Full state proportional controller for Adaptive Cruise Control System

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***Abstract*—The paper presents a proposal of using a full state proportional controller in Adaptive Cruise Control (ACC) systems. A full state proportional controller is one of the simplest controllers used in automation systems. Its advantage is based on the fact that there is no additional dynamics, which makes the state of a closed system unchanged. The article proposes a method of a numerical search for optimum parameters of the controller to ensure asymptotic stability of a closed system. The correctness of the proposed solutions was verified via computer simulations.**

***Index Terms*—adaptive cruise control, optimal controler, optimization, proportional controller**

I. Introduction

Controlling a car is a complex process and it requires great involvement of the driver and focus of attention. Covering long distances is very exhaustive for the driver and may result in making mistakes causing road accidents. Since 1990s extensive research has been carried out, resulting in the development of various systems improving drivers' safety and comfort. The systems are commonly referred to as *Advanced Driving Assistance Systems* (ADAS) [1], [2]. The *Adaptive Cruise Control* (ACC) system [3], [4] is one of such examples and makes further development of *Cruise Control* (CC) systems. The ACC is supposed to maintain the set distance between moving vehicles, which helps reduce the driver's load in a significant way and to optimise other factors, e.g. fuel consumption, improve road capacity etc. [5], [6], [7], [2].

ACC systems are based on various control theory methods. Many papers on ACC present a review of the applied methods, e.g. [1], [8]. PI and LQ controllers [9], B-BA type control algorithm (*Balance-Based Adaptive Control method*) [7], quadratic optimal control [10], [11], PID controllers [12], sliding mode control [13], [14] can be enumerated as examples of control algorithms. Model predictive control is a very popular control method of ACC systems[2], [6], [15]. Neuron controllers [16], [17] and fuzzy control [18], [19] are also used. The majority of ACC control systems contain, besides the controller itself, a logical part responsible for appropriate classification of an occurrence and for using a relevant controller for such a situation [3], [20], [21].

The main purpose of the article is to investigate the possibilities of using a full state proportional controller in ACC systems. Moreover, the authors intend to develop a method of selecting optimum parameters of such a controller to guarantee asymptotic stability of a closed system. The issue of stability in ACC systems, in particular when several vehicles are moving, is of great importance and has been subject to consideration in a number of scientific studies [11], [22]. Cars produced nowadays contain many advanced control systems, hence it is very important to verify their correct operation [23], [24]. The parameters of the proposed controller will be selected numerically by solving a properly defined task of cost function optimisation alongside with limitations for decisive variables. Each stage of the controller design process complies with the common method of control systems design [25], [26].

The paper is structured in the following way. Section II presents a simplified mathematical model of the ACC system. The model has a form of simultaneous ordinary differential equations and makes a model that is commonly used. Section III presents a proportional state controller used in the control system of ACC system. The section contains a description of the proposed controller and its properties and popular quality indexes used in control systems. Moreover, the section III describes additional algebraic conditions for the controller parameters whose meeting guarantees an asymptotic stability of a closed control system with the proposed controller. Finally,

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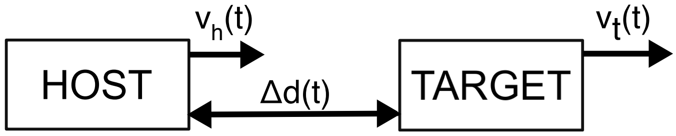

Fig. 1. The structure of ACC system

Section III presents the results of a numerical optimisation that helped identify optimum parameters of the controller. Section IV describes test scenarios that were used to check the proposed controller for correct operation. The section contains the results of the performed simulations with a short presentation of each test scenario. The last Section V contains conclusions and directions for further research. The appendix A summarises all parameters of the employed models and simulations.

II. Simplified ACC Model

The analysed dynamic system is composed of two moving vehicles that can be referred to as the *host* vehicle and *target* vehicle. The *host* vehicle where the ACC system works, should maintain a constant distance $\Delta d(t)$ to the *target* vehicle, regardless of the vehicle speed changes.

