Math 323 HW2

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1. Problem 4: Be careful reading these formula

   (a) Prove that for all natural numbers $n, \sum_{k=1}^{n}(2k-1) = n^2$

   (b) Prove that for all natural numbers $n, \sum_{k=1}^{n} 2k - 1 = n^2 + n - 1$

   Solution:

   (a) *Proof:* Assume $n \in \mathbb{N}$, we will prove part (a) using a proof by induction on n.

      i. Base case: If $n = 1$, then $\sum_{k=1}^{n}(2k-1) = n^2$. Assume $n = 1$, we consider

      $$\sum_{k=1}^{n}(2k-1) = \sum_{k=1}^{1}(2k-1) = 1 \text{ and } n^2 = 1^2 = 1$$

      So for $n = 1$, $\sum_{k=1}^{n}(2k-1) = n^2$.

      ii. Inductive case: If for $n = n_0, \sum_{k=1}^{n}(2k-1) = n^2$, then for $n = n_0 + 1$, $\sum_{k=1}^{n}(2k-1) = n^2$.
      Assume $n = n_0$ and $\sum_{k=1}^{n}(2k-1) = n^2$, meaning

      $$\sum_{k=1}^{n_0}(2k-1) = n_0^2 \tag{1}$$

      Equation (1) is our inductive hypothesis. We add $2(n_0+1)-1$ to both side of (1).

      $$\sum_{k=1}^{n_0}(2k-1) + 2(n_0+1) - 1 = n_0 + 2(n_0+1) - 1$$

      $$\begin{aligned}\sum_{k=1}^{n_0+1}(2k-1) &= n_0^2 + 2n_0 + 2 - 1 \\ &= n_0^2 + 2n_0 + 1 \\ &= (n_0+1)^2\end{aligned}$$

      So we have proved: If $n = n_0, \sum_{k=1}^{n}(2k-1) = n^2$, then if for $n = n_0 + 1$ then $\sum_{k=1}^{n}(2k-1) = n^2$.

Proving claim (i) and (ii) completes our proof by induction on n. $\square$

(b) *Proof:* Assume $n \in \mathbb{N}$, we will prove (b) using a proof by induction on n.

i. Base case: If $n = 1$, then $\sum_{k=1}^{n} 2k - 1 = n^2 + n - 1$. Assume $n = 1$, we have

$$\sum_{k=1}^{n} 2k-1 = \sum_{k=1}^{1} 2k-1 = 2(1)-1 = 1 \text{ and } n^2+n-1 = 1^2+1-1 = 1$$

So for $n = 1$, $\sum_{k=1}^{n} 2k - 1 = n^2 + n - 1$.

ii. Inductive case: If for $n = n_0, \sum_{k=1}^{n} 2k - 1 = n^2 + n - 1$, then for $n = n_0 + 1, \sum_{k=1}^{n} 2k - 1 = n^2 + n - 1$. Assume that $n = n_0, \sum_{k=1}^{n} 2k - 1 = n^2 + n - 1$. We consider our inductive hypothesis:

$$\sum_{k=1}^{n_0} 2k - 1 = n_0^2 + n_0 - 1$$

$$\sum_{k=1}^{n_0} 2k - 1 + 2(n_0 + 1) = n_o^2 + n_0 - 1 + 2(n_0 + 1)$$

$$\begin{aligned}\sum_{k=1}^{n_0+1} 2k - 1 &= n_0^2 + n_0 - 1 + 2n_0 + 2 \\ &= n_0^2 + 2n_0 + 1 + n_0 + 1 - 1 \\ &= (n_0 + 1)^2 + (n_0 + 1) - 1\end{aligned}$$

So proving (i) and (ii) completes our proof by induction on n. $\square$

2. Problem 7: Prove that: $\forall n \in \mathbb{N}, \prod\limits_{k=1}^{n} (1 + \frac{1}{k}) = n + 1$

Proof. Assume that $n \in \mathbb{N}$, we will prove that $\prod\limits_{k=1}^{n} (1 + \frac{1}{k}) = n + 1$ by induction on n.

(a) Base case: If $n = 1$, then $\prod\limits_{k=1}^{n} (1 + \frac{1}{k}) = n + 1$. Assume $n = 1$, consider

$$\prod_{k=1}^{n} (1 + \frac{1}{k}) = \prod_{k=1}^{1} (1 + \frac{1}{k}) = 1 + \frac{1}{1} = 2 \text{ and } n + 1 = 1 + 1 = 2$$

So for $n = 1$, $\prod\limits_{k=1}^{n} (1 + \frac{1}{k}) = n + 1$.

(b) Inductive case: If for $n = n_0$, $\prod\limits_{k=1}^{n} (1 + \frac{1}{k}) = n + 1$, then for $n = n_0 + 1$, $\prod\limits_{k=1}^{n} (1 + \frac{1}{k}) = n + 1$. Assume $n = n_0$, by our inductive hypothesis, we have

$$\prod_{k=1}^{n_0}(1+\frac{1}{k}) = n_0 + 1 \tag{2}$$

Multiplying both side of (2) by $(1+\frac{1}{n_0+1})$ we get

$$\begin{aligned}
\prod_{k=1}^{n_0}(1+\frac{1}{k})(1+\frac{1}{n_0+1}) &= (n_0+1)(1+\frac{1}{n_0+1}) \\
\prod_{k=1}^{n_0+1}(1+\frac{1}{k}) &= (n_0+1)1+\frac{n_0+1}{n_0+1} \\
&= n_0+1+1 = (n_0+1)+1
\end{aligned}$$

Proving claim (a) and (b) completes the proof by induction on n. $\square$