The Shape of a Program

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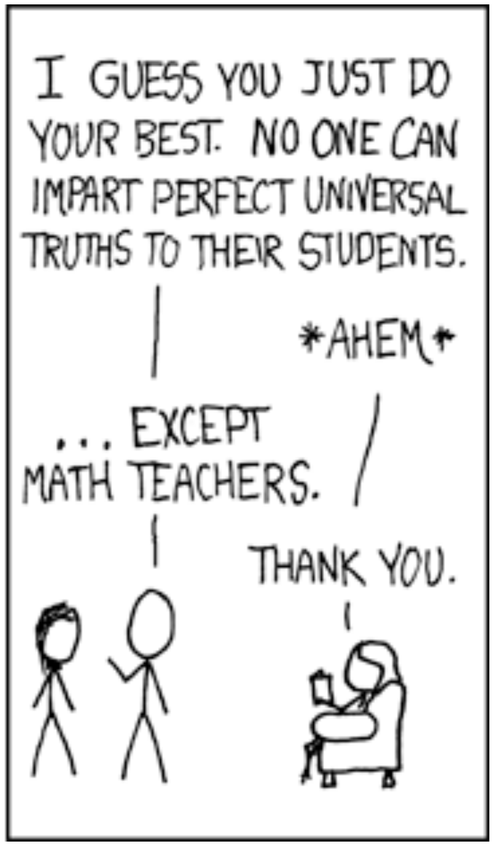
C++Now, May 2018

Why don’t we routinely write down the reasoning behind our programs in a formal way, and have computers check it?

The mathematical tools we use for proofs present a poor user interface for procedural programming.



xkcd.com/263

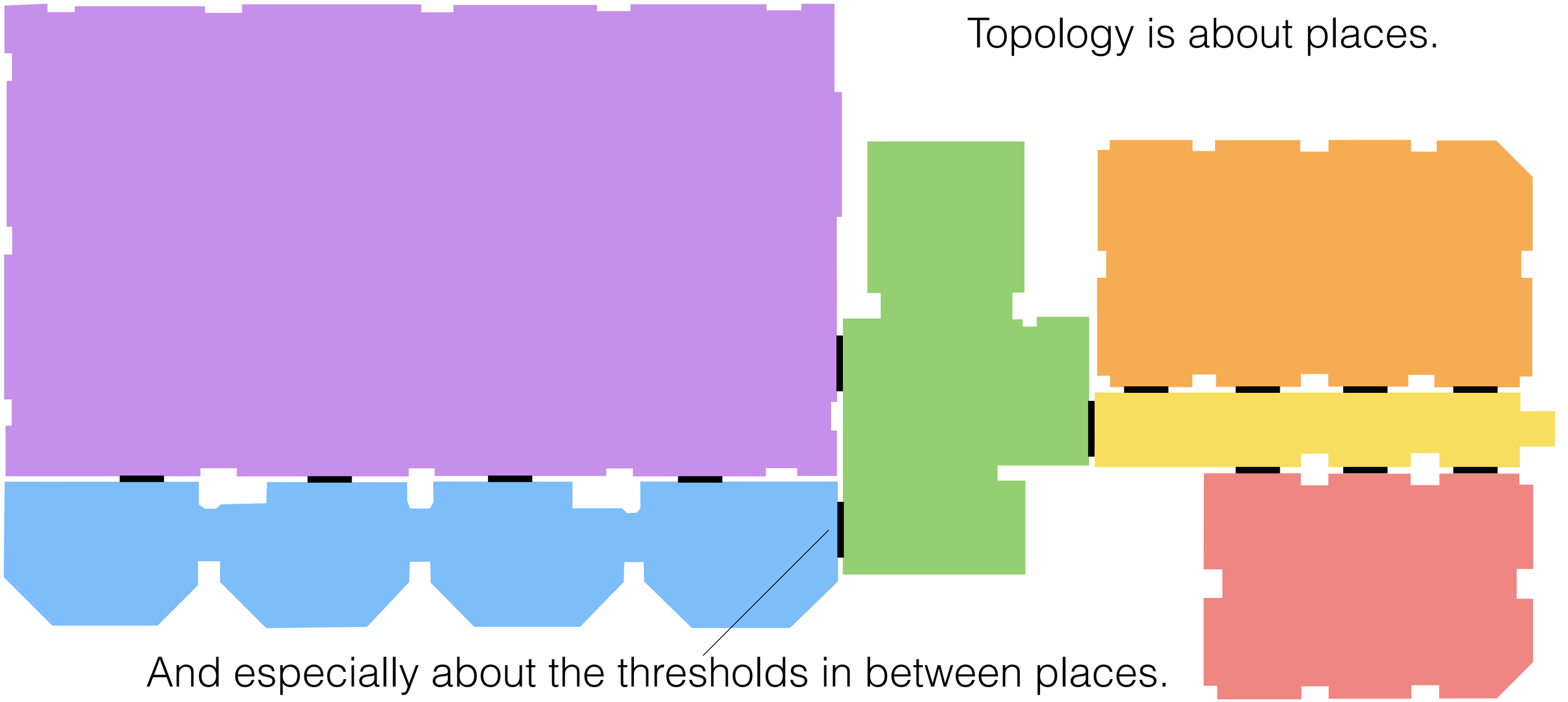
Many people understand mathematics as describing timeless, universal truths.

Local

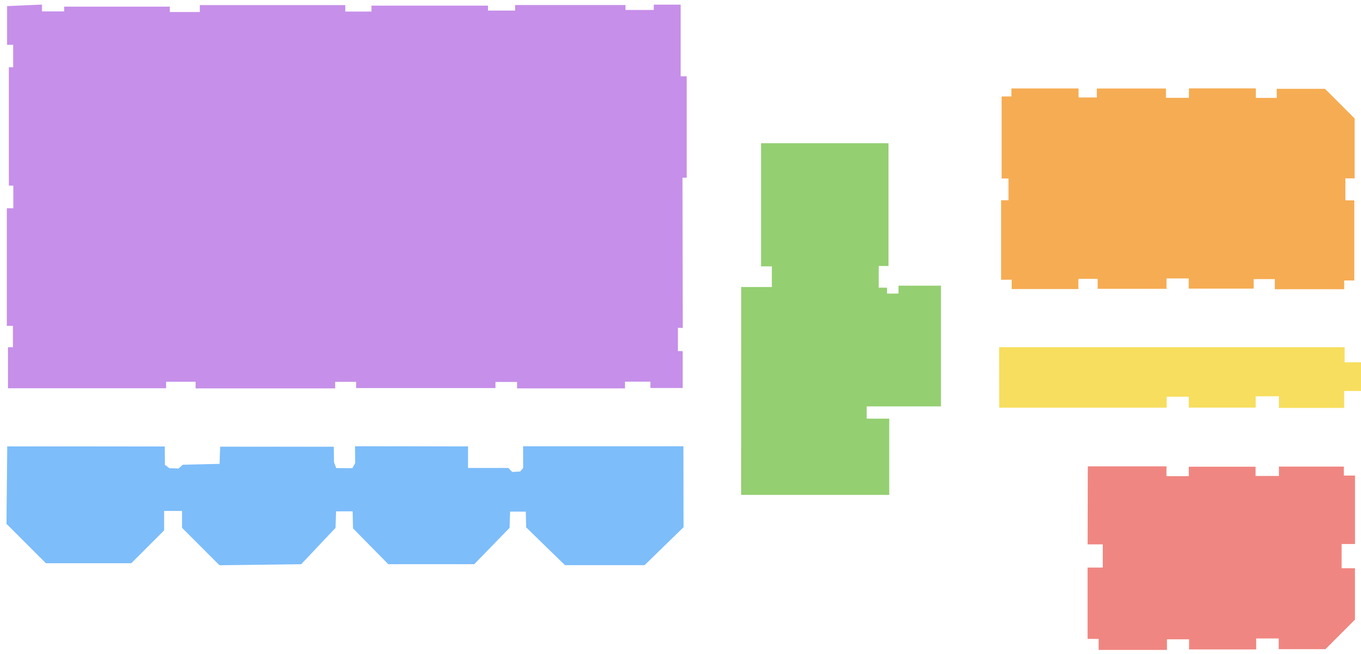
I’m going to be talking about things you already know, perhaps with language you don’t know.

The code here is written in a fantasy C++, with extensions that make proofs fit into the code.

Topology is about places.

