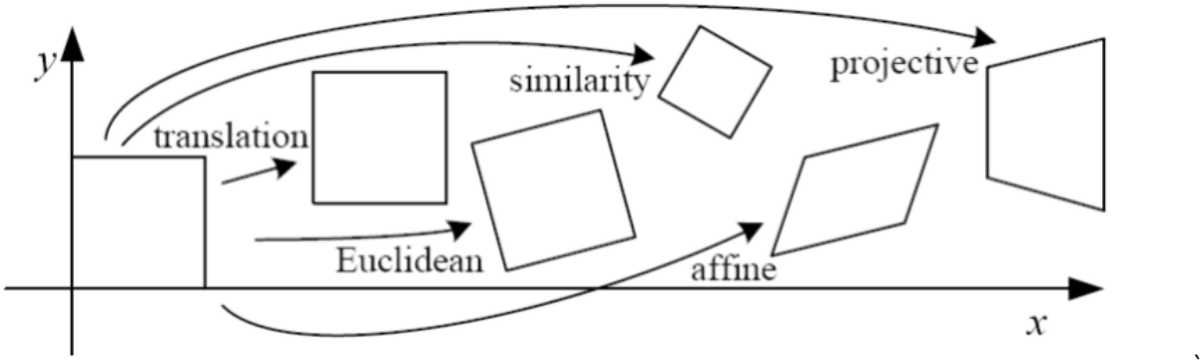
EE 046746 - Technion - Computer Vision

Tutorial 06 - Homography, Alignment & Panoramas



Agenda

The largest panorama in the world (2014): Mont Blanc

In2WHITE Video

In2WHITE Full Image

Homographic usage examples:

http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/

https://www.instagram.com/albumplusart/

In [2]:
```
%%html
<iframe src="http://www.in2white.com/" width="700" height="600"></iframe>
```

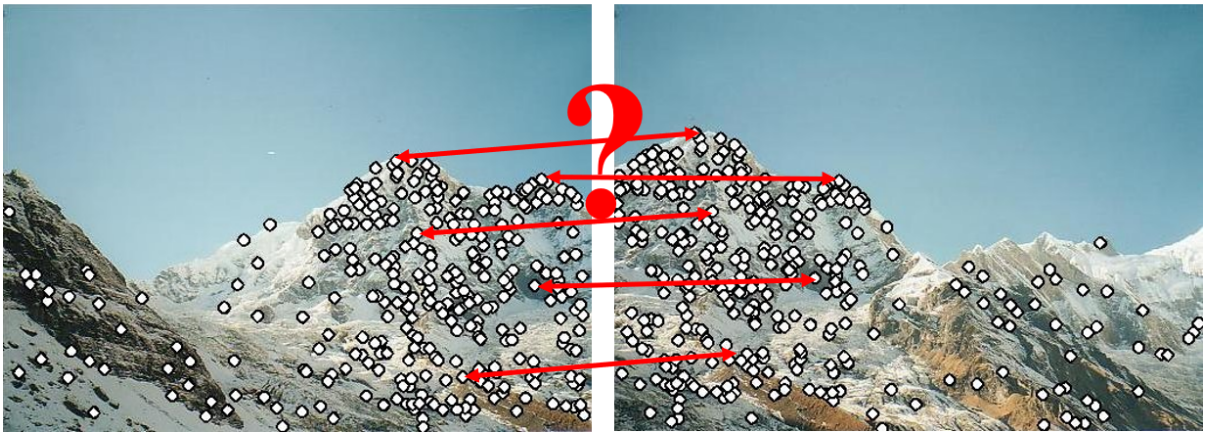
In [1]:
```
# imports for the tutorial
import numpy as np,sys
import matplotlib.pyplot as plt
from PIL import Image
import cv2
from scipy import signal
%matplotlib inline
```

In [3]:
```
# plot images function
def plot_images(image_list, title_list, subplot_shape=(1,1), axis='off', fontsize=30, fi
    plt.figure(figsize=figsize)
    for ii, im in enumerate(image_list):
        c_title = title_list[ii]
        if len(cmap) > 1:
            c_cmap = cmap[ii]
        else:
            c_cmap = cmap[0]
        plt.subplot(subplot_shape[0], subplot_shape[1],ii+1)
        plt.imshow(im, cmap=c_cmap)
        plt.title(c_title, fontsize=fontsize)
        plt.axis(axis)
```

Matching Local Features

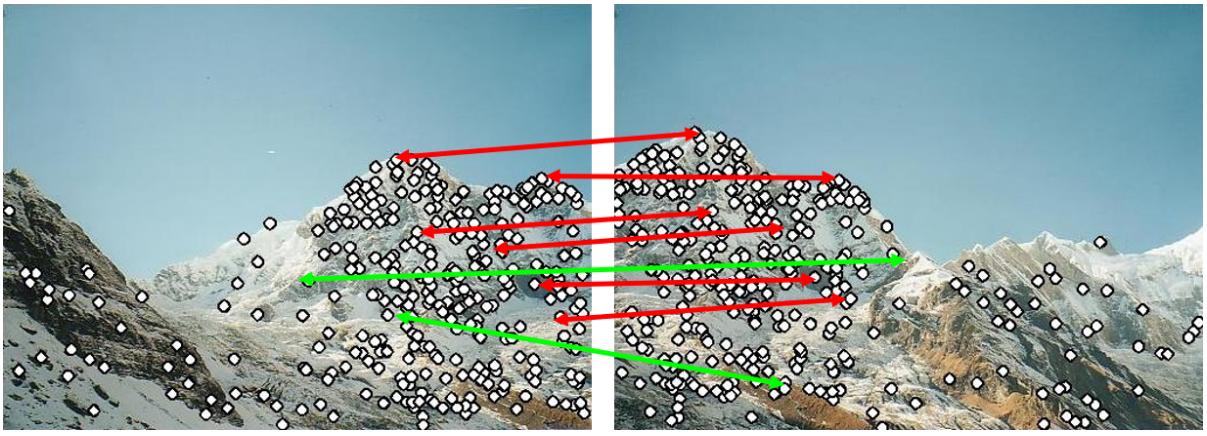
Feature matching

- We know how to detect and describe good points
- Next question: How to match them?



Typical feature matching results

- Some matches are correct
- Some matches are incorrect

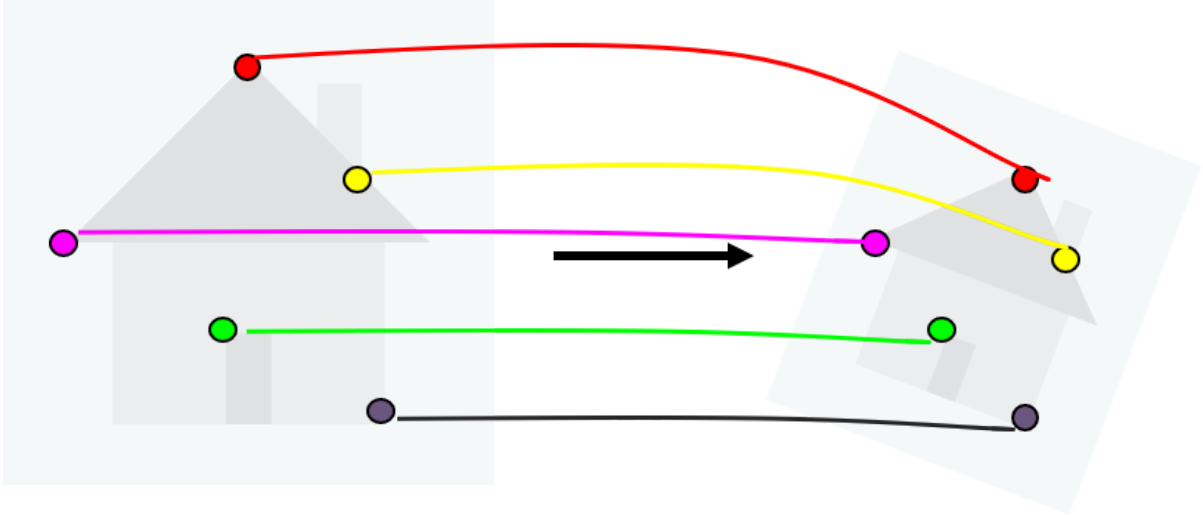


- Solution: search for a set of geometrically consistent matches

Parametric Transformations

Image Alignment

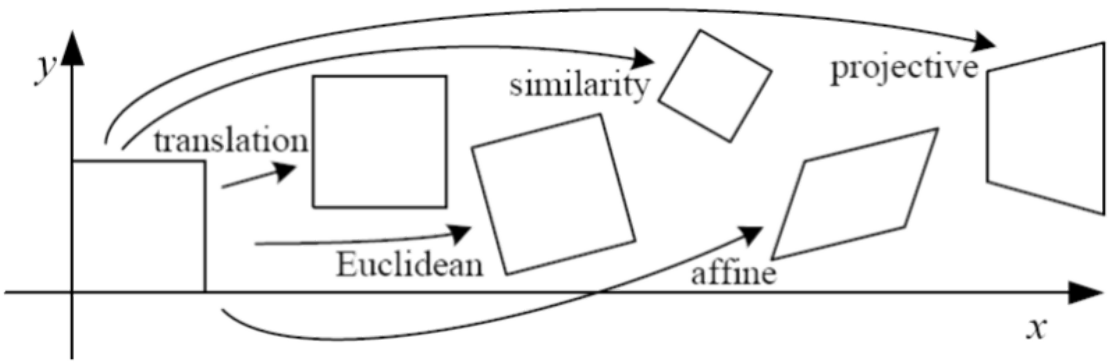
Given a set of matches, what parametric model describes a geometrically consistent transformation?



