Deep Reinforcement Learning

Overview of main articles
Part 1. Value-based algorithms

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MSU

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Reinforcement Learning [reminder]

MDP

MDP is $\{\mathbb{S}, \mathbb{A}, \mathbb{T}, r\}$:

$\mathbb{S}$ — set of states

$\mathbb{A}$ — set of actions

$\mathbb{T}$ — probability $p(s' \mid s, a)$, where $s, s' \in \mathbb{S}, a \in \mathbb{A}$

r — function $\mathbb{S} \to \mathbb{R}$

RL Goal

We search for policy $\pi : \mathbb{S} \to \mathbb{A}$ which maximizes[1]

$$\mathbb{E} \sum_t r(s_t)$$

[1] over what probability distributions is this expectation?

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This goal does not suit infinite horizon case, so for generalization purposes goal is substituted with

$$\mathbb{E} \sum_t \gamma^t r(s_t)$$

for $\gamma \in (0, 1)$.

[1] over what probability distributions is this expectation?

Definitions

For convenience[2]:

$$R = \sum_t \gamma^t r(s_t)$$

[2] *What does it depend on?*

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For given policy π:

$$V^\pi(s) = \mathbb{E} R \mid s_0 = s$$

$$Q^\pi(s, a) = \mathbb{E} V(s') \mid s, a$$

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Let π^* be optimal policy.

[2] *What does it depend on?*

Bellman Equation

For every π it's true:

$$Q^{\pi}(s,a) = \mathbb{E}\left[r(s') + Q^{\pi}(s',\pi(s'))\right]$$

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For finite-state case Q^{π^*} is finite vector of unknown values.
Bellman equations can be solved using point iteration:

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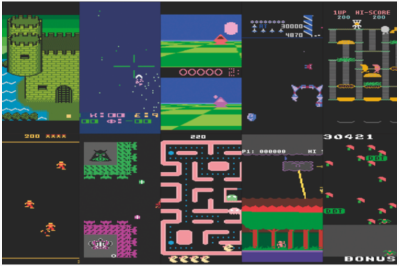
✓ Is a *contraction mapping* $\Rightarrow$ converges.

Deep Q-learning (2014)

Atari

* No prepared features for each game.
* Screen image as input.
* Finite-state case... not quite finite.



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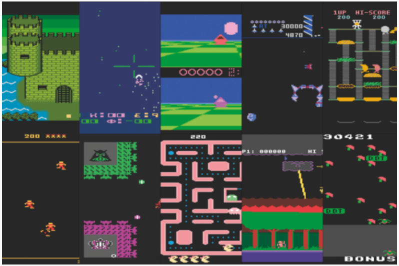
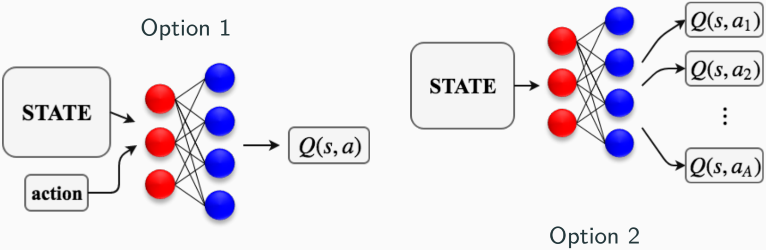


Atari games

> We want to approximate $Q(s, a)$ with neural net.

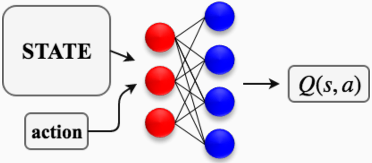
Q-network

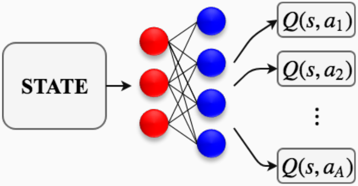

Option 1
STATE
action
$Q(s, a)$
STATE
$Q(s, a_1)$
$Q(s, a_2)$
$\vdots$
$Q(s, a_A)$
Option 2

Q-network

Option 1

Requires forward pass for each action[1]





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Number of actions must be adequate

[1] Is there a case when option 1 might be better?

