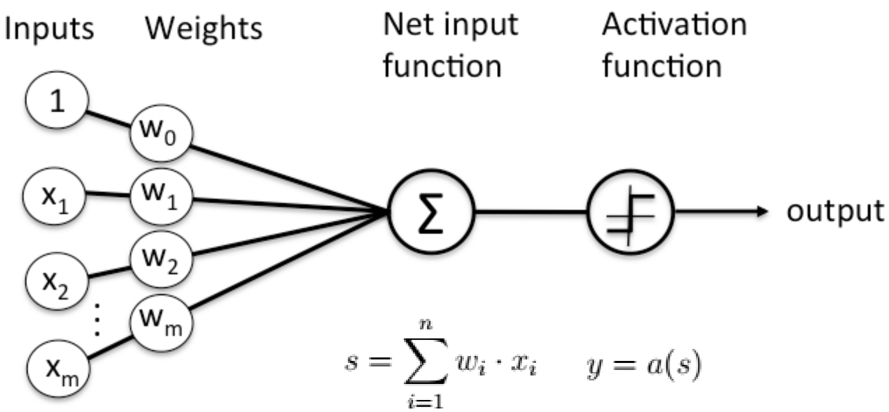
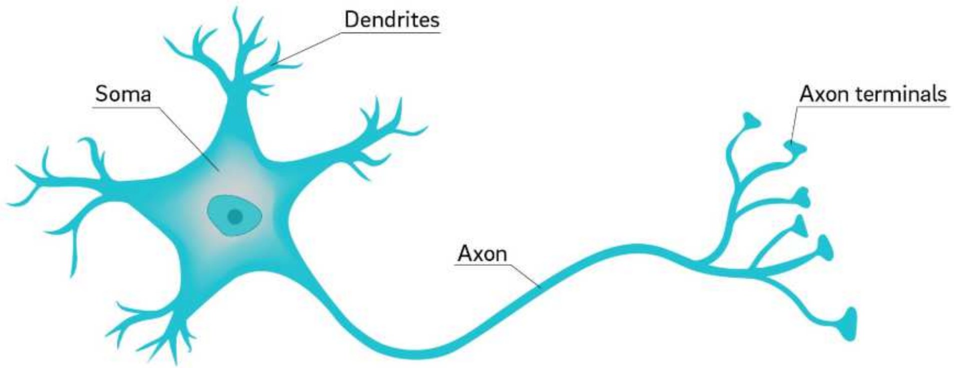
Labor_07

NEURAL NETWORK TRAIN

Neuron


Inputs
Weights
Net input function
Activation function
1
w_0
x_1
w_1
x_2
w_2
w_m
x_m
output
$s = \sum_{i=1}^{n} w_i \cdot x_i$
$y = a(s)$


