# Deep Reinforcement Learning

Overview of main articles
Part 2. Policy gradient algorithms


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MSU

# Table of contents i

# Basic policy gradient methods

# Direct optimization

Recall RL goal:

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**Options:**

* Metaheurisics
* Log-derivative trick[1].

[1] aka REINFORCE

# Stochasitc estimators optimization via log-derivative trick

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$$\nabla_\theta \pi(\theta) = \pi(\theta) \nabla_\theta \log \pi(\theta)$$

$$= \int \pi(x,\theta) \nabla_\theta \log \pi(x,\theta) f(x) dx = \mathbb{E}_{x \sim \pi(x,\theta)} \nabla_\theta \log \pi(x,\theta) f(x)$$

# From importance sampling point of view

Recall Importance Sampling. For arbitrary distribution $\phi(x)$:

$$\mathbb{E}_{x \sim \pi(x,\theta)} f(x) = \mathbb{E}_{x \sim \phi(x)} \frac{\pi(x,\theta)}{\phi(x)} f(x)$$

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**Note:** that is the same gradient as with log-derivative trick[2].

[2] really? Could it even happen otherwise?

# REINFORCE

Let's apply log-derivative trick to our goal!

$$\nabla_\theta \mathbb{E}_{\pi(\theta)} \mathbb{E}_{\mathcal{T}} R = \mathbb{E}_{\pi(\theta)} \nabla_\theta \log \pi(\theta) \mathbb{E}_{\mathcal{T}} R$$

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$$\nabla_\theta \mathbb{E}_{\pi(\theta)} \mathbb{E}_{\mathcal{T}} R = \mathbb{E}_{\pi(\theta)} \nabla_\theta \log \pi(\theta) \mathbb{E}_{\mathcal{T}} R \approx$$

We can estimate this gradient using Monte-Carlo by playing, let's say, one game:

$$\approx \sum_t^T \nabla_\theta \log \pi(a_t \mid s_t, \theta) R$$

# Problems of REINFORCE

- $\times$ Doesn't work.

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    - **Reason:** *high variance* of Monte-Carlo gradient estimation.

# Problems of REINFORCE

× Doesn't work.

- **Reason:** *high variance* of Monte-Carlo gradient estimation.
- you can play more than one game for one gradient step, but that doesn't help much.

# Baseline

**Proprosition**

For arbitrary distribution $\pi(\theta)$:

$$\mathbb{E}\nabla_\theta \log \pi(\theta) = \int \nabla_\theta \pi(\theta) = \nabla_\theta \int \pi(\theta) = \nabla_\theta 1 = 0$$

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Adding $\mathbb{E}\nabla_\theta \log \pi(\theta) b$ for some $b$ to gradient estimate will not lead to bias, but may change variance.

# Lowest variance baseline

**Theorem**

$$b = \frac{\mathbb{E}(\nabla_\theta \log \pi(\theta))^2 R}{\mathbb{E}(\nabla_\theta \log \pi(\theta))^2}$$

is the baseline which leads to the lowest variance.

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**Theorem**

$$b = \frac{\mathbb{E}(\nabla_\theta \log \pi(\theta))^2 R}{\mathbb{E}(\nabla_\theta \log \pi(\theta))^2}$$

is the baseline which leads to the lowest variance.

* similar to average reward, which is also a good baseline.

# Carefull REINFORCE

**Strange thing:** our gradient estimate depends on $R$, which includes reward in the first state $r(s_0)$, where we haven't performed any actions.[3]

[3] did we make any mistake?

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Let's untangle our goal:

$$\nabla_\theta \mathbb{E}_{p(s_1)} \left( r(s_1) + \mathbb{E}_{a_1 \sim \pi(s_1, \theta)} \mathbb{E}_{p(s_2 | s_1, a)} \left[ r(s_2) + \ldots \right] \right)$$

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After carefully applying log-derivative trick:

$$= \mathbb{E} \sum_t^T \nabla_\theta \log \pi(a_t \mid s_t, \theta) \left( \sum_{t'=t+1}^T r(s_{t'}) \right)$$

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✓ that's much better!

