Deep Learning: Why and What

Case Study: Image Classification

Hao Zhang

haomoodzhang@gmail.com

LAMDA Group
National Key Laboratory for Novel Software Technology
Department of Computer Science and Technology
Nanjing University

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Hao Zhang
haomoodzhang@gmai

Image Classification

Image Classification

- Given a fixed categories, such as {cat, dog, plane, trunk, ...},

Image Classification

- Given a fixed categories, such as {cat, dog, plane, trunk, ...},
- For each image, assign one of those categories as its label.

Semantic Gap

Image recognition task

Semantic Gap

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- Easy for human,

Semantic Gap

Image recognition task

- Easy for human,
- Hard for computer,

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Semantic Gap

Image recognition task

- Easy for human,
- Hard for computer,
- Computer can only see an array of numbers.

Other Challenges

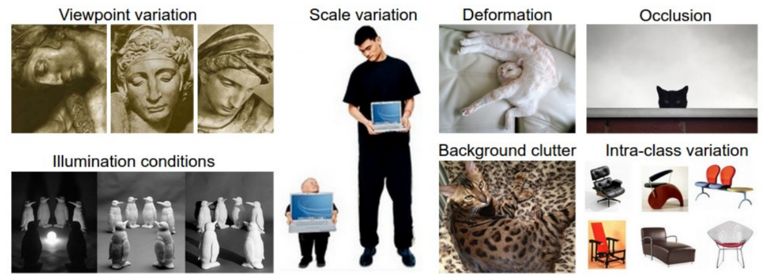

Figure: Other challenges for image recognition. This figure is reproduced from [3].

Data-Driven Approach

How to do image classification?

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- Can not be solved by rule-based methods,

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haomoodzhang@gmai

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Data-Driven Approach

How to do image classification?

- Can not be solved by rule-based methods,
- So we mimic the human learning process,
- Let machine learn from data.

Hao Zhang
haomoodzhang@gmai

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Training Data

$$D = \{(\vec{x}^{(i)}, y^{(i)})\}_{i=1}^{m} \,, \tag{1}$$

$$\vec{x}^{(i)} \in \mathbb{R}^n, \forall i \,, \tag{2}$$

$$y^{(i)} \in \{0, 1, 2, 3, \ldots, K-1\}, \forall i \,. \tag{3}$$

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- $y^{(i)}$: label corresponds to $\vec{x}^{(i)}$.

Hypothesis Function

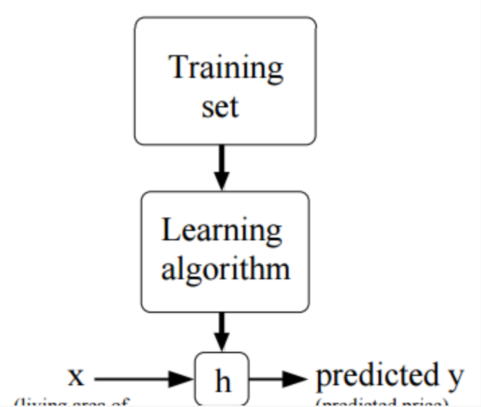

Figure: $h(\vec{x})$ gives confidence or score of $\vec{x}$ belonging to different classes. This figure is reproduced from [8].

Hypothesis of Linear Classifier

h: Linear mapping

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$$h(\vec{x}; W, \vec{b}) = W\vec{x} + \vec{b}, \tag{4}$$

$$W \in \mathbb{R}^{K \times n}, \vec{b} \in \mathbb{R}^{K}. \tag{5}$$

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haomoodzhang@gmai

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- W: weights,
- $\vec{b}$: bias vector.

Parameters Learning

Goal

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haomoodzhang@gmai

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- Set $(W, \vec{b})$ to let the score computed match the ground truth,

Parameters Learning

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haomoodzhang@gmai

Goal

- Set $(W, \vec{b})$ to let the score computed match the ground truth,
- Once learning done, we can throw away the training set, and make prediction based on $(W, \vec{b})$.

Understanding of Hypothesis Function

$$
\begin{aligned}
h(\vec{x}; W, \vec{b}) &= W\vec{x} + \vec{b} && (6)\\
&= \begin{bmatrix} \vec{w}_0^T \\ \vec{w}_1^T \\ \vdots \\ \vec{w}_{K-1}^T \end{bmatrix} \vec{x} + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{K-1} \end{bmatrix} && (7)\\
&= \begin{bmatrix} \vec{w}_0^T \vec{x} + b_0 \\ \vec{w}_1^T \vec{x} + b_1 \\ \vdots \\ \vec{w}_{K-1}^T \vec{x} + b_{K-1} \end{bmatrix}. && (8)
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- Simultaneously has K classifiers,
- For each $0 \leq d \leq K-1$, $\vec{w}_d^T \vec{x} + b_d$ computes the confidence or score $\vec{x}$ belonging to the d-th class.