Basic 2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	3	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$	4	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times3}$	8	straight lines	

(**x** is in *homogeneous* coordinates)



Parametric (Global) Warping

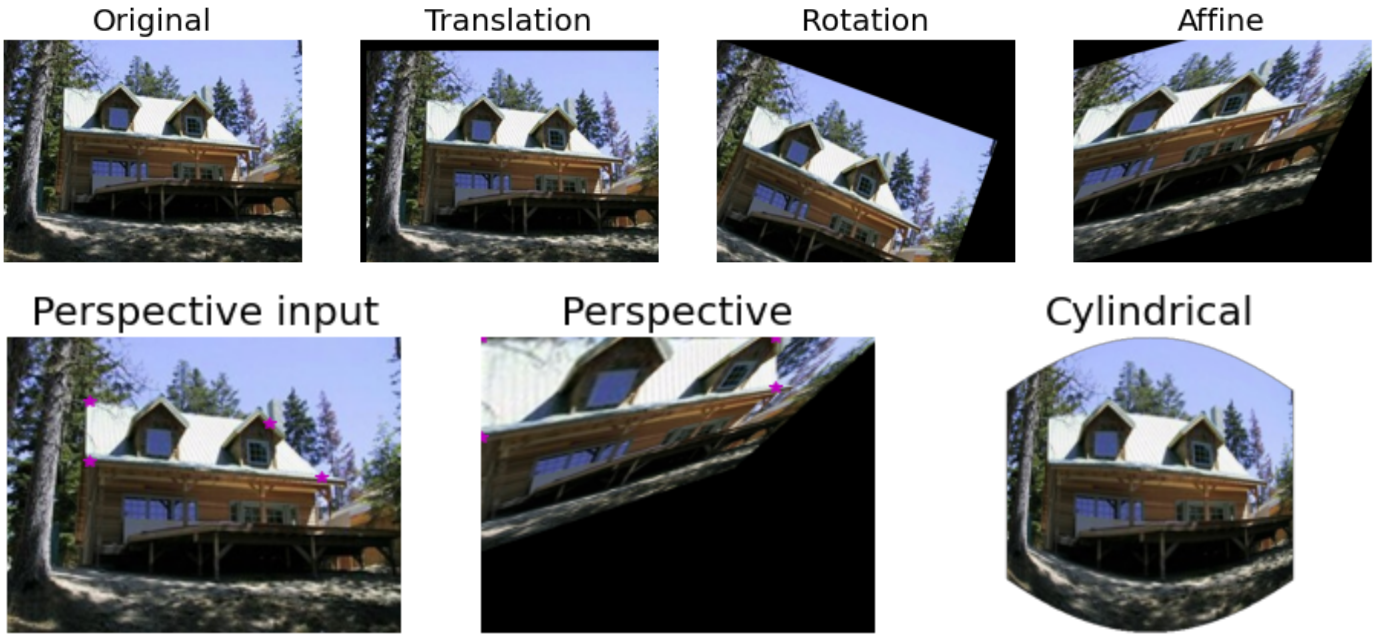
Examples of parametric warps:

In [4]:
```
im = cv2.imread('/Users/yiftachedelstain/ee046746-computer-vision/assets/tut_8_exm.jpg')
im = cv2.cvtColor(im,cv2.COLOR_BGR2RGB)
rows,cols,d = im.shape
print(im.shape)

# Translation:
tx,ty = [10,20]
h_T = np.float32([[1,0,tx],[0,1,ty],[0,0,1]])

# Rotation
theta = np.deg2rad(20)
h_R = np.float32([[np.cos(theta),-np.sin(theta),0],[np.sin(theta),np.cos(theta),0],[0,0,

# Affine
h_AFF = np.array([[  1.26666667,-0.5,-60],[ -0.33333333,1,66.66666667],[0,0,1]])

H = [h_T,h_R,h_AFF]

img = [im]
for h in H:
    img.append(cv2.warpPerspective(im, h,(cols,rows)))
Titles = ['Original','Translation','Rotation','Affine']


# Perspective
pts1 = np.float32([[100,77],[320,105],[100,150],[385,170]])
pts2 = np.float32([[0,0],[300,0],[0,100],[300,50]])
M = cv2.getPerspectiveTransform(pts1,pts2)
im_perspective = cv2.warpPerspective(im,M,(400,300))

# Cylindrical (non linear)
def cylindricalWarp(img, K):
    """This function returns the cylindrical warp for a given image and intrinsics matri
    h_,w_ = img.shape[:2]
    # pixel coordinates
    y_i, x_i = np.indices((h_,w_))
    X = np.stack([x_i,y_i,np.ones_like(x_i)],axis=-1).reshape(h_*w_,3) # to homog
    Kinv = np.linalg.inv(K)
    X = Kinv.dot(X.T).T # normalized coords
    # calculate cylindrical coords (sin\theta, h, cos\theta)
    A = np.stack([np.sin(X[:,0]),X[:,1],np.cos(X[:,0])],axis=-1).reshape(w_*h_,3)
    B = K.dot(A.T).T # project back to image-pixels plane
    # back from homog coords
    B = B[:,:-1] / B[:,[-1]]
    # make sure warp coords only within image bounds
    B[(B[:,0] < 0) | (B[:,0] >= w_) | (B[:,1] < 0) | (B[:,1] >= h_)] = -1
    B = B.reshape(h_,w_,-1)

    img_rgba = cv2.cvtColor(img,cv2.COLOR_RGB2RGBA) #BGR2BGRA for transparent borders...
    # warp the image according to cylindrical coords
    return cv2.remap(img_rgba, B[:,:,0].astype(np.float32), B[:,:,1].astype(np.float32),
                     cv2.INTER_AREA, borderMode=cv2.BORDER_TRANSPARENT)
# Camera parameters:
K = np.array([[200,0,cols/2],[0,400,rows/2],[0,0,1]]) # mock intrinsics
img_cyl = cylindricalWarp(im, K)
```

```
(362, 484, 3)
```

In [5]:
```
plot_images(img,Titles,(1,4),figsize=(16,4),fontsize=20)

plt.figure(figsize=(12,4))
plt.subplot(131)
plt.imshow(im),plt.title('Perspective input',fontsize=20)
plt.plot(pts1[:,0],pts1[:,1],'m*')
plt.axis('off')
```

```
plt.subplot(132),plt.imshow(im_perspective),plt.title('Perspective',fontsize=20)
plt.plot(pts2[:,0],pts2[:,1],'m*')
plt.axis('off')
plt.subplot(133),plt.imshow(img_cyl)
plt.title('Cylindrical',fontsize=20)
_ = plt.axis('off')
```



- Code source - OpenCV
- Cylinder code source - More Than Technical
- Cylinder coordinates

Basic 2D Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective (Homography)

When can we use a homography matrix to describe the transformation of the scene?

Homography maps between:

- Points that lay on a plane in the real world and their positions in an image.
- Points in two different images that lay on the **same** plane.
- Two images of a 3D object where the camera has been rotated but has not been translated.

Homographies are used to approximate the transformations of far away objects:

- Works fine for small viewpoint changes.

Excercise 1 (Spring 2020 Q1A)

Write down the homography H ($3x3$), that maps from image-1 to image-2 in the following cases:

1. Image-2 is a translated image 1 by 100 pixels to the left.
2. Image-2 is a $3\times$ zoom of image 1, so that the pixel indexed $(0,0)$ stays in its original position.

Answer:

1. Image-2 is a translated image 1 by 100 pixels to the left.

We know that for a translation the homography matrix becomes:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

The translation of the image is to the left, which means

$$x_2 = x_1 - 100$$

and the position in the y axis stays the same:

$$y_2 = y_1$$

Therefore, the homography that describes the given translation is given by:

$$\begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sanity check:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 - 100 \\ y_1 \\ 1 \end{bmatrix}$$

In [6]:
```
im = cv2.imread('/Users/yiftachedelstain/ee046746-computer-vision/assets/tut_8_exm.jpg')
im = cv2.cvtColor(im,cv2.COLOR_BGR2RGB)
rows,cols,d = im.shape
h_T = np.float32([[1, 0, -100],
                  [0, 1, 0],
```

```
                    [0, 0, 1]])
im_T = cv2.warpPerspective(im, h_T,(cols,rows))
Titles = ['Original','Translation']

plot_images([im,im_T],Titles,(1,2),figsize=(16,4),fontsize=20)
```

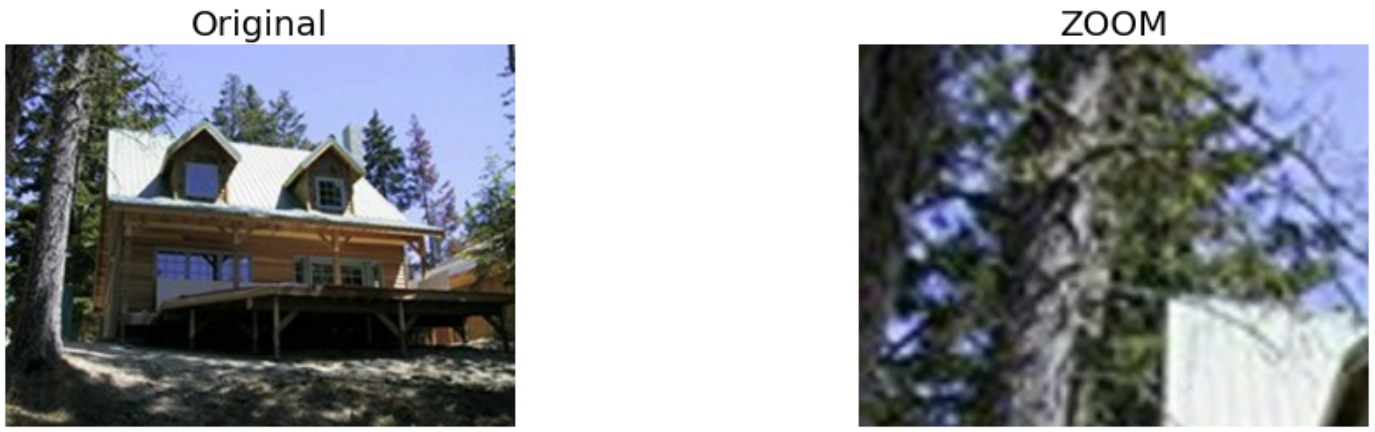
1. Image-2 is a $3\times$ zoom of image 1, so that the pixel indexed $(0,0)$ stays in its original position.

