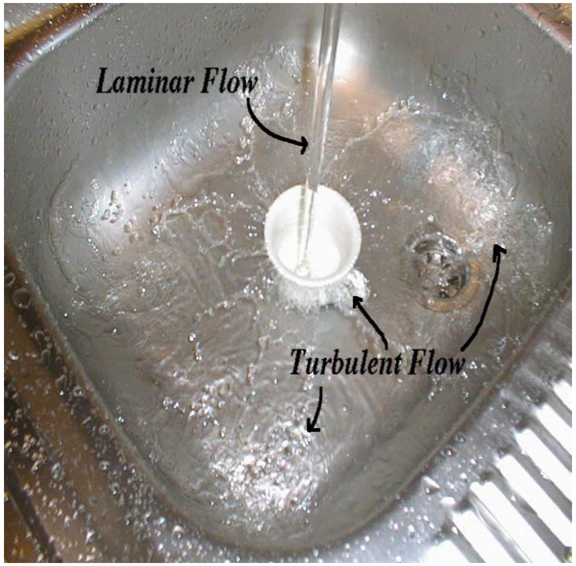
- **Two types of viscous flows:**

➢ **Laminar flow:**

- Where the fluid moves slowly in layers for instance in a pipe, without much mixing among the layers (typically occurs when the velocity is low or the fluid is very viscous).

➢ **Turbulent flow**

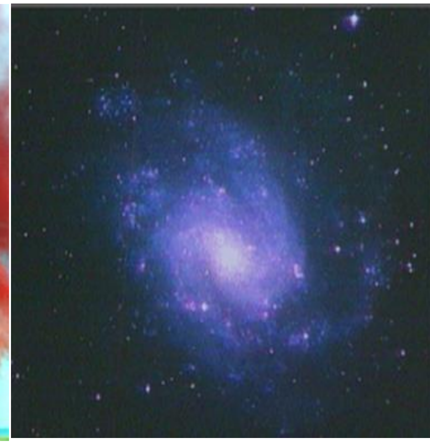
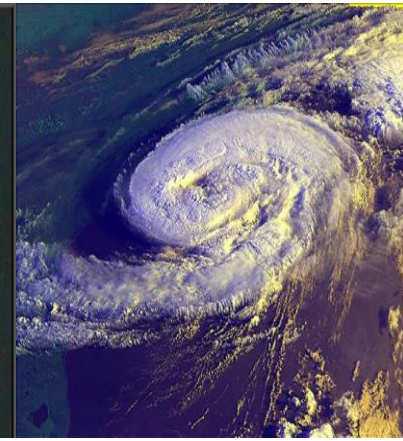
- Opposite of laminar, where considerable mixing occurs, velocities are high.



Laminar and Turbulent flows can be characterized and **quantified** using a ***Reynolds Number (Re) or Rayleigh number (Ra).***

Why study turbulence?

- Fluids and fluid instabilities, including turbulence, appear in a wide range of natural contexts as well as engineering systems.


- The problem of turbulence has been studied by many of the greatest physicists and engineers of the 19th and 20th centuries, and yet we do not understand in complete detail how or why turbulence occurs, nor can we predict turbulent behavior with any degree of reliability, even in very simple (from an engineering perspective) flow situations. Thus, study of turbulence is motivated both by its inherent intellectual challenge and by the practical utility of a thorough understanding of its nature.

To study turbulent flow…

Analytically

Solutions are available for only very few problems.

Experimentally

Combined with empirical correlations have traditionally been the main tool – an expensive one

Numerically

Potentially provides an unlimited power for solving any flow problems

➢ 

- Flow visualization/how many probes you like!
- Continual variations of parameters (e.g. *Re*, *Pr*)
- Unconditional validity of the approximations (e.g. Boussinesq approx.)
- Precise assignment of boundary conditions (especially temperature)

➢ 

- Enough spatial resolution to solve numerical equations:

  Thermal and viscous boundary layers

  Bulk smallest scales

  Using (really) stretched grid, increase time of computation
- Enough temporal resolution to simulate

  The fastest flow scales

  Long time integration to accumulate enough statistics

Validation of numerical modelling

Numerical modelling results always need validation. They can be:

- Compared with experiments
- Compared with analytical solutions
- Checked by intuition/common sense
- Compared with other codes (only for coding validation!)

What is Computational Fluid Dynamics (CFD)?

CFD is the analysis, by means of computer-based simulations, of systems involving fluid flow, heat transfer and associated phenomena such as chemical reactions.

- **The main issues involved in CFD, including those of:**
  - Numerical methods
  - Turbulence modelling

CFD applications

Aerospace

Energy

Biomedical

Automobile

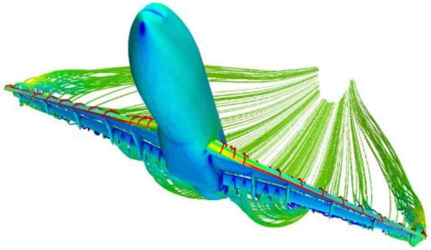
Marine

Turbo-machines

climate

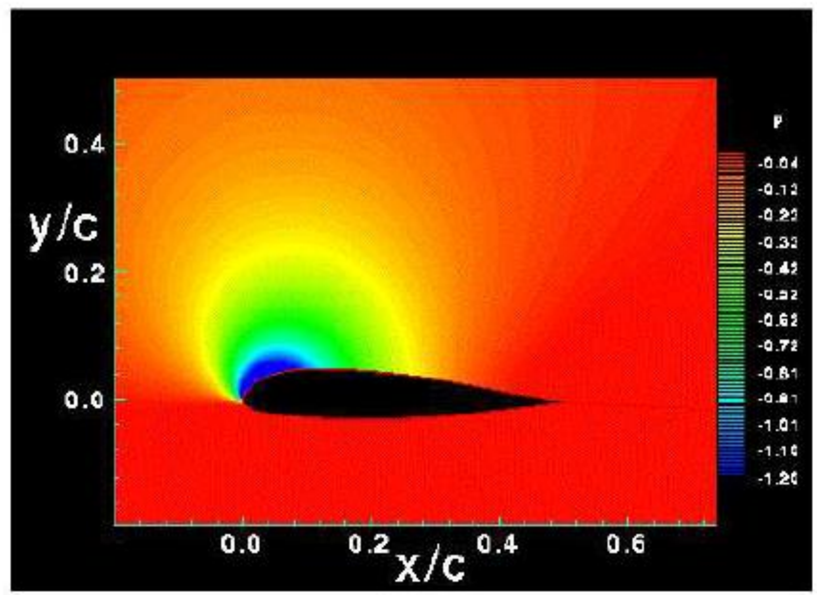
Fluid structure interaction

Electronics



...which enables the airplane to fly!!!!