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haomoodzhang@gmai

Hypothesis of Softmax Classifier

$$\vec{s} = h(\vec{x}) = W\vec{x} + \vec{b}\,. \tag{9}$$

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Probability interpretation

$$\Pr(y = k|\vec{x}) = \frac{\exp(s_k)}{\sum_{j=0}^{K-1}\exp(s_j)} \in [0, 1], \forall k\,. \tag{10}$$

Loss Function

- Given h, for each $\vec{x}^{(i)}$, we can compute scores $\vec{s}^{(i)}$ belonging to each class,

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Loss Function

- Given h, for each $\vec{x}^{(i)}$, we can compute scores $\vec{s}^{(i)}$ belonging to each class,
- We need a measure $\vec{s}^{(i)}$ does or does not match ground truth $y^{(i)}$,
- This measure tells us goodness of parameters $(W, \vec{b})$,
- Loss function measures how much $\vec{s}^{(i)}$ does not match $y^{(i)}$.

Cross Entropy Loss

Loss function of softmax classifier can be computed by maximum likelihood estimate

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haomoodzhang@gmai

Cross Entropy Loss

Loss function of softmax classifier can be computed by maximum likelihood estimate

$$
\begin{aligned}
W^\star, \vec{b}^\star &= \arg\max_{W,\vec{b}} \prod_{i=1}^{m} p(y^{(i)} | \vec{x}^{(i)}; W, \vec{b}) && (11) \\
&= \arg\min_{W,\vec{b}} \frac{1}{m} \sum_{i=1}^{m} \left(-s_{y^{(i)}}^{(i)} + \log\left(\sum_{j=0}^{K-1} \exp(s_j^{(i)})\right) \right) && (12) \\
&\stackrel{\text{def}}{=} \arg\min_{W,\vec{b}} \frac{1}{m} \sum_{i=1}^{m} \mathrm{err}(W, \vec{b}; \vec{x}^{(i)}, y^{(i)}) && (13) \\
&\stackrel{\text{def}}{=} \arg\min_{W,\vec{b}} J(W, \vec{b}) \,. && (14)
\end{aligned}
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Cross Entropy Loss

Loss function of softmax classifier can be computed by maximum likelihood estimate

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$$\stackrel{\text{def}}{=} \arg\min_{W,\vec{b}} \frac{1}{m} \sum_{i=1}^{m} \mathrm{err}(W, \vec{b}; \vec{x}^{(i)}, y^{(i)}) \tag{13}$$

$$\stackrel{\text{def}}{=} \arg\min_{W,\vec{b}} J(W, \vec{b})\,. \tag{14}$$

For simplicity, write $\mathrm{err}(W, \vec{b}; \vec{x}^{(i)}, y^{(i)})$ as $\mathrm{err}^{(i)}$.

Regularization

Good $(W, \vec{b})$ min.

$$J(W, \vec{b}) = \frac{1}{m} \sum_{i=1}^{m} - \log \frac{\exp(s_{y^{(i)}}^{(i)})}{\sum_{j=0}^{K-1} \exp(s_j^{(i)})} . \tag{15}$$

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- Suppose $(W^\star, \vec{b}^\star)$ is the opt. sol.,
- Then the result of adding const. $c, c > 0$ to every element of $(W^\star, \vec{b}^\star)$ is also opt.,
- We handle this problem by make preference of $(W, \vec{b})$, this is achieved by adding regularization term $\Omega(W)$,

Regularization (cont'd)

- Commonly used regularization term is ℓ_2 norm

$$\Omega(W) = \|W\|_F^2 = \sum_{d=0}^{K-1} \sum_{j=0}^{n-1} W_{dj}^2 \,. \tag{16}$$

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- ℓ_2 prefers small and diffused weights, which is good for generalization.

Then the total loss function becomes

$$J(W, \vec{b}) = \frac{1}{m} \sum_{i=1}^{m} -\log \frac{\exp(s_{y^{(i)}}^{(i)})}{\sum_{j=0}^{K-1} \exp(s_j^{(i)})} + \frac{\lambda}{2} \|W\|_F^2 \qquad (17)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \mathrm{err}^{(i)} + \frac{\lambda}{2} \|W\|_F^2 . \qquad (18)$$

Optimization

Goal:

$$W^\star, \vec{b}^\star = \arg\min_{W, \vec{b}} J(W, \vec{b})\,. \tag{19}$$

Hao Zhang
haomoodzhang@gmai

Optimization

Goal:

$$W^\star, \vec{b}^\star = \arg\min_{W, \vec{b}} J(W, \vec{b}) \,. \tag{19}$$

This is an unconstrained optimization problem.

