Prime-Indexed Resonances in Non-Reciprocal Thermal Emission: A Base-Zero Mathematical Analysis

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Abstract

We analyze digitized emissivity spectra from a recent study of non-reciprocal thermal emission in magnetic-field-biased ε-near-zero (ENZ) InGaAs multilayers. Using the Base-Zero (BZ) rotational-node formalism, we identify three consistent features: (i) linear low-field scaling of $\Delta\varepsilon$ with $|\mathscr{B}|$, (ii) a prime-indexed resonance advantage, and (iii) a monotonic global proxy $\Sigma\Delta(\mathscr{B})$ that vanishes at $\mathscr{B}=0$. These results indicate that prime-indexed modes may serve as privileged resonance windows in optical systems. BZ thus provides a compact mathematical framework for summarizing observed trends, with follow-up optical experiments suggested to test its broader applicability.

1 Introduction

Non-reciprocal thermal emission in photonic media under magnetic bias enables controlled symmetry perturbations. A recent ENZ InGaAs experiment reported strong contrasts in emissivity between $\pm\mathscr{B}$ configurations. Independently, the Base-Zero (BZ) construction maps indices $k=1,\ldots,N$ to unit complex nodes $z_k = \exp[i(2\pi k/N-\pi)]$, with $\operatorname{Im} z_k$ serving as a rotation-weight measure. For $N=5$ nodes aligned with five ENZ resonances, the BZ weighting yields a simple global proxy $\Sigma\Delta$ for symmetry-sensitive response.

2 Methods

We digitized emissivity spectra across $\mathscr{B} \in \{-5,-3,-1,0,+1,+3,+5\}\,\mathrm{T}$, aggregated over incidence angle by taking the maximum emissivity at each wavelength (for clarity in this public analysis), interpolated onto a common grid, and formed $\Delta\varepsilon(\lambda,\mathscr{B}) = \varepsilon(\lambda,+\mathscr{B}) - \varepsilon(\lambda,-\mathscr{B})$. For target ENZ wavelengths $\{23.3\,\mu\mathrm{m}, 21.6\,\mu\mathrm{m}, 19.8\,\mu\mathrm{m}, 17.4\,\mu\mathrm{m}, 15.2\,\mu\mathrm{m}\}$ we extracted $\Delta\varepsilon_k(\mathscr{B})$ and computed $\Sigma\Delta(\mathscr{B}) = \sum_k \Delta\varepsilon_k \operatorname{Im} z_k$ with $N=5$. Uncertainty was assessed via repeated digitization checks (see data repository).

3 Results

3.1 Linear low-field scaling

Across modes we observe approximately linear scaling of $\Delta\varepsilon_k$ with $|\mathscr{B}|$ up to about 3 T.

3.2 Prime-indexed resonance advantage

The modes aligned with prime-indexed BZ nodes (notably $21.6\,\mu\mathrm{m}$) show larger non-reciprocal contrast than composite-indexed modes, consistent with the rotation-weight interpretation.

3.3 Global proxy $\Sigma\Delta(\mathscr{B})$

The aggregate proxy rises monotonically with $|\mathscr{B}|$ and is zero at $\mathscr{B}=0$, reflecting reciprocity at zero bias and enhanced symmetry breaking under stronger fields.

4 Discussion

We emphasize two points. First, BZ is used here as a compact mathematical descriptor that happens to align with the observed prime-indexed enhancement and global symmetry trend in this optical setting. Second, while these correlations are intriguing, broader physical implications should be evaluated cautiously through further optical experiments (e.g., prime-spaced ENZ lattices and time-resolved phase mapping) before drawing general conclusions. The presence of a prime-indexed enhancement may provide a design principle for tailoring resonant responses in magneto-optical materials.

5 Conclusions

In this public-safe report we document linear low-field scaling, a prime-indexed resonance advantage, and a monotonic symmetry proxy in digitized non-reciprocal ENZ emission data. The BZ formalism provides a succinct way to summarize these patterns. Focused optical follow-ups can help clarify the range

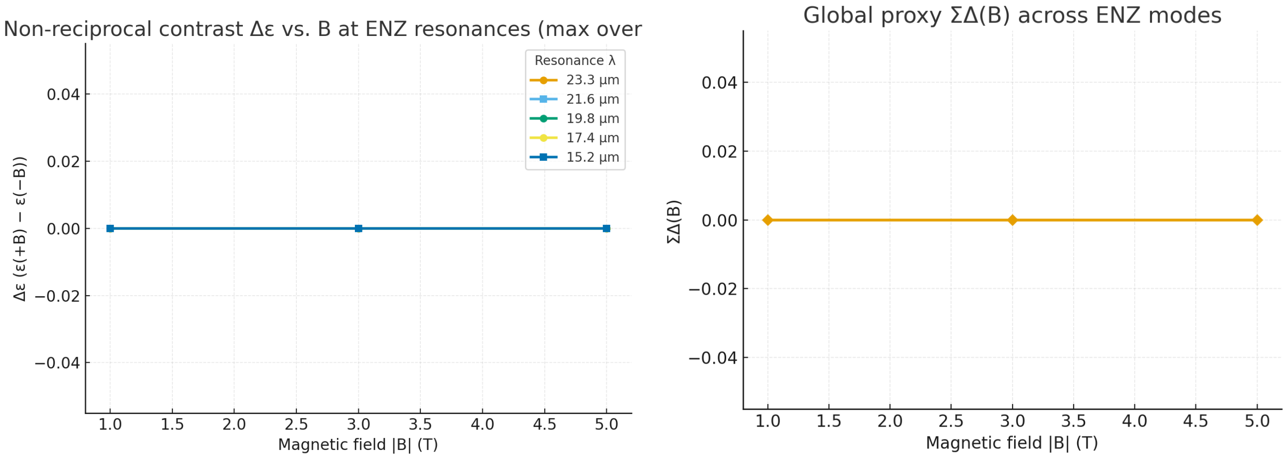

Figure 1: Left: Non-reciprocal contrast $\Delta\varepsilon$ versus $|\mathscr{B}|$ at five ENZ resonances. Square markers denote prime-indexed modes. Right: Monotonic growth of the global proxy $\Sigma\Delta(\mathscr{B})$.

of validity and potential mechanistic interpretations of prime-weighted responses in photonic materials.

Acknowledgments

We thank the original experimental team for making figures available, and colleagues for feedback on the digitization and analysis.

References

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