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# Actor-critic

**Note:** $\sum_{t'=t+1}^{T} r(s_{t'})$ is estimation of $Q^{\pi}(s_t, a_t)$!

$$\nabla = \mathbb{E} \sum_{t}^{T} \nabla_{\theta} \log \pi(a_t \mid s_t, \theta) \left( \sum_{t'=t+1}^{T} r(s_{t'}) \right)$$

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\begin{aligned}
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Better estimation of $Q^{\pi}(s, a)$
should lead to lower variance.

* $\pi$ is an *actor*
* estimate of $Q^{\pi}(s, a)$ is a *critic*

# Advantage Actor Critic

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Let's insert some baseline:

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Critic can be a second neural net!

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$$Q^\pi(s_t, a_t) - V^\pi(s_t) \approx r(s_{t+1}) + V^\pi(s_{t+1}) - V^\pi(s_t)$$

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  ✓ the least complex one! [5]

[4] can we just use Q-learning for this?
[5] why?

# Critic learning

For given state $s$ we can calculate a target $y = V^{\pi}(s) \approx \sum_{t'=t+1}^{T} r(s_{t'})$.

At the end of the game, make a step of gradient descent to teach critic.

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**Alternative:** $y = V^{\pi}(s) \approx r(s') + V^{\pi}(s')$

# A2C: Complete algorithm

**Advantage Actor-Critic (A2C) Algorithm:**

- get $(s, a, r, s')$

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Check out this comic about A2C!

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  - ✓ do gradient descent step every $N$ game steps.

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# A2C: Resume

- ✓ lower variance!
- × yet policy gradient estimates are not unbiased anymore![6]
- × batch_size = 1
  - ✓ do gradient descent step every $N$ game steps.
  - ✓ play several games in parallel.

[6] why?

# Eligibility trace

We may use critic only for baseline:

$$\nabla = \mathbb{E} \sum_{t}^{T} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \left( \sum_{t'=t+1}^{T} r(s_{t'}) - V^\pi(s_t) \right)$$

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Or use a compromise:

$$\nabla = \mathbb{E} \sum_{t}^{T} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \left( \sum_{t'=t+1}^{t+N} r(s_{t'}) + V^\pi(s_{t+N}) - V^\pi(s_t) \right)$$

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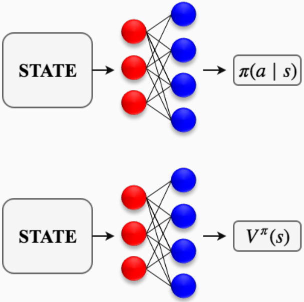
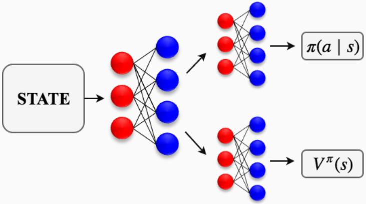
× new hyperparameter $N$

✓ regulates trade-off between variance and bias

# Dealing with two networks

Option 1: just two neural nets



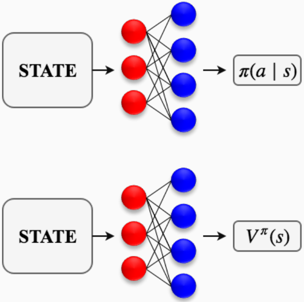



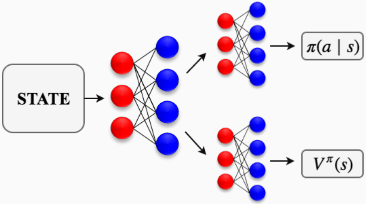

Option 2: shared feature extractor

# Dealing with two networks

Option 1: just two neural nets

- × obviously redundant




Option 2: shared feature extractor

- × may be unstable

# NEXT: A3C