Visualing Loss Function

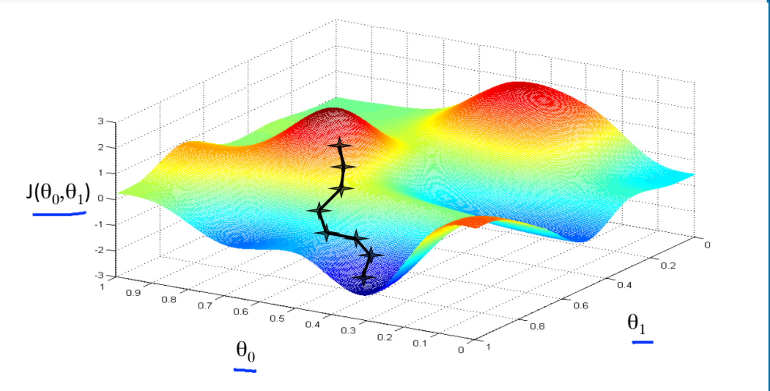

Figure: Visualizing J wrt $(W, \vec{b})$ is hard, but we can choose 2 directions (θ_0, θ_1) in parameter space, and draw J along that 2 directions. This figure is reproduced from [9].

Gradient Descent

How to find opt. $(W, \vec{b})$

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$$W \leftarrow W - \alpha \nabla_W J(W, \vec{b}) = W - \alpha \frac{1}{m} \sum_{i=1}^{m} \nabla_W \mathrm{err}^{(i)} - \alpha \lambda W \,, \tag{20}$$

$$\vec{b} \leftarrow \vec{b} - \alpha \nabla_{\vec{b}} J(W, \vec{b}) = \vec{b} - \alpha \frac{1}{m} \sum_{i=1}^{m} \nabla_{\vec{b}} \mathrm{err}^{(i)} \,. \tag{21}$$

Softmax Update Rule

$$\vec{s}^{(i)} = W\vec{x}^{(i)} + \vec{b}, \forall i \,, \tag{22}$$

$$\mathrm{err}^{(i)} = -\log \frac{\exp(s^{(i)}_{y^{(i)}})}{\sum_{j=0}^{K-1} \exp(s^{(i)}_j)} \,. \tag{23}$$

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By caculus,

$$\nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)} = \frac{\exp(\vec{s}^{(i)})}{\sum_{j=0}^{K-1} \exp(s^{(i)}_j)} - \vec{e}_{y^{(i)}}\,. \tag{24}$$

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- $\vec{e}_{y^{(i)}}$ is a K-dimension vector,
- it has 1 in $y^{(i)}$'s position, and 0 otherwise.

Softmax Update Rule (cont'd)

By chain rule

$$\nabla_W \mathrm{err}^{(i)} = (\nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)}) \vec{x}^{(i)T}, \tag{25}$$

$$\nabla_{\vec{b}} \mathrm{err}^{(i)} = \nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)}. \tag{26}$$

Prediction and Evaluation

Prediction and Evaluation

- Let the classifier model to predict on an unseen test set,

Prediction and Evaluation

- Let the classifier model to predict on an unseen test set,
- A good model will have many predictions match the true label.

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haomoodzhang@gmai

Prediction

For unseen $\vec{x}$, the classifier make a prediction by computing

$$\hat{y} = \arg\max_k h(\vec{x})_k = \arg\max_k s_k \,. \tag{27}$$

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Prediction

For unseen $\vec{x}$, the classifier make a prediction by computing

$$\hat{y} = \arg\max_k h(\vec{x})_k = \arg\max_k s_k \,. \tag{27}$$

I.e., assign $\vec{x}$ with label whose score is the highest.

Evaluation

Given test set

$$D_{test} = \{(\vec{x}^{(i)}, y^{(i)})\}_{i=1}^{m_{test}}, \tag{28}$$

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For each $\vec{x}^{(i)}$, compute prediction $\hat{y}^{(i)}$.

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Given test set

$$D_{test} = \{(\vec{x}^{(i)}, y^{(i)})\}_{i=1}^{m_{test}}, \tag{28}$$

For each $\vec{x}^{(i)}$, compute prediction $\hat{y}^{(i)}$.
Accuracy of the model is

$$Acc = \frac{1}{m_{test}} \sum_{i=1}^{m_{test}} 1\{\hat{y}^{(i)} = y^{(i)}\}. \tag{29}$$

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Linear Models

Linear Models

- Softmax classifier represents a series of linear models,

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Linear Models

- Softmax classifier represents a series of linear models,
- Training is convex optimization,
- Guaranteed to converge to global optimal,
- But it can only use multiple hyperplanes to seperate input space into some simple regions,
- For many problems, the raw pixels are not linearly separable.

Feature Engineering

Figure: Find proper representation $\phi(\vec{x})$ to take place $\vec{x}$, such that $\phi(\vec{x})$'s are linearly separable. This figure is reproduced from [6].

SIFT Feature

Figure: SIFT can handle occlusion. This figure is reproduced from [7].