Dendrites
Soma
Axon terminals
Axon

Neural Network: Forward Step

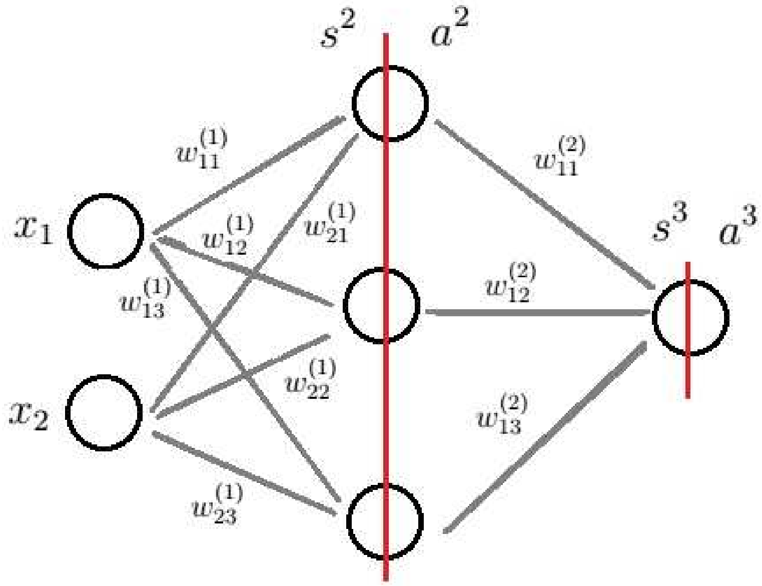
$$xw^{(1)} = s^{(2)}$$

$$a^{(2)} = f(s^{(2)}) = sigmoid(s^{(2)})$$

$$s^{(3)} = a^{(2)}w^{(2)}$$

$$\hat{y} = f(s^{(3)}) = sigmoid(s^{(3)})$$

$$C = \sum \frac{1}{2}(y - \hat{y})^2$$



Neural Network: Cost function

$$C = \sum\{\frac{1}{2}(y - \hat{y})^2\}$$

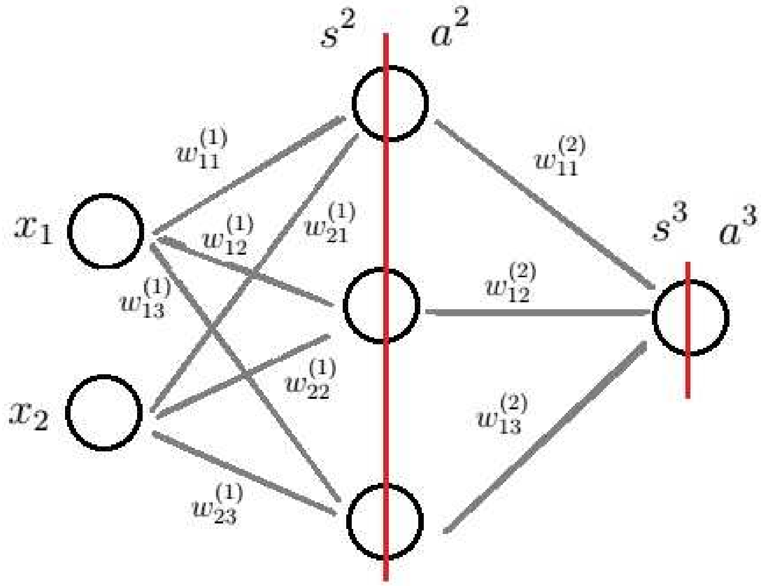
$$C = \sum\{\frac{1}{2}(y - a^{(3)})^2\}$$

$$C = \sum\{\frac{1}{2}(y - f(s^{(3)}))^2\}$$

$$C = \sum\{\frac{1}{2}(y - f(a^{(2)}w^{(2)}))^2\}$$

$$C = \sum\{\frac{1}{2}(y - f(f(s^{(2)})w^{(2)}))^2\}$$

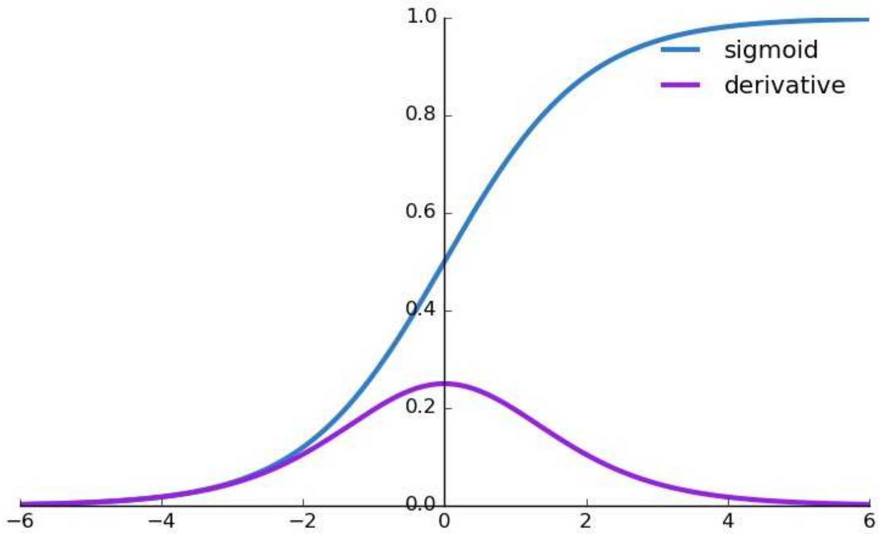
$$C = \sum\{\frac{1}{2}(y - f(f(xw^{(1)})w^{(2)}))^2\}$$