It is best to use the following parameters as the state variables: error between the set distance value d_{des} and the *target* vehicle:

$$\Delta d(t) = d_{\mathrm{des}} - d(t) \tag{1}$$

$$\Delta v(t) = v_{\mathrm{h}}(t) - v_{\mathrm{t}}(t) \tag{2}$$

the difference in the speed of both vehicles and $a_{\mathrm{h}}(t)$ – the *host* vehicle acceleration. There are the following differential equations describing the dynamics of the assumed state variables [1], [6], [27]:

$$\Delta \dot{d}(t) = \Delta v(t) + \tau_{\mathrm{h}} a_{\mathrm{h}}(t) \tag{3}$$

$$\Delta \dot{v}(t) = a_{\mathrm{h}}(t) \tag{4}$$

$$\dot{a}_{\mathrm{h}}(t) = -\frac{1}{\tau_{\mathrm{b}}} a_{\mathrm{h}}(t) + \frac{1}{\tau_{\mathrm{b}}} a_{\mathrm{des}}(t) \tag{5}$$

where τ_{h} – time headway the parameter helps model the driver's driving style, τ_{b} – time constant used to model the car dynamics. The equations (3), (4) and (5) can be presented in a matrix-vector form for further analysis:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \tag{6}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \tau_{\mathrm{h}} \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau_{\mathrm{b}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1/\tau_{\mathrm{b}} \end{bmatrix},$$

$$\mathbf{x}(t) = \begin{bmatrix} \Delta d(t) \\ \Delta v(t) \\ a_{\mathrm{h}}(t) \end{bmatrix}, \quad u(t) = a_{\mathrm{des}}(t)$$

In a model defined this way (6) the zero point makes the balance $\mathbf{x} = [0\ 0\ 0]^{\mathsf{T}}$ which means that the vehicles move at the same distance from one another d_{des} and at the same speed $v_{\mathrm{h}}(t) = v_{\mathrm{t}}(t)$.

When designing ACC systems, it is usually divided into two independent subsystems: an *upper controller* that generates the control signal $a_{\mathrm{h}}(t)$ and a *bottom controller* that is supposed to transfer the control signal $a_{\mathrm{h}}(t)$ onto the actual car acceleration [6], [27]. All elaborations in the paper apply only to the analysis of the *upper controller*, i.e. identifying the optimum acceleration of the *host* vehicle. The proposed model is a simplified one, in particular when it comes to the vehicle dynamics. It was assumed in the paper that the vehicle comes with a combustion engine. All proposed solutions can also be used in vehicles with an electric drive. The dynamics of electric engines, and DC engines in particular which are used in electric vehicles, can also be modelled approximately using linear ordinary differential equations [28], [29]. Descriptions of more complex models can cover a number of factors, such as non-linear air resistance and engine dynamics, [3], [7], [16], [30].

III. Optimal proportional controller

A state proportional controller is one of the simplest controllers used in control systems. The equation describing such a controller is as follows:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{e}(t) \tag{7}$$

where $\mathbf{K}$ is the controller gain matrix, $\mathbf{e}(t)$ is the difference between the set value and the current state of the system $\mathbf{x}(t)$:

$$\mathbf{e}(t) = \mathbf{x}_{\mathrm{des}} - \mathbf{x}(t) \tag{8}$$

Control in linear dynamic systems can be re-formulated as the task of leading the system to the zero point. In such a case, the controller equation (7) comes in the following form:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{e}(t) = \mathbf{K}(0 - \mathbf{x}(t)) = -\mathbf{K}\mathbf{x}(t) \tag{9}$$

Substituting (9) to the equation (6) the following equation results:

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}(t) \tag{10}$$

If a system described by the equations (6) is controllable, by selecting correct values of the controller matrix $\mathbf{K}$ it is possible to modify eigenvalues of a closed loop system. The eigenvalues can be in particular located in the left semi-plane of a complex plane, ensuring asymptotic stability of a closed loop system (10) [31], [32].