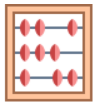
Answer:

The scaled image is different by 1 DOF, since we scale the X-axis and the Y-axis the same amount (same zoom). Going back to the homography matrix, this means only the s parameter changes:

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In [7]:
```
h_S = np.float32([[3,0,0],[0,3,0],[0,0,1]])
im_S = cv2.warpPerspective(im, h_S,(cols,rows))
Titles = ['Original','ZOOM']

plot_images([im,im_S],Titles,(1,2),figsize=(16,4),fontsize=20)
```





Computing Parametric Transformations

- Affine
- Projective

Computing Affine Transformation

- Assuming we know correspondences, how do we get the transformation?

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix} = \begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix}$$

$$b = Ah$$

- Solve with Least-squares $||Ah - b||^2$

$$h = (A^T A)^{-1} A^T b$$

  In Python:

```
A_inv = pinv(A)
```

```
h = np.linalg.pinv(A)@b
```

- How many matches (correspondence pairs) do we need to solve?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for any pixel (x_{new}, y_{new})?

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x'_{new} = Hx_{new}$$

Computing Projective Transformation

- Recall working with homogenous coordinates

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x_i' = u_i / w_i$$

$$y_i' = v_i / w_i$$

- We get the following non-linear equation:

$$x_i' = \frac{h_1 x_i + h_2 y_i + h_3}{h_7 x_i + h_8 y_i + h_9}$$

$$y_i' = \frac{h_4 x_i + h_5 y_i + h_6}{h_7 x_i + h_8 y_i + h_9}$$

- We can re-arrange the equation

$$\begin{bmatrix} & & & & & \cdots & & & \\ x_i & y_i & 1 & 0 & 0 & 0 & -x_i x_i' & -y_i x_i' & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -x_i y_i' & -y_i y_i' & -y_i' \\ & & & & & \cdots & & & \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} \cdots \\ 0 \\ 0 \\ \cdots \end{bmatrix}$$

- We want to find a vector h satisfying

$$Ah = 0$$

where A is full rank. We are obviously not interested in the trivial solution $h = 0$ hence we add the constraint

$$||h|| = 1$$

- Thus, we get the homogeneous Least square equation:

$$arg\min_h ||Ah||_2^2, s.t||h||_2^2 = 1$$

Compute Projective transformation using SVD:

$$arg\min_h ||Ah||_2^2, s.t||h||_2^2 = 1$$

- Let decompose A using SVD: $A = UDV^T$, where U and V are orthonomal matrix, and D is a diagonal matrix.
  - Need a reminder on SVD? Click Here

- From orthonormality of U and V:

$$||UDV^T h|| = ||DV^T h||$$

$$||V^T h|| = ||h||$$

Hence, we get the following minimization problem:

$$arg\min_h ||DV^T h|| \text{ s.t. } ||V^T h|| = 1$$

- Subsitute $y = V^T h$:

$$arg\min_h ||Dy|| \text{ s.t. } ||y|| = 1$$

- D is a diagonal matrix with decreasing values. Then, it is clear that $y = [0, 0, \ldots, 1]^T$.

- Therefore, choosing h to be the last column in V will minimize the equation.

In Python:

```
(U,D,Vh) = np.linalg.svd(A,False)
```

```
h = Vh.T[:,-1]
```

Some more options to find h:

- Lagrange multipliers - Least–squares Solution of Homogeneous Equations
- Using EVD (eigenvalue decomposition) on $A^T A$.
- If we know our transformation is nearly Affine we can get an approximate solution using linear least squares

- As we said, some of the points may be a good match, and some won't.
- If we use a wrong point in the algorithm, we will get the wrong transformation.
- How can we solve that major problem?

- We can use our prior knowledge on the type of the trasformation, and use points that have strong evidence that are correct.
- For example, we know that homographys preserve lines, so we can try to look for keypoints that lay in a line, and use them to find the transformation.
- How would we find keypoints that lay on a line?

RANSAC

RANdom SAmple Consensus [Fischler & Bolles 1981]

- Key ideas:
  - Look for “inliers” and use only them
  - If we fit a model to “outliers” we will not get a good fitting

RANSAC Algorithm

Loop:

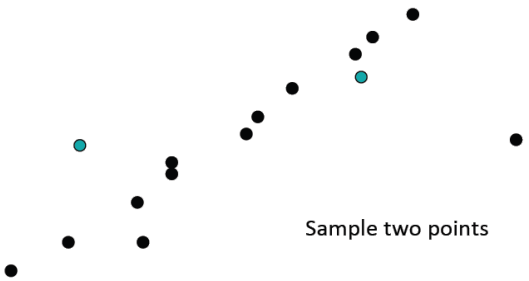
1. Randomly select a group of points
2. Fit a model to the selected group
3. Find the inliers of the computed model
4. If number of inliers is large enough re-compute model using only inliers
5. Compute number of inliers of updated model

The winner: model with the largest number of inliers

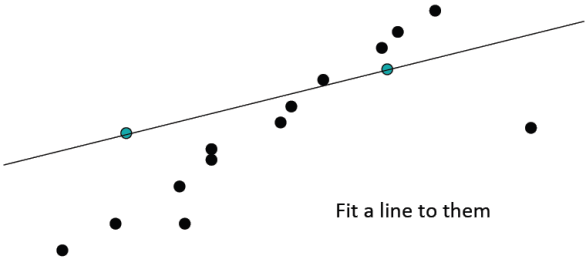
Line Fitting with RANSAC

- Input: A set of edge points

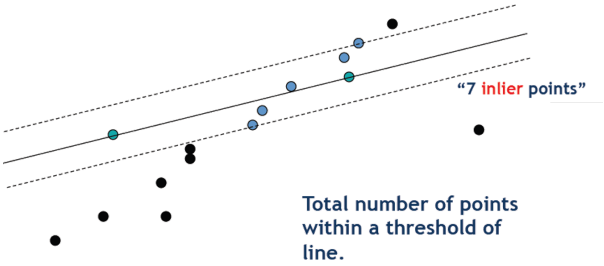
- **Step 1**: Select two points



- **Step 2**: Fit a line to the selected points

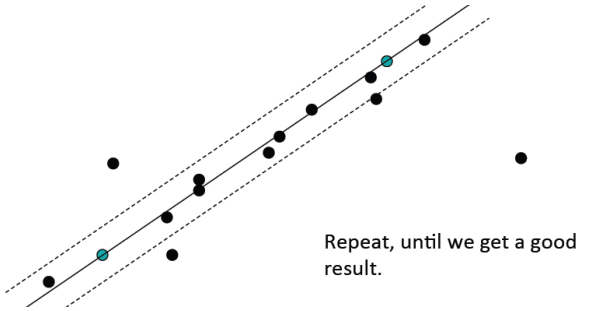


- **Step 3**: Identify inliers

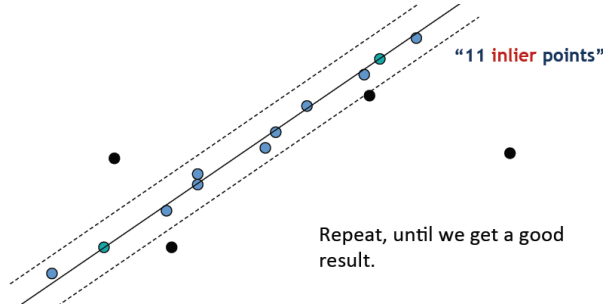


- **Step 4**: Repeat Steps 1-3 until convergence

- **Iteration 2**: Steps 1-2:
  - Sample new pts and fit a line



- **Iteration 2**: Step 3:
  - Count number of new inliers



RANSAC – Stopping Criteria

- **Option 1**: when the model is good enough:
  - By the number of inliers
  - By its fitting error
- **Option 2**: according to probability
  - Let K be the number of iterations
  - Let n be the number of points needed to compute the model
  - Let f be the fraction of inliers of a model
  - Then the probability that a single sample is correct: f^n

- The probability that all K samples fail is $(1-f^n)^K$
- Choose K high enough to keep the failure rate low enough

RANSAC – For Multiple Models?

- How can we use RANSAC to compute multiple models?
  - Apply RANSAC multiple times, each time we remove the detected inliers.

RANSAC - Summary

- **Pros**
  - General method that works well for lots of model fitting problems
  - Easy to implement
- **Cons**
  - When the percentage of outliers is high too many iterations are needed and failure rate increases
- **Implementation**
  - OpenCV usage with Homographies (Tutorial 7): `cv2.findHomography(src_pts, dst_pts, cv2.RANSAC)`
  - Scikit-image: `skimage.measure.ransac()`
  - Scikit-learn: `sklearn.linear_model.RANSACRegressor()`
  - and many more..