In order for an aircraft to rise into the air, a force must be created that equals or exceeds the force of gravity. This force is called lift. In heavier-than-air craft, lift is created by the flow of air over an airfoil. The shape of an airfoil causes air to flow faster on top than on bottom. The fast flowing air decreases the surrounding air pressure. Because the air pressure is greater below the airfoil than above, a resulting lift force is created



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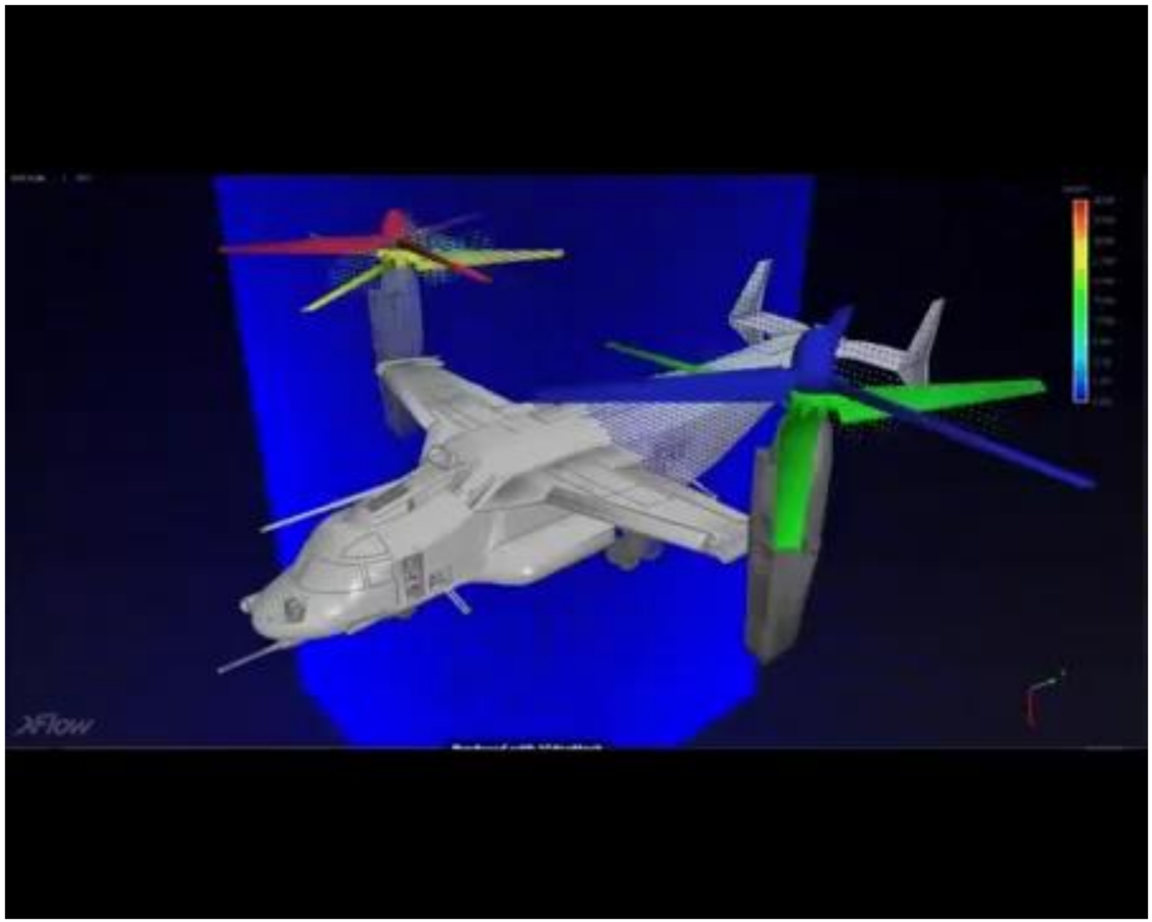
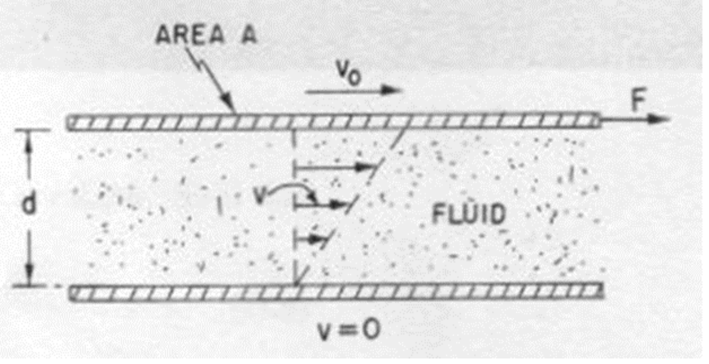
Aircraft simulation



Shear stress

Fluids are like graduate students….constantly under stress!!! (Joe Niemela)

- If you apply a shearing force to a fluid it will move— the shear forces are described by the viscosity. Consider a layer of fluid between two plates, one stationary and one moving at a slow speed v_0.



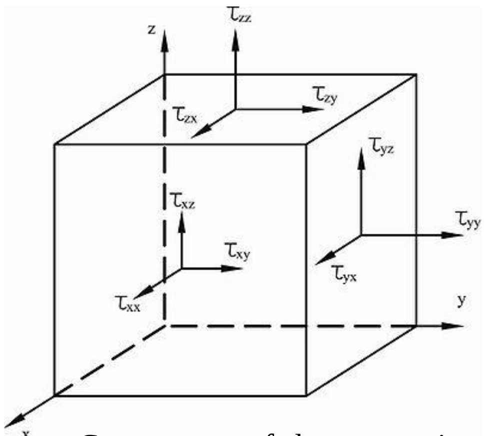
shear stress $\tau = \frac{F}{A} = \mu \frac{\partial u}{\partial y}$ Newtonian fluid

Symmetric and antisymmetric tensors

A tensor **B** is called symmetric if $B_{ij} = B_{ji}$

A tensor **B** is called antisymmetric if $B_{ij} = -B_{ji}$

For an *incompressible* fluid a *stress tensor* is given by



Components of shear stress in a fluid

$$\tau_{ij} = -\mathrm{p}\delta_{ij} + 2\mu e_{ij}$$

p is the thermodynamic pressure

(e.g., the thermodynamic pressure for a perfect gas p=ρRT)

$$\tau_{ij} = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix} \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Strain rate tensor