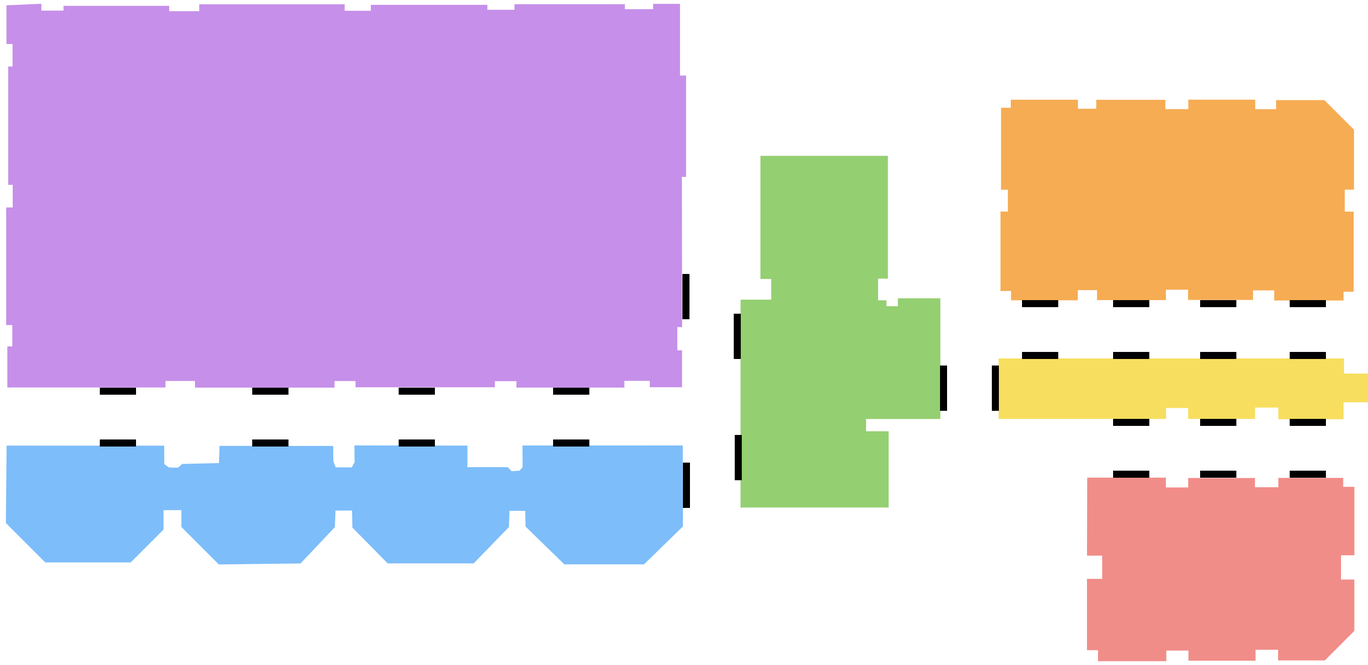


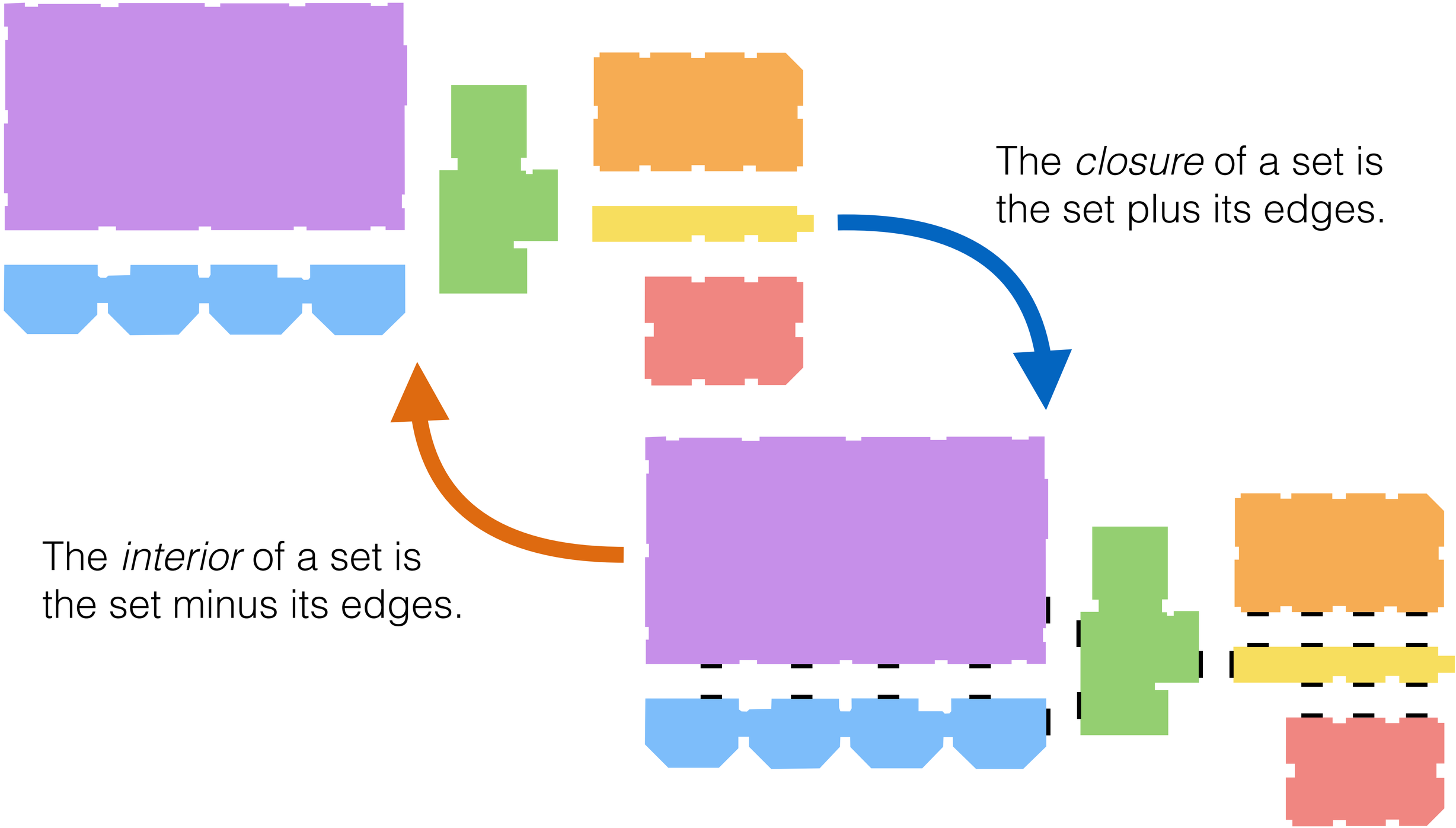
And especially about the thresholds in between places.

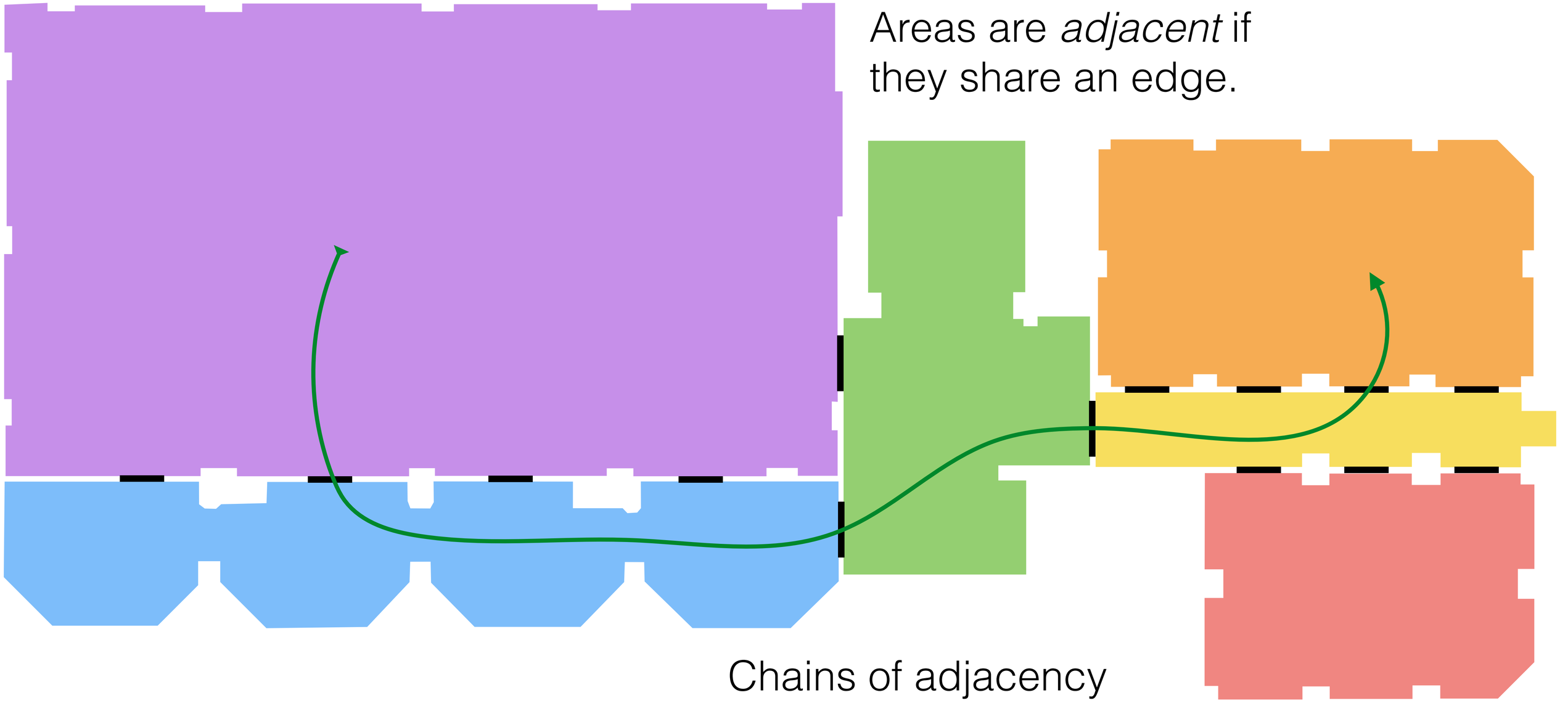


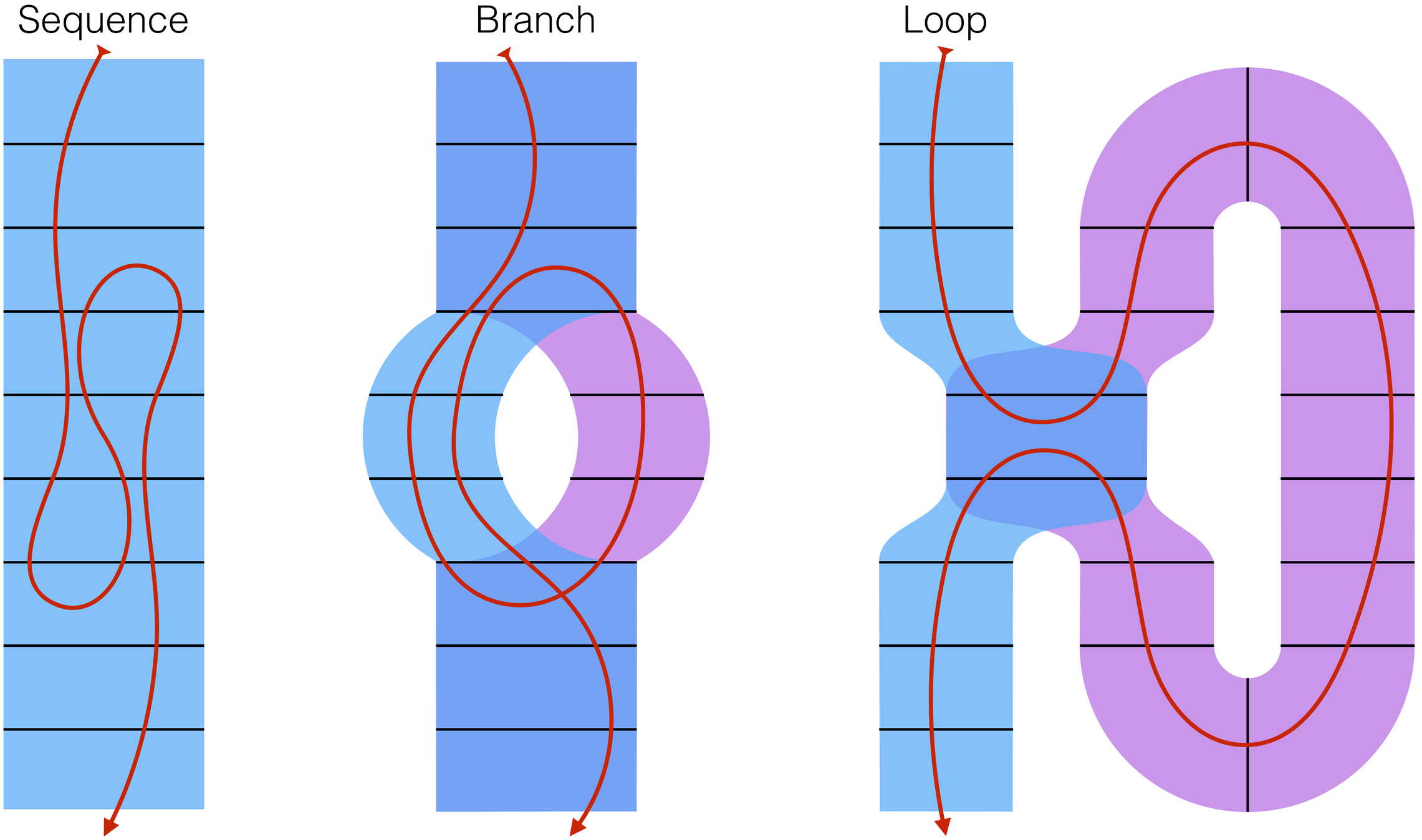
Open sets are meaningful areas, not including their edges.

Closed sets are meaningful areas, including their edges.

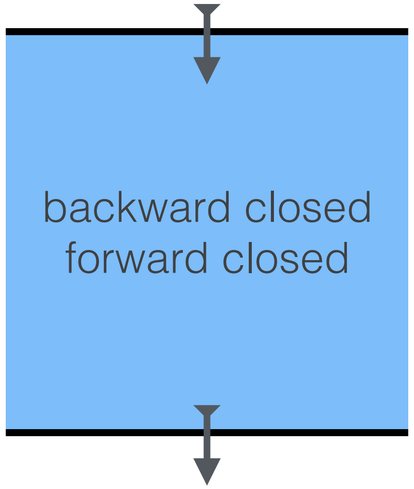



The *closure* of a set is
the set plus its edges.
The *interior* of a set is
the set minus its edges.

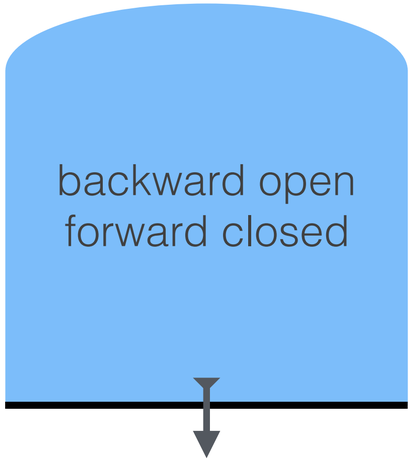

Areas are *adjacent* if
they share an edge.
Chains of adjacency
connect areas together.