Many improved algorithms:

- PROSAC
  - Key idea is to assume that the similarity measure predicts correctness of a match
- Randomized RANSAC
  - Each step take a random subset of the query points and perform RANSAC
- KALMANSAC
- and more...
- Estimating homogrpahy with RANSAC in OpenCV: `cv2.findHomography(src_pts, dst_pts, cv2.RANSAC)`

Excercise 2 (Spring 2020 Q1D) - Ransac

We'd like to fit an ellipse to a given set of 200 points, $X_i = (x_i, y_i), i = 1, \ldots, 200$.
As you know, an ellipse can be described by the implicit equation $a^T(x^2, xy, y^2, x, y, 1) = 0$ where $a = (a_1, a_2, a_3, a_4, a_5, a_6)$ is a coefficients vector. The data points might be noisy, such that some of the points may not reside on the perimeter of the ellipse (outliers).

- Describe the RANSAC algorithm needed to fit the ellipse (and find a).

Answer:

- to define an ellipse, we need at least 5 points (Why?). This means a is defined up to a scale factor.

Step 1: Sample 5 points from X_i randomly. **Step 2**: Fit an ellipse to the points with Least Squares.

- $$\begin{bmatrix} & & \cdots & & & \\ x_i^2 & x_i y_i & y_i^2 & x_i & y_i & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = 0$$

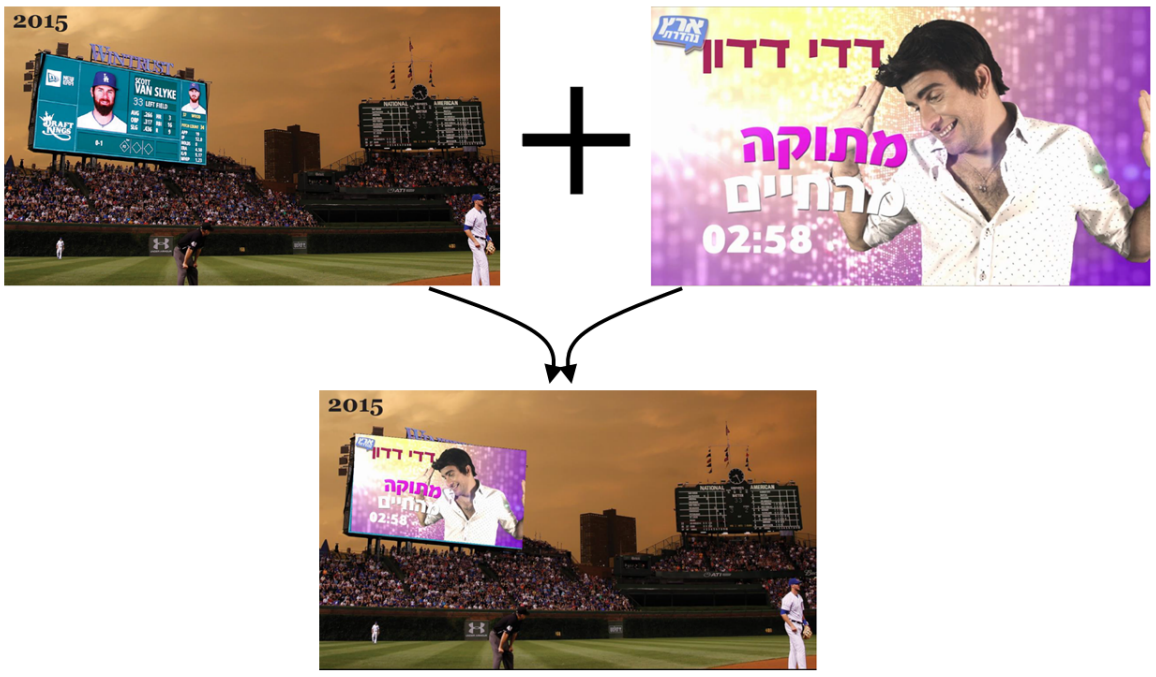
- This can be denoted as $Sa = 0$, and solved via SVD under the constraint $||a|| = 1$

Step 3: Find the points that fit to the model (inliers), i.e. points with a fit error below a threshold chosen in advanced.
Step 4: Refine our model according to all the inliers, and count them. The number of inliers is the "score" of the current model (from this iteration).
Step 5: Repeat 1-4, until the stop criteria has been met.

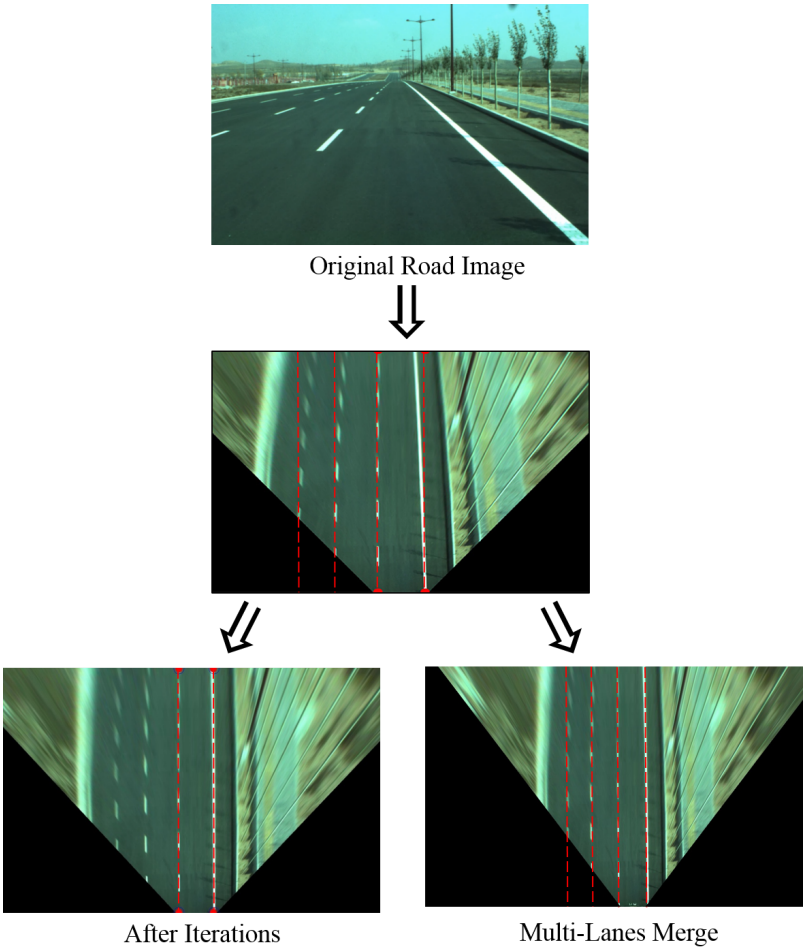
When do we stop?

- The percentage of inliers out of all the points is above a certain percentage.
- The amount of inliers is above a certain amount.
- An error threshold (for the LS) is reached.
- A predefined amount of iterations is reached.