From the definition of curl of a vector it follows that the vorticity vector of a fluid element is related to the velocity vector by $\vec{\omega} = \nabla \times \vec{u}$

$$\omega_i = \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j} \qquad \omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

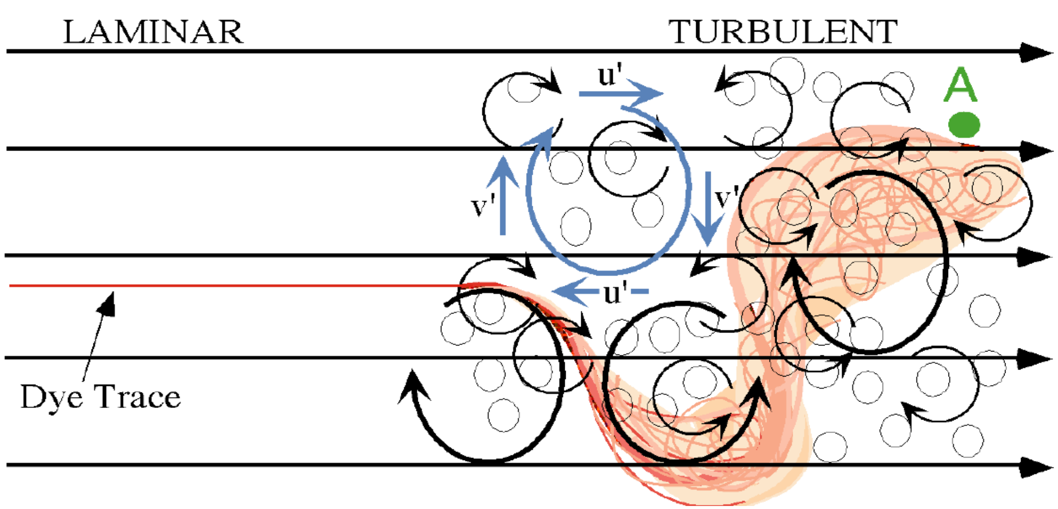
$$\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right), \qquad \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right), \qquad \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right)$$

Reynolds number (*Re*)

- This is not an universal definition of turbulent field, rather it is known from experiments and observations that a flow becomes turbulent when *"Re"* is large enough;

$$Re = \frac{inertia\ force}{viscous\ force}, \qquad Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu}, \text{ where } \qquad \upsilon = \frac{\mu}{\rho}$$

- U: typical inertial velocity scale of the flow
- L: typical inertial length scale of the flow
- ν: kinematic viscosity of the fluid
- μ: dynamic viscosity of the fluid
- ρ: density of the fluid



An example…

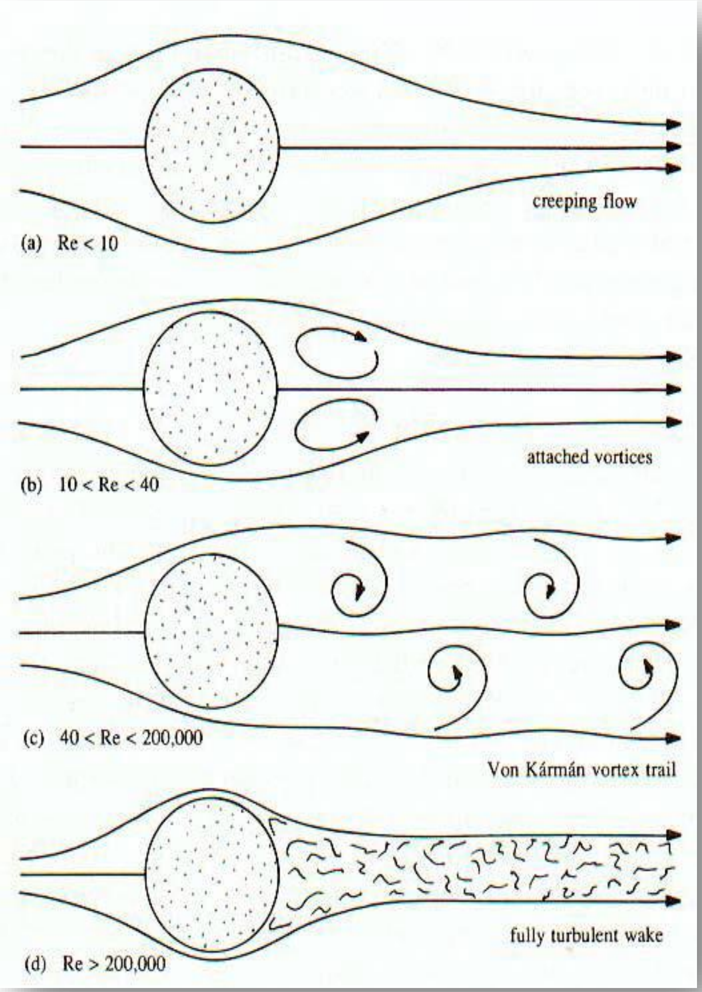
✓ The flow pattern around some an obstruction in the flow depends on the Reynolds Number, *Re*, and on the shape of an object. For a cylindrical obstruction, the following patterns are observed.

– At **low *Re*** (*Re* < 10) the flow is laminar and the streamlines are smooth.

– At **higher *Re*** (> 10), eddies start to develop, but the flow pattern is steady and not chaotic.

