

Original Paper

### MEASUREMENT AND CONTROL

Measurement and Control 2020, Vol. 53(1-2) 214–221 © The Author(s) 2019 Article reuse guidelines: sagepub.com/journals-permissions DOI: 10.1177/002034019847704 journals.sagepub.com/home/mac



# Robust path planning for avoiding obstacles using time-environment dynamic map

Xiaolin Zhao by, Yu Zhang and Boxin Zhao by

#### **Abstract**

Small unmanned aerial vehicles are widely used in urban space because of its flexibility and maneuverability. However, there are full of dynamic obstacles and immobile obstacles which will affect safe flying in urban space. In this paper, a novel integrated path planning approach for unmanned aerial vehicles is presented, which is consisted of three steps. First, a time-environment dynamic map is constructed to represent obstacles by introducing time axis. Second, unmanned aerial vehicles' flyable paths are explored based on breadth-first algorithm. Third, a path planning method using A\* algorithm and local trace-back model is designed in order to discover sub-optimal feasible path rapidly in unmanned aerial vehicles' field of view. Finally, the simulation results have illustrated that the proposed method can ensure unmanned aerial vehicles' autonomous path planning safely and effectively in urban space crowded with obstacles.

#### **Keywords**

Path planning, unmanned aerial vehicles, time-environment dynamic map

Date received: 31 January 2019; accepted: 22 March 2019

#### Introduction

Since unmanned aerial vehicles (UAVs) have great potential to complete missions without human intervention, large number of applications of UAVs for both military and commercial purposes have been emerged. However, in the urban space, the UAVs usually encounter with various obstacles, such as buildings, electrical wires and other flying UAVs, which may affect UAVs' safety. Therefore, designing a safe as well as flyable path for UAVs in the environment crowded with obstacles is needed.

In recent years, many approaches about path planning have been investigated for UAVs. McLain and Beard<sup>2</sup> constructed a Voronoi diagram based on the known locations of threats to generate an initial path and then took different combinations of the edges so as to choose the lowest-cost flyable path. As pointed out by Wang et al.,<sup>3</sup> this approach did not take the threats' effective range into consideration, and thus a UAV may go through some of the threat zones. Wang et al. developed a path planning method, which built a Laguerre diagram based on Delaunay graphs, to overcome the drawbacks of Voronoi diagram. The graph-based methods such as Voronoi diagrams,<sup>2,4,5</sup> Laguerre diagram<sup>3,6</sup> or visibility graphs<sup>7</sup> all assume that the entire environment remains unchanged when a UAV is

flying, so that a safe path from start-point to end-point can be designed according to the collected environmental information. However, it is inaccessible to obtain reliable information of the entire environment, for instance, pop-up obstacles will never be localized in advance until a UAV is approaching them. Thus, the traditional graph-based approaches can hardly meet the requirements of planning a safe path in dynamic environment.

It turns out that UAVs must have the ability to deal with new obstacles that burst into environments. Furthermore, there are many works in the literature dealing with path planning for UAVs in dynamic environments, such as A\* and D\* algorithms, 9-11 potential fields, 12 model predictive control 13 and evolutionary algorithmic techniques. 14,15 Many efforts have been made in the field of UAV path planning, and these approaches can be mainly classified in two thoughts:

<sup>1</sup>Air Force Engineering University, Xi'an, China

#### Corresponding author:

Boxin Zhao, Air Force Engineering University, Xi'an 710038, China. Email: boxin.zhao@nudt.edu.cn

<sup>&</sup>lt;sup>2</sup>Department of Electronic Engineering, Tsinghua University, Beijing, China

- 1. Assuming that the number of obstacles is limited and obstacles are far from each other so that a safe path can be easily designed.<sup>2–7</sup> This assumption is not suitable for urban environments because irregular obstacles are densely placed in the environment. Thus, flyable areas are limited.
- 2. Calculating the obstacles' probability of occurrence at some points in the future via dynamic prediction algorithms so as to plan a path without obstacles for UAVs. 16,17 However, these methods highly depend on the accuracy of location and the efficiency of algorithms, and thus are not practical for real-time path planning in the dynamically changing environments.