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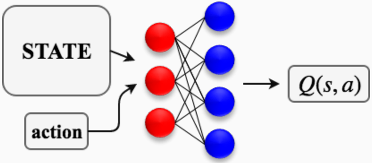

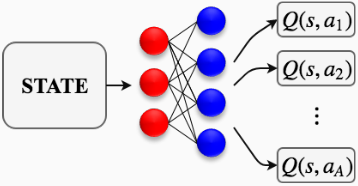

Option 2

Number of actions must be adequate

Atari: up to 18 discrete actions. Use option 2.

[1]Is there a case when option 1 might be better?

TD-learning to gradient descent

TD-learning is «similar» to gradient descent.

$$Q_{t+1}(s, a) = \alpha Q_t(s, a) + (1 - \alpha) \left[r(s') + \max_a Q_t(s', a) \right]$$

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Let $y = r(s') + \max_a Q_t(s',a)$.
If dependence of y from Q is ignored:

$$L = (Q_t(s,a) - y)^2$$

Q-learning

With $Q(s, a)$ as neural net, its parameters θ determine function.

$$Q_{t+1}(s, a, \theta) = Q_t(s, a, \theta) - \eta \nabla_Q L$$

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Let's move gradient descent from space of Q functions to θ!

$$\theta_{t+1} = \theta_t - \beta \nabla_\theta L$$

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× batch_size = 1. Wow.

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Problems:

- × batch_size = 1. Wow.
- × Target y changes after each step.
- × All theoretical guarantees are lost.

ExperienceReplay

Utilize all experienced transitions $(s, a, s', r, done)$ for generating a batch for stochastic optimization step.

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Pretend on each step that loss function is

$$\mathbb{E}_{(s,a,s',r,done)}(Q(s,a,\theta) - y(s',r,done))^2$$

Batch of transitions is sampled uniformly from memory.

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✓ Decorellates samples.

* Target y can be calculated only for this batch.
* Only last N observed transitions may be stored

ε-greedy exploration

Problem: at the very beginning trajectories generated by $\pi(s) = \underset{a}{argmax}\, Q(s, a, \theta)$ are very similar.

ε-greedy exploration

Problem: at the very beginning trajectories generated by $\pi(s) = \underset{a}{argmax}\, Q(s, a, \theta)$ are very similar.

Choose random actions sometimes.

For example, with probability ε.

ε annealing

ε should be big at the beginning and small at the end.

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Atari: $\varepsilon(i) = 0.01 + 0.99\, exp\{-\frac{i}{30000}\}$ where i is frames counter.

Details

- Gray-scale frames were downsampled and cropped to 84x84.
- Last 4 frames[3] were considered as state to satisfy MDP Markov's property.
- Same NN architecture was used for all games: 3 convolutional[4] and 2 feedforward layers.

[3] 3 for Space Invaders cause of laser blinking period
[4] why no max pooling here?

More details

Playing Atari with Deep Reinforcement Learning (2014)

- Reward was restricted to {+1, 0, -1}. Allowed to use same learning rate for all games.

:(50 hours per game / 10 000 000 frames per game.

:} Bought by Google after 7 games.

Stabilizing Q-learning

Unstability

Recall our target on each step:

$$y(s', r) = r + \max_{a'} Q(s', a', \theta)$$

- Changes each frame
- Formally depends on θ
- "Correlates" with actions chosen during playing
- Tends to overestimate true $V(s')$

$\Rightarrow$ loss is completely unstable and can even diverge.

Target network (2015)

Change the target not every step, but each K-th step.

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For this purpose:

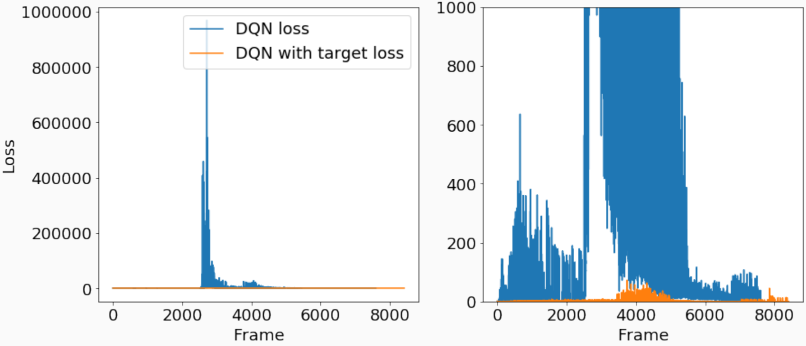
- Make a copy of Q-network, *target network*, with parameters θ^-
- Use it on every step to calculate

$$y(s', r) = r + \max_{a'} Q^{\text{target}}(s', a', \theta^-)$$

- Each K-th step update θ^- with current Q-network's weights θ.

