Math 323 HW20

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Let $f : \mathbb{R} \to \mathbb{R}$ be a function. We say f is bounded if: $\exists m > 0$ s.t $\forall x \in \mathbb{R}, |f(x)| \leq m$.

1. Prove that $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{1}{x^2+1}$ is bounded.

   Proof. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{1}{x^2+1}$ is bounded. Let $x \in \mathbb{R}$. So then $x^2 + 1 \geq 1$ and thus $\frac{1}{x^2+1} \leq 1$. We know $\frac{1}{x^2+1} > 0$ and thus $\frac{1}{x^2+1} \geq -1$. So we have $-1 \leq \frac{1}{x^2+1} \leq 1$. This means $|\frac{1}{x^2+1}| \leq 1$. $\square$

2. Prove that $g : \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $g(x) = \frac{1}{x-1}$ is not bounded.

   Proof. Let $g : \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $g(x) = \frac{1}{x-1}$. We want to show: $\forall m > 0$, $\exists x \in \mathbb{R}$ s.t $|\frac{1}{x-1}| > m$.
   Let $m > 0$. Let $x = \frac{m+2}{m+1} \in \mathbb{R}$.

   $$|g(\frac{m+2}{m+1})| = |\frac{1}{\frac{m+2}{m+1} - 1}| = |\frac{1}{\frac{m+2-m-1}{m+1}}| = m + 1 > m$$

   $\square$