An advantage of a proportional controller is that it does not introduce extra dynamics into a closed loop control system with a controller. The disadvantage is however that a constant deviation occurs $\mathbf{e}(t) \neq 0$ if the set value $\mathbf{x}_{\mathrm{des}} \neq 0$. It results directly from the controller equation (7) which, in order to generate a control signal $\mathbf{u}(t)$, needs the deviation value other than zero $\mathbf{e}(t)$. The disadvantage is not observed, if the set value $\mathbf{x}_{\mathrm{des}} = 0$.

In an ACC system control system, the control signal $\mathbf{u}(t)$ comes as a single-dimensional vector. In such a case, the proportional controller equation (9) has the following form:

$$\mathbf{u}(t) = u(t) = -\mathbf{K}\mathbf{x}(t) \tag{11}$$

And the controller matrix $\mathbf{K}$ is a row vector:

$$\mathbf{K} = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \tag{12}$$

A. Cost Functions

Quality indexes in control systems allow for comparing the strategies and kinds of control systems. Most commonly they come as transformations $f : \mathbb{R}^m \to \mathbb{R}$ and are defined in the following way: the better the control, the lower the value of the quality index. Different quality indexes are used in control systems, and the most popular ones are presented below [26], [32]:

$$J_{\mathrm{ITnAE}} = \int_0^\infty t^n \sum_i |e_i(t)| \mathrm{d}t \tag{13}$$

$$J_{\mathrm{ITnSE}} = \int_0^\infty t^n \mathbf{e}(t)^\mathsf{T} \mathbf{e}(t) \mathrm{d}t \tag{14}$$

$$J_{\mathrm{ITnSEU}} = \int_0^\infty \Big(t^n \mathbf{e}(t)^\mathsf{T} \mathbf{e}(t) + \mathbf{u}(t)^\mathsf{T} \mathbf{u}(t) \Big) \mathrm{d}t \tag{15}$$

Based on the performed numerical simulations a decision was made to use the quality index (15) with parameter $n = 2$. For the quality index selected this way the most satisfactory results and action of the ACC system corresponding to the actual behaviour of a driver were obtained. Optimum $\mathbf{u}(t)$ control for the quality index (15) minimise the system deviation from the state $\mathbf{e}(t)$ and the control signal energy $\mathbf{u}(t)$. In the first control stage the control energy is optimised $\mathbf{u}(t)$ (and the control system does not generate signals with high amplitude). As the control time t goes by, the control $\mathbf{u}(t)$ optimises the system deviation from the state more and more $\mathbf{e}(t)$, which results from the ever faster increase in t^2.

B. Criterion of asymptotic stability

Every control system has to be stable. Stability is the characteristic that guarantees that if any interference occurs, the system will be brought to the set operation point within a finite time. There are many ways and methods of testing stability of control systems. The Hurwitz criterion [33], [34] makes one of algebraic criteria. For a control system (10) with a proportional controller (9) we have a direct impact on the eigenvalues of a closed loop system via a vector, and their position on a complex plane can be examined using the Hurwitz criterion. For the ACC system, the limitations for the controller vector $\mathbf{K}$ can be defined using the Hurwitz criterion, so that a closed loop system (10) becomes a stable system.