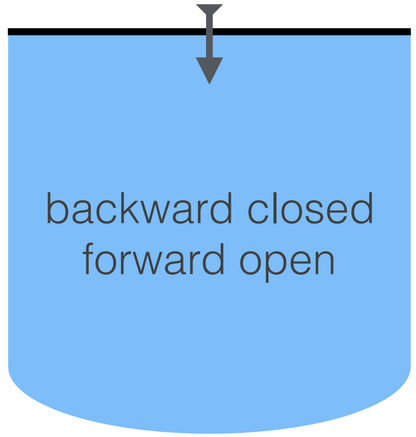

Sequence
Branch
Loop

Forward closed sets include their exits.



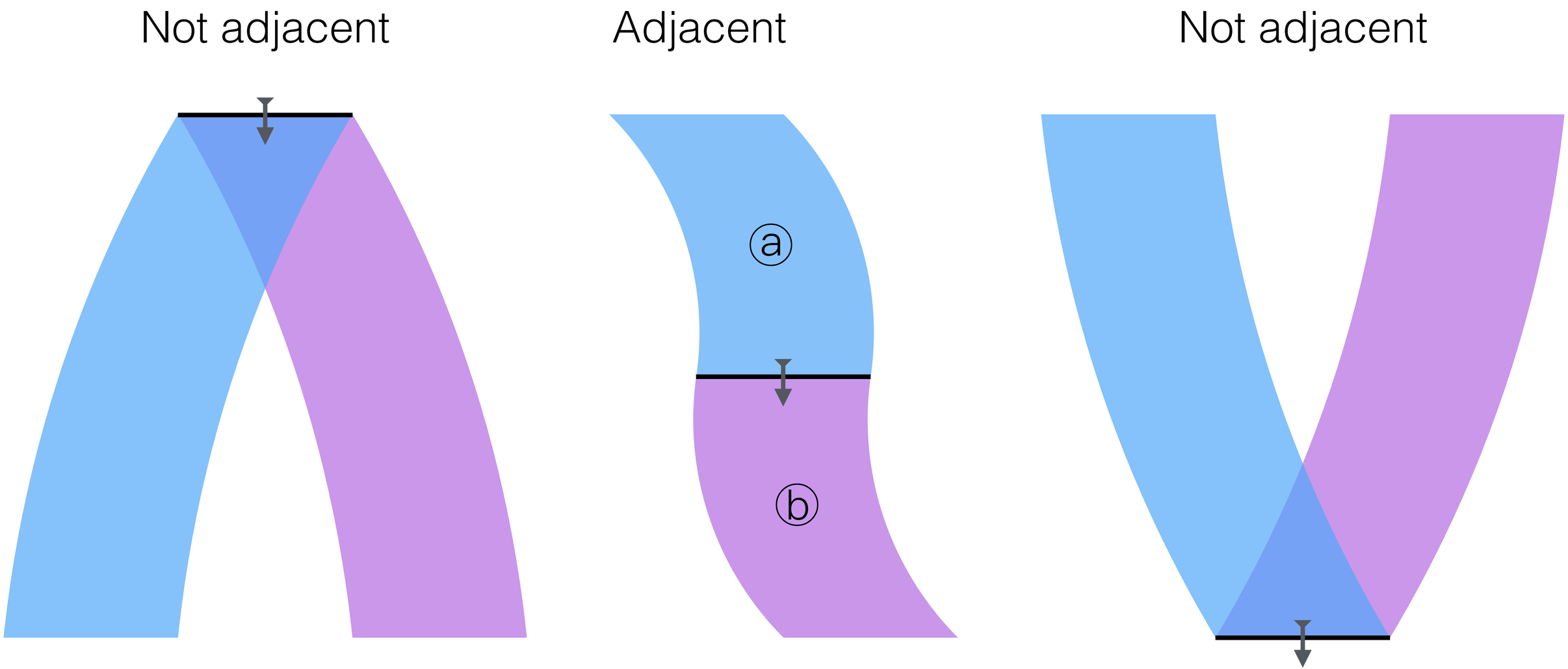
Backward closed sets include their entrances.



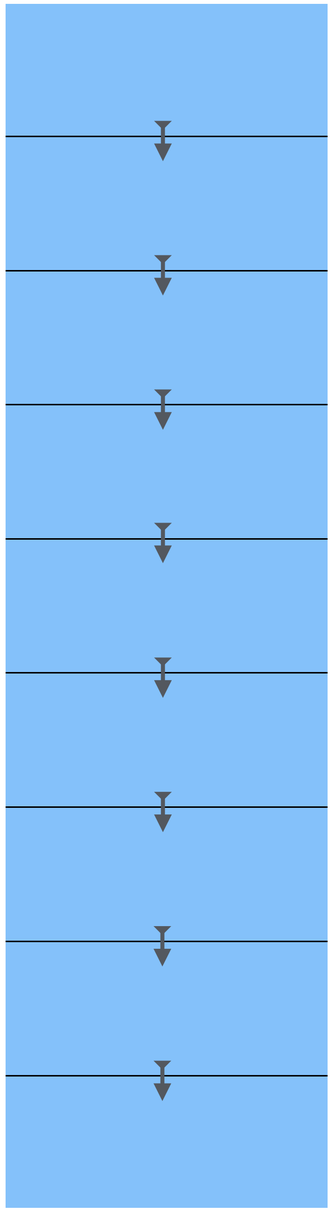


Backward open sets do not include their entrances.

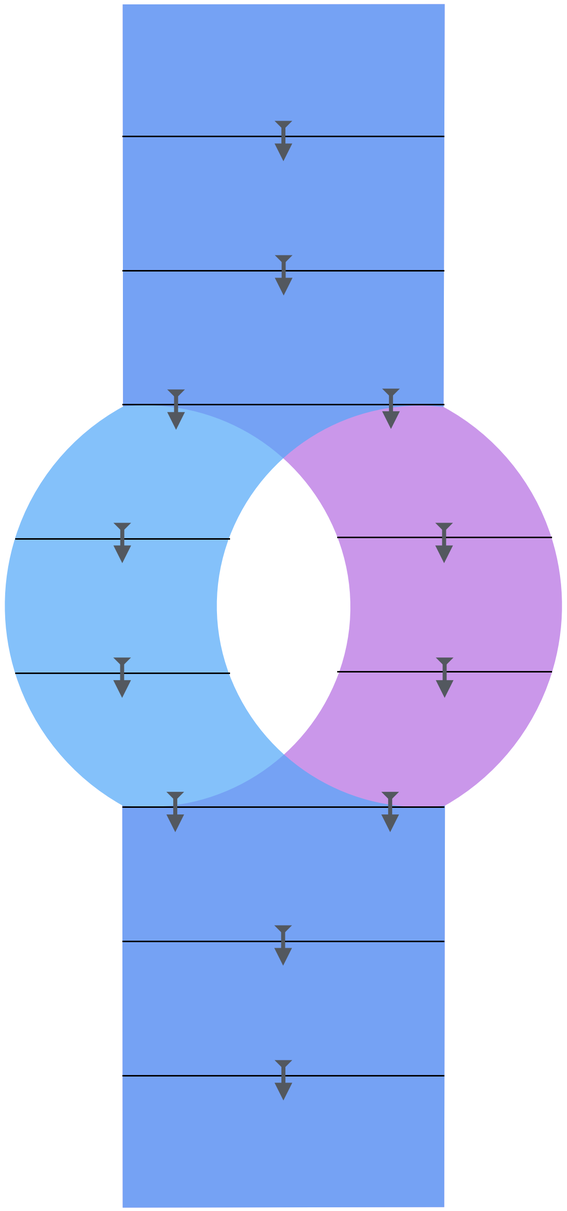
Forward open sets do not include their exits.