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HOG feature

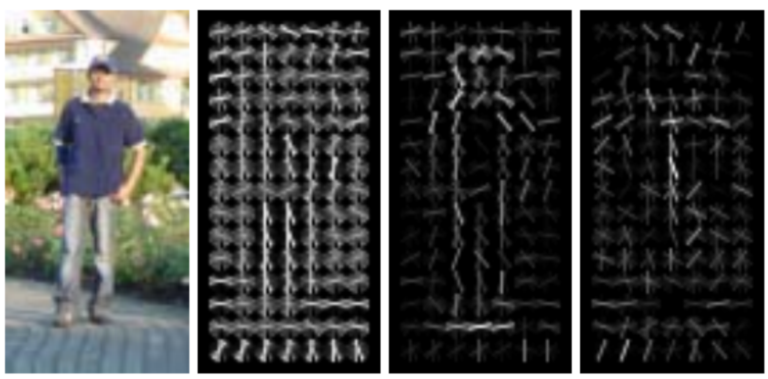

Figure: HOG feature. This figure is reproduced from [1].

Kernel Methods

Hao Zhang
haomoodzhang@gmai

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Kernel Methods

- Use fixed ϕ to map $\vec{x}$ to a higher dimension space,
- The optimization is still convex,
- The choice of ϕ is kernel engineering,
- Commonly used kernel is Gauss kernel: $k(\vec{x}, \vec{x}') = \exp(-\gamma \|\vec{x} - \vec{x}'\|^2)$.

Representation Learning

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Machine learning can learn the mapping from representation to output.

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Machine learning can learn the mapping from representation to output.
Can we also let machine learn appropriate representation?

Representation Learning

Machine learning can learn the mapping from representation to output.
Can we also let machine learn appropriate representation?
Autoencoder is one example of representation learning.

Deep Learning

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Deep Learning

Learn appropriate mapping directly from raw pixel is still very hard.

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Learn appropriate mapping directly from raw pixel is still very hard.

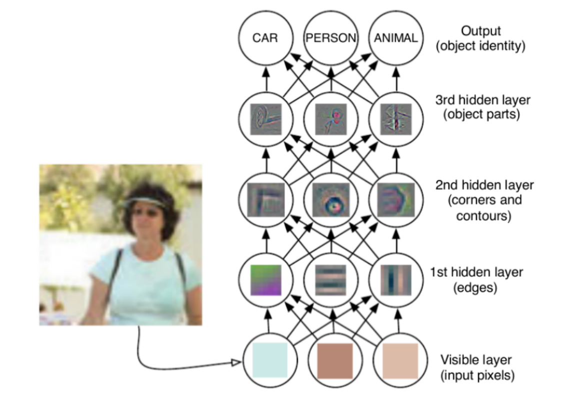
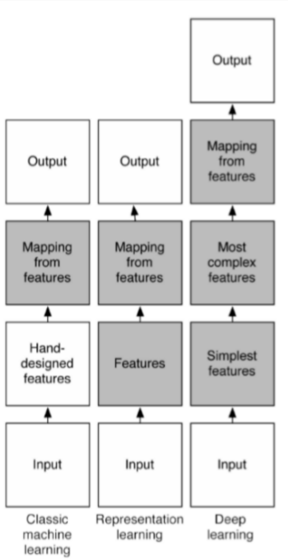


Figure: Deep learning divides it into a series of easy learning problems. $\phi(\vec{x}) = \phi_3(\phi_2(\phi_1(\vec{x})))$. This figure is reproduced from [4].

Comparation



Data Representation

Linear hypothesis function

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Changes to

$$h(\vec{x}) = W\phi(\vec{x}) + \vec{b}. \tag{31}$$

Data Representation

Linear hypothesis function

$$h(\vec{x}) = W\vec{x} + \vec{b}\,. \tag{30}$$

Changes to

$$h(\vec{x}) = W\phi(\vec{x}) + \vec{b}\,. \tag{31}$$

Nested representation learning

$$\phi(\vec{x}) = \phi_{L-1}(\phi_{L-2}(\ldots\phi_2(\phi_1(\vec{x}))))\,. \tag{32}$$

Composition of each $\phi_l(\vec{a})$

Linear part

$$\vec{z} = W^{(l)}\vec{a} + \vec{b}^{(l)}, \forall l = 1, 2, \ldots, L-1\,. \tag{33}$$

Composition of each $\phi_l(\vec{a})$

Linear part

$$\vec{z} = W^{(l)}\vec{a} + \vec{b}^{(l)}, \forall l = 1, 2, \ldots, L-1\,. \tag{33}$$

Non linear part (activation function)

$$\phi_l(\vec{a}) = \max(0, \vec{z}), \forall l = 1, 2, \ldots, L-1\,. \tag{34}$$

Activation Function

Rectified linear units, ReLU

$$\max(0, s) = 1\{s > 0\}s\,. \tag{35}$$

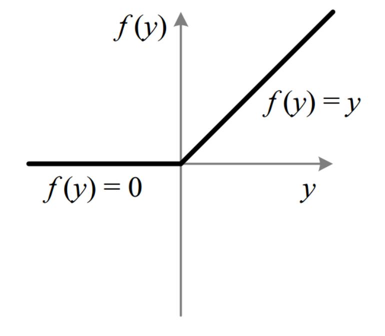

Figure: ReLU. This figure is reproduced from [5].