In this paper, we propose a collision avoidance algorithm focusing on dense and static threats in uncertain environments. In order to plan a safe flyable path in urban space crowded with irregular buildings and moving obstacles, we introduce a time axis to the collision avoidance algorithm. In this manner, the environments where UAVs fly are extended to time-environment dynamic map (TEDM). Based on TEDM, we designed an integrated approach which combines breadth-first algorithm, A\* algorithm and local trace-back model to plan an optimal path in each short time interval, thus avoiding irregular buildings without simplification. Our proposed method is detailed in the following sections.

#### Problem statements and UAV modeling

#### Problem statement

With popularity and proven success of UAVs in various situations, the demand of UAVs used in urban space is increasing. The remarkable character of urban space is that, as we all know, it is filled with obstacles which are irregular and hazardous, and the flyable region between buildings is restricted. To avoid flightrisks of obstacle collision, we design a novel path planning algorithm for UAVs in urban space crowded with irregular obstacles, which makes use of the cost function to build an optimal local trace-back model within the range of sensors. The dynamic obstacles in environment should also be considered, which is an enormous challenge for path planning in such a strict circumstance. Figure 1 depicts a representative urban space, in which the gray boxes indicate building obstacles, the black spots mean other dynamic UAVs which represent moving obstacles and the arrows mean the corresponding direction. Given such an environment, our mission is to plan a safe path for a UAV from the start position to the final position, which passes through the path from the target position 1 to the target position 2 sequentially. The red spots in Figure 1 stand for the target positions, respectively.

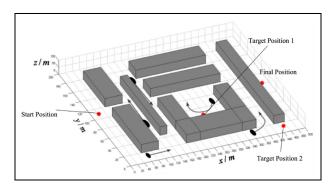


Figure 1. Sketch map of urban space crowded with obstacles.

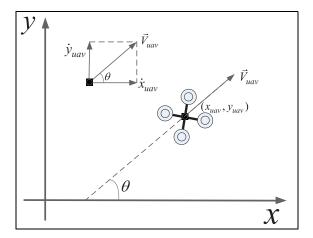


Figure 2. The kinematic model of UAV.

#### **UAV** modeling

The results we present in this paper can be applied to different kinds of rotorcrafts. In this paper, we focus on quadrotor UAVs for it is an intuitive model that is often used in literature.

The key features of quadrotor aircrafts lie in its flexibility and maneuverability so it comes out that we do not need to think much about the restrictions of the minimum turning radius and the overload problem while turning. As a result, a four-rotor aircraft can be regarded as a controllable particle, <sup>19</sup> without considering the rotation around center of mass. So, the kinematic model of UAV is defined by its position  $(x_{uav}, y_{uav})$ , orientation  $\theta$  and velocity vector  $\overrightarrow{V}_{uav}$  (see Figure 2).

Therefore, the UAV's kinematic equation is given by

$$\begin{cases} \dot{x}_{uav} = \overrightarrow{V}_{uav} \cos \theta \\ \dot{y}_{uav} = \overrightarrow{V}_{uav} \sin \theta \end{cases}$$
 (1)

Note that  $V_{\min} \leq 0$  and that a negative velocity corresponds to moving backward. Because we will apply the UAV gridding model later, it is convenient to choose the symmetric bounds for the velocity term.

For convenience, many works assume that the UAVs maintain constant velocity;<sup>20,21</sup> however, it is

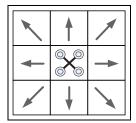


Figure 3. Gridding model of UAV.

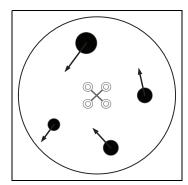


Figure 4. Sketch map of FOV.

still not easy for UAV's controller to compute because constant variables are often inestimable. Hence, the UAV's kinematic equation based on particle is further simplified using gridding model. As one can observe from Figure 3, a gridding model of UAV is able to move in eight geographical directions in case they are free.