A dynamic system (10) is a third order system with a typical polynomial in the following form:

$$a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0 \tag{16}$$

From the first part of the Hurwitz criterion we obtain conditions necessary for the values of the polynomial coefficients (16):

$$\forall_{i=0,1,2,3} \quad a_i > 0 \tag{17}$$

The second part of the Hurwitz criterion involves examining the sign of the Hurwitz matrix determinants that have the following form for the polynomial (16):

$$\mathbf{\Delta}_1 = \begin{bmatrix} a_1 \end{bmatrix}, \quad \mathbf{\Delta}_2 = \begin{bmatrix} a_1 & a_3 \\ a_0 & a_2 \end{bmatrix} \tag{18}$$

A dynamic system (10) is asymptotically stable, if the following is true for all Hurwitz matrix determinants (18):

$$\forall_{i=1,2} \quad |\mathbf{\Delta}_i| > 0 \tag{19}$$

If the following marking is assumed $a = \tau_h > 0$ and $b = 1/\tau_b > 0$, then:

$$\begin{aligned}(\mathbf{A} - \mathbf{BK}) &= \begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 1 \\ 0 & 0 & -b \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ bK_1 & bK_2 & bK_3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 1 \\ -bK_1 & -bK_2 & -b(1+K_3) \end{bmatrix}\end{aligned} \tag{20}$$

The characteristic equation of the system (10):

$$\begin{aligned}|\mathbf{A} - \mathbf{BK} - \lambda \mathbf{I}| &= \begin{vmatrix} -\lambda & 1 & a \\ 0 & -\lambda & 1 \\ -bK_1 & -bK_2 & -b(1+K_3) - \lambda \end{vmatrix} \\ &= -\lambda^3 + \lambda^2(-b(1+K_3)) \\ &\quad + \lambda b(-aK_1 - K_2) + (-bK_1)\end{aligned} \tag{21}$$

Hence the distribution of the characteristic equation elements has to be examined:

$$\lambda^3 + \lambda^2(b(1+K_3)) + \lambda b(aK_1 + K_2) + (bK_1) = 0 \tag{22}$$

depending on the $K_1,\ K_2,\ K_3$ of the controller vector (12). From the first part of the Hurwitz criterion we get the following limitations for the controller vector $\mathbf{K}$:

$$K_1 > 0 \ \wedge K_3 > -1 \ \wedge \ aK_1 + K_2 > 0 \tag{23}$$

From the second part of the Hurwitz criterion, we get subsequent limitations for the controller vector $\mathbf{K}$:

$$b(aK_1 + aK_1K_3 + K_2)(1+K_3) - K_1 > 0 \tag{24}$$

Summarising, it can be said that if the controller vector $\mathbf{K}$ parameters are selected so that the conditions (23) and (24) are simultaneously fulfilled, a closed loop system (10) will be an asymptotically stable system, which means that the λ_i elements of a characteristic equation (22) will be located in the left half of the complex plane $\mathbb{Z}$.

C. Numerical Optimization Results of Optimal Proportional Controller

The controller parameters (11) can be identified using numerical optimisation methods. The optimisation task involves finding such values of the vector $\mathbf{K}$ so that the value of the quality index (15) was as low as possible, i.e.:

$$J^*(\mathbf{K}) = \min_{\mathbf{K}} J(\mathbf{K}) \tag{25}$$

Numerical methods implemented in the MATLAB package were used to search for optimum parameters of the controller. The optimisation is subject to linear and non-linear limitations that is why the *fmincon* procedure was use to search for the minimum of the set function taking the two kinds of limitations into account. The *fmincon* procedure searches for the function minimum at the limitations that are defined in the following way [35]:

$$\min_{\mathbf{K}} J(\mathbf{K}) \text{ such that } \begin{cases} c(\mathbf{K}) & \leq & 0 \\ c_{\mathrm{equ}}(\mathbf{K}) & = & 0 \\ \mathbf{A}_{\mathrm{neq}} \cdot \mathbf{K} & \leq & \mathbf{b}_{\mathrm{neq}} \\ \mathbf{A}_{\mathrm{equ}} \cdot \mathbf{K} & = & \mathbf{b}_{\mathrm{equ}} \\ \mathrm{lb} & < \mathbf{K} < & \mathrm{ub} \end{cases} \tag{26}$$