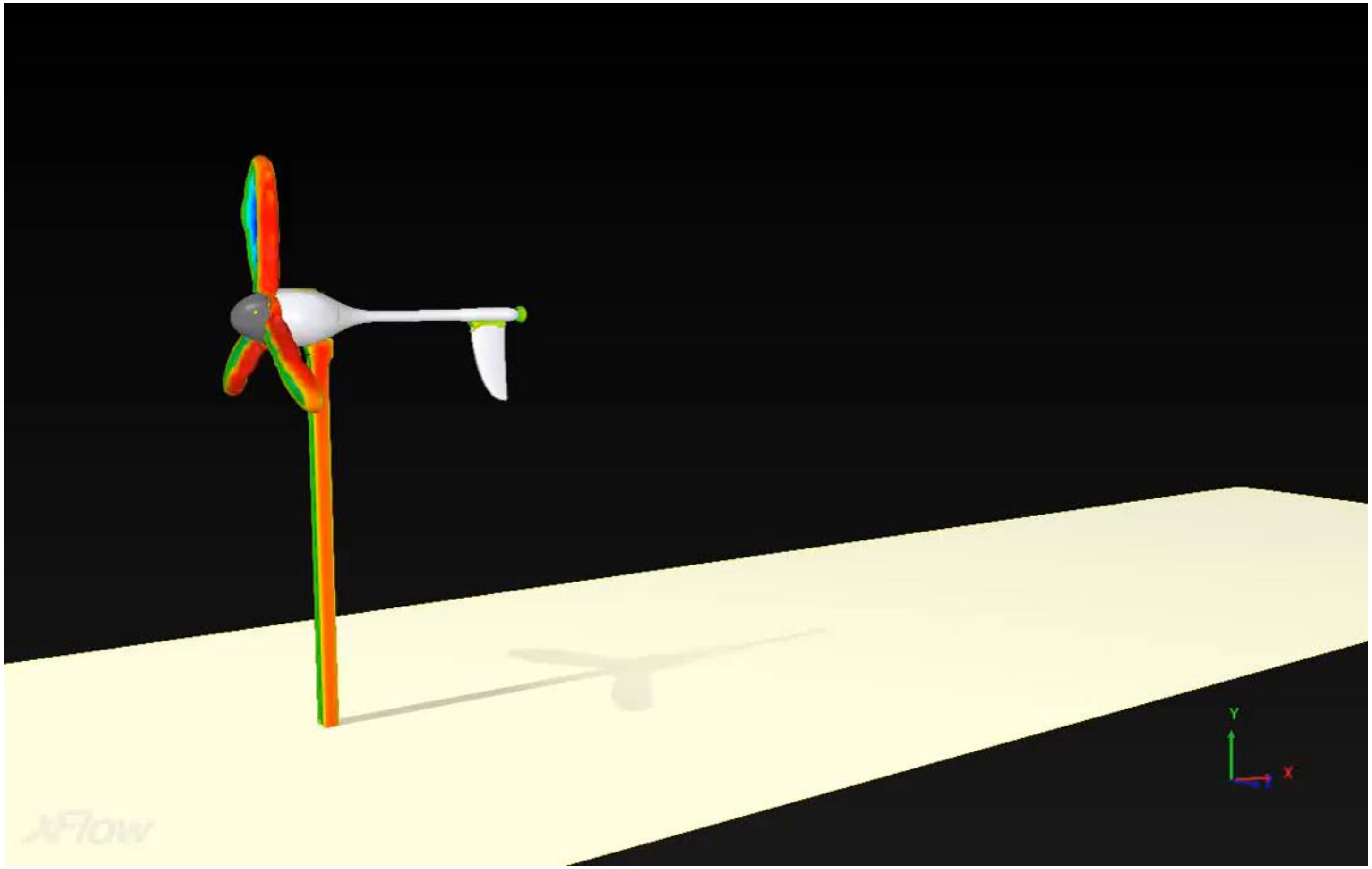
– At *Re* > 40, the eddies repeatedly grow and are shed periodically to form a "vortex street".

– Turbulence starts to develop at around *Re* ~ 1000, and the flow in the wake of the cylinder becomes more and more chaotic.



