Abstract

The influence of the different thermal boundary conditions at the bottom and top plates on the dynamics and statistics of a turbulent Rayleigh-Bénard convection flow is studied in three-dimensional direct numerical simulations. The flow evolves in a closed cylinder with an aspect ratio of $\Gamma = 1/2$, for Prandtl number $Pr = 0.7$ in air, at a Rayleigh number $Ra = 10^7$.

1 Simulation model

1.1 Equations and parameters

In completion to 2 Dimensional conjugate heat transfer study, which can be find at NEK5000 official website (https://nek5000.mcs.anl.gov/), here we solve the three-dimensional equations of motion of thermal convection with the Boussinesq approximation. We consider a fluid of depth H, with a kinematic viscosity ν_f and a thermal diffusivity κ_f at constant mass density ρ_f. The thermal expansion coefficient α, the specific heat at constant pressure $c_{p,f}$, and the thermal conductivity $\lambda_f = \rho_f c_{p,f} \kappa_f$ are given. The equations for non-dimensional fluid velocity $\boldsymbol{u}(\boldsymbol{x}, t)$ and fluid temperature $T(\boldsymbol{x}, t)$ are given by

$$\boldsymbol{\nabla . u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u.\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} P + \sqrt{Pr/Ra}\nabla^2 \boldsymbol{u} + T\boldsymbol{e_z} \tag{2}$$

$$\frac{\partial T}{\partial T} + (\boldsymbol{u.\nabla})T = \frac{1}{\sqrt{RaPr}}\nabla^2 T \tag{3}$$

with $\boldsymbol{e}_z$ as the unit vector pointing opposite to the direction of acceleration due to gravity g. The kinematic pressure field is denoted by $p(\boldsymbol{x}, t)$. The equations (1)–(3) are made dimensionless by the fluid depth H, the free fall velocity $U_f = (g\alpha\tilde{\Theta}H)^{1/2}$, and the characteristic temperature scale $\tilde{\Theta}$ for the specific thermal boundary conditions (details are given below). For clarity in our notation, we set a tilde for the temperature scale with a physical dimension.

For turbulent RBC there are three main input parameters which control the system; the aspect ratio $\mathit{\Gamma}$, the Prandtl number Pr and the Rayleigh number Ra. The specific definition of Ra, which quantifies the strength of thermal driving of the turbulence, depends on the prescribed boundary conditions is given by

$$Ra = \frac{\alpha g H^3}{\nu_f \kappa_f}\tilde{\Theta}. \tag{4}$$

In this case studied here, we consider the equilibrium state of pure conductive heat transfer as the initial condition. It takes the form

$$T = 1 - z, \qquad \text{for} \quad 0 < z < 1 \,. \tag{5}$$

The specification of above equations is completed once the conditions of the temperature at the boundaries ($z = 0$ and $z = 1$) are specified.

1.2 Fluid bounded by finitely thick plates at bottom and top

In order to study the boundary layer structure with more realistic conditions and get as close as possible to laboratory results , we also studied the fluid bounded by two finitely thick copper plates, above and below, with Dirichlet BCs applied at the top of upper plate and the bottom of lower plate. The solid plates have the same thickness $h_s = 0.2H$ for the present case with a thermal diffusivity κ_s and thermal conductivity $\lambda_s = \rho_s c_{p,s} \kappa_s$. For the following, it is more convenient to define the ratio of solid-to-fluid thickness as

$$h = \frac{h_s}{H} \,. \tag{6}$$

In this case, the heat conduction equations in the solid parts, $-h < z < 0$ and $1 < z < 1 + h$, for the temperature T_s have to be coupled to the advection-diffusion equation in the fluid domain , $0 < z < 1$. This configuration is denoted as the conjugated heat transfer (CHT) case. We thus have to solve the additional heat conduction equation in the plates.

We apply temperature at the plates such that the tempereture drop at the interfaces are 1 and 0. and it changes with different Ra and Pr number. As an instance the additionally imposed temperature increment at $Ra = 10^7$ found to be $\delta T_1 = 2.51$, in which at $Ra = 10^8$ is 4.99, yet in case of $\delta T_2 = 4.72$ (see Figure 1).

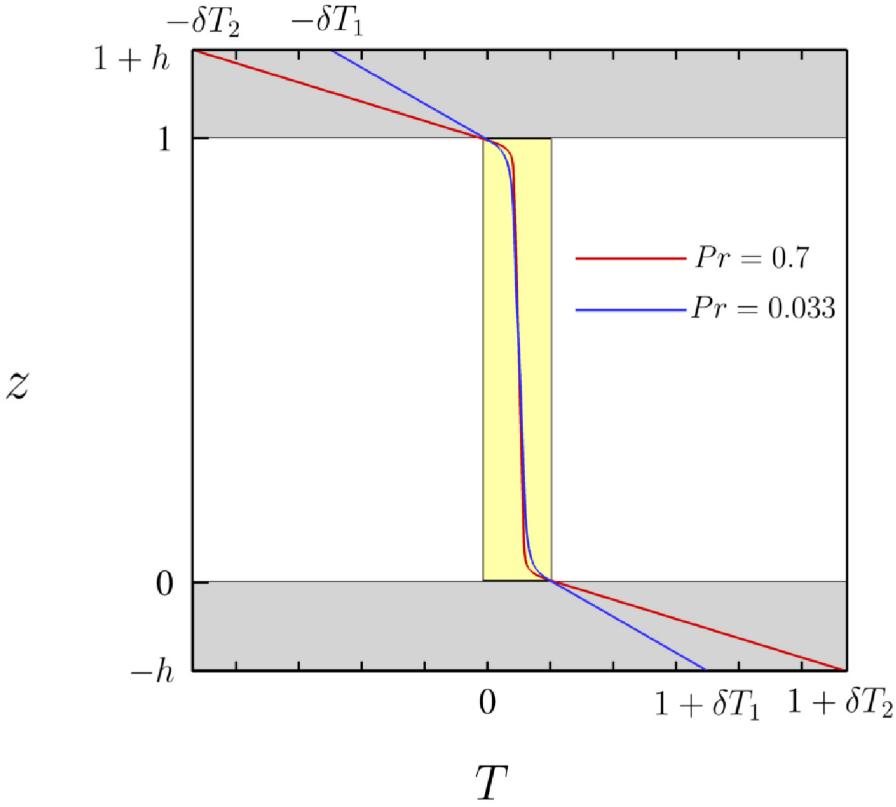

Figure 1: Mean dimensionless temperature profiles in the conjugate heat transfer (CHT) setting for $Ra = 10^7$, $\Gamma = 1/2$, $Pr = 0.7$ (red-solid line) and $Pr = 0.033$ (blue-solid line). Temperature distributions are adjusted at the outer solid boundaries such that the mean temperature at the bottom and top interfaces reaches $\langle T(z=0)\rangle_{A,t} \approx 1$ and $\langle T(z=1)\rangle_{A,t} \approx 0$, respectively. The additional imposed temperature for $Pr = 0.033$ is $\delta T_1 = 2.51$ and for $Pr = 0.7$ is $\delta T_2 = 4.72$. The solid plates are shaded in gray. The yellow box indicates the range of the mean temperature profile in the corresponding RBC cases.