Limitations resulting from the first condition of the Hurwitz criterion (23) have a form of inequality:

$$\mathbf{A}_{\mathrm{neq}} \cdot \mathbf{K} \leq \mathbf{b}_{\mathrm{neq}} \tag{27}$$

with matrices:

$$\mathbf{A}_{\mathrm{neq}} = \begin{bmatrix} -1 & 0 & 0 \\ -a & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \mathbf{b}_{\mathrm{neq}} = \begin{bmatrix} -\epsilon \\ -\epsilon \\ 1-\epsilon \end{bmatrix} \tag{28}$$

The ϵ constant has a low value and was added since the *fmincon* function gains non-strict limitations and strict limitations are acquired from the Hurwitz theorem.

Limitations resulting from the second Hurwitz criterion (24) have the following form $c(\mathbf{K}) \leq 0$:

$$c(\mathbf{K}) = -(b(aK_1 + K_2)(1 + K_3) - K_1) + \epsilon \leq 0 \tag{29}$$

The ϵ constant has the same function as in the previous limitations. Additionally, the applied *fmincon* optimisation procedure can use the value of the $c(\mathbf{K})$ limitation gradient, if it can be identified analytically. In the case of the task under consideration, the vector $\mathbf{K} = [K_1\ K_2\ K_3]$ is the decisive variable, that is why the $c(\mathbf{K})$ gradient can be easily identified and comes as:

$$\nabla c(\mathbf{K}) = - \begin{bmatrix} ab(1+K_3) - 1 \\ b(1+K_3) \\ b(aK_1 + K_2) \end{bmatrix} \tag{30}$$

One of the methods of selecting optimum parameters of the controller is to minimise the values of the selected quality index in response to the unit step [26], [32]. If the ACC system controller parameters are selected according to the equations (10) instead of a unit step of the control signal $u(t)$ values of the point $\mathbf{x}(0) \neq 0$ different than zero were assumed. The optimum parameters of the controller (11) were obtained by minimising the quality index (15) for $n = 2$. For the $\mathbf{K}$ vector of the controller, the initial value was set $\mathbf{K}_{\mathrm{init}} = [0\ 0\ 0]$, which means that at the beginning of the simulation, the controller (11) does not work. The following initial values $\mathbf{x}(0)$ of the model (10)were assumed:

$$\begin{aligned} \Delta d(0) &= 100.0 \quad \mathrm{m} \\ \Delta v(0) &= 8.330 \quad \mathrm{m/s} \\ a_{\mathrm{h}}(0) &= 0.000 \quad \mathrm{m/s^2} \end{aligned}$$

Such values mean that at the initial moment $t = 0$ in front of the *host* vehicle a *target* vehicle occurs at the distance of 0 metres, which is moving at a speed lower than the speed of the *host* vehicle by 30 km/h. Acquiring the zero value $a_{\mathrm{h}}(0)$ means that the *host* vehicle was moving at a constant speed $v_{\mathrm{h}} = \mathrm{const}$. Such a situation is not very likely in reality and it can be theoretically treated as the worst case scenario.

The searched $\mathbf{K}$ vector of the controller (11) was identified for all types of drivers for whom the parameters are presented in Table III and assuming that $d_{\mathrm{des}} = 100$ m. The results of simulations are given in Table I.

TABLE I
CONTROLLER PARAMETERS FOR DIFFERENT TYPES OF DRIVERS

Driver	K		
1	0.1157	0.5223	0.2115
2	0.1172	0.5835	0.1548
3	0.1207	0.6764	0.1103
4	0.0953	0.3357	0.1790