Can be seen on loss

✓ Loss really stabilized!



Value overestimation

Recall our target is proxy of $V^{\pi^*}(s', a')$

$$y(s', r) = r + \max_{a'} Q(s', a', \theta)$$

Practice: this proxy overestimates true value of states.

Intuition: this max operator will prefer actions, for which $Q(s', a', \theta)$ is overestimating true value due to approximation or luck.

Action Selection vs Evaluation

Recall Bellman Equation derivation and untangle our target:

$$y(s', r) = r + \max_{a'} Q(s', a', \theta) = r + Q(s', argmax_{a'} Q(s', a', \theta), \theta)$$

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General idea:

Use different approximations for evaluation and for selection to avoid *max*.

Two Q-learnings

Basic way to do this:
run two Q-learning algorithms with two approximations of Q^{π^*}:
$Q_1(s, a, \theta_1)$ and $Q_2(s, a, \theta_2)$.

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Targets for Q-learnings:

$$y_1 = r + Q_2(s', \underset{a'}{argmax}\, Q_1(s', a', \theta_1), \theta_2)$$

$$y_2 = r + Q_1(s', \underset{a'}{argmax}\, Q_2(s', a', \theta_2), \theta_1)$$

Double DQN (2015)

Deep Reinforcement Learning with Double Q-learning (2015)

- **more convenient way to do this:**

Use target network as one of two approximations.

[5] how many backwards?

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- **more convenient way to do this:**

Use target network as one of two approximations.

$$y = r + Q^{\text{target}}(s', \underset{a'}{argmax}\, Q(s', a', \theta), \theta^-)$$

* Keep ignoring dependence of y from θ.
* Requires three forward passes on each step[5].

[5] how many backwards?

Comparing DQNs

Table 1: DQN targets

DQN	target y
Classic Deep Q-learning	$r + Q(s', \underset{a'}{argmax}\, Q(s', a', \theta), \theta)$
With target-network	$r + Q^{\text{target}}(s', \underset{a'}{argmax}\, Q^{\text{target}}(s', a', \theta^{-}), \theta^{-})$
Double Deep Q-learning	$r + Q^{\text{target}}(s', \underset{a'}{argmax}\, Q(s', a', \theta), \theta^{-})$

Dueling DQN: Motivation

Note:

* In most states our choice of action does not affect future value.
* After finding $Q(s, a)$ Q-learning still gains no information about $Q(s, a')$ for $a' \neq a$.

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Learning $Q(s, a)$ should lead to learning $V(s)$

Advantage function

Define *advantage* function:

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$$

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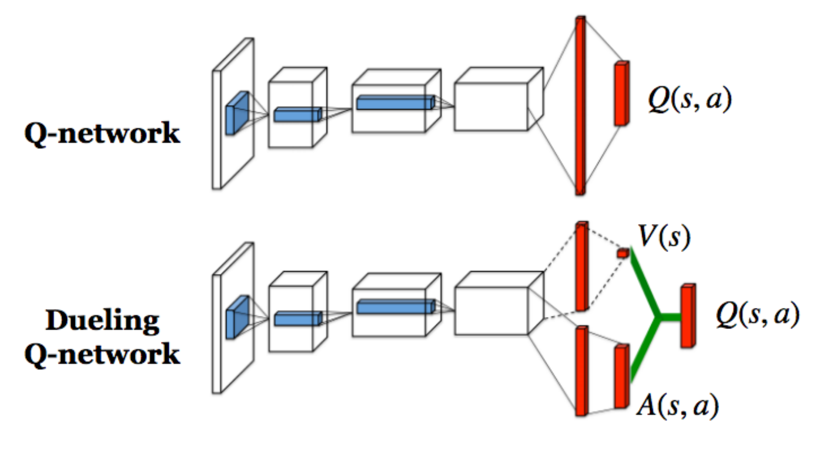
Rewrite Q-function in terms of value of state:

$$Q^\pi(s,a) = V^\pi(s) + A^\pi(s,a)$$

Dueling DQN (2016)

Dueling Network Architectures for Deep Reinforcement Learning (2016)



Dueling Q-network architecture

Struggling with identifiability

Problem: $A(s,a)$ is not arbitrary. Recall $\mathbb{E}_{a\sim\pi}A^{\pi}(s,a) = 0$.