application 1: Planting images into other images



application 2: BirdEye

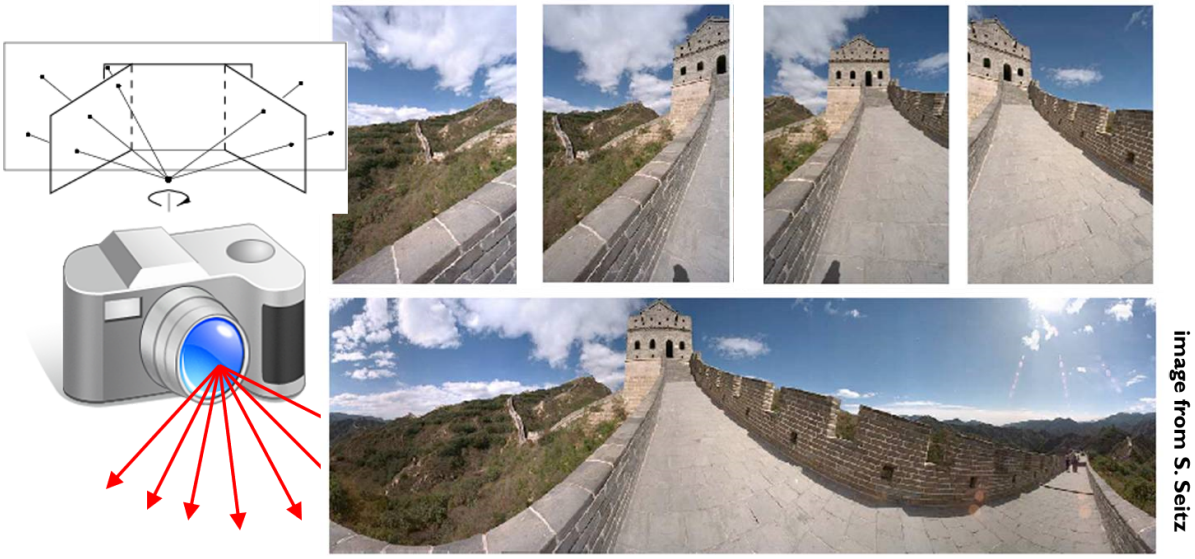


- Image Source

application 3: Looking into the past

- Image Source from Flickr

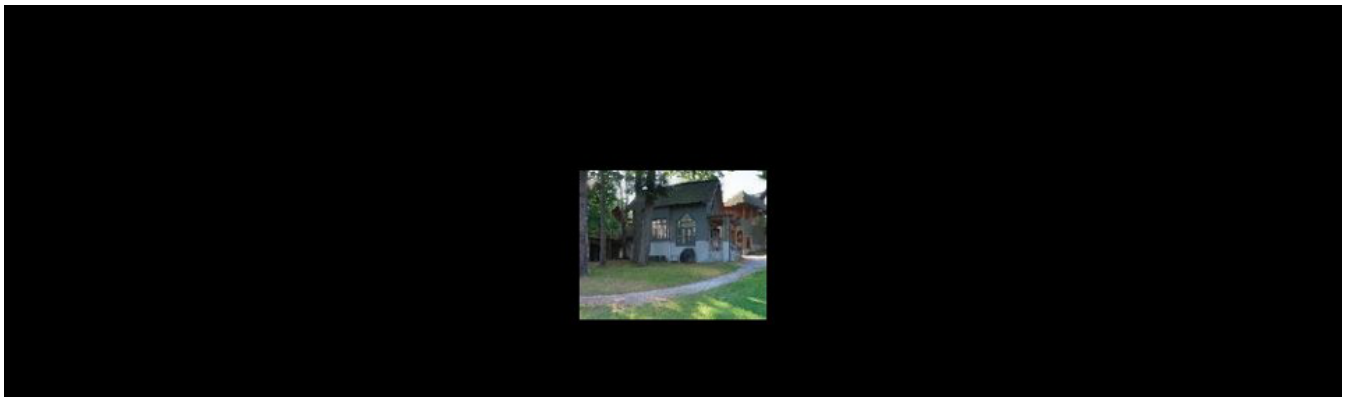
application 4: Panorama



Obtain a wider angle view by combining multiple images

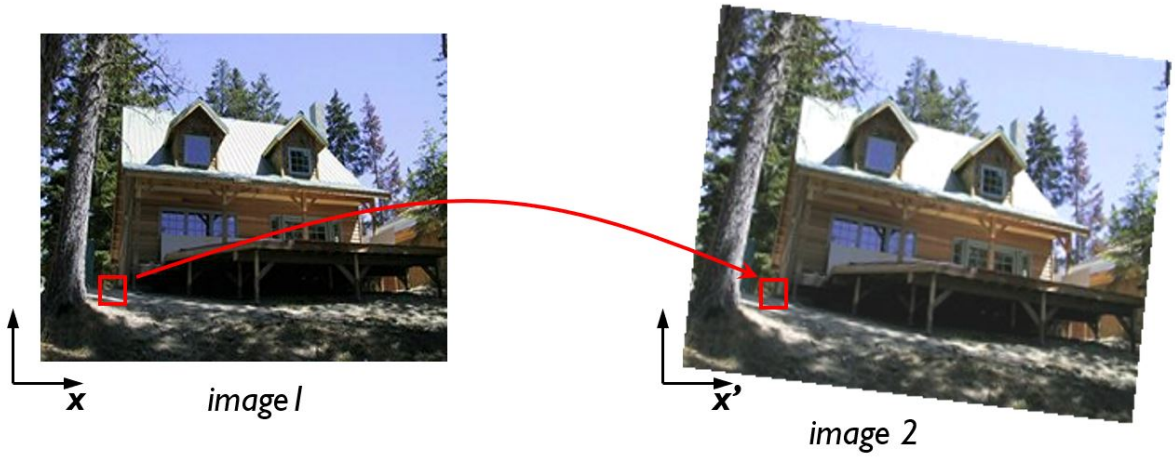
Panorama

- Warping
- Image Blending (Feathering)



Warp - What we need to solve?

- Given source and target images, and the transformation between them, how do we align them?
- Send each pixel x in image1 to its corresponding location x' in image 2



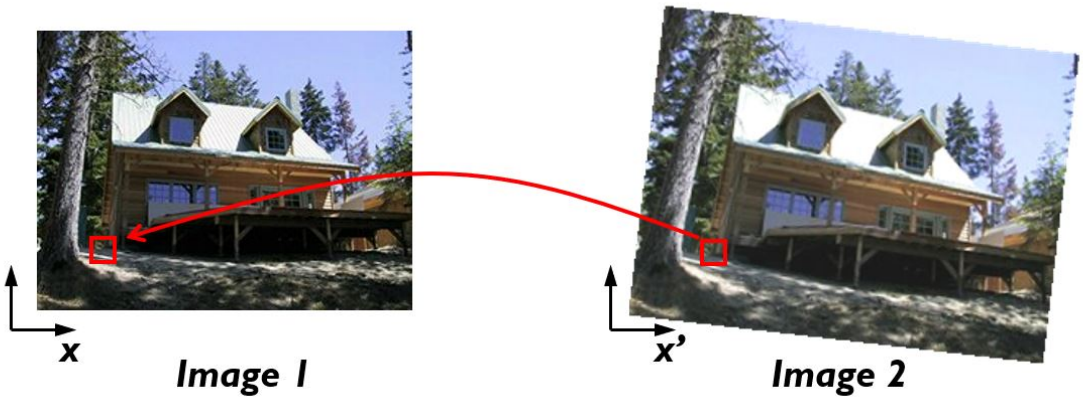
Forward Warping

- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels and normalize (splatting)

- Limitation: Holes (some pixels are never visited)

Inverse Warping

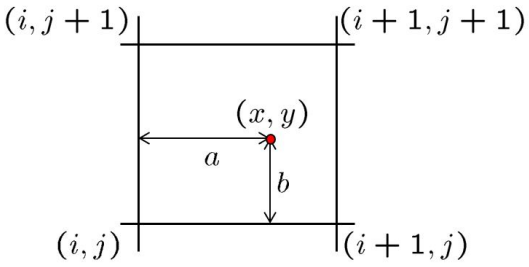
- For each pixel x′ in image 2 find its origin x in image 1
- Problem: What if pixel comes from “between” two pixels?



- Answer: interpolate color value from neighbors

Bilinear Interpolation

Sampling at $f(x,y)$:



$$
\begin{aligned}
f(x,y) = (1-a)(1-b)f[i,j] \\
+ a(1-b)f[i+1,j] \\
+ abf[i+1,j+1] \\
+ (1-a)bf[i,j+1]
\end{aligned}
$$

Python:

- `interp2d()` - https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp2d.html
- Inverse warping in OpenCV: `cv2.warpPerspective(im,*,*,cv2.WARP_INVERSE_MAP)`

Excercise 3 (Spring 2020 Q1B)

---

1. Describe forward and backward wraping.
2. For which out of the two cased before will there be a difference between using inverse or forward warping?
3. For the case which a difference exists, what type of warping should be used and why?

reminder:

- Image-2 is a translated image 1 by 100 pixels to the left
- Image-2 is a 3x zoom of image 1, so the index (0,0) stay in place

Answer

1. Read the tutorial :)
2. For the translation there will be no difference between forward/inverse warping, while for the zooming there will be a difference - we'll get holes.

To understand why, let's denote the pixel of image k at the indices (i,j) as x_{ij}^k.

In the first case, each pixel is moved to the left by a whole number, therefore for forward warping we follow the warp defined by:

$$x_{(i-100)j}^2 = x_{ij}^1$$

And for inverse warping we use:

$$x_{ij}^2 = x_{(i+100)j}^1$$

There's no need to interpolate anything here - because the pixel moved by an integer, and thus there is no difference.
In the second case we get for forward warping:

$$x^2_{(3i)(3j)} = x^1_{ij}$$

And for inverse warping we get an interpolation dependant on α, β:

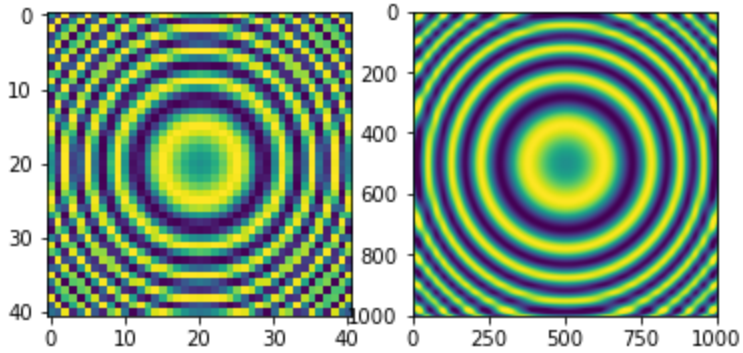
$$x^2_{ij} = (1-\alpha)(1-\beta)x^1_{\left\lfloor\frac{i}{3}\right\rfloor\left\lfloor\frac{j}{3}\right\rfloor} + \alpha(1-\beta)x^1_{\left(\left\lfloor\frac{i}{3}\right\rfloor+1\right)\left\lfloor\frac{j}{3}\right\rfloor} + \alpha\beta x^1_{\left\lfloor\frac{i}{3}\right\rfloor\left(\left\lfloor\frac{j}{3}\right\rfloor+1\right)} + (1-\alpha)\beta x^1_{\left(\left\lfloor\frac{i}{3}\right\rfloor+1\right)\left(\left\lfloor\frac{j}{3}\right\rfloor+1\right)}$$

We can see that for the forward case we get "holes" in image 2, for example $x^2_{1,1}$ has no origin in image 1.