Activation Function (cont'd)

Sigmoid

$$\sigma(s) = \frac{1}{1 + \exp(-s)} \,. \tag{36}$$

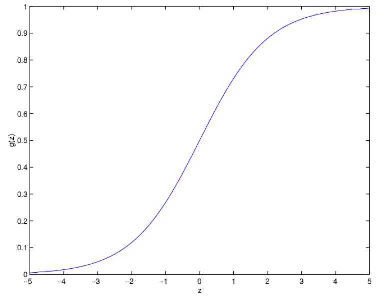

Figure: Sigmoid. This figure is reproduced from [8].

In All

Hao Zhang
haomoodzhang@gmai

If define

$$\vec{a}^{(0)} = \vec{x} \,. \tag{37}$$

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$$\vec{z}^{(l)} = W^{(l)}\vec{a}^{(l-1)} + \vec{b}^{(l)}, \forall l = 1, 2, \ldots, L-1\,, \tag{38}$$

$$\vec{a}^{(l)} = \max(0, \vec{z}^{(l)}), \forall l = 1, 2, \ldots, L-1\,, \tag{39}$$

$$h(\vec{x}) = W^{(L)}\vec{a}^{(L-1)} + \vec{b}^{(L)}\,. \tag{40}$$

In All

If define

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$$h(\vec{x}) = W^{(L)} \vec{a}^{(L-1)} + \vec{b}^{(L)} \,. \tag{40}$$

- $\vec{a}^{(L-1)}$ is the representation $\phi(\vec{x})$.

Artificial Neural Networks

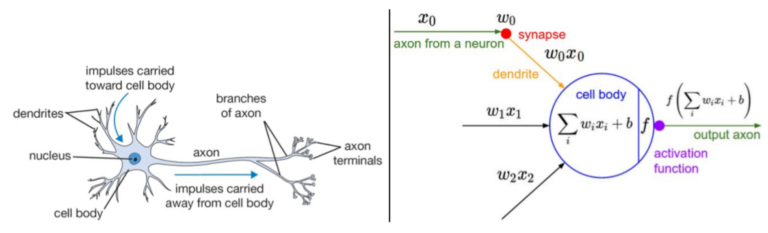

Figure: Biology model and math model. This figure is reproduced from [3].

Architecture

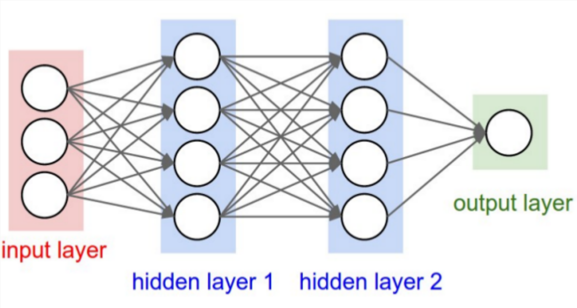

Figure: Multilayer feedforward nueral network. This figure is reproduced from [3].

Loss Function

$$J(W, \vec{b}) = \frac{1}{m} \sum_{i=1}^{m} -\log \frac{\exp(s_{y^{(i)}}^{(i)})}{\sum_{j=0}^{K-1} \exp(s_j^{(i)})} + \sum_{l=1}^{L} \frac{\lambda}{2} \| W^{(l)} \|_F^2 \tag{41}$$

$$= \frac{1}{m} \sum_{i=1}^{m} \mathrm{err}^{(i)} + \sum_{l=1}^{L} \frac{\lambda}{2} \| W^{(l)} \|_F^2 \,. \tag{42}$$

$$\vec{s}^{(i)} = h(\vec{x}^{(i)}) \,. \tag{43}$$

Gradient Descent

Update rule

$$W^{(l)} \leftarrow W^{(l)} - \alpha \nabla_{W^{(l)}} J = W^{(l)} - \alpha \frac{1}{m} \sum_{i=1}^{m} \nabla_{W^{(l)}} \mathrm{err}^{(i)} - \alpha \lambda W^{(l)}, \forall l, \quad (44)$$

$$\vec{b}^{(l)} \leftarrow \vec{b}^{(l)} - \alpha \nabla_{\vec{b}^{(l)}} J = \vec{b}^{(l)} - \alpha \frac{1}{m} \sum_{i=1}^{m} \nabla_{\vec{b}^{(l)}} \mathrm{err}^{(i)}, \forall l. \quad (45)$$