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Proposition:

$$Q(s, a) = V(s) + A(s, a) - \max_a A(s, a)$$

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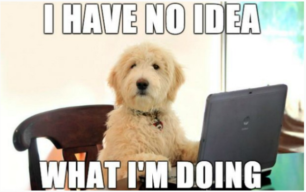

$$Q(s,a) = V(s) + A(s,a) - \underset{a}{mean}\, A(s,a)$$

suddenly worked better.

Results

✓ Learning $Q(s, a)$ leads to correcting $V(s)$.

* Only network architecture is changed.
* Double DQN still works for dueling architecture.

Prioritized replay memory (2015)

Motivation

In standard DQN we sample batch of transitions from replay memory uniformly.

- × Some transitions are more important than others
- × Replay memory is full of almost useless transitions

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$\delta = |y(s', r, done) - Q(s,a)|$ is
a good proxy of transition importance

Prioritized Replay Memory (2015)

Prioritized Experience Replay (2015):

$$p(\mathcal{T}) \propto \delta(\mathcal{T})^{\alpha}$$

Authors found $\alpha \approx 0.6$ is a good universal value.

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- $\times$ On each step this probability changes for all the replay memory [6]

[6] which capacity is on the order of 1M transitions

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Authors found $\alpha \approx 0.6$ is a good universal value.

Problems:

- $\times$ On each step this probability changes for all the replay memory [6]
  - $\approx$ on each step update δ only for the sampled batch used for learning

[6] which capacity is on the order of 1M transitions

Prioritized Replay Memory (2015)

Prioritized Experience Replay (2015):

$$p(\mathcal{T}) \propto \delta(\mathcal{T})^{\alpha}$$

Authors found $\alpha \approx 0.6$ is a good universal value.

Problems:

- $\times$ On each step this probability changes for all the replay memory [6]
  - $\approx$ on each step update δ only for the sampled batch used for learning
- $\times$ Introduces bias (transitions are now sampled from hell knows what distribution).

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Background: Importance Sampling

For arbitrary distribution $q(x)$:

$$\mathbb{E}_{p(x)} f(x) = \int p(x) f(x) dx = \int \frac{q(x)}{q(x)} p(x) f(x) dx =$$

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That's exactly what we want: to substitute expectation of loss ($f(x)$) over uniform sampling from experience replay ($p(x)$) to expectation over our own prioritized distribution ($q(x)$) !

Applying Importance Sampling

If N is replay memory capacity:

$$L = \mathbb{E}_{\mathcal{T} \sim uniform}(y - Q(s, a))^2 = \mathbb{E}_{\mathcal{T} \sim prioritized} \frac{1}{Np(\mathcal{T})}(y - Q(s, a))^2$$

IS just adds weights to our batch:

$$w_i = \frac{1}{Np(\mathcal{T}_i)}$$

Annealing weights

Problem: at the beginning these weights might not be that relevant, yet slowing down learning.

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Let's smooth them at the beginning of learning:

$$L = \mathbb{E}_{\mathcal{T} \sim \textit{prioritized}} \left(\frac{1}{Np(\mathcal{T})} \right)^{\beta} (y - Q(s, a))^2,$$

where β changes from 0.4 to 1 linearly during first 100 000 frames.

Hints

* Weights significantly vary scale of loss function. Constant learning rate might be inappropriate.

  Hint:[7] normalize weights by dividing on $\max_i w_i$.

[7] max taken over all replay memory. Yet in some implementations it is taken over current batch

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* Weights significantly vary scale of loss function. Constant learning rate might be inappropriate.