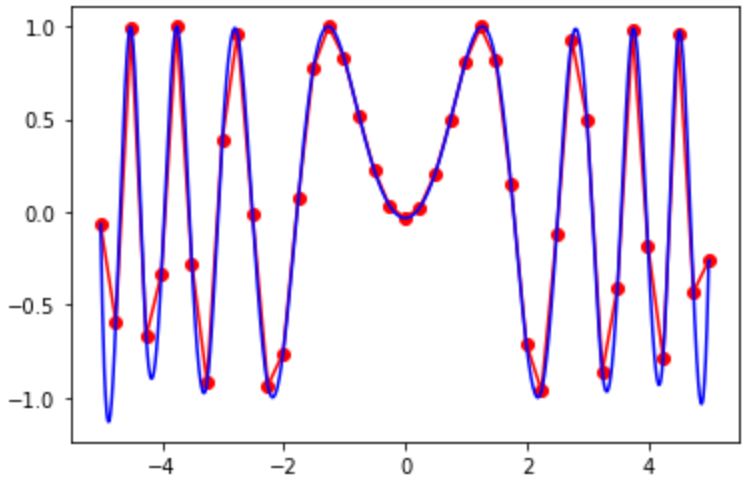
1. The backward wraping will be of course better, since as we saw no holes will be created.

In [8]:
```
import numpy as np
from scipy import interpolate
import matplotlib.pyplot as plt

# original samples
x = np.arange(-5.01, 5.01, 0.25)
y = np.arange(-5.01, 5.01, 0.25)
xx, yy = np.meshgrid(x, y)
z = np.sin(xx**2+yy**2)

# interpolated samples
xnew = np.arange(-5.01, 5.01, 1e-2)
ynew = np.arange(-5.01, 5.01, 1e-2)
```

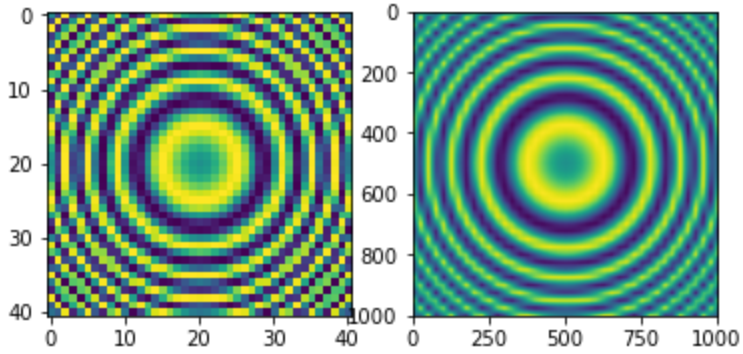
In [9]:
```
## Cubic
f = interpolate.interp2d(x, y, z, kind='cubic')
znew = f(xnew, ynew)

plt.figure()
plt.subplot(121)
plt.imshow(z,vmin=-1,vmax=1)
plt.subplot(122)
plt.imshow(znew,vmin=-1,vmax=1)
plt.show()
```



In [10]:
```
plt.figure()
plt.plot(x, z[0, :], 'ro-', xnew, znew[0, :], 'b-')
plt.show()
```



In [11]:
```
##linear
f2 = interpolate.interp2d(x, y, z, kind='linear')
znew2 = f2(xnew, ynew)

plt.figure()
plt.subplot(121)
plt.imshow(z,vmin=-1,vmax=1)
plt.subplot(122)
plt.imshow(znew2,vmin=-1,vmax=1)
plt.show()
```



In [12]:
```
plt.figure()
plt.plot(x, z[0, :], 'ro-', xnew, znew2[0, :], 'b-')
plt.show()
```

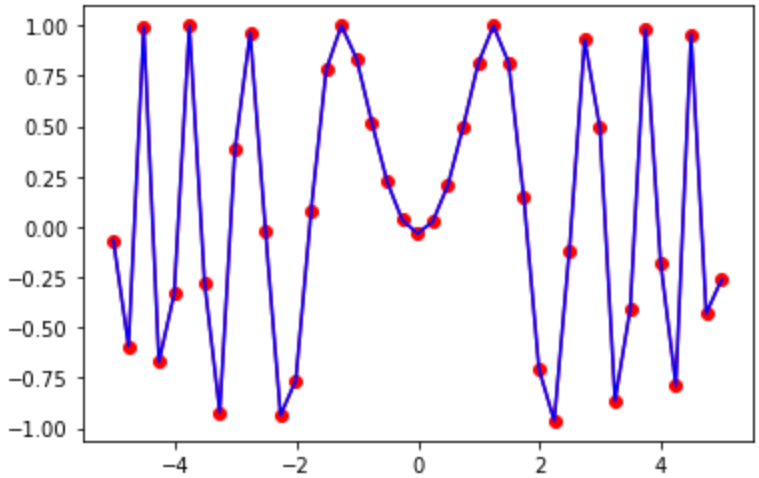
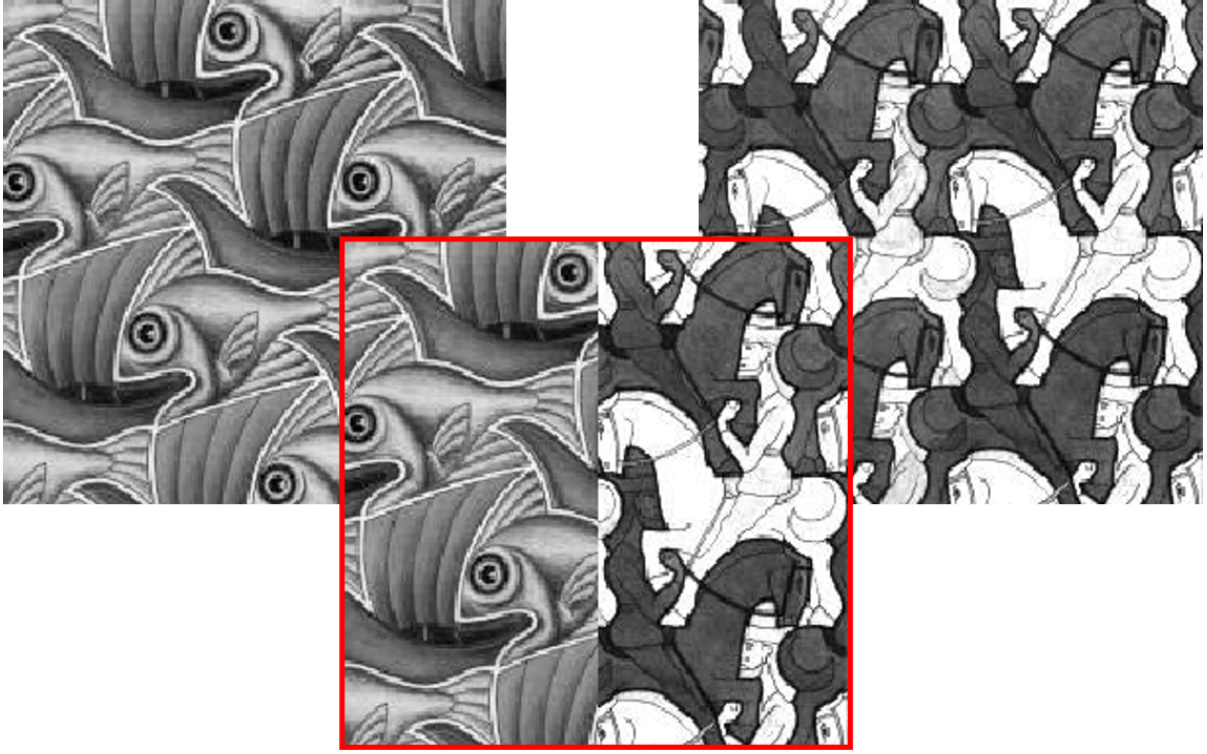
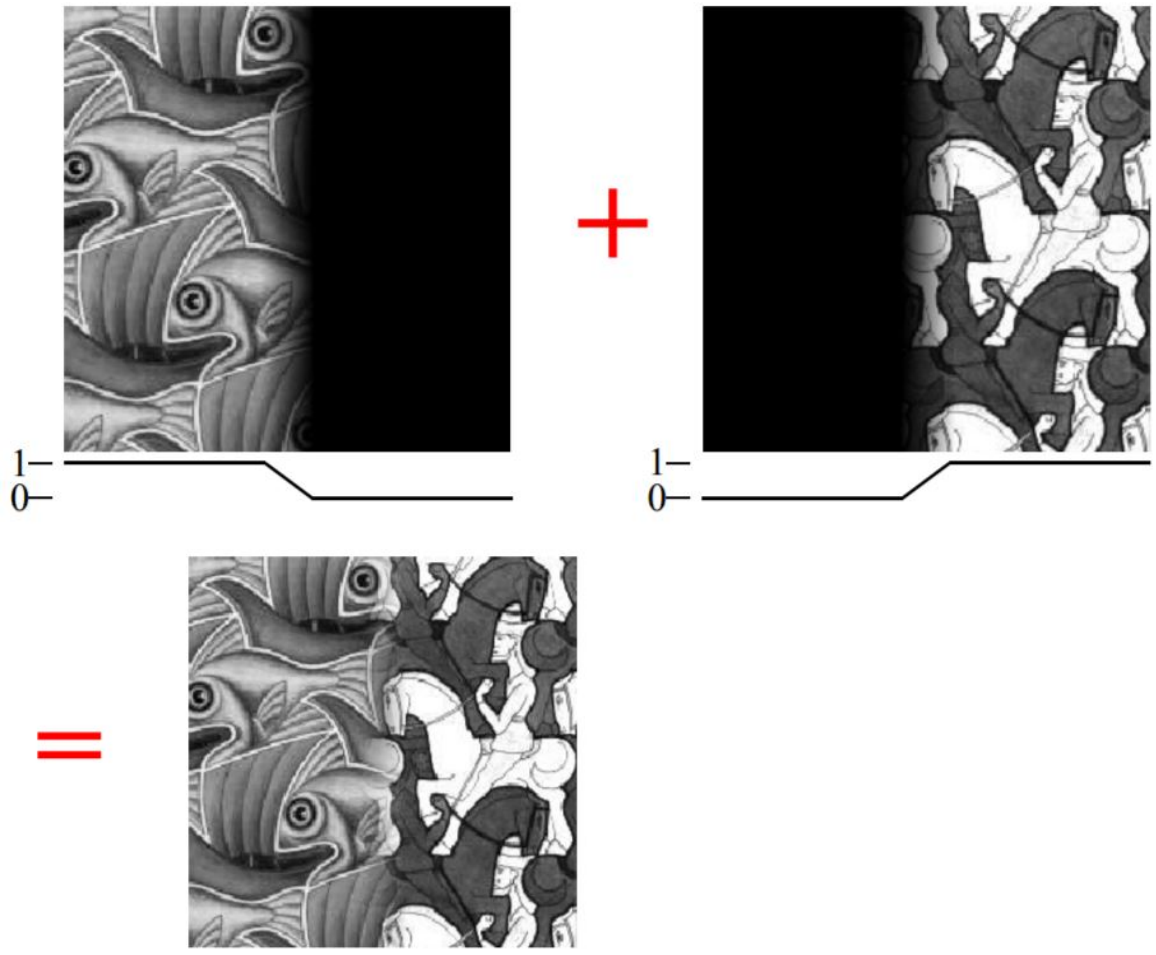