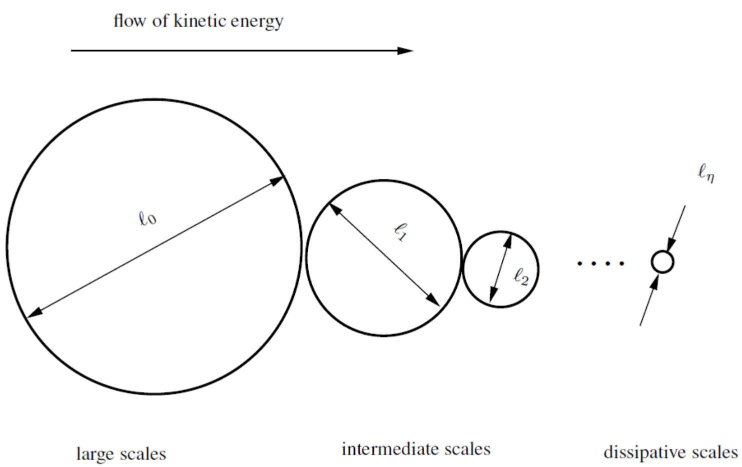
Flow past a cylinder

Wind turbine

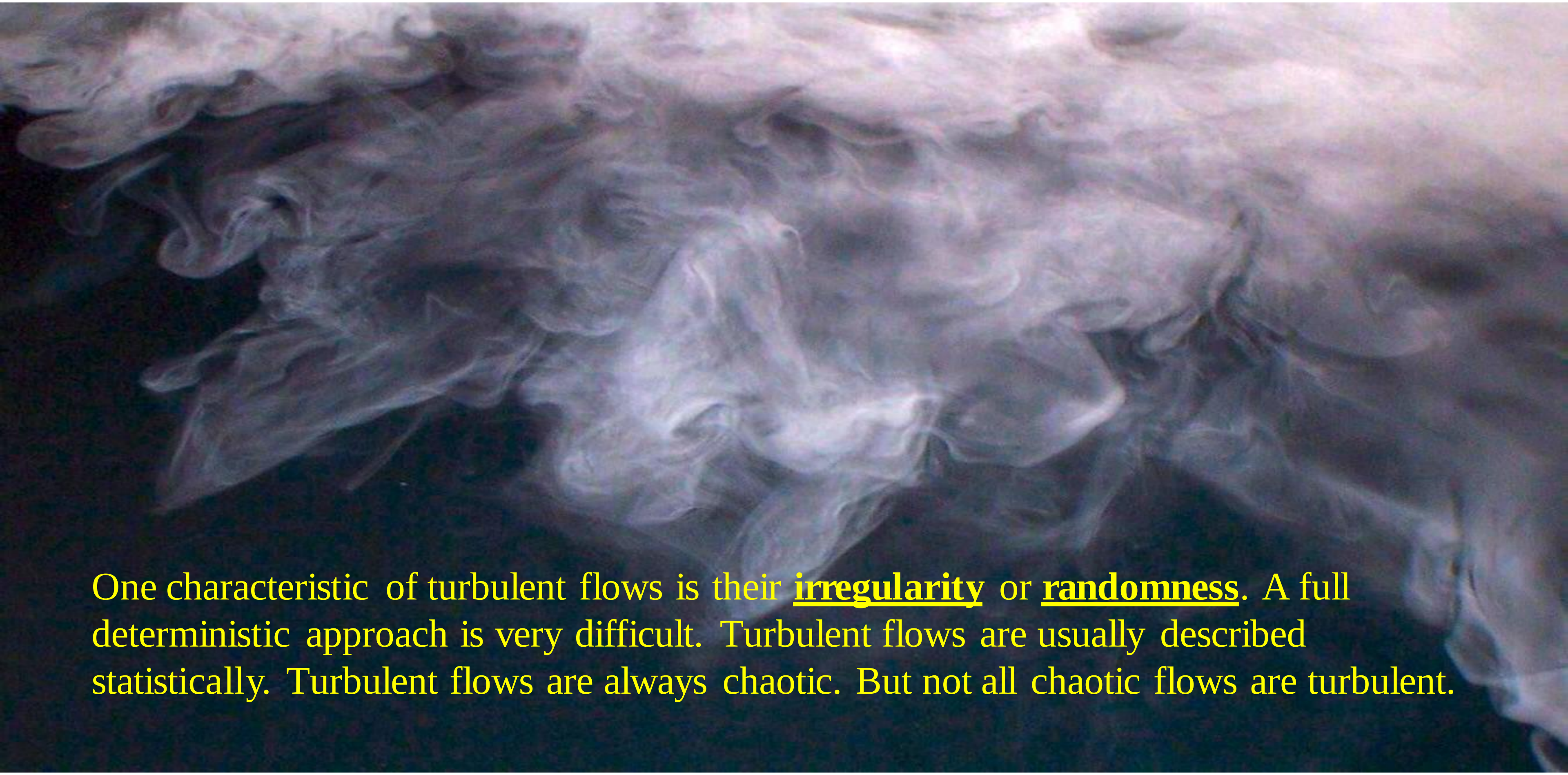


Some defining characteristics of turbulence

- ✓ It is chaotic
- ✓ It is characterized by the presence of large amount of vorticity
- ✓ It is dissipative
- ✓ It is characterized by strong mixing
- ✓ A turbulent field is also continuum (in the continuum mechanics sense)