Gradient Descent (cont'd)

$$\nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)} = \frac{\exp(\vec{s}^{(i)})}{\sum_{j=0}^{K-1} \exp(s_j^{(i)})} - \vec{e}_{y^{(i)}} , \tag{46}$$

$$\vec{s}^{(i)} = h(\vec{x}^{(i)}) = W^{(L)} \vec{a}^{(L-1)} + \vec{b}^{(L)} . \tag{47}$$

Gradient Descent (cont'd)

$$\nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)} = \frac{\exp(\vec{s}^{(i)})}{\sum_{j=0}^{K-1} \exp(s_j^{(i)})} - \vec{e}_{y^{(i)}} \,, \tag{46}$$

$$\vec{s}^{(i)} = h(\vec{x}^{(i)}) = W^{(L)} \vec{a}^{(L-1)} + \vec{b}^{(L)} \,. \tag{47}$$

Easy one

$$\nabla_{W^{(L)}} \mathrm{err}^{(i)} = (\nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)}) \vec{x}^{(i)T} \,, \tag{48}$$

$$\nabla_{\vec{b}^{(L)}} \mathrm{err}^{(i)} = \nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)} \,. \tag{49}$$

Gradient Descent (cont'd)

$$\nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)} = \frac{\exp(\vec{s}^{(i)})}{\sum_{j=0}^{K-1} \exp(s_j^{(i)})} - \vec{e}_{y^{(i)}} \,, \tag{46}$$

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Easy one

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$$\nabla_{\vec{b}^{(L)}} \mathrm{err}^{(i)} = \nabla_{\vec{s}^{(i)}} \mathrm{err}^{(i)} \,. \tag{49}$$

Hard One: $\nabla_{W^{(l)}} \mathrm{err}^{(i)}$ and $\nabla_{\vec{b}^{(l)}} \mathrm{err}^{(i)}$, $l < L$?

Error Back-Propagation

At layer l

$$\vec{z}^{(l)} = W^{(l)}\vec{a}^{(l-1)} + \vec{b}^{(l)}, \tag{50}$$

$$\vec{a}^{(l)} = \max(0, \vec{z}^{(l)}). \tag{51}$$

Error Back-Propagation

At layer l

$$\vec{z}^{(l)} = W^{(l)}\vec{a}^{(l-1)} + \vec{b}^{(l)}\,, \tag{50}$$

$$\vec{a}^{(l)} = \max(0, \vec{z}^{(l)})\,. \tag{51}$$

Assume $\nabla_{\vec{a}^{(l)}}\mathrm{err}^{(i)}$ is known, then

$$\nabla_{\vec{z}^{(l)}}\mathrm{err}^{(i)} = \left(\frac{\partial \vec{a}^{(l)}}{\partial \vec{z}^{(l)}}\right)^T \nabla_{\vec{a}^{(l)}}\mathrm{err}^{(i)} \tag{52}$$

$$= 1\{\vec{z}^{(l)} > 0\} \odot \nabla_{\vec{a}^{(l)}}\mathrm{err}^{(i)}\,. \tag{53}$$

Deep Learning: Why and What

Hao Zhang
haomoodzhang@gmai

Error Back-Propagation

At layer l

$$\vec{z}^{(l)} = W^{(l)}\vec{a}^{(l-1)} + \vec{b}^{(l)}\,, \tag{50}$$

$$\vec{a}^{(l)} = \max(0, \vec{z}^{(l)})\,. \tag{51}$$

Assume $\nabla_{\vec{a}^{(l)}}\mathrm{err}^{(i)}$ is known, then

$$\begin{aligned}\nabla_{\vec{z}^{(l)}}\mathrm{err}^{(i)} &= \left(\frac{\partial \vec{a}^{(l)}}{\partial \vec{z}^{(l)}}\right)^T \nabla_{\vec{a}^{(l)}}\mathrm{err}^{(i)} &(52)\\ &= 1\{\vec{z}^{(l)} > 0\} \odot \nabla_{\vec{a}^{(l)}}\mathrm{err}^{(i)}\,. &(53)\end{aligned}$$

$$\begin{aligned}\nabla_{\vec{a}^{(l-1)}}\mathrm{err}^{(i)} &= \left(\frac{\partial \vec{z}^{(l)}}{\partial \vec{a}^{(l-1)}}\right)^T \nabla_{\vec{z}^{(l)}}\mathrm{err}^{(i)} &(54)\\ &= W^T \nabla_{\vec{z}^{(l)}}\mathrm{err}^{(i)}\,, &(55)\\ \nabla_{W^{(l)}}\mathrm{err}^{(i)} &= (\nabla_{\vec{z}^{(l)}}\mathrm{err}^{(i)})\vec{a}^{(l-1)T}\,, &(56)\\ \nabla_{\vec{b}^{(l)}}\mathrm{err}^{(i)} &= \nabla_{\vec{z}^{(l)}}\mathrm{err}^{(i)}\,. &(57)\end{aligned}$$

Prediction and Evaluation

Prediction and Evaluation

- The same as linear models,

Prediction and Evaluation

- The same as linear models,
- But $\vec{s}^{(i)} = h(\vec{x}^{(i)})$ has different form.