Image Blending



- Alpha blending
- Pyramid blending

Alpha Blending



Pyramid Blending:

1. Build a Gaussian pyramid for each image
2. Build the Laplacian pyramid for each image
3. Decide/find the blending border (in the example: left half belongs to image 1, and right half to image 2 -> the blending border is `cols/2`)
    - Split by index, or
    - Split using a 2 masks (can be weighted masks)
4. Constract a new mixed pyramid - mix each level seperatly acording to (3)
5. Reconstract a blend image from the mixed pyramid

In [13]:
```
def create_pyrs(A,B):
    # generate Gaussian pyramid for A
    G = A.copy()
    gpA = [G]
    for i in range(6):
        G = cv2.pyrDown(G)
        gpA.append(G)
    # generate Gaussian pyramid for B
    G = B.copy()
    gpB = [G]
    for i in range(6):
        G = cv2.pyrDown(G)
        gpB.append(G)
    # generate Laplacian Pyramid for A
    lpA = [gpA[5]]
    for i in range(5,0,-1):
        GE = cv2.pyrUp(gpA[i])
        L = cv2.subtract(gpA[i-1],GE)
        lpA.append(L)
    # generate Laplacian Pyramid for B
    lpB = [gpB[5]]
    for i in range(5,0,-1):
        GE = cv2.pyrUp(gpB[i])
        L = cv2.subtract(gpB[i-1],GE)
        lpB.append(L)
    return lpA,lpB
```

In [1]:
```
def blend_images(A,B):
    lpA,lpB = create_pyrs(A,B)
    # Now add left and right halves of images in each level
    LS = []
    for la,lb in zip(lpA,lpB):
        rows,cols,dpt = la.shape
        ls = np.hstack((la[:,0:int(cols/2)], lb[:,int(cols/2):])) #mixing can also be do
        LS.append(ls)
    # now reconstruct
    ls_ = LS[0]
    for i in range(1,6):
        ls_ = cv2.pyrUp(ls_)
        ls_ = cv2.add(ls_, LS[i])
    # image with direct connecting each half
    real = np.hstack((A[:,:int(cols/2)],B[:,int(cols/2):]))
    return real, ls_
```

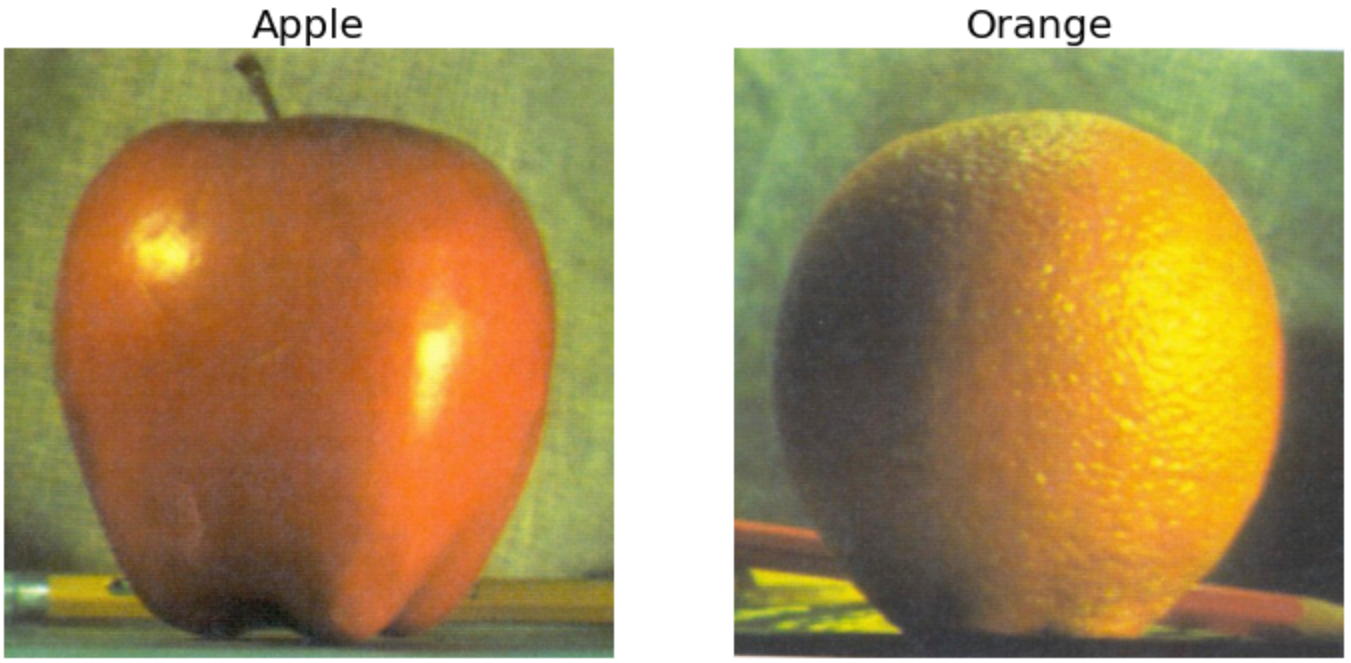
In [15]:
```
def switch_texture(A,B):
    lpA,lpB = create_pyrs(A,B)
    # Now add left and right halves of images in each level
    LS = []
#     for la,lb in zip(lpA,lpB):
#         rows,cols,dpt = la.shape
#         ls = np.hstack((la[:,0:int(cols/2)], lb[:,int(cols/2):])) #mixing can also be
```

```
#         LS.append(ls)
    # now reconstruct
    ls_ = lpA[0]
    for i in range(1,6):
        ls_ = cv2.pyrUp(ls_)
        ls_ = cv2.add(ls_, lpB[i])
    # image with direct connecting each half
    real = np.hstack((A[:,:int(cols/2)],B[:,int(cols/2):]))
    return real, ls_
```

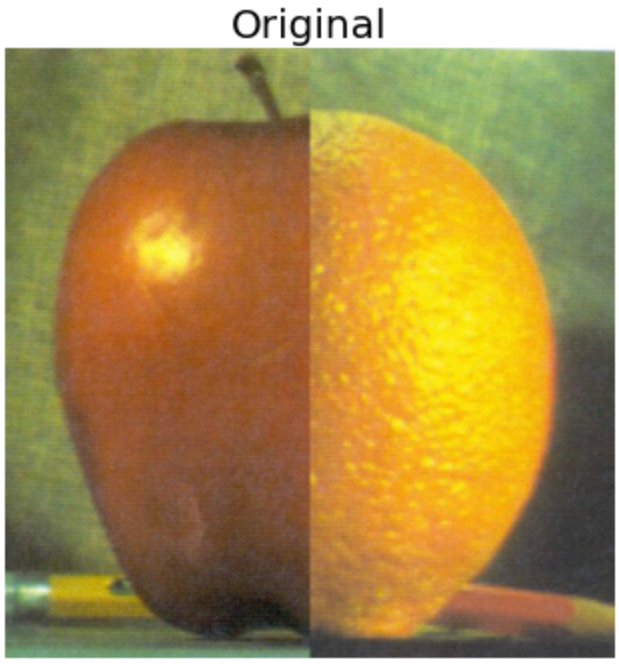
In [16]:
```
def alpha_blend(A,B,MASK):
    A = np.float32(A)
    B = np.float32(B)
    return np.uint8(A*MASK), np.uint8(A*MASK+B*(1-MASK))
```