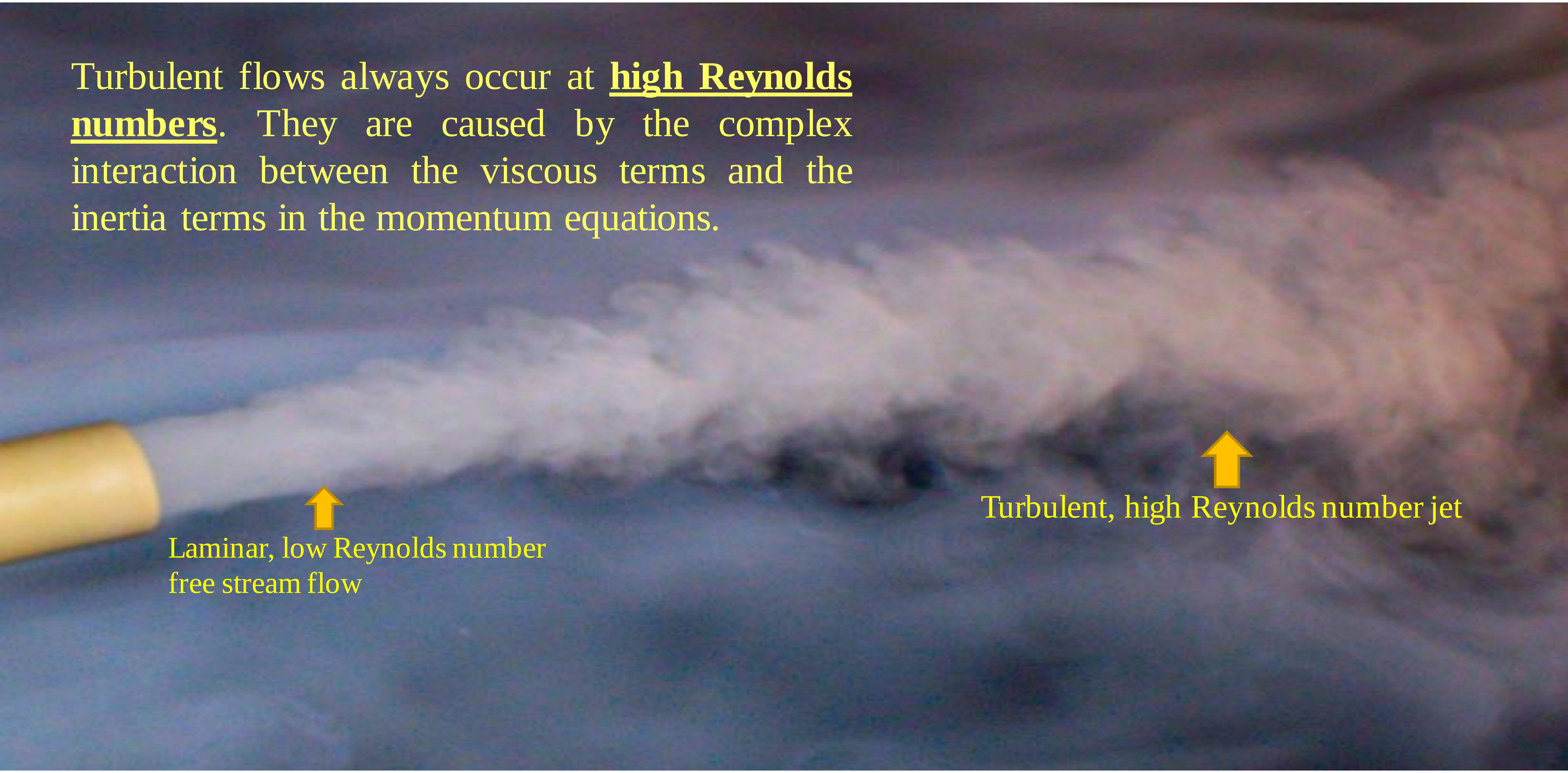
Turbulent flows are chaotic



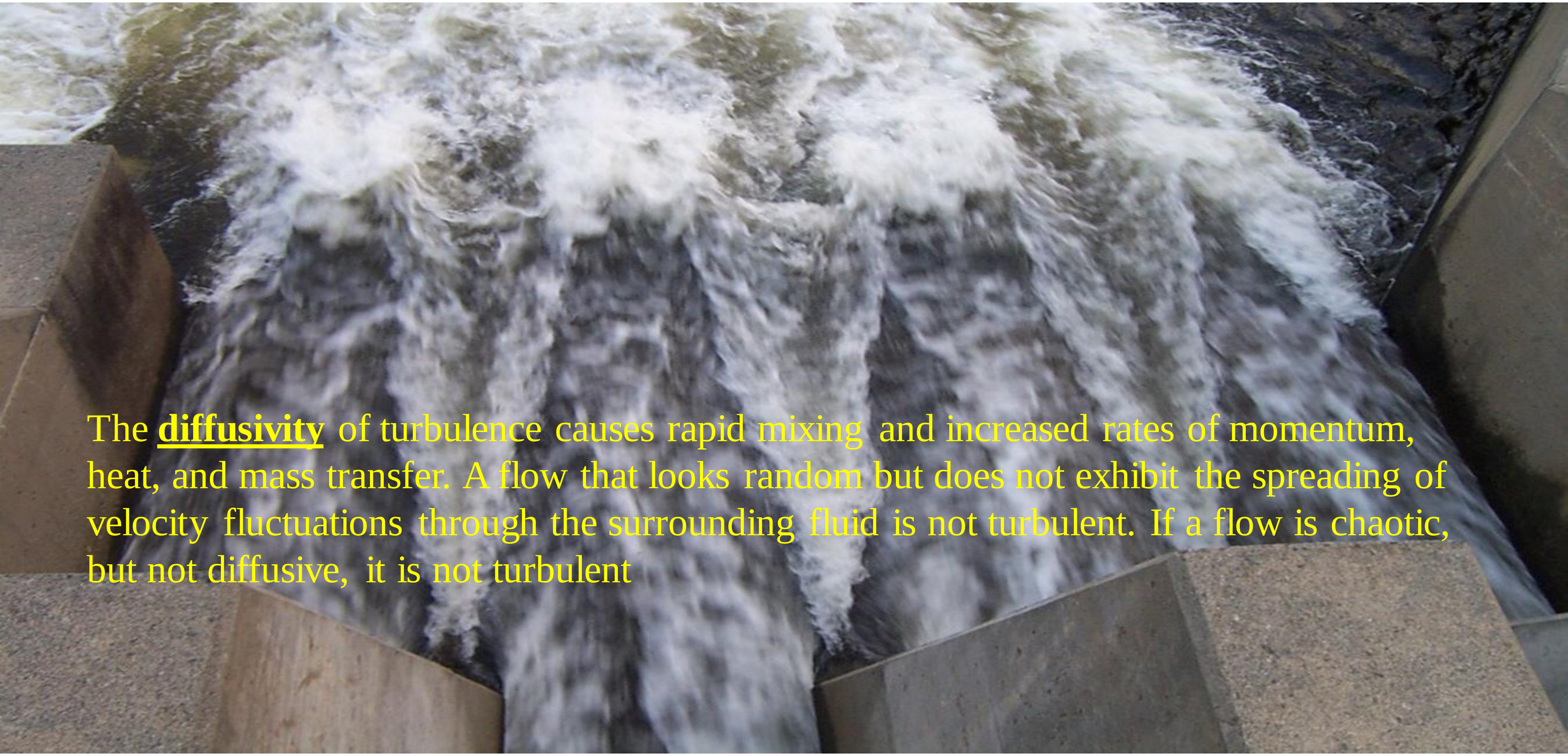
One characteristic of turbulent flows is their **irregularity** or **randomness**. A full deterministic approach is very difficult. Turbulent flows are usually described statistically. Turbulent flows are always chaotic. But not all chaotic flows are turbulent.

Turbulence: high Reynolds numbers

Turbulent flows always occur at **high Reynolds numbers**. They are caused by the complex interaction between the viscous terms and the inertia terms in the momentum equations.