IV. SIMULATIONS

Analysing the obtained values of the $\mathbf{K}$ vector (Table I), it can be observed that their values are similar for different types of drivers. Therefore, the mean values of the coordinate system of the $\mathbf{K}$ vector can be assumed so that a controller operating correctly in different scenarios is obtained. Such a procedure can also be justified by the need to develop one controller whose operation will be approved by a large group of drivers. The $\mathbf{K}$ vector whose elements are the mean values of the data from Table I will be used for further simulations:

$$\mathbf{K} = \begin{bmatrix} 0.1122 & 0.5295 & 0.1639 \end{bmatrix} \tag{31}$$

Such a $\mathbf{K}$ vector meets the limitations (23) and (24), which means that the closed loop system (10) is asymptotically stable. Depending on the difference in the distance (1) and speed (2) of the *host* and *target* vehicles, 5 scenarios that can occur while the vehicles are moving can be distinguished. All possible scenarios are presented in Table II:

TABLE II
SIMULATION SCENARIOS

No.	Distance	Speed
1.	$d(0) < d_{\mathrm{des}}$	$v_h(0) = v_t(0)$
2.	$d(0) < d_{\mathrm{des}}$	$v_h(0) > v_t(0)$
3.	$d(0) < d_{\mathrm{des}}$	$v_h(0) < v_t(0)$
4.	$d(0) = d_{\mathrm{des}}$	$v_h(0) > v_t(0)$
5.	$d(0) = d_{\mathrm{des}}$	$v_h(0) < v_t(0)$

In the scenarios under consideration, the ACC system in the *host* vehicle is supposed to generate such a control signal $u(t)$ so that there is no collision, the set distance $d(T) = d_{\mathrm{des}}$ is reached and the vehicles move at the same speed. It means that the system (10) shall be brought to the point $\mathbf{x}(T) = [0\ 0\ 0]^{\mathsf{T}}$ in the finite control time T. The following sections contain the results of numerical simulations of the proposed controller

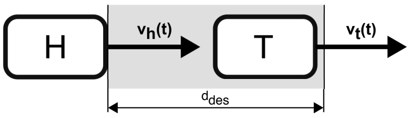

Fig. 2. Test scenario 1.

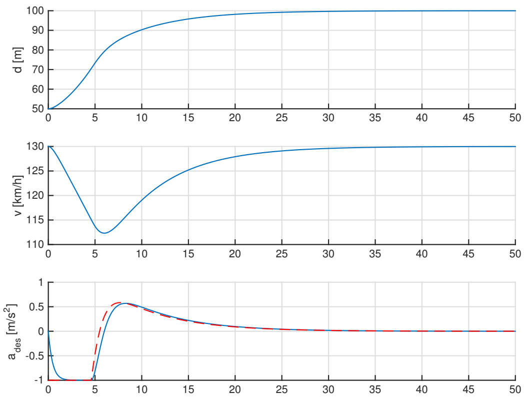

Fig. 3. Trajectories of the *host* vehicle variables in test scenario 1.

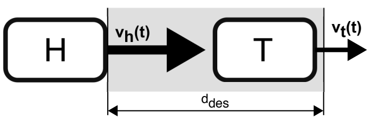

Fig. 4. Test scenario 2.

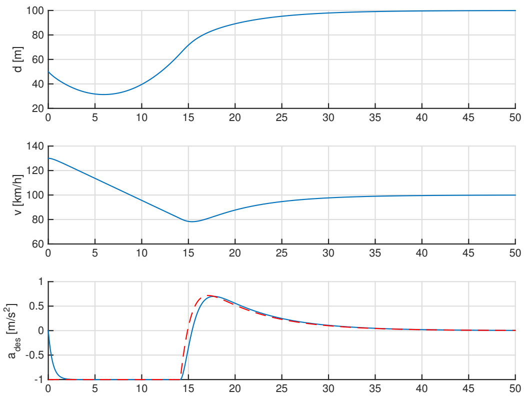

Fig. 5. Trajectories of the *host* vehicle variables in test scenario 2.

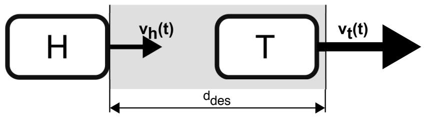

Fig. 6. Test scenario 3.