Not adjacent
Adjacent
Not adjacent
ⓐ
ⓑ
ⓐ immediately precedes ⓑ

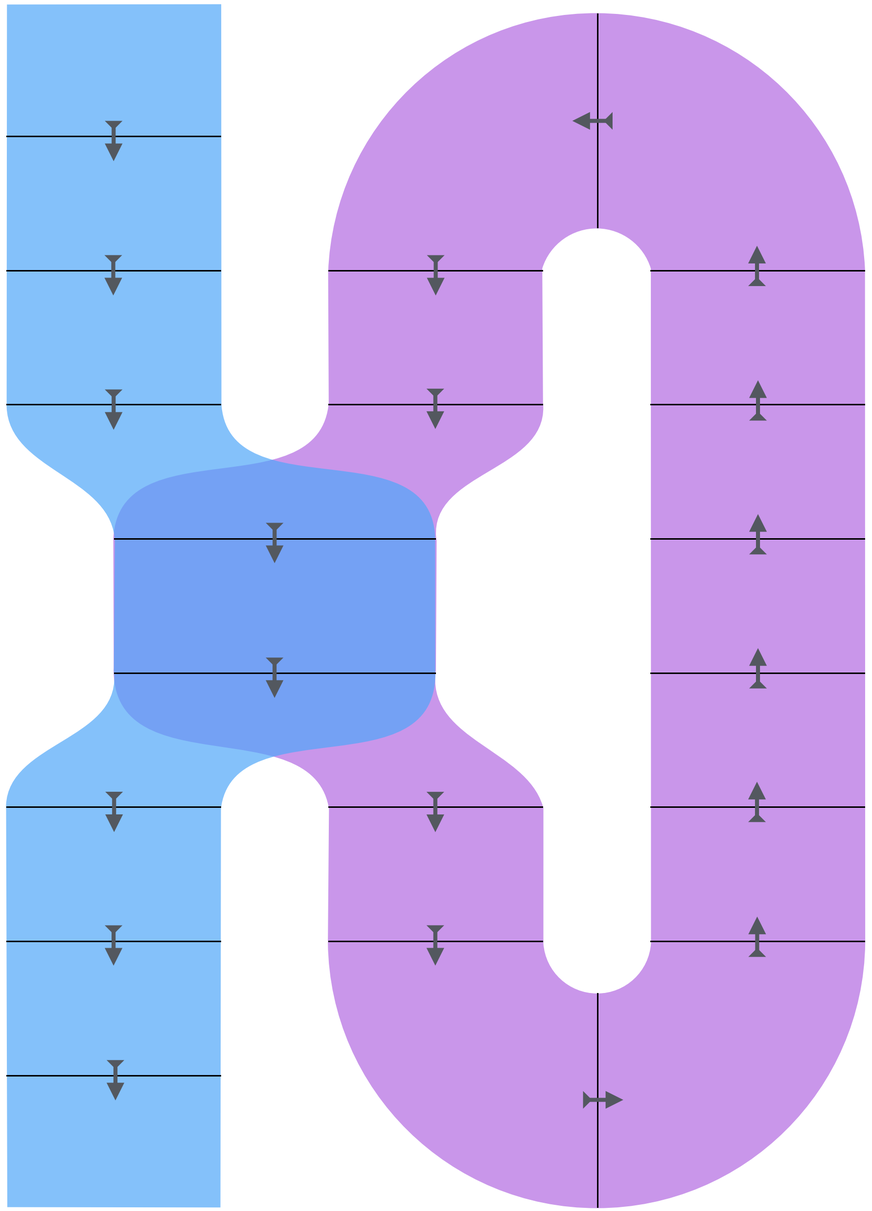
Sequence

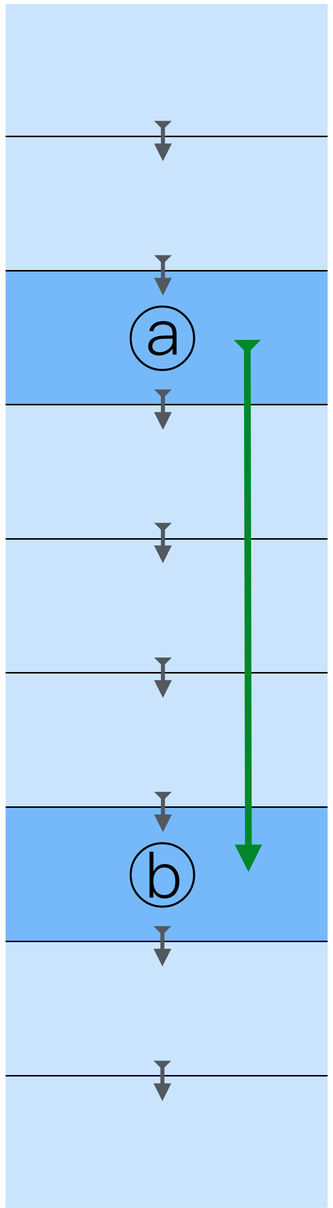


Branch



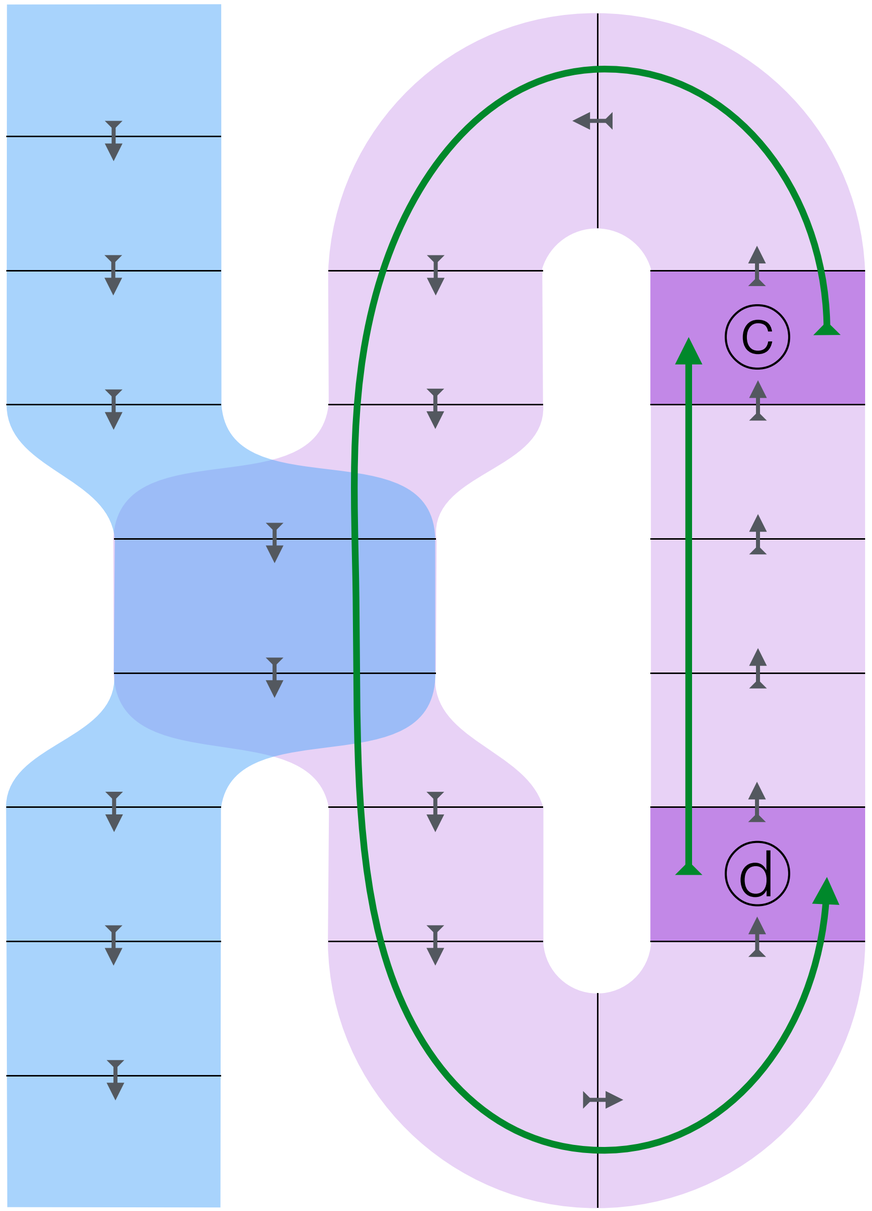
Loop



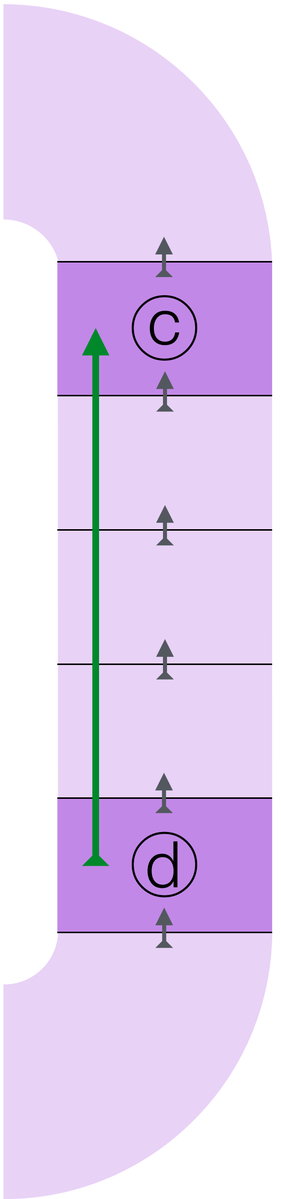


ⓐ is before ⓑ:
there is a connection from ⓐ to ⓑ, but
there is no connection from ⓑ to ⓐ.

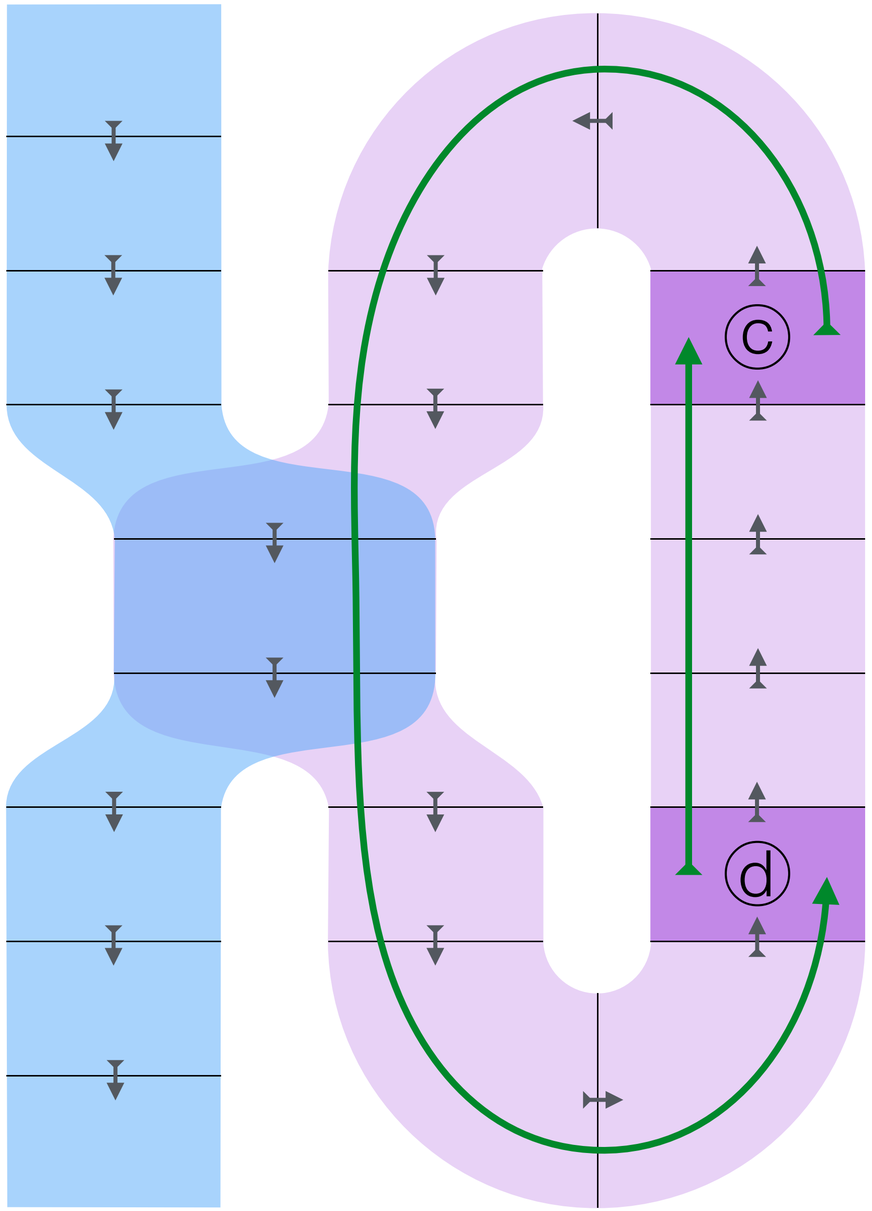
ⓒ is both before and after ⓓ:
there is a connection from ⓒ to ⓓ, and
there is also a connection from ⓓ to ⓒ.

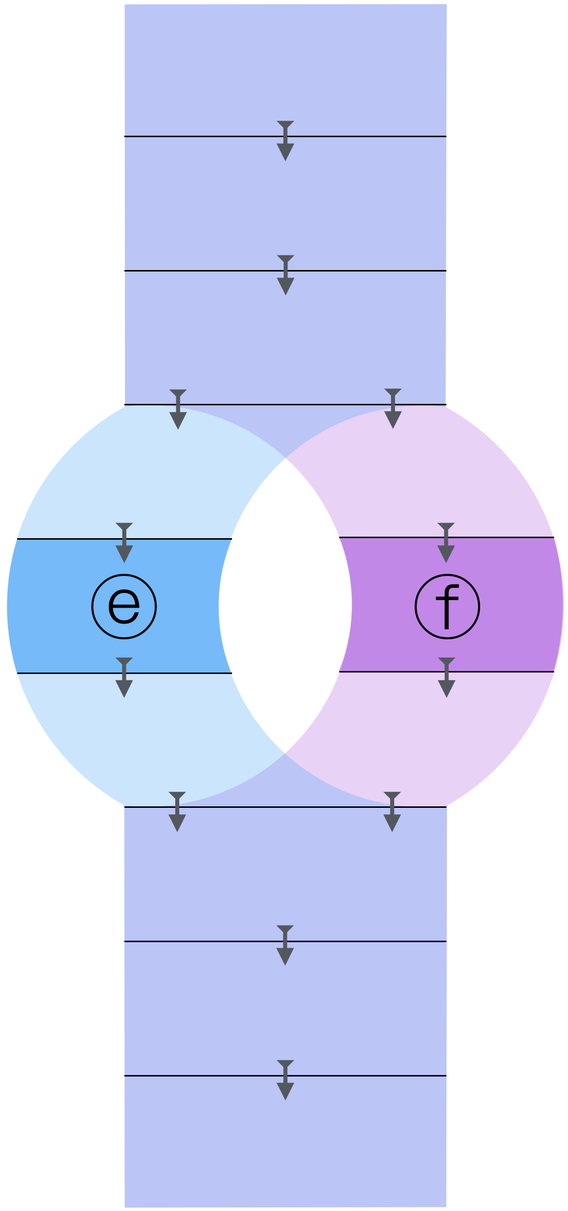


But in this smaller neighborhood, ⓒ is after ⓓ:
there is no connection from ⓒ to ⓓ, but
there is a connection from ⓓ to ⓒ.