Hao Zhang
haomoodzhang@gmai

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Training Data

Require image input

$$D = \{(\mathbf{X}^{(i)}, y^{(i)})\}_{i=1}^{m} \,, \tag{58}$$

$$\mathbf{X}^{(i)} \in \mathbb{R}^{H \times W \times D}, \forall i \,, \tag{59}$$

$$y^{(i)} \in \{0, 1, 2, 3, \ldots, K-1\}, \forall i \,. \tag{60}$$

Hypothesis Function

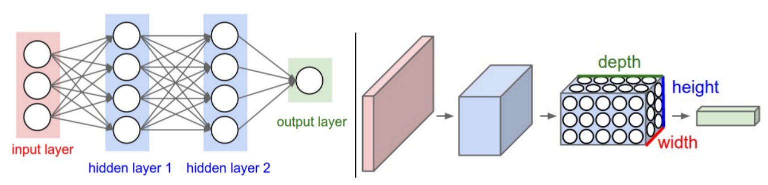


Figure: Normal neural networks and CNN. This figure is reproduced from [3].

Expand Fully Connected Affine to Tensor

$$\vec{a} = W\vec{x} + \vec{b} \tag{61}$$

$$= \begin{bmatrix} \vec{w}_0^T \\ \vec{w}_1^T \\ \vdots \\ \vec{w}_{n_l-1}^T \end{bmatrix} \vec{x} + \vec{b} \tag{62}$$

$$= \begin{bmatrix} \vec{w}_0^T \vec{x} + b_0 \\ \vec{w}_1^T \vec{x} + b_1 \\ \vdots \\ \vec{w}_{n_l-1}^T \vec{x} + b_{n_l-1} \end{bmatrix}. \tag{63}$$

Expand Fully Connected Affine to Tensor

$$\vec{a} = W\vec{x} + \vec{b} \tag{61}$$

$$= \begin{bmatrix} \vec{w}_0^T \\ \vec{w}_1^T \\ \vdots \\ \vec{w}_{n_l-1}^T \end{bmatrix} \vec{x} + \vec{b} \tag{62}$$

$$= \begin{bmatrix} \vec{w}_0^T\vec{x} + b_0 \\ \vec{w}_1^T\vec{x} + b_1 \\ \vdots \\ \vec{w}_{n_l-1}^T\vec{x} + b_{n_l-1} \end{bmatrix} . \tag{63}$$

$$a_{d_l} = \vec{w}_{d_l}^T\vec{x} + b_{d_l} = \vec{w}_{d_l} \odot \vec{x} + b_{d_l}, \forall d_l . \tag{64}$$

Expand Fully Connected Affine to Tensor (cont'd)

Hao Zhang
haomoodzhang@gmai

When $\mathbf{X} \in \mathbb{R}^{H_{l-1} \times W_{l-1} \times D_{l-1}}$,

Expand Fully Connected Affine to Tensor (cont'd)

When $\mathbf{X} \in \mathbb{R}^{H_{l-1} \times W_{l-1} \times D_{l-1}}$,
$\mathbf{W}_{d_l} \in \mathbb{R}^{H_{l-1} \times W_{l-1} \times D_{l-1}}$,

$$a_{d_l} = \mathbf{W}_{d_l} \odot \mathbf{X} + b_{d_l} = \sum_{i=0}^{H_{l-1}-1} \sum_{j=0}^{W_{l-1}-1} \sum_{d=0}^{D_{l-1}-1} \mathbf{X}(i,j,d) \mathbf{W}_{d_l}(i,j,d) + b_{d_l}, \forall d_l . \tag{65}$$

Two Simplifications of Convolutional Layer

- Sparse Connection
- Parameter Sharing

Sparse Connection

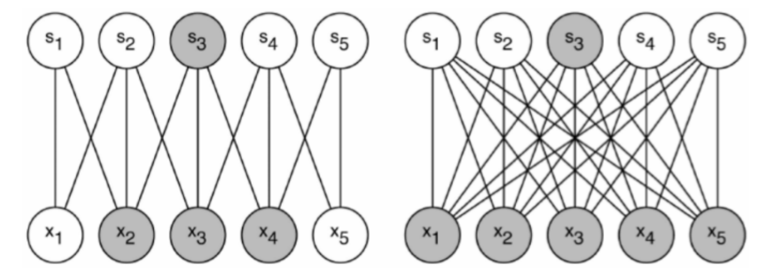


Figure: Sparse connection in 1 dimension. This figure is reproduced from [4].

Sparse Connection (cont'd)

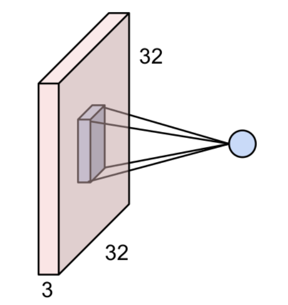


Figure: Sparse connection in 3 dimensions. This figure is reproduced from [3].