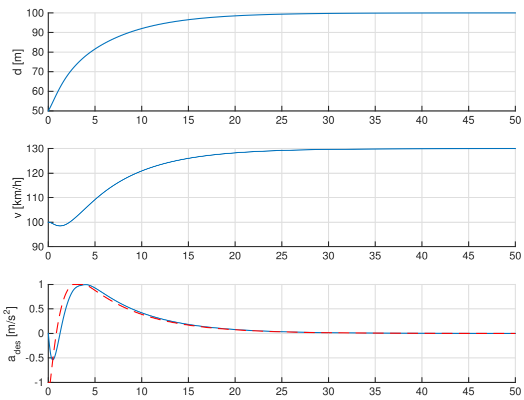

Fig. 7. Trajectories of the *host* vehicle variables in the test scenario 3.

(11) whose $\mathbf{K}$ vector has the value of (31). Other parameters of all simulations are presented in appendix A. Diagrams 3 to 11 present as follows: top diagram – distance $d(t)$ between the vehicles, diagram in the middle – speed $v_{\mathrm{h}}(t)$ of the *host* vehicle, bottom diagram – acceleration $a_{\mathrm{des}}(t)$ (continuous line) of the *host* vehicle and the control signal $a_{\mathrm{des}}(t) = u(t)$ (dashed line).

A. Test scenario 1

In front of the *host* vehicle there is the *target* vehicle at the distance $d(0) = 50$ m smaller than d_{des}, both vehicles move at the same speed $v_{\mathrm{h}}(0) = v_{\mathrm{t}}(0) = 130$ km/h.

In the first control phase the *host* vehicle slows down rapidly and then reaches the set distance d_{des} between the vehicles by accelerating slowly.

B. Test scenario 2

In front of the *host* vehicle there is the *target* vehicle at the distance $d(0) = 50$ m smaller than d_{des}, the vehicles are driving at different speed, which is $v_{\mathrm{h}}(0) = 130$ km/h and $v_{\mathrm{t}}(0) = 100$ km/h respectively.

In the first control stage the *host* vehicle has to reduce its speed suddenly. For some time the distance $d(t)$ between the vehicles is decreasing and then reaches the set value d_{des}. In the final stage, the *host* vehicle control system adapts the speed to the speed of the *target* vehicle.

C. Test scenario 3

In front of the *host* vehicle there is the *target* vehicle at the distance $d(0) = 50$ m smaller than d_{des}, the vehicles are driving at different speed, which is $v_{\mathrm{h}}(0) = 100$ km/h and $v_{\mathrm{t}}(0) = 130$ km/h respectively.

In the first control stage the *host* vehicle slows down for a while to accelerate afterwards and reach the same speed $v_{\mathrm{h}}(t)$ as the speed of the *target* vehicle $v_{\mathrm{t}}(t)$. After reaching the set distance d_{des} the vehicles move at the same speed.

Fig. 8. Test scenario 4.

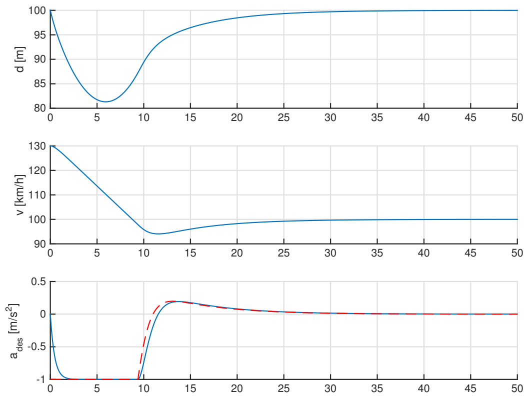

Fig. 9. Trajectories of the *host* vehicle variables in test scenario 4.

Fig. 10. Test scenario 5.

D. Test scenario 4

In front of the *host* vehicle there is the *target* vehicle at the distance of $d(0) = d_{\mathrm{des}}$, the speed of the vehicles is different and amounts to $v_{\mathrm{h}}(0) = 130$ km/h and $v_{\mathrm{t}}(0) = 100$ km/h respectively.