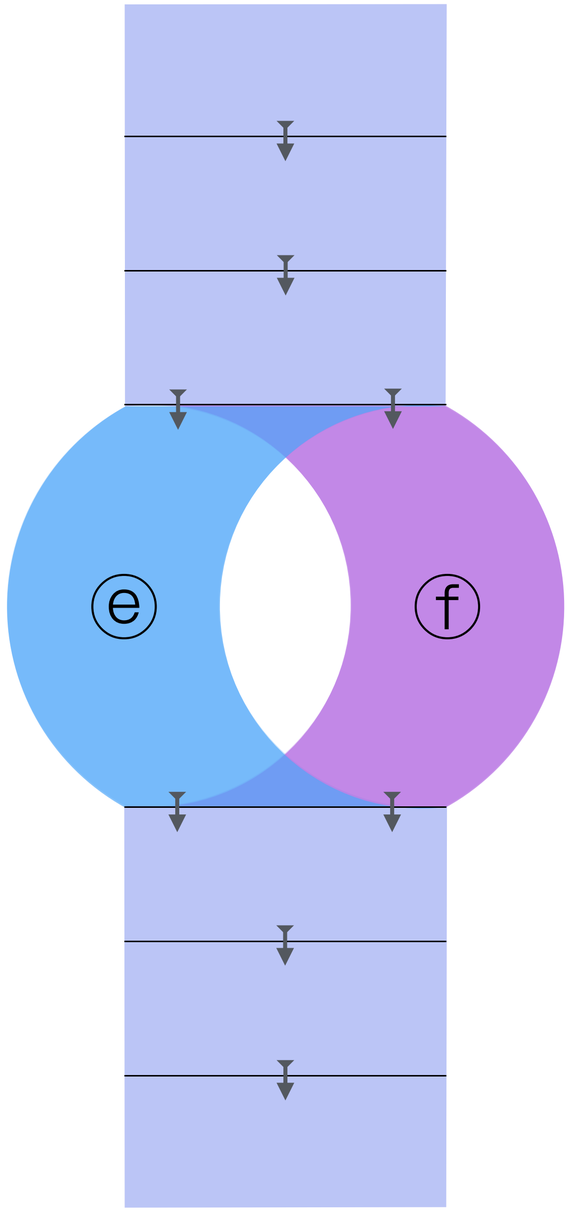


ⓒ is both before and after ⓓ:
there is a connection from ⓒ to ⓓ, and
there is also a connection from ⓓ to ⓒ.



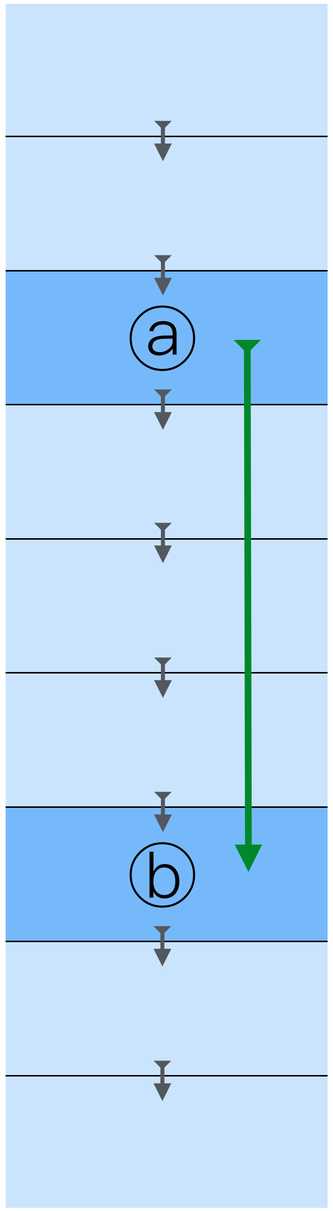


ⓔ and ⓕ are alternative possibilities:
there is a connection neither
from ⓔ to ⓕ nor from ⓕ to ⓔ.

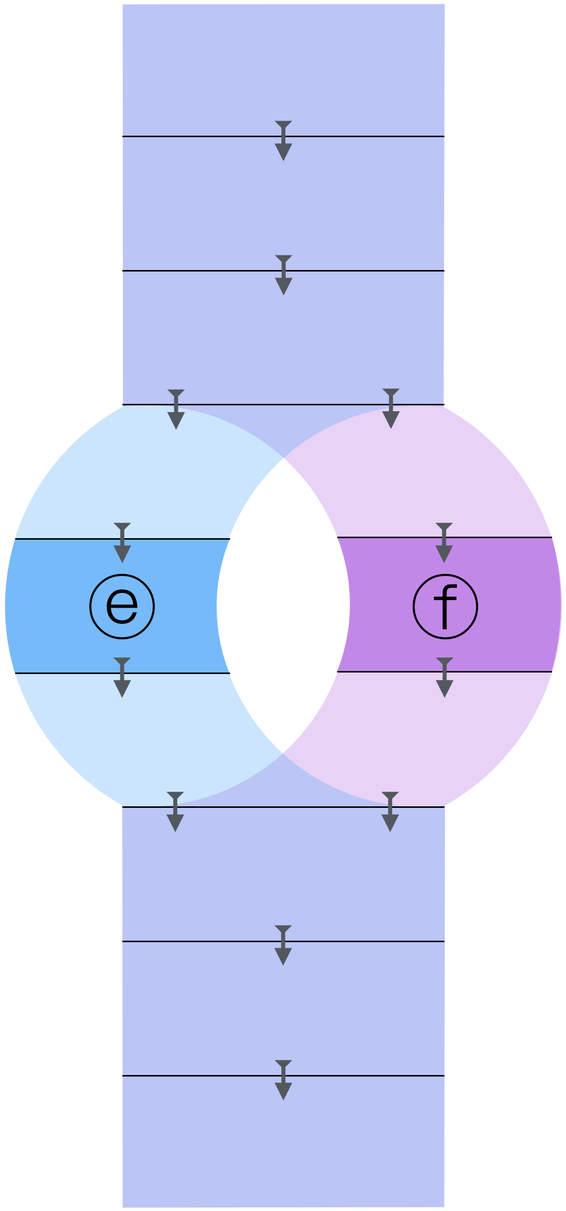


If we expand ⓔ and ⓕ to share entrances and exits, they remain alternative possibilities.