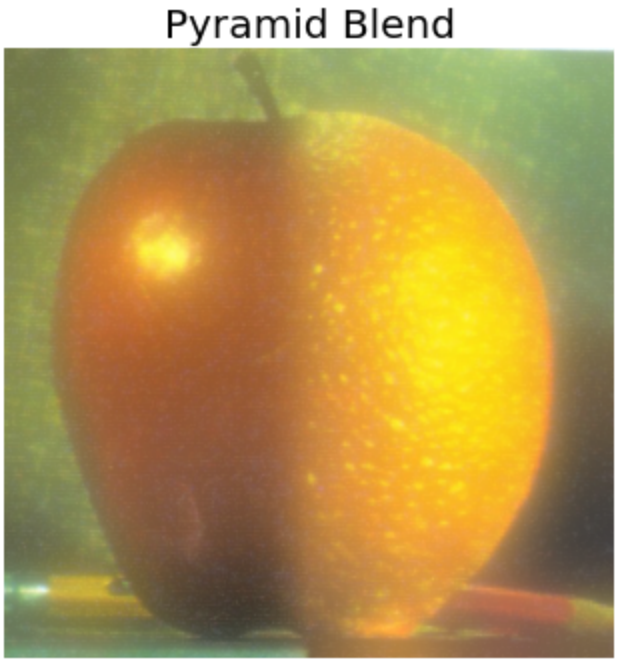
In [17]:
```
A = cv2.imread('/Users/yiftachedelstain/ee046746-computer-vision/assets/apple.jpg')
B = cv2.imread('/Users/yiftachedelstain/ee046746-computer-vision/assets/orange.jpg')
real,ls_ = blend_images(A,B)

A_ = cv2.cvtColor(A,cv2.COLOR_BGR2RGB)
B_ = cv2.cvtColor(B,cv2.COLOR_BGR2RGB)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

In [18]:
```
plot_images([A_,B_],['Apple','Orange'],(1,2),figsize=(12,12),fontsize=20)
```



In [19]:
```
plot_images([real,ls_],['Original','Pyramid Blend'],(1,2),figsize=(12,12),fontsize=20)
```

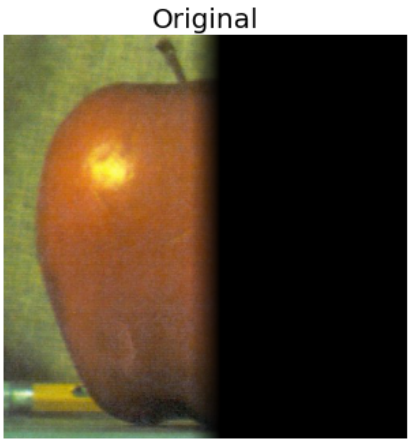


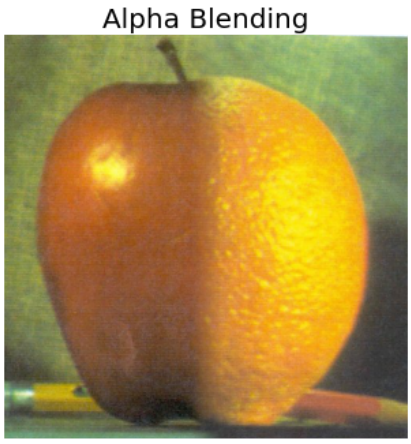


- Alpha blending example:

In [20]:
```
MASK = np.ones_like(A,np.float32)
rows,cols,dpt = MASK.shape
w=20
v_dec = np.linspace(1,0,2*w)
MASK[:,int(cols/2):]=0
MASK[:,(int(cols/2)-w):(int(cols/2)+w)]=np.tile(np.reshape(v_dec,[1,-1,1]),[rows,1,3])
real,ls_ = alpha_blend(A,B,MASK)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

In [21]:
```
plot_images([real,MASK,ls_],['Original','Mask','Alpha Blending'],(1,3),figsize=(18,12),f
```

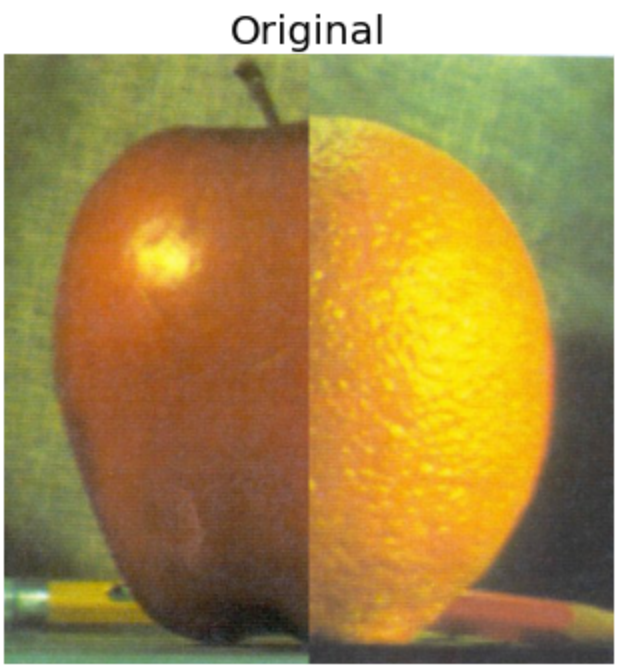


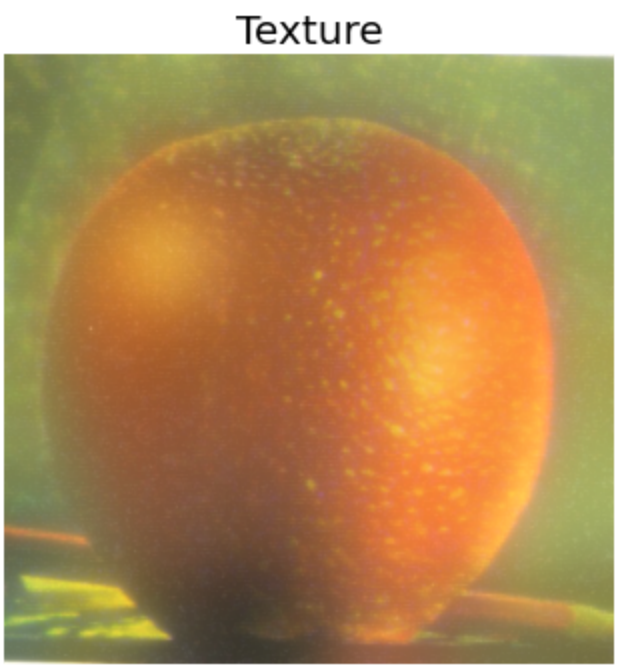


- Stylize image using pyramids:

In [22]:
```
real,ls_ = switch_texture(A,B)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

In [23]:
```
plot_images([real,ls_],['Original','Texture'],(1,2),figsize=(12,12),fontsize=20)
```





Blending Improvments

- Many algorithms have different variations of combining alpha and pyramid blending (different masks for different frequencies)
- Find the boundaries using **segmentation**

Panorama - Summary

- Detect features
- Compute transformations between pairs of frames
- Can Refine transformations using RANSAC
- Warp all images onto a single coordinate system
- Find mixing borders (e.g. using segmentation)
- Blend

Transformations in Deep Learning

- Can we incorporate transformations in the pipeline of a deep learning algorithm?
  - Moreover, can we accelerate these transformations by performing them on a GPU?
- **YES!**

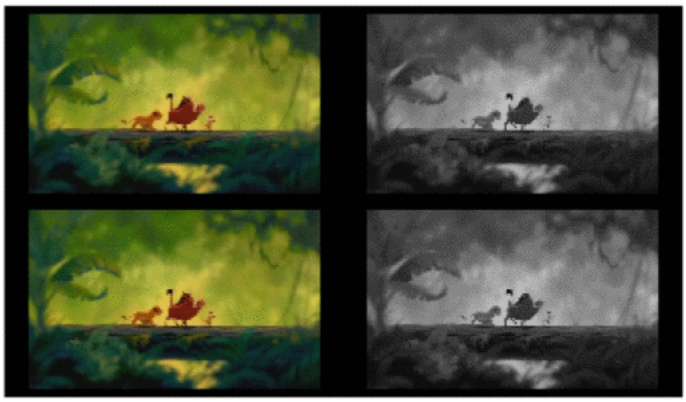
Kornia - Computer Vision Library for PyTorch

- Kornia is a differentiable computer vision library for PyTorch
  - That means you can have gradients for the transformations!
- Inspired by OpenCV, this library is composed by a subset of packages containing operators that can be inserted within neural networks to train models to perform image transformations, epipolar geometry, depth estimation, and low-level image processing such as filtering and edge detection that operate directly on tensors.
- Check out `kornia.geometry` - https://kornia.readthedocs.io/en/latest/geometry.html
- Warp image using perspective transform

kornia

PyTorch

```
import torch
import kornia

frame: torch.Tensor = load_video_frame(...)

out: torch.Tensor = (
    kornia.rgb_to_grayscale(frame)
)
```



Recommended Videos

Warning!

- These videos do not replace the lectures and tutorials.
- Please use these to get a better understanding of the material, and not as an alternative to the written material.

Video By Subject

- Homography
  - Image geometry and planar homography - ENB339 lecture 9: Image geometry and planar homography
  - Homography - Homography in computer vision explained
- Transformations - Lect. 5(1) - Linear and affine transformations
- Matching Local Features
  - SIFT - CSCI 512 - Lecture 12-1 SIFT

Credits

- EE 046746 Spring 2022 - Moshe Kimhi, Hila Manor
- EE 046746 Spring 2020 - Dahlia Urbach

- Slides - Elad Osherov (Technion), Simon Lucey (CMU)
- Multiple View Geometry in Computer Vision - Hartley and Zisserman - Section 2
- Least–squares Solution of Homogeneous Equations - Center for Machine Perception - Tomas Svoboda
- Computer vision: models, learning and inference , Simon J.D. Prince - Section 15.1
- Computer Vision: Algorithms and Applications - Richard Szeliski - Sections 2,4,6, 9 (Free for Technion students via remote library)

- Icons from Icon8.com - https://icons8.com