Taking the factor into consideration that UAV needs to explore obstacles around to get information about the environment, we define the boundary of detected coverage as field of view (FOV, which is similar to the field we can see through our view). Let  $R_{sensor}$  be the max detection distance of UAV's sensors onboard while it is flying, and hence, FOV can be described as a circular region (as shown in Figure 4) where the position of UAV's particle ( $x_{uav}$ ,  $y_{uav}$ ) represents the center of circle and  $R_{sensor}$  means the radius of circular region

$$(x_{FOV} - x_{uav})^2 + (y_{FOV} - y_{uav})^2 \leqslant R^2_{sensor}$$
 (2)

#### **TEDM**

We should avoid not only static but also dynamic obstacles while planning path in such a limited environment as illustrated in Figure 1; thus, a new approach is put forward using TEDM, aiming to fulfilling the demand of real-time path planning in harsh and special environments.

In order to simplify the static obstacles, a horizontal plane is chosen from stereoscopic space. As most of the buildings in urban space are vertical from bottom to top and their shape and position in horizontal plane are time-invariant, we can use two-dimensional space

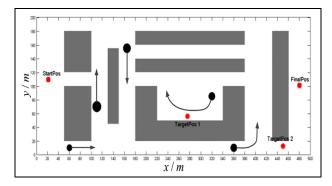


Figure 5. Horizontal plane of urban space.

to replace three-dimensional space while constructing the model of static obstacles (see the gray regions in Figure 5).

To moving obstacles (see Figure 5, the black area), time axis is introduced on the basis of the horizontal plane we built before to discretize the environment, forming a three-dimensional TEDM. It consists of state  $\langle x, y, v, \theta, t \rangle$ , where  $D\_obstacle_i = \langle x_i, y_i, v_i, \theta_i, t \rangle$  contains a finite number of dynamic obstacles with i = 1, 2, ..., N, specific time point t, the ith dynamic obstacle's position  $(x_i, y_i)$ , its velocity  $v_i$  and orientation  $\theta_i$ . Set the update rate of TEDM as  $\Delta t_{TEDM}$ , and for the dynamic obstacles

$$\lim_{\Delta t_{TEDM} \to 0} (v_{t + \Delta t_{TEDM}} - v_{t}, \theta_{t + \Delta t_{TEDM}} - \theta_{t}) = (0, 0) \quad (3)$$

which means  $v_i$  and  $\theta_i$  remain unchanged in the short time period  $[t, t + \Delta t_{TEDM}]$  where  $\Delta t_{TEDM}$  is infinitesimal; here, we define  $\Delta t_{TEDM}$  equals to the quotient value between the velocity of the quadrotor and the length from the UAV to the edge of FOV. It can be seen that if we locate the dynamic obstacles in UAV's FOV at t moment, then the motional tendency in the period  $[t, t + \Delta t_{TEDM}]$  can be predicted and located. What's more, if we regard the regions that obstacles will (probably) reach as threat regions, a path without any collision threat can be designed in  $[t, t + \Delta t_{TEDM}]$ only if we avoid the threat regions. This method is applied through the path planning in urban space, thus a safe and flyable path can be generated to avoid dynamic obstacles, and the TEDM is constructed by sensing the obstacles in FOV as shown in Figure 6.

#### Path planning based on TEDM

#### Reasons for the methods

Traditional methods often regard obstacles are far from each other. They assume the obstacles are small and slow. When encounter with irregular obstacles, it is hard to simplify them. Furthermore, the fast dynamic obstacles require efficient algorithm. The A\* algorithm can guide the UAV, but it often has a collision with irregular obstacles. The breadth-first algorithm always

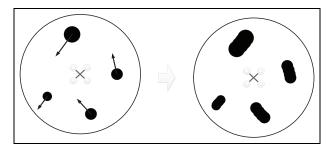


Figure 6. Time-environment dynamic map.

searches the entire environment to find the optimal path, causing a waste of calculating time.

We take the advantages of A\* and breadth-first algorithms to solve the problems. The trace-back model is applied to find the optimal local path, purposing on reducing the time-consumption and improving the safety. Finally, the method for path planning based on TEDM is built, which can not only reflect to dynamic obstacles fast but also avoid collision into irregular obstacles.

#### Principium of path planning based on TEDM

Generally speaking, the purpose of path planning is to design a safe path for UAV; however, as we analyzed in section "Introduction," the existing methods can hardly meet the requirements in such complex environments with crowded obstacles. As shown in Figure 7, an integrated approach can be planned as follows. First, we adopt the breadth-first algorithm to explore the surrounding environment and then find out an optimal waypoint in FOV with A\* algorithm, and finally apply the trace-back model to plan an optimal path in each short time interval. In this way, we can get a path avoiding obstacle collisions over the urban space.