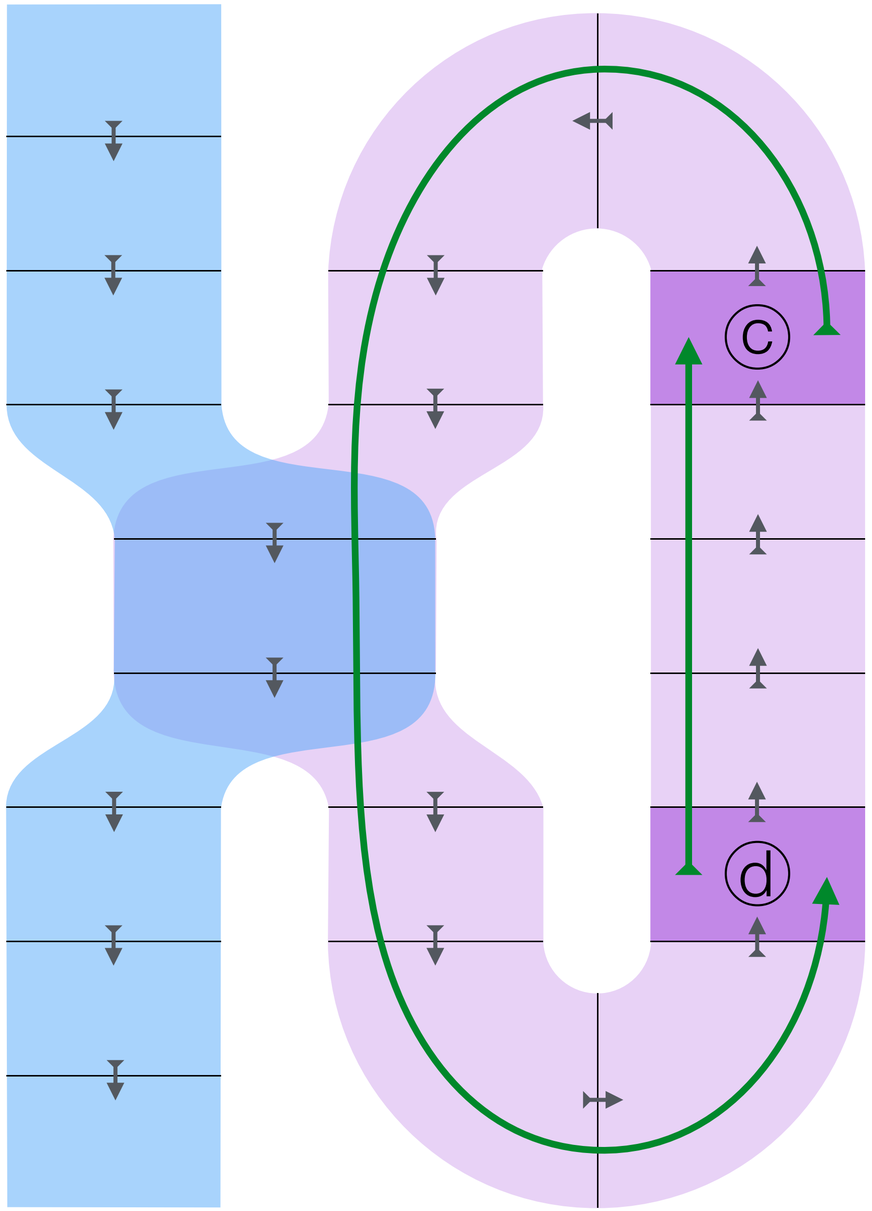
ⓐ is before ⓑ

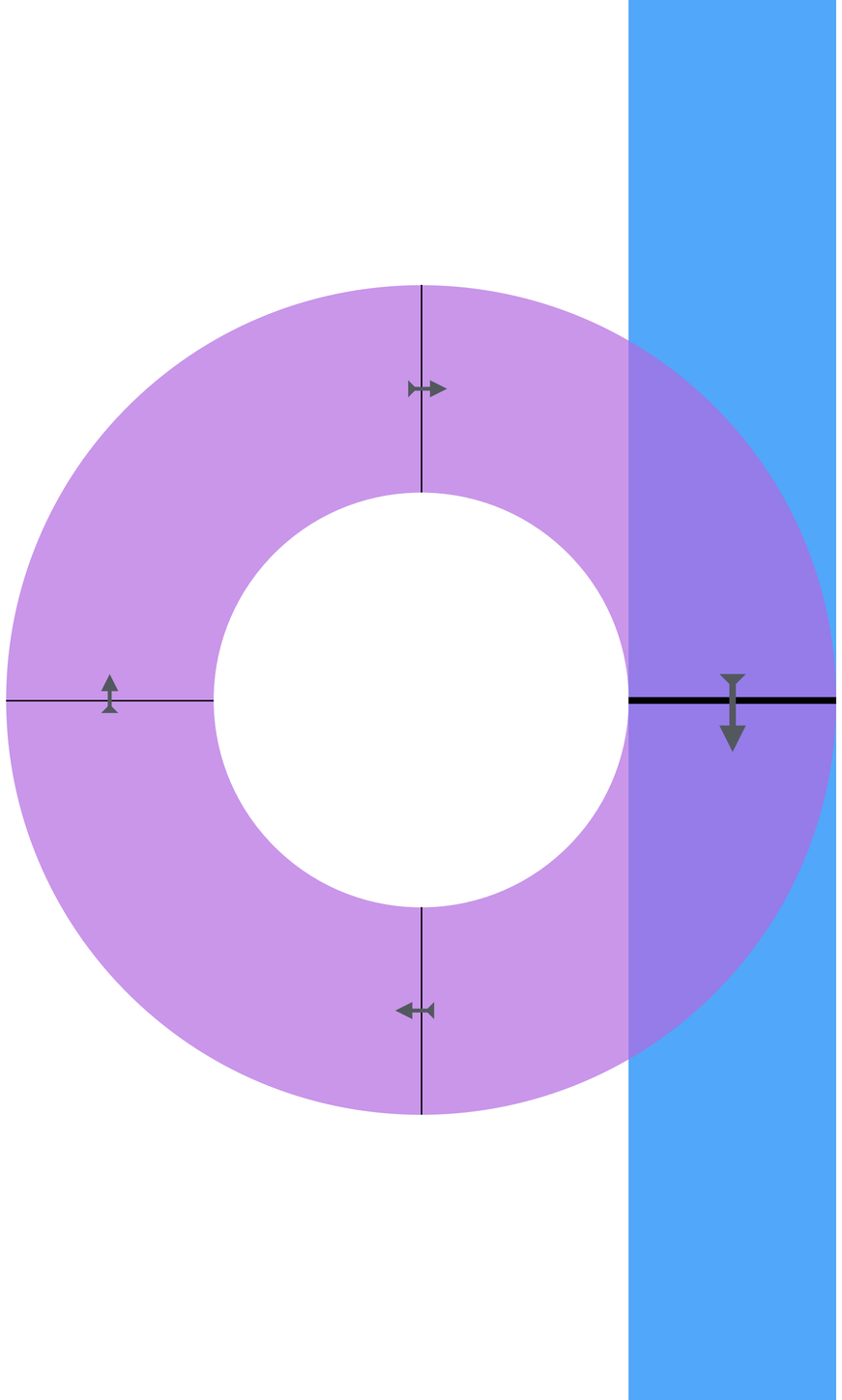


ⓔ is alternative to ⓕ



ⓒ is both before and after ⓓ





Assertions are experiments.

Successful assertions are repeatable, and have no meaningful effect.

Assertion edge

An assertion describes its edge, in dimensions of space and possibility.

Some things need to be asserted, but not govern branches:

readable(const T&)
writable(T&)

destructible(T&)

deallocatable(void *, size_t)
array_deallocatable(void *, size_t)

exception_is_rethrowable()

dynamic_type_identifiable(T&)

dereferencable(Iterator)
reachable(Iterator, Iterator)

resizable(vector<T>&)
reallocatable(vector<T>&)

fclosable(int)

in_the_past(time_point<steady_clock>)

proper(T&)

Capabilities can be asserted, but can't govern branches:

readable(const T&)
writable(T&)

destructible(T&)

deallocatable(void *, size_t)
array_deallocatable(void *, size_t)

exception_is_rethrowable()

dynamic_type_identifiable(T&)

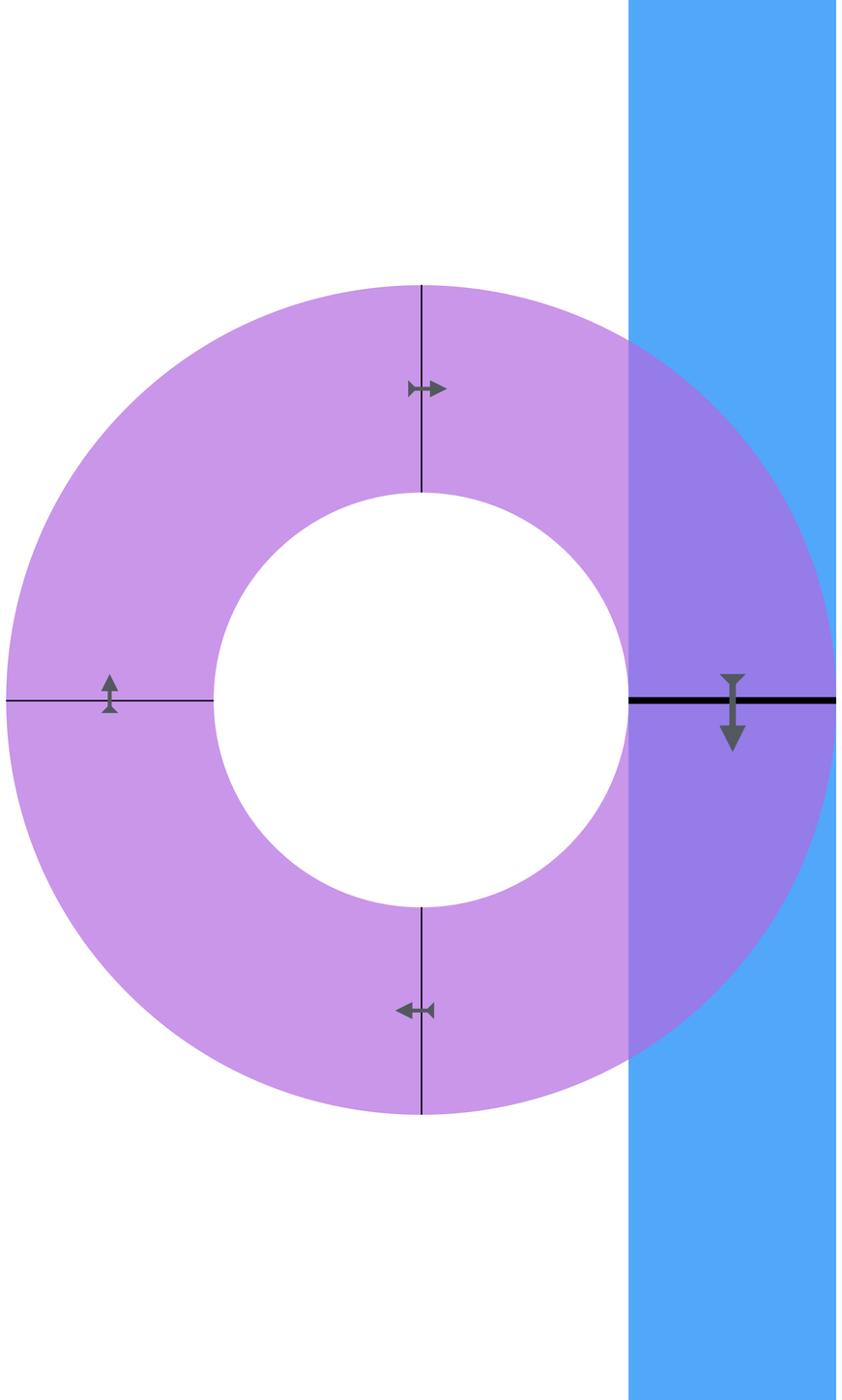
dereferencable(Iterator)
reachable(Iterator, Iterator)

resizable(vector<T>&)
reallocatable(vector<T>&)

fclosable(int)

~~in_the_past(time_point<steady_clock>)~~
memorable(time_point<steady_clock>)

~~proper(T&)~~
usable(T&)



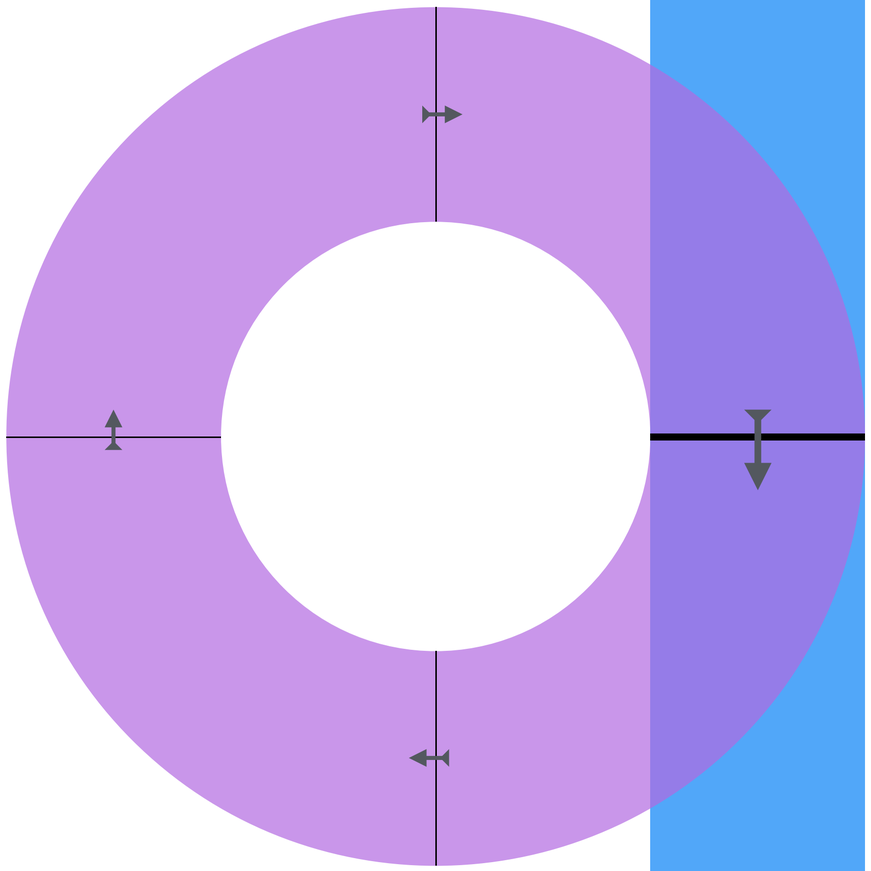
Assertions are experiments.

Successful assertions are repeatable, and have no meaningful effect.