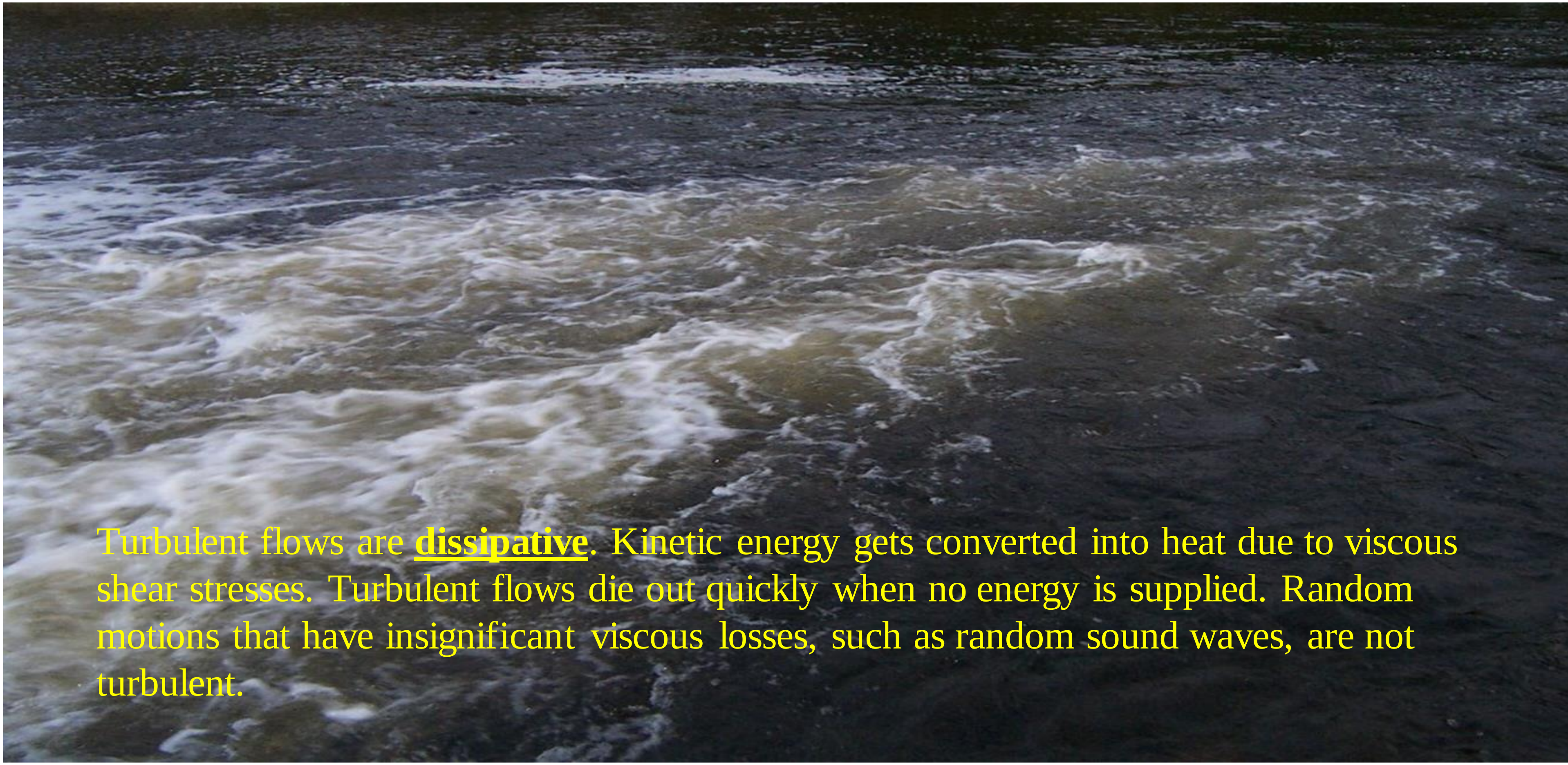


Turbulence: diffusivity



The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent. If a flow is chaotic, but not diffusive, it is not turbulent