  Hint:[7] normalize weights by dividing on $\max_i w_i$.
* $\min(1, |\delta|)$ is used instead of $|\delta|$ for stabilization purposes.
* new transitions are stored with maximum priority.

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Noisy networks for exploration (2017)

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Noisy Nets for Exploration (2017):

$$w_i = \mu_i + \sigma_i * \varepsilon_i, \quad \varepsilon \sim \mathcal{N}(0, 1)$$

* μ_i, σ_i are both learnable parameters.
* all weights are independent random variables
* use policy $\pi(s) = \underset{a}{argmax}\, Q(s, a, \mu, \sigma, \varepsilon)$

Optimized Loss

Formally, our loss[8] is now:

$$\mathbb{E}_{\varepsilon}\mathbb{E}_{\mathcal{T}}(Q(s, a, \theta, \varepsilon) - y(\mathcal{T}))^2$$

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* use different noise samples for it:

$$y = r + Q(s', \underset{a'}{argmax}\, Q(s', a', \varepsilon''), \varepsilon')$$

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Noise generation optimization

Problem: noise generation turns to be a bottleneck in terms of wall-clock time.

- $MN + N$ samples required for linear layer mapping M features to N.

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  * for whole batch![10]

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[10] is this theoretically coherent?

Application tips

- ✓ No hyperparameters
  - * Except where to put noise in the network... Convolution layers better leave deterministic[11].

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- ✓ almost random behavior at the beginning
  - * yet ε-greedy strategy may also be used

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Categorical DQN (2017)

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Consider a state where you get 1000 or -1000 with probabilities 0.5.

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Learn a distribution over future reward instead of it's expectation.

Value Distribution

Recall

$$Q^\pi(s, a) = \mathbb{E} \sum_t r(s_t) \mid s, a$$

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For fixed policy π let's define *value distribution*:

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! It's a random variable!

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Value distribution satisfies a *recursive distributional equation*:

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✓ yes, for $d(Z_1, Z_2) = \sup\limits_{s,a} \mathcal{W}(Z_1(s,a), Z_2(s,a))$, where $\mathcal{W}$ is Wasserstein distance between two random variables.

Distributional Bellman Equation

Analogically: $\pi^*(s) = \max_a \mathbb{E} Z^{\pi^*}(s, a)$

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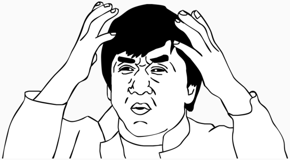
- $\times$ and existence of one doesn't guarantee convergence to it



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Let's do point iteration anyway! Our wish:

$$p(Z_{t+1}(s,a)) \leftarrow p\left(r(s,a) + \gamma Z_t\left[s', \max_{a'} \mathbb{E} Z_t(s',a')\right]\right)$$

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Loss

$Z_{t+1} \stackrel{\mathrm{D}}{=} r(s,a) + \gamma Z_t(s', \max_a \mathbb{E} Z_t(s,a))$ is a convolution involving MDP transition probability $p(s' \mid s, a)$:

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Our loss is $L = \mathcal{D}(p(Z_{t+1}) \parallel Z_\theta) = \mathcal{D}(\mathbb{E}_{\mathcal{T}}\, p(y \mid s') \parallel Z_\theta)$.

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* KL requires Z_{t+1} and Z_θ share domain.

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Options:

- Gaussian mixture
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Let's $\mathcal{P}$ be a family of categorical distribution on the grid from $V_{\min}$ to $V_{\max}$ with N atoms (outcomes).

Parametrization:
For each action our neural network $Z(s, a)$ outputs N numbers, summing into 1

Calculating target

Suppose you have transition $(s, a, r, s', done)$, $Z(s, a) \in \mathcal{P}$. Then:

$$y(s') = r + \gamma Z(s', \max_{a'} \mathbb{E}Z(s', a'))$$

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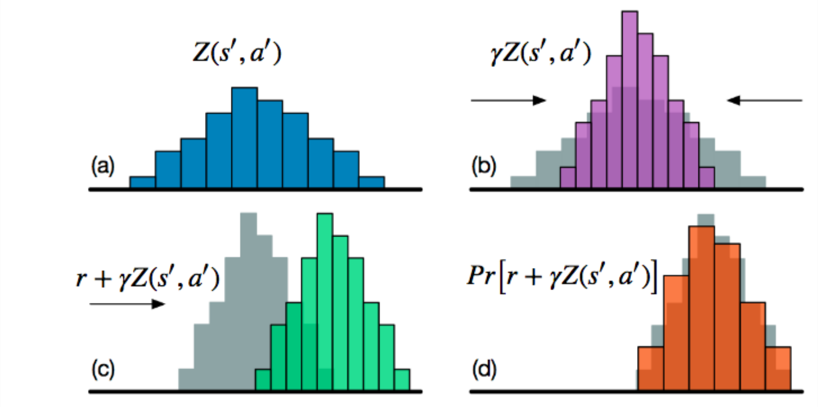

How it looks like

Failed to insert video into beamer ;o)

Rainbow DQN (2018)

Blend them all!

Multistep DQN: Motivation

Recall our target in classic DQN:

$$y = r + \gamma \max_{a'} Q(s', a')$$

If we have nonzero reward at the end of M-step game, we need at least M iterations of Q-learning to «propagate» this reward to all visited states.

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Look more than one step ahead!

Multistep DQN: Realisation

- work with transitions $(s, a, r, r', r'', \ldots, r^{(M-1)}, s^{(M)}, done)$

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$\times$ the further we look the worser y approximates $Q^{\pi^*}(s, a)$

$\Rightarrow$ number of steps should be chosen carefully.

[14] why?

Multistep Categorical DQN

Recall categorical DQN target:

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Loss stays the same:

$$L = \mathrm{KL}(p(y) \parallel p(Z))$$

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Recall dueling DQN:

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$$Z(s,a) = V_N(s) + A_N(s,a) - \underset{a}{mean}\, A_N(s,a)$$

where $V_N(s)$ and $A_N(s,a)$ are categorical N-atomed distributions.

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Let's make our $Z(s,a)$ (modeling categorical distribution with N atoms) in dueling way:

$$Z(s,a) = \ softmax(V_N(s) + A_N(s,a) - \underset{a}{mean}\, A_N(s,a))$$

where $V_N(s)$ and $A_N(s,a)$ are arbitrary N numbers[15].

[15] why couldn't we only add softmax?

Double dueling multi-step noised categorical DQN with prioritized replay AKA Rainbow

Rainbow: Combining Improvements in Deep Reinforcement Learning (2018):

Dueling + Multistep + Categorical + DQN +

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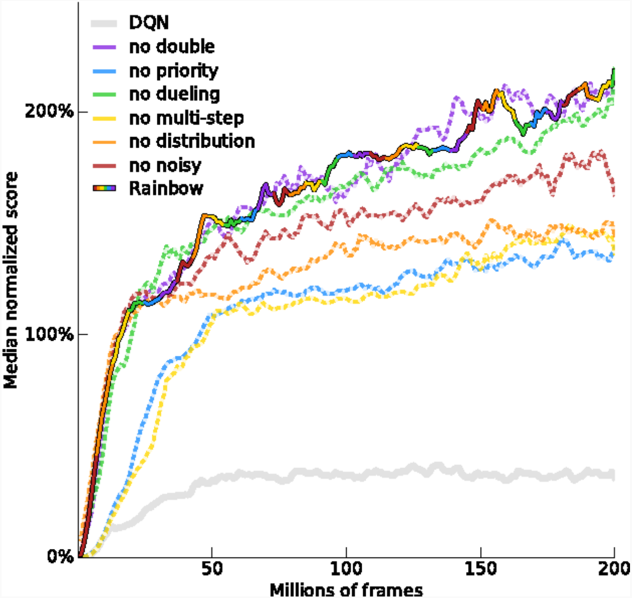
- **Noisy:** add noise to all fully connected layers
- **Prioritized Replay:** just use it[16]

[16] guess proxy of transition priority

Do we really need all this?



Rainbow: resume

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$\times$ a lot of hyperparameters

Rainbow: resume

- * all improvements are important as they address different problems
- $\times$ a lot of hyperparameters
- ? Allegedly 10 hours for 7M frames on single GPU
  - :(I can't reproduce [17]

[17] 10 hours for 3M. Noise generation seems to be a problem!

NEXT: see pt.2 for Policy Gradient algorithms