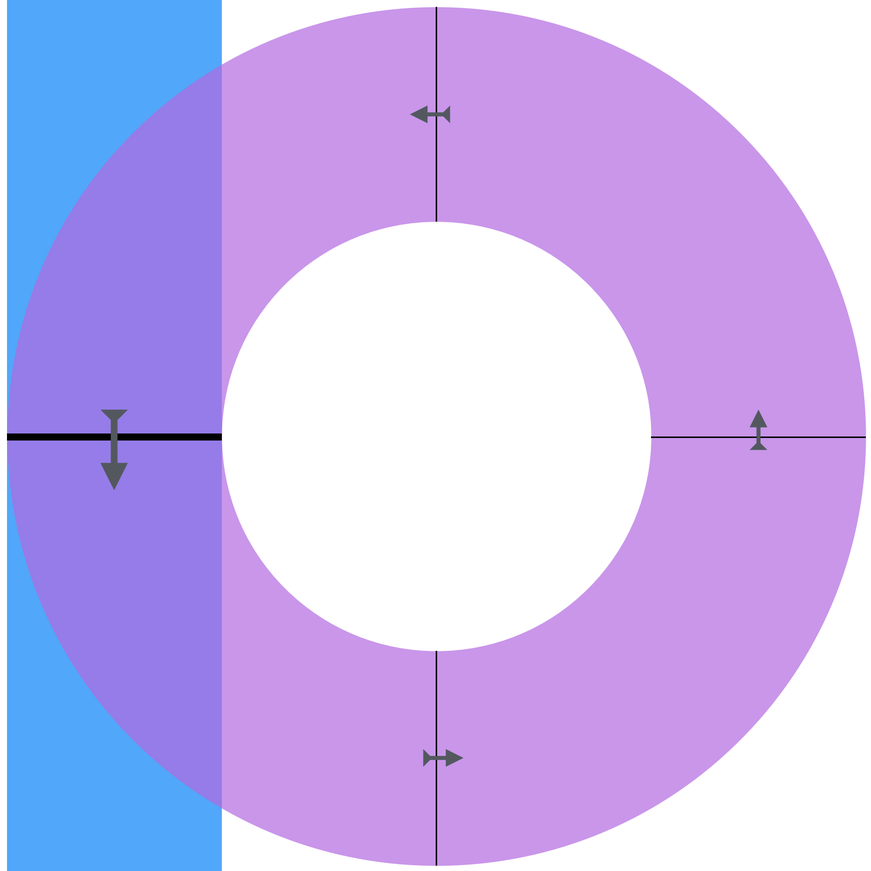
Assertion edge

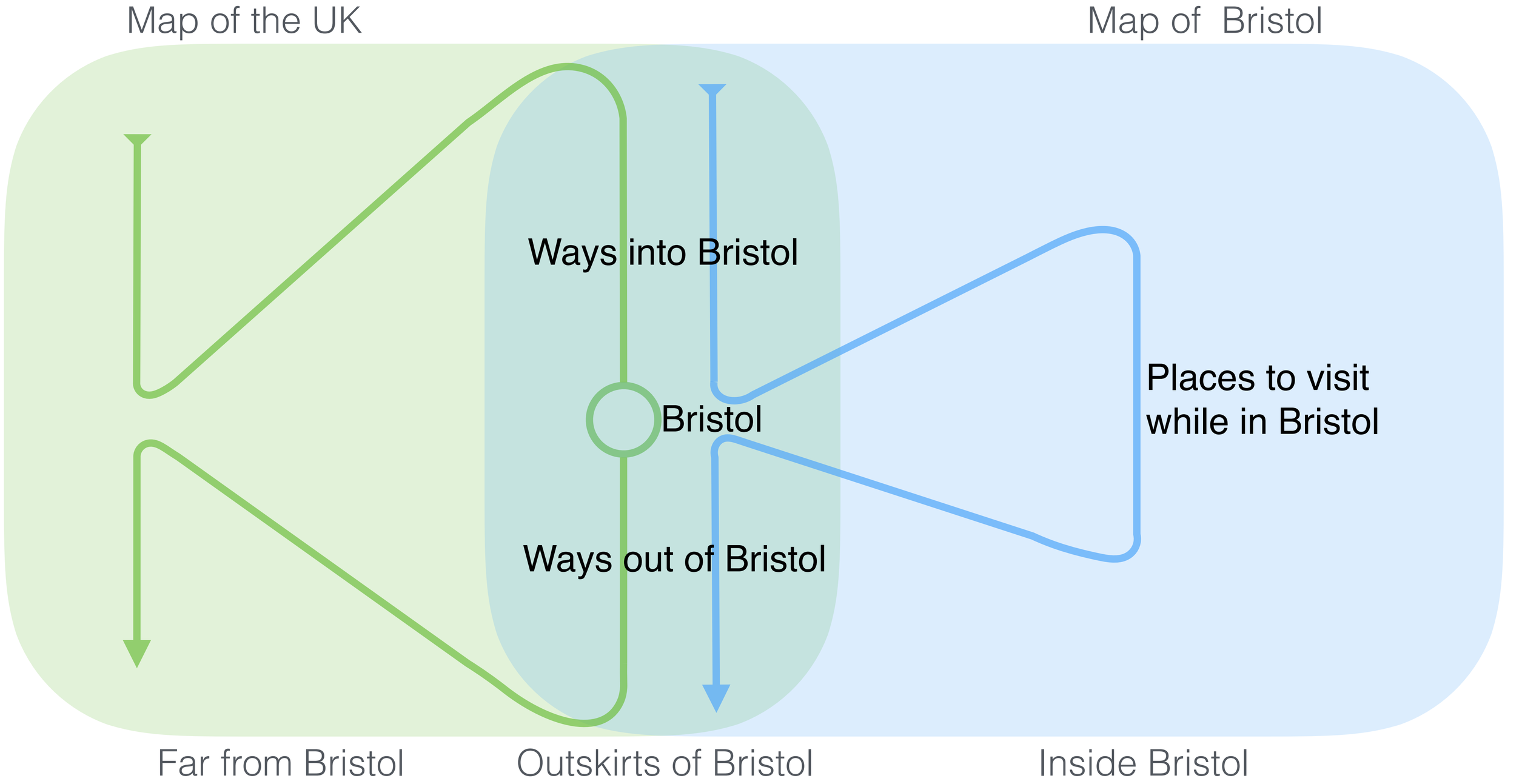
An assertion describes its edge, in dimensions of space and possibility.

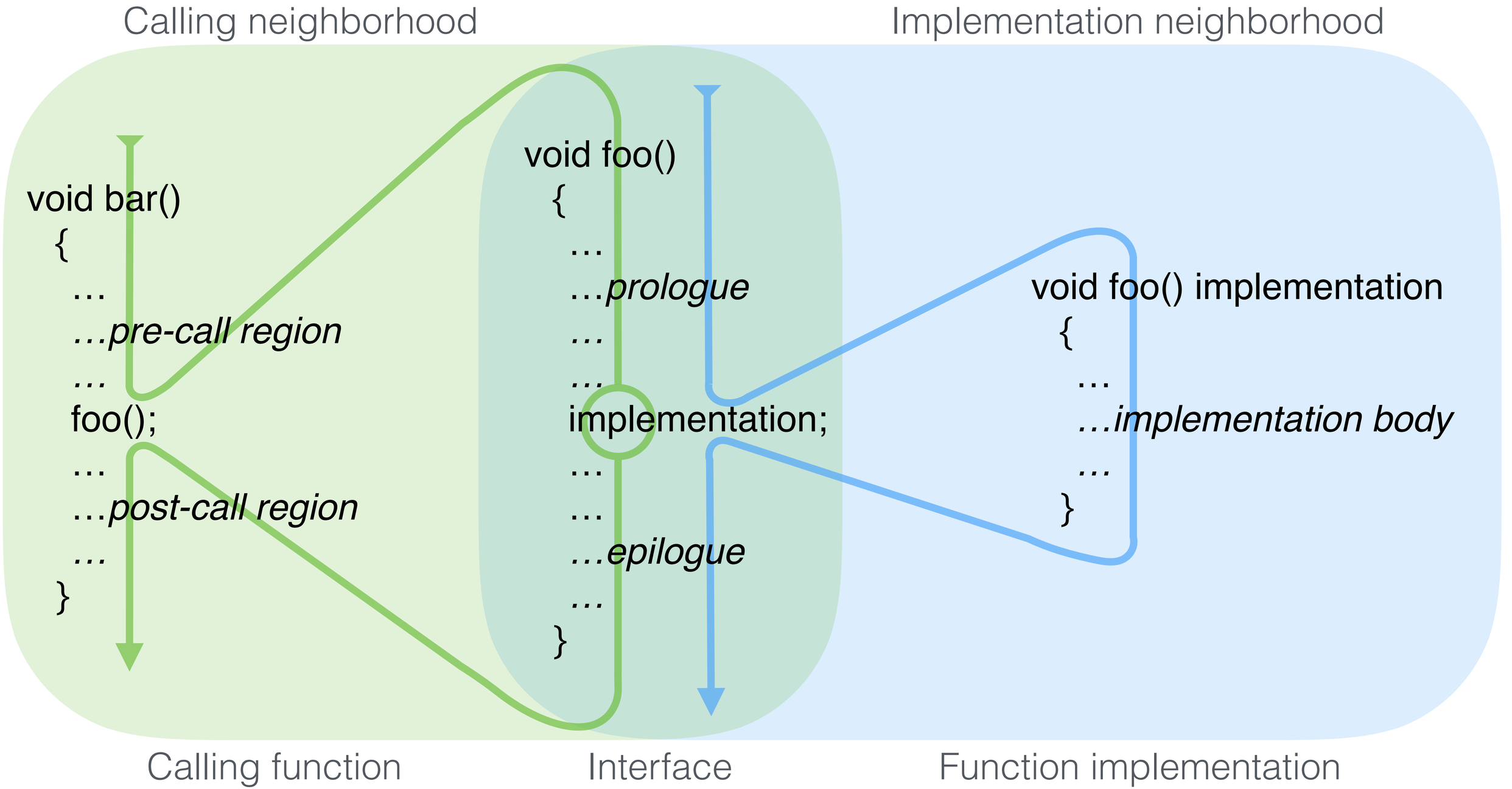
Claimed assertion
(proof is local)



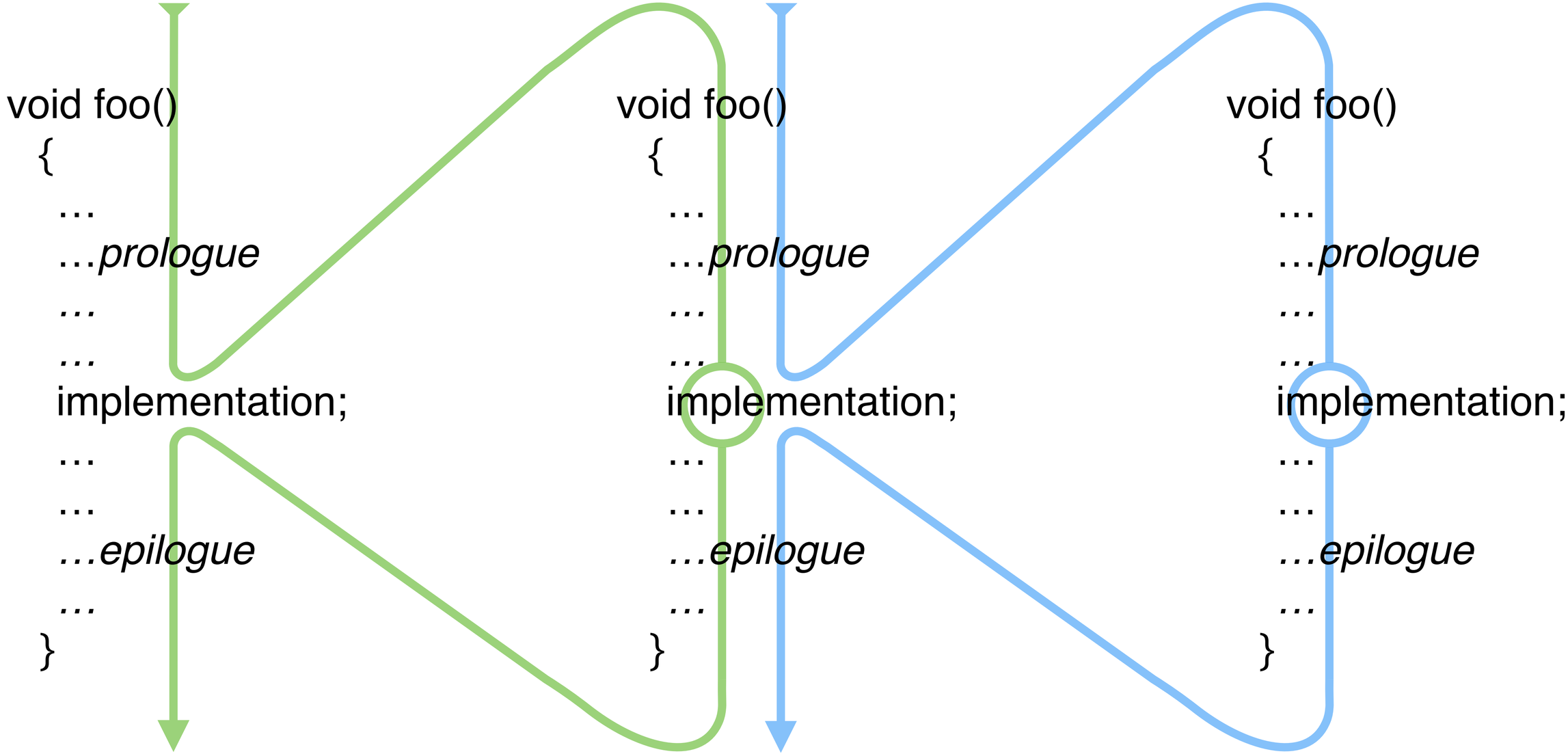
Posited assertion
(proof is elsewhere)