Sigmoid and Derivate

$$g(z) = \frac{1}{1+e^{-z}}$$

$$\begin{aligned} g'(z) &= \frac{d}{dz}\frac{1}{1+e^{-z}} \\ &= \frac{1}{(1+e^{-z})^2}\left(e^{-z}\right) \\ &= \frac{1}{(1+e^{-z})}\cdot\left(1-\frac{1}{(1+e^{-z})}\right) \\ &= g(z)(1-g(z)). \end{aligned}$$



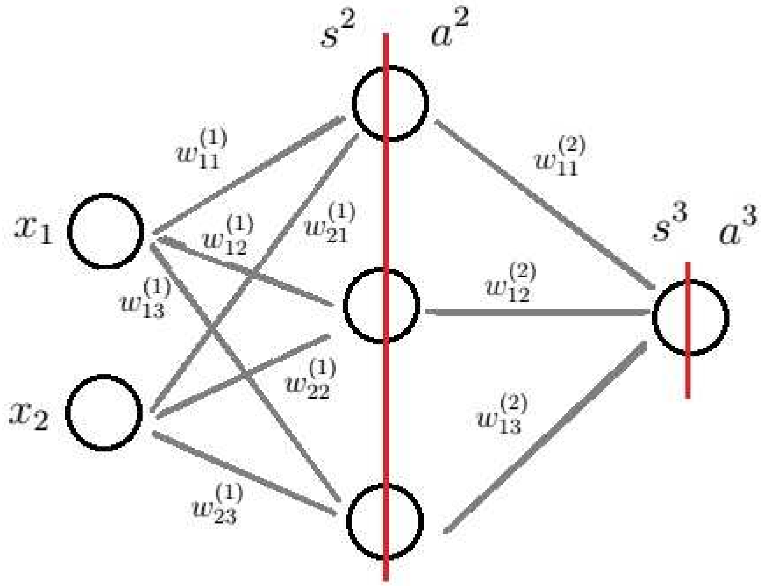
Neural Network: Back Propagation

$$\frac{\partial C}{\partial w^{(2)}} = \frac{\partial \sum \frac{1}{2}(y-\hat{y})^2}{\partial w^{(2)}} = \sum\left(\frac{\partial \frac{1}{2}(y-\hat{y})^2}{\partial w^{(2)}}\right)$$

$$\begin{aligned}\frac{\partial \frac{1}{2}(y-\hat{y})^2}{\partial w^{(2)}} &= (y-\hat{y})\left(-\frac{\hat{y}}{\partial w^{(2)}}\right) \\ &= -(y-\hat{y}) \cdot \frac{\partial \hat{y}}{\partial s^{(3)}} \cdot \frac{\partial s^{(3)}}{\partial w^{(3)}} \\ &= -(y-\hat{y}) \cdot f'(s^{(3)}) \cdot \frac{\partial a^{(2)} w^{(2)}}{\partial w^{(2)}} \\ &= \delta^{(3)} \cdot a^{(2)}\end{aligned}$$

Dimension check:

$$(a^{(2)})^T \delta^{(3)}$$



Neural Network: Training

$$-(y-\hat{y})\cdot f'(s^{(3)}) = \delta^{(3)}$$

$$(a^{(2)})^T\delta^{(3)} = \frac{\partial C}{\partial w^{(2)}}$$

$$\delta^{(3)}\cdot (w^{(2)})^T \cdot f'(s^{(2)}) = \delta^{(2)}$$

$$x^T\delta^{(2)} = \frac{\partial C}{\partial w^{(1)}}$$

$$w^{(1)} = w^{(1)}\cdot \mu\frac{\partial C}{w^{(1)}} + regularization$$

$$w^{(2)} = w^{(2)}\cdot \mu\frac{\partial C}{w^{(2)}} + regularization$$