First, a global evaluation function is constructed by A\* algorithm

$$f^k = g^k + h^k \tag{4}$$

$$h^{k} = |end.x - now.x| + |end.y - now.y|$$
 (5)

where (x, y) presents the position in two-dimensional space. *end.x* and *end.y* represents target position. *now.x* and *now.y* represents the current position of UAV. k is the sequence number when building TEDM.

Thus, the local environment in FOV can be depicted in TEDM, and the motion tendency of obstacles in FOV can be predicated in a short time interval  $\Delta t_{TEDM}$ 

$$\begin{bmatrix} x_{t} + \Delta t_{TEDM} \\ y_{t} + \Delta t_{TEDM} \end{bmatrix} = \begin{bmatrix} x_{t} \\ y_{t} \end{bmatrix} + \Delta t_{TEDM} \cdot \begin{bmatrix} V_{x} \\ V_{y} \end{bmatrix}$$
 (6)

Then, the TEDM in FOV is gridded and each grid represents a minimum node when flying, so breadth-first algorithm is used to explore the environment in FOV. Given kth local cost function  $g^k$  as

$$g^k = \sum_{i=m}^n g_i^k(s_i, u_i) \tag{7}$$

where m stands for the UAV's node now, n means the number of nodes in FOV,  $s_i$  shows the position of UAV and  $u_i$  represents the decision-making when it comes to ith step.

When the start node m is set, all flyable paths can be determined in FOV

$$g^k = g_m^k(s_m, u_m) + G_{m+1, n}^k(s_{m+1}, \dots, s_n)$$
 (8)

where  $G_{m+1,n}^k$   $(s_{m+1}, \ldots, s_n)$  is the gathering that made decisions from m+1 stage to final n stage at kth path planning, which can be denoted as  $G_{m+1,n}^k$ .

The breadth-first algorithm is used to compute the cost values of marginal nodes. When the decision-making process reaches FOV's marginal nodes, the optimal evaluation function can be selected by choosing the minimum  $f^k$  of marginal nodes

$$opt(f^k) = opt(g^k) + opt(h^k)$$
 (9)

where opt is short for optimization, which means the minimum  $f^k$  we choose.

Take a step back from  $opt(g^k)$ , so the former optimal step  $G_{n-1,n}$  can be defined as

$$opt(g^k) = \sum_{i=m}^{n-1} g_i^k(s_i, u_i) + opt(G_{n-1,n}^k)$$
 (10)

After equation (10), the optimal step  $G_{n-1,n}^k$  can be obtained, but the step between m+1 and n-1 is not sure. In the next step, we can infer opt  $g^k$  equals to the sum of  $\sum_{i=m}^{n-2} g_i^k(s_i, u_i)$  and opt  $G_{n-2,n}^k$ . Then, the traceback model can be applied by this analogy. And, an optimal path in FOV can be designed until it meets the m node

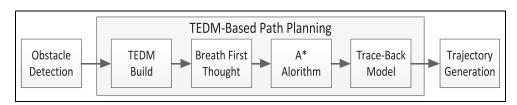


Figure 7. TEDM-based path planning system.

Table 1. Path planning step sequence.

Step	The applied strategies
1	Build TEDM
2	Grid and label FOV
3	Search and record environment by breadth-first algorithm
4	Select the optimal node using A* algorithm
5	Get path in FOV with trace-back model algorithm

TEDM: time-environment dynamic map; FOV: field of view.

$$opt(g^{k}) = \sum_{i=m}^{n-1} g_{i}^{k}(s_{i}, u_{i}) + opt(G_{n-1, n}^{k})$$

$$= \sum_{i=m}^{n-2} g_{i}^{k}(s_{i}, u_{i}) + opt(G_{n-2, n}^{k})$$

$$\vdots$$

$$= g_{m}^{k}(s_{m}, u_{m}) + opt(G_{m+1, n}^{k})$$

$$(11)$$

Finally, the above procedures are repeated until a safe path in the global urban space is obtained. The distance of the final path can be expressed as

$$Dis\_UAV = \sum_{k} Dis\_UAV_{k,k+1}$$
 (12)

# Implementation of path planning based on TEDM

As described in section "Path planning based on TEDM," the proposed method aims to plan an obstacle-free path using the TEDM in urban space, and that can be achieved as following five steps (Table 1).