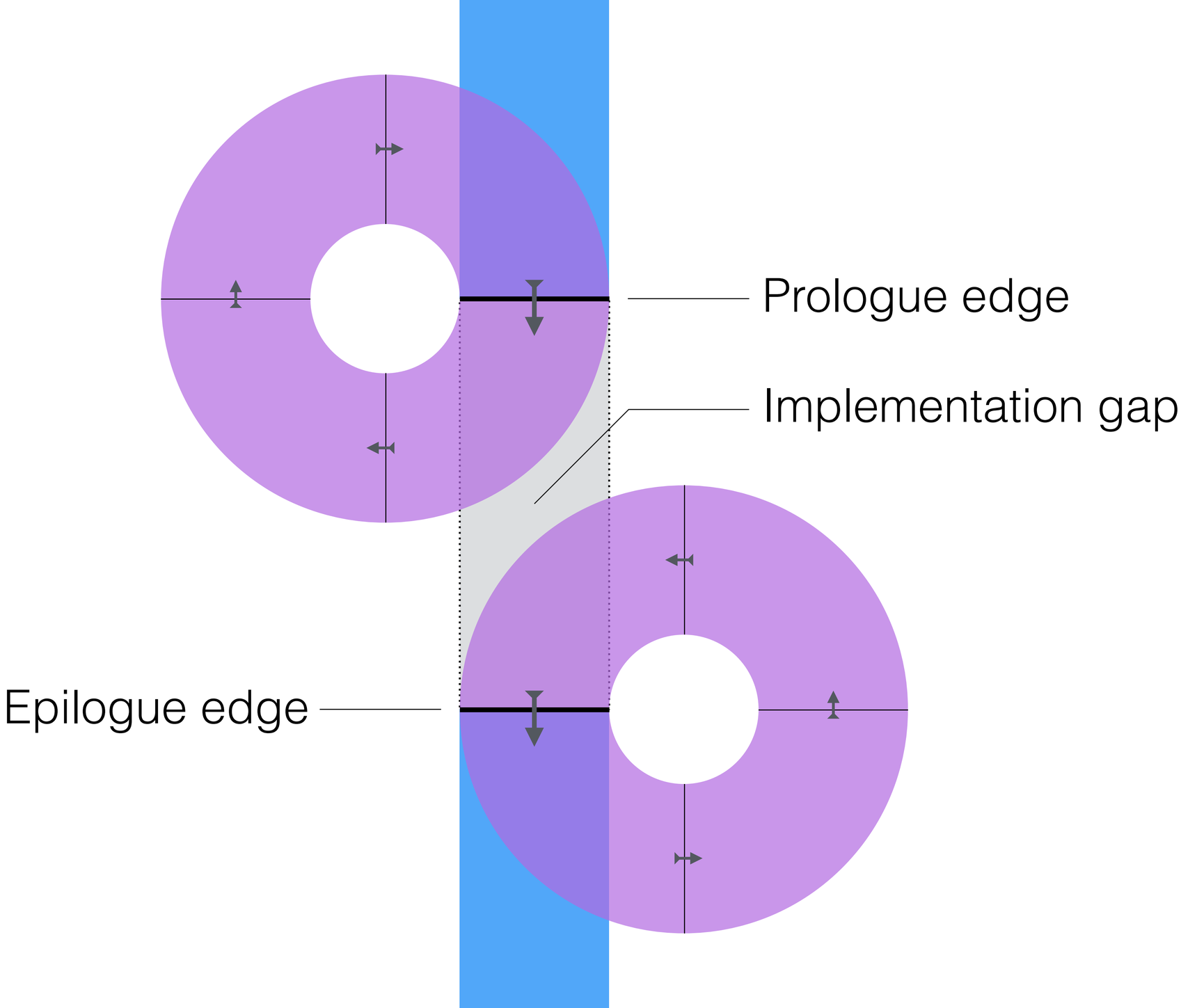


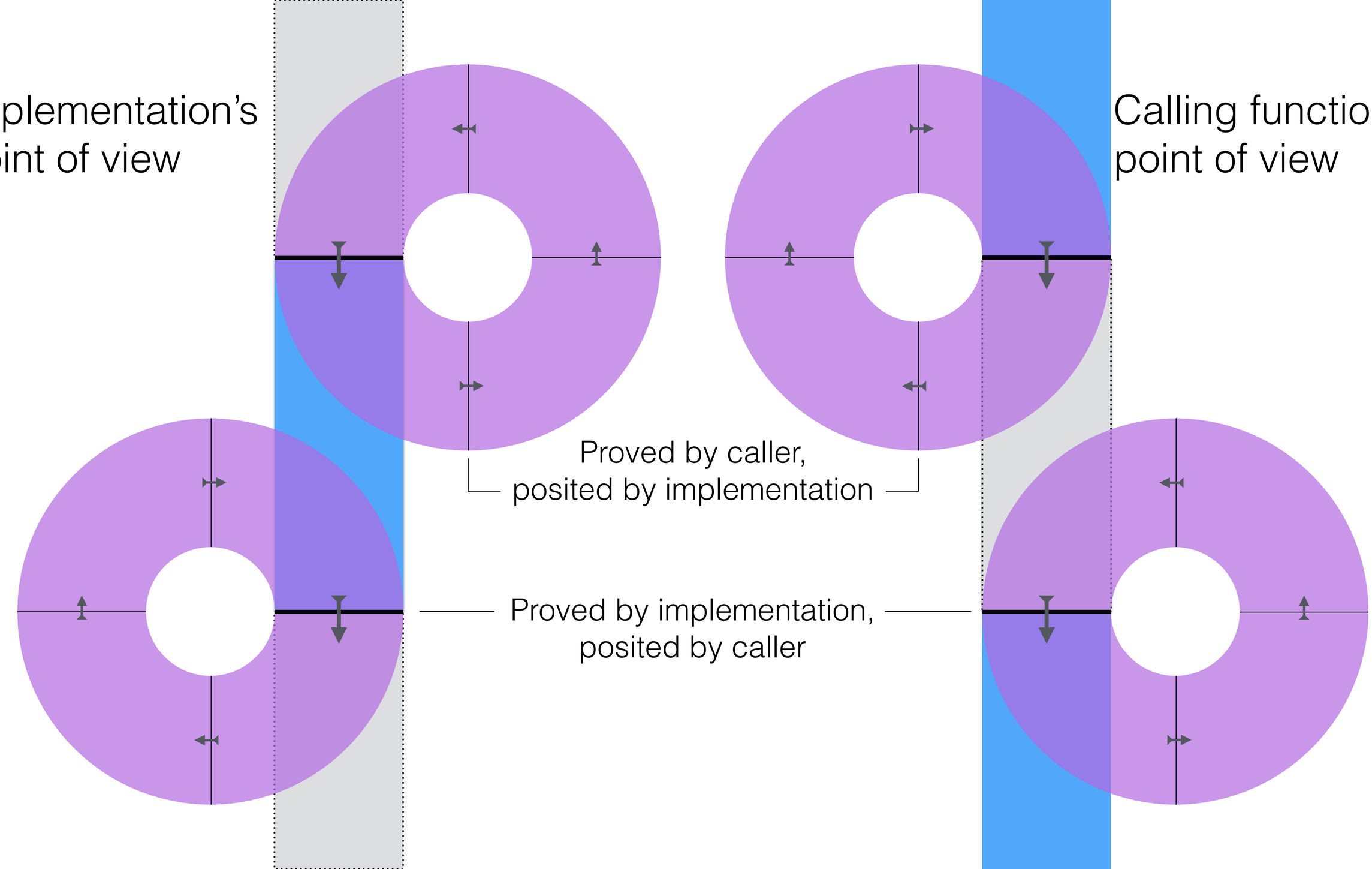

Map of the UK
Map of Bristol
Ways into Bristol
Bristol
Ways out of Bristol
Places to visit
while in Bristol
Far from Bristol
Outskirts of Bristol
Inside Bristol


Calling neighborhood
Implementation neighborhood
void bar()
{
…
…pre-call region
…
foo();
…
…post-call region
…
}
void foo()
{
…
…prologue
…
…
implementation;
…
…
…epilogue
…
}
void foo() implementation
{
…
…implementation body
…
}
Calling function
Interface
Function implementation

A function interface is an experiment in two parts, nestable within itself.

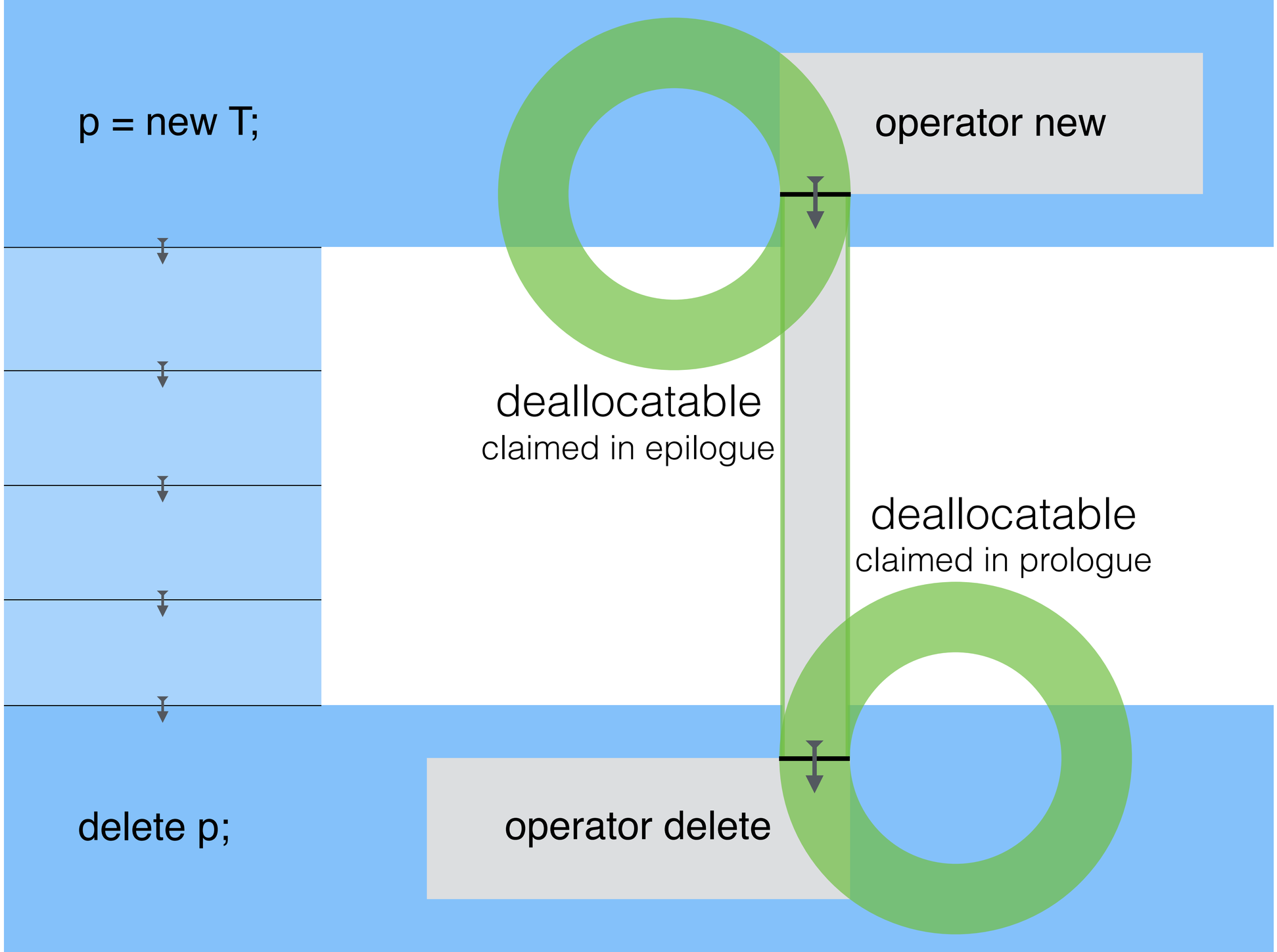



Prologue edge
Implementation gap
Epilogue edge