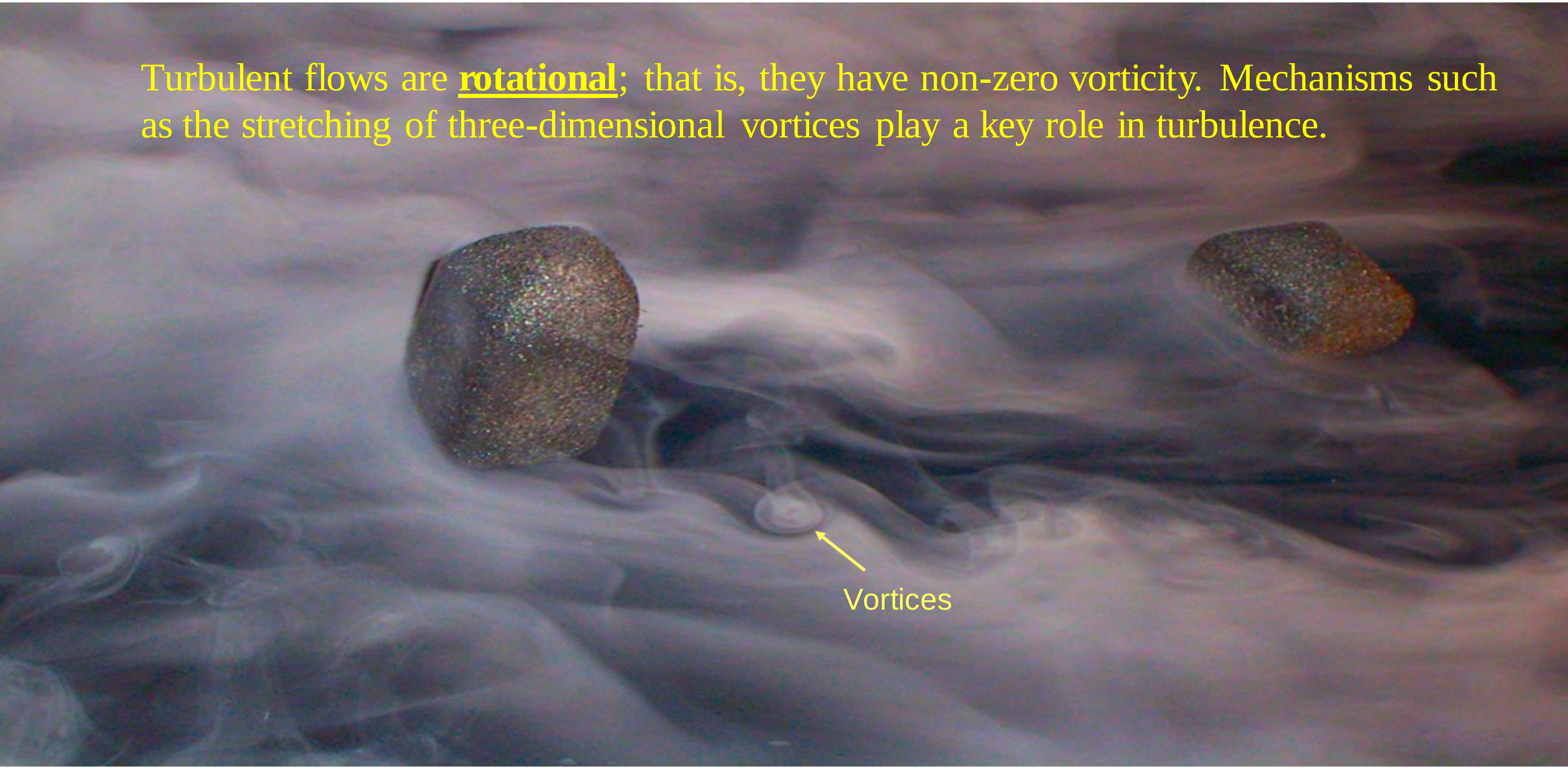
Turbulence: dissipation



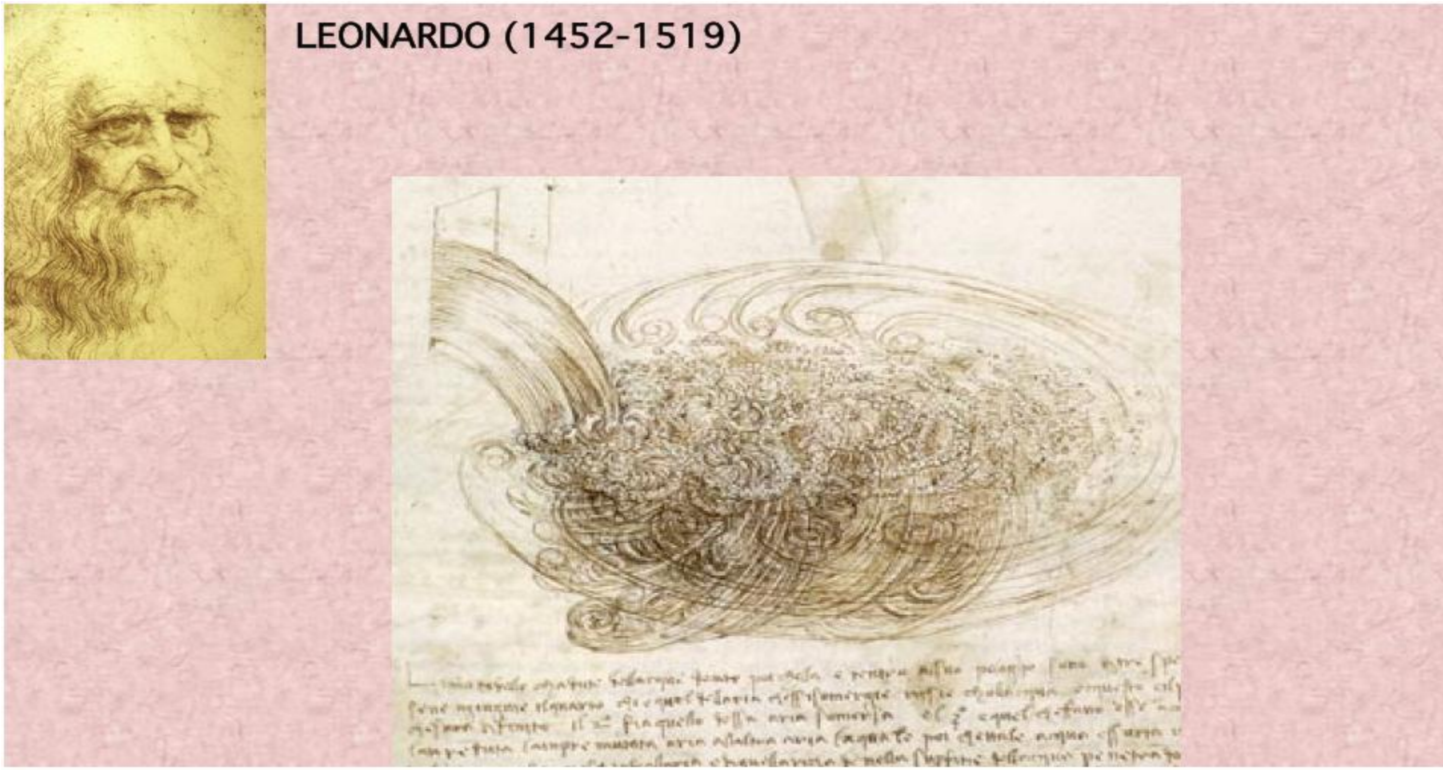
Turbulent flows are **<u>dissipative</u>**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied. Random motions that have insignificant viscous losses, such as random sound waves, are not turbulent.

Turbulence: rotation and vorticity

Turbulent flows are **rotational**; that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.



Statistical description of turbulence



Leonardo first had the intuition that a turbulent field could be treated in a statistical sense (the motion considered as the sum of a mean and a fluctuating field)

Fundamental equations of motion;

- The fundamental equations of fluid dynamics are based on the following universal laws of conservation:

1) Conservation of mass
2) Conservation of momentum (Newton's 2nd law)
3) Conservation of energy

1) Physical principle: Mass is conserved

$$\frac{D\rho}{Dt} + \rho(\boldsymbol{\nabla} . \boldsymbol{u}) = 0,$$

A fluid is usually called incompressible if its density does not change with pressure

$$(\frac{D\rho}{Dt} = 0)$$

$$\nabla . \boldsymbol{u} = 0,$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Whether or not the flow is steady.

2) Physical principle: Energy is conserved

Applying the continuity equation, the simplified NS equation for incompressible fluids are then:

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{\rho} F_i$$

Tensor form

$$\frac{D\boldsymbol{u}}{Dt} = \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}.\boldsymbol{\nabla}\boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla}p + \boldsymbol{\nu}\boldsymbol{\nabla}^2\boldsymbol{u} + \frac{1}{\rho}\boldsymbol{F}$$

Vector form

3) Physical principle: Energy is conserved

$$\frac{\partial(\rho e_t)}{\partial t} + \boldsymbol{\nabla}.(\rho e_t \vec{u}) = k\boldsymbol{\nabla}.\boldsymbol{\nabla}T - \boldsymbol{\nabla}p.\boldsymbol{u} + (\boldsymbol{\nabla}.\tau).\boldsymbol{u}$$

Boundary conditions

- The NS equations govern the flow of a fluid. They are the same equations whether the flow is, for example, over an aircraft, through a subsonic wind tunnel or past a windmill. However, the flow fields are quite *different* for these cases, although the governing equations are the *same*. Why? Where does the difference enter? The answer is through the ***boundary conditions***, which are quite different for each case. The boundary conditions, and sometimes the initial conditions, dictate the particular solutions to be obtained from the governing equations. For a viscous fluid, the boundary condition on a surface assumes no relative velocity between the surface and the gas immediately at the surface. This is called the *no-slip* condition. If the surface is stationary, with the flow moving past it, then

$u=v=w=0$ At the surface (for a viscous flow)

- For an inviscid fluid, the flow slips over the surface (there is no friction to promote its 'sticking' to the surface); hence, at the surface, the flow must be *tangent* to the surface.

$\vec{V}.\vec{n} = 0$ At the surface (for an inviscid flow)

- where $\vec{n}$ is a unit vector perpendicular to the surface. The boundary conditions elsewhere in the flow depend on the type of problem being considered, and usually pertain to inflow and outflow boundaries at a finite distance from the surfaces, or an 'infinity' boundary condition infinitely far from the surfaces. The boundary conditions discussed above are *physical boundary conditions* imposed by nature. In computational fluid dynamics we have an additional concern, namely, the *proper numerical implementation of the boundary conditions*. In the same sense as the real flow field is dictated by the physical boundary conditions, the computed flow field is driven by the numerical boundary conditions. The subject of proper and accurate boundary conditions in CFD is very important, and is the subject of much current CFD research.

In summary

- ✓ In fluid dynamics , turbulence or turbulent flow is a fluid regime characterized by chaotic, stochastic property changes.
- ✓ This includes low momentum diffusion, high momentum convection and rapid variation of pressure and velocity in space and time.
- ✓ The phenomenon of turbulence reveals that their solutions can become very complex if a critical parameter e.g., the Reynolds number (Re) or the Rayleigh number (Ra), becomes large
- ✓ In the past, two approaches in science:

- Theoretical (old)
- Experimental (old)
- Computer (new) ⇒ Numerical simulation

Computational Fluid Dynamics (CFD)

Expensive experiments are being replaced by numerical simulations :

- cheaper and faster
- simulation of phenomena that can not be experimentally reproduced (weather, ocean, ...)

Thank you