As we discussed in section "TEDM," all obstacles in FOV can be predicated in  $[t, t + \Delta t_{TEDM}]$  (see in Figure 8) and then translated to TEDM. It is because that  $\Delta t_{TEDM}$  is infinitesimal so that the UAV can avoid all obstacles with avoiding the obstacles sensed in  $[t, t + \Delta t_{TEDM}]$ .

After translating the environment to TEDM, we grid the environment. The A\* algorithm, breadth-first algorithm and trace-back model rely on grids to design the path. Therefore, we transfer the environment to a matrix at the beginning, and labeled 0 and 1 to the safe area and obstacles, respectively. To TEDM, the labels are as shown in Figure 9.

Then, breadth-first algorithm is used to explore and record all nodes in FOV from UAV's start node, and for every node at the margin of FOV, the cost function is calculated to define the optimal margin node (see Figure 10).

After that, an optimal node on FOV's edge can be obtained using A\* algorithm, and take a step back from this optimal node, then it is the second optimal node.

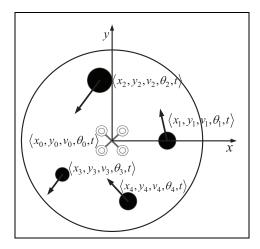


Figure 8. Obstacle sensing by sensors onboard.

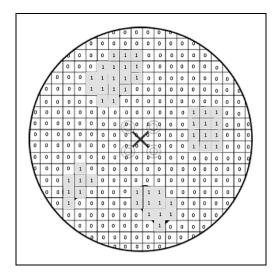
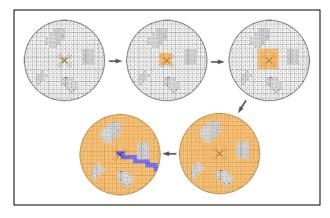


Figure 9. Grid map of obstacles in UAV's FOV.



**Figure 10.** Explore and record environment based on breadth-first algorithm.

Tracing back step by step, we can get the optimal path in FOV at *t* moment.

Finally, this integrated method is applied over the environment along with the position of target and then

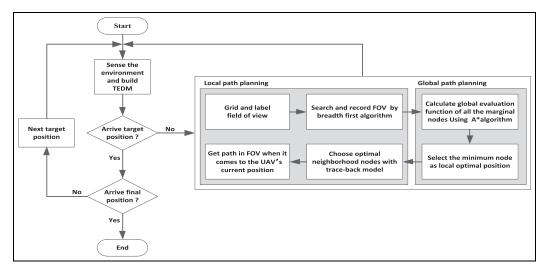


Figure 11. Functional architecture of TEDM-based path planning.

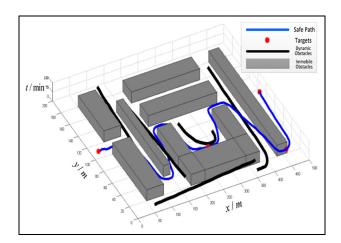


Figure 12. TEDM-based path planning.

a safe flyable path can be designed to avoid collisions in the urban space crowded with different kinds of obstacles.

Figure 11 shows the functional architecture for path planning based on TEDM and illustrates how this integrated method operates.

#### Simulations and analysis

Suppose the complex environment in urban space is a rectangular with 500 m height and 200 m width, the of UAV velocity is 2 m/s, and its mission is to fly from the start (2, 109) position, with passing through the first target (76, 55) and the second target (450, 13), to the final position (480, 101). Figure 12 shows the simulated results.

To further depict the safety of the UAV, some flying moments are shown in Figure 13, where the brown disk

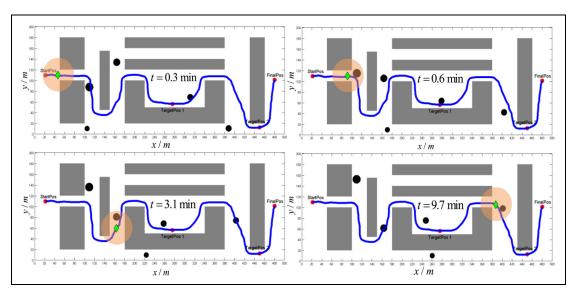


Figure 13. Some moments when designing path in urban space.