Since the speed of the *host* vehicle at the beginning is greater than the speed of the *target* vehicle, for some time the *host* vehicle has to move at a lower speed to reach the set distance d_{des} between the vehicles. After reaching the set distance d_{des} , the vehicles move at the same speed.

E. Test scenario 5

In front of the *host* vehicle there is the *target* vehicle at the distance $d(0) = d_{\mathrm{des}}$, the speed of the vehicles is different and amounts to $v_{\mathrm{h}}(0) = 100$ km/h and $v_{\mathrm{t}}(0) = 130$ km/h respectively.

Since the speed of the *host* vehicle is initially lower than the speed of the *target* vehicle, for some time the *host* vehicle has to move at a greater speed to reach the set distance d_{des} between the vehicles. After reaching the set distance d_{des} , the vehicles move at the same speed.

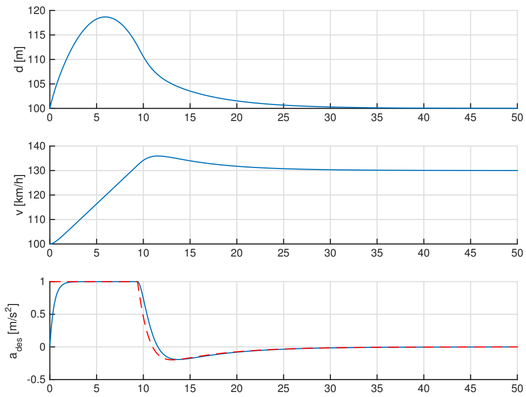

Fig. 11. Trajectories of the *host* vehicle variables in test scenario number 5.

V. Conclusion

The paper discusses a state proportional controller for ACC systems. The parameters of the proposed controller are determined numerically as a search of the minimum defined quality index. Additionally, adding some extra limitations for the searched parameters of the controller guarantees an asymptotic stability of a closed loop system. The controller parameters were identified for four types of drivers. The mean value of the controller parameters obtained this way was tested by simulation in different situations that can occur when using the ACC system. In each of the tested scenarios the controller was operating correctly.

The article proposes a proportional for the ACC system. The authors intend to extend the controller for further studies by designing two separate controllers for braking and accelerating, with a relevant switching function and nonlinear controllers like [36], [37]. The simple model of the vehicle will be replaced with a more complex one that will model the vehicle dynamics in a more precise way. Research on using the ACC systems in autonomic electric vehicles is also planned.

Appendix A
Simulation Parameters

The following model parameters were assumed for numerical simulations, presented in the paper [1]. The parameters for all types of drivers are presented in table III. Parameters for

TABLE III
Model Parameters

i-ty Human Driver	τ_{h} [s]	τ_{b} [s]	d_0 [m]	d_{des} [m]
Driver 1	1.70	0.45	1.64	57.74
Driver 2	1.25	0.45	4.30	47.21
Driver 3	0.67	0.45	2.25	24.58
Driver 4	2.85	0.45	5.00	100.00

driver type 4 were identified from the equation:

$$d_{\mathrm{des}} = d_0 + v_{\mathrm{h}}(t)\tau_{\mathrm{h}} \tag{32}$$

assuming that the distance between the vehicles is $d_{\mathrm{des}} = 100$ m. In all simulations the control signal $u(t)$ was limited to the following value:

$$u(t) \in [-1,\ 1] \tag{33}$$

The positive values of the $u(t)$ signal are interpreted as the vehicle acceleration, while the negative ones stand for braking. The maximum speed at which the vehicles should move was assumed as 130 km/h, which corresponds to the acceptable speed limit on motorways in the majority of the world. The assumed simulation time was $T = 50$ s, and the distance between the vehicles was $d_{\mathrm{des}} = 100$ m for all simulations.

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