Sparse Connection (cont'd)

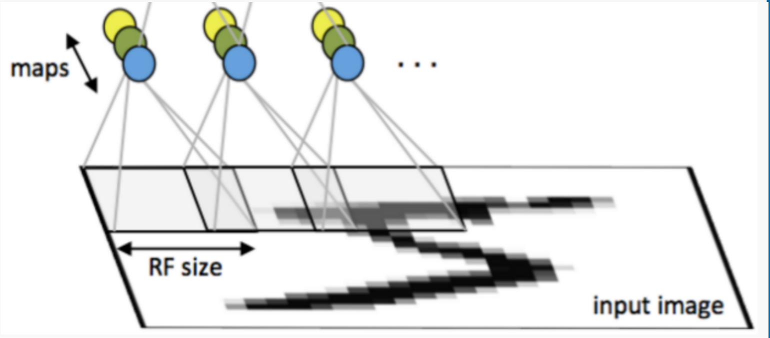


Figure: Activation map. This figure is reproduced from [2].

Sparse Connection (cont'd)

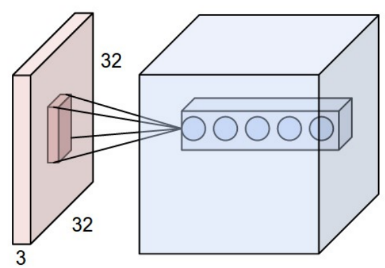


Figure: Input and output tensor. This figure is reproduced from [3].

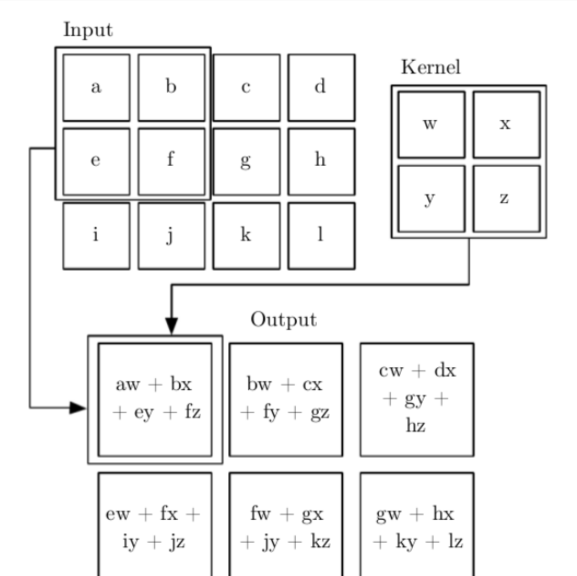
Parameter Sharing

If a motif can appear in one part of the image, it could appear anywhere,

Parameter Sharing

If a motif can appear in one part of the image, it could appear anywhere,
Hence units at different locations sharing the same weights

Convolution



Convolution (cont'd)

$$\mathbf{A}_{d_l} = \mathbf{X} * \mathbf{W}_{d_l} + b_{d_l}, \forall d_l \,. \tag{66}$$

$$\mathbf{A}(i_l, j_l, d_l) = \sum_{i=0}^{F_1-1} \sum_{j=0}^{F_2-1} \sum_{d=0}^{D_{l-1}-1} \mathbf{X}(i_l+i, j_l+j, d)\mathbf{W}(i, j, d, d_l)+b_{d_l}, \forall i_l, j_l, d_l \,. \tag{67}$$

Pooling Layer

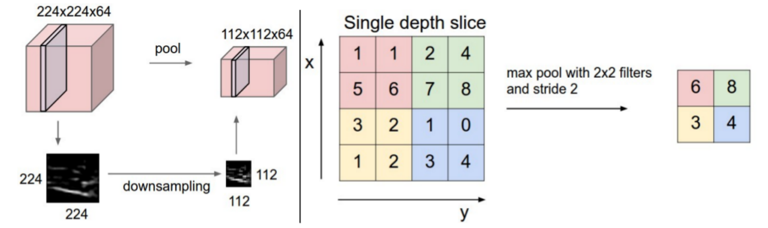


Figure: Max pooling. This figure is reproduced from [3].

Pooling Layer (cont'd)

$$\mathbf{A}(i_l, j_l, d_l) = \max_{0 \le i < F_1, 0 \le j < F_2} \mathbf{X}(i_l S + i, j_l S + j, d_l), \forall i_l, j_l, d_l \,. \tag{68}$$

ReLU Layer

$$\mathbf{A}(i_l, j_l, d_l) = \max(0, \mathbf{X}(i_l, j_l, d_l)), \forall i_l, j_l, d_l\,. \tag{69}$$

Fully Connected Layer

The same as normal feedforward neural networks.

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Thanks

Questions Please!

Hao Zhang
haomoodzhang@gmail.com