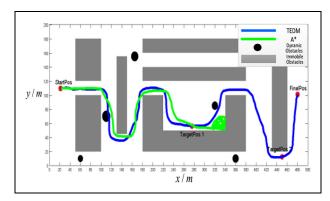
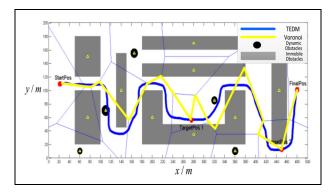


Figure 14. Path planning based on A\* algorithm compared with TFDM.



**Figure 15.** Path planning based on Voronoi compared with TEDM.

stands for the FOV, and the gray blocks and the black spots represent static and moving obstacles, respectively.

As shown in Figure 13, a safe path can be obtained with the method of TEDM. However, the effectiveness of TEDM is not discussed. So, we introduce A\*- and Voronoi-based approaches to verify the effectiveness of the method of TEDM.

As shown in Figure 14, the TEDM algorithm performs well, but A\* algorithm cannot arrive to the final position. Because A\* algorithm tries to look for the final position and attempts to get an optimal path from the UAV to final position, and when there are threats between the UAV and final position, it will probably fall into dead zone.

In Figure 15, though the Voronoi graph can plan a path from start position to final position, but the path often goes through buildings. The TEDM algorithm can avoid crowded obstacles safely, and the path obtained by TEDM is smoother than Voronoi graph.

More detailed information can be found in Table 2. The results show that TEDM costs 14.8 min in getting a safe path, but the other two never arrive to the final position. To the average memory consumption, TEDM performs better than A\* and Voronoi, which spends less than half memory consumption. To the UAV, the

Table 2. Comparisons of A\*, Voronoi and TEDM.

Methods	Time consumption (min)	Average memory consumption (%)	Collision amounts
A*	_	82	1
Voronoi	-	91	14
TEDM	14.8	46	0

TEDM: time-environment dynamic map.

most important factor is safety; here, we use collision amounts to judge the safety. We can infer from Table 2 that TEDM has no collision, while A\* and Voronoi get 1 and 14, respectively. The results show that TEDM has a better performance compared with the other two traditional algorithms.

#### Conclusion

In terms of complex environment, we have argued how the UAV can avoid collision in urban space safer and faster. After reviewing the studies in the context of immobile and dynamic obstacles, a conceptual constructive model which combines A\* and breadth-first algorithms has been proposed. Following are some concluding remarks:

- A TEDM is built, which divides the whole target into two parts, global target and local target. A\* and breadth-first algorithms are applied to global and local environments, which improve the safety in path planning.
- Small time interval is introduced to transfer dynamic obstacles to instantaneous immobile obstacles, making it more efficient to avoid dynamic obstacles.
- The proposed method can avoid irregular buildings without simplification, using A\* algorithm to guide itself globally and breadth-first method to avoid local obstacles.

Still, there are several issues in need of attention and further investigations, including practical studies and the study on three-dimensional path planning.

#### **Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

#### **Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The research was supported by the National Natural Science Foundation of China (grant nos 61503405 and 61703428) and National Aeronautical Science Foundation of China (grant nos 20160896007 and 20160896008).

#### **ORCID iD**

Xiaolin Zhao (b) https://orcid.org/0000-0002-5094-9615

#### References

- Amin JN, Boskovi JD and Mehra RK. A fast and efficient approach to path planning for unmanned vehicles. In: *Proceedings of the AIAA guidance, navigation and control conference*, Keystone, CO, 21–24 August 2006.
- McLain TW and Beard RW. Trajectory planning for coordinated rendezvous of unmanned air vehicles. In: Proceedings of the AIAA guidance, navigation, and control conference and exhibit, Denver, CO, 14–17 August 2000.
- 3. Wang SL, Wei RX, Shen D, et al. Laguerre diagram construction algorithm for path planning. *Syst Eng Electron* 2013; 35(3): 552–556.
- Bortoff SA. Path planning for UAVs. In: Proceedings of the American control conference, Chicago, IL, 28–30 June 2000.
- Chandler PR, Rasmussen S and Pachter M. UAV cooperative path planning. In: Proceedings of the AIAA guidance, navigation, and control conference and exhibit, Denver, CO, 14–17 August 2000.
- Wei RX, Xu ZF, Wang SL, et al. Self-optimization Astar algorithm for UAV path planning based on Laguerre diagram. Syst Eng Electron 2015; 37(3): 577– 582
- Bellingham J, Richards A and How JP. Receding horizon control of autonomous aerial vehicles. In: *Proceedings of the American control conference*, Anchorage, AK, 8–10 May 2002.
- 8. Beard RW, McLain TW and Goodrich M. Coordinated target assignment and intercept for unmanned air vehicles. In: *Proceedings of the IEEE international conference on robotics and automation*, Washington, DC, May 2002, http://citeseerx.ist.psu.edu/viewdoc/download?doi = 10.1. 1.6.6229&rep=rep1&type=pdf
- 9. Russell S and Norvig P. *Artificial intelligence: a modern approach.* Upper Saddle River, NJ: Prentice Hall, 2003.
- Mitchell JS and Keirsey DM. Planning strategic paths through variable terrain data. In: *Proceedings of the SPIE* conference on applications of artificial intelligence, vol. 485, Arlington, VA, 14 June 1984, pp. 172–179. Bellingham, WA: SPIE.
- 11. Stentz A. Optimal and efficient path planning for partially-known environments. In: *Proceedings of the 1994 international conference on robotics and automation, vol. 4*, Los Alamitos, CA, 8–13 May 1994, pp. 3310–3317. New York: IEEE.
- 12. Khatib O. Real-time obstacle avoidance for manipulators and mobile robots. *Int J Robot Res* 1986; 5(1): 90–98.
- 13. Vanek B, Péni T, Bokor J, et al. Practical approach to real-time trajectory tracking of UAV formations. In:

- Proceedings of the American control conference, Portland, OR, 8–10 June 2000.
- Fogel DB and Fogel LJ. Optimal routing of multiple autonomous underwater vehicles through evolutionary programming. In: *Proceedings of the 1990 symposium on* autonomous underwater vehicles technology, Washington, DC, 5–6 June 1990, pp. 44–47.
- 15. Capozzi BJ and Vagners J. Evolving (semi)autonomous vehicles. In: *Proceedings of the AIAA guidance, navigation, and control conference and exhibit, Montreal, QC, 6 August 2001.*
- 16. Lai CK and Whidborne JF. Real-time trajectory generation for collision avoidance with obstacle uncertainty. In: *Proceedings of the AIAA guidance, navigation and control conference*, Portland, OR, 8–11 August 2011.
- 17. Van den Berg J and Overmars M. Kinodynamic motion planning on roadmaps in dynamic environments. In: *Proceedings of the 2007 IEEE/RSJ international conference on intelligent robots and systems*, San Diego, CA, 29 October–2 November 2007.
- Zhang Q, Wei R, He R, et al. Path planning for unmanned aerial vehicle in urban space crowded with irregular obstacles. *Control Theor Appl* 2015; 32(10): 1407–1413.
- 19. Greatwood C and Richards A. Implementation of fast MPS with quadrotor for obstacle avoidance. In: *Proceedings of the AIAA guidance, navigation and control conference*, Boston, MA, 19 August 2013.
- 20. Techy L and Woolsey CA. Minimum-time path planning for unmanned aerial vehicles in steady uniform winds. *J Guid Control Dyn* 2009; 32(6): 1736–1746.
- 21. Singh SN, Pachter M, et al. Input—output invertibility and sliding mode control for close formation flying. In: *Proceedings of the AIAA guidance, navigation and control conference*, Denver, CO, 14 August 2000.

#### Appendix I

## Notation

f,g	global and local cost function
$G_{m+1,n}^k$	decision-making gather from $m + 1$ stage
	to final <i>n</i> stage at the <i>k</i> th path planning.
$R_{sensor}$	maximum detection distance of UAV's
	sensors onboard
$\overrightarrow{V}_{uav}$	UAV's velocity vector
x, y	x and y positions of UAV
$\Delta t_{TEDM}$	update rate of time-environment dynamic
	map

orientation of UAV