HW4 Report

1. Results

Parameters: K = 40, r = 6%, 50 exercisable points per year, N= 100,000 (50,000 plus 50,000 antithetic)

S	s (%)	T	American put price	standard error	European put price
36	20	1	4.4752	0.0093	3.8411
36	20	2	4.8382	0.0109	3.7727
36	40	1	7.1003	0.0192	6.7182
36	40	2	8.5221	0.0226	7.6874
38	20	1	3.2468	0.0094	2.8541
38	20	2	3.7459	0.0113	3.0132
38	40	1	6.1562	0.0186	5.8338
38	40	2	7.6794	0.0222	6.9847
40	20	1	2.3120	0.0086	2.0591
40	20	2	2.8785	0.0105	2.3604
40	40	1	5.3070	0.0179	5.0378
40	40	2	6.9188	0.0218	6.3629
42	20	1	1.6138	0.0076	1.4626
42	20	2	2.2144	0.0097	1.8455
42	40	1	4.5654	0.0171	4.3452
42	40	2	6.2327	0.0214	5.7435
44	20	1	1.1019	0.0066	1.0145
44	20	2	1.6963	0.0088	1.4297
44	40	1	3.9599	0.0164	3.7979
44	40	2	5.6446	0.0208	5.2194

The setting above is from the paper "Valuing American Options by Simulation: A Simple Least-Squares Approach"

2. Implementation

Generate stock price paths using Brownian motion

- Stock prices $S_1, S_2, S_3, \ldots$ at times $\Delta t, 2\Delta t, 3\Delta t, \ldots$ can be generated via
  - $S_{i+1} = S_i e^{(\mu - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}\xi}$
  - where $\xi \sim N(0,1)$
- We will generate N price paths
  - Half the paths are generated using the standard Brownian motion
  - The other half are antithetic paths, which are the negations of the corresponding standard normal values

Put option payoff function

- Payoff function
  - $max(K - S_t, 0)$
  - Will be used to determine the cash flow of the last time period

Decide whether to early exercise or not at each point

- Compare the immediate exercise value with the expected cash flows from continuing, and then exercise if immediate exercise is more valuable
- We iterate backward from time period $n - 1$, assume the current time period is t
  - let x be the stock prices of different paths at t, where x are smaller than the strike price K
  - let y be the corresponding expected value of continuing the option, i.e., the discounted cash flow
  - Regressing on y on the basis functions $1, x, x^2, x^3$

    - Then we substitute the stock prices back into the polynomial obtained from regression to determine whether to early exercise or not. That is, if the current exercise payoff is greater than the value obtained by substituting stock prices into the polynomial, then we early exercise.

Calculate the average option price and standard error

- We can get the price of American put by discounting each cash flow back to time zero, and averaging over all paths
- Standard error
    - $\frac{\sigma}{\sqrt{n}}$
    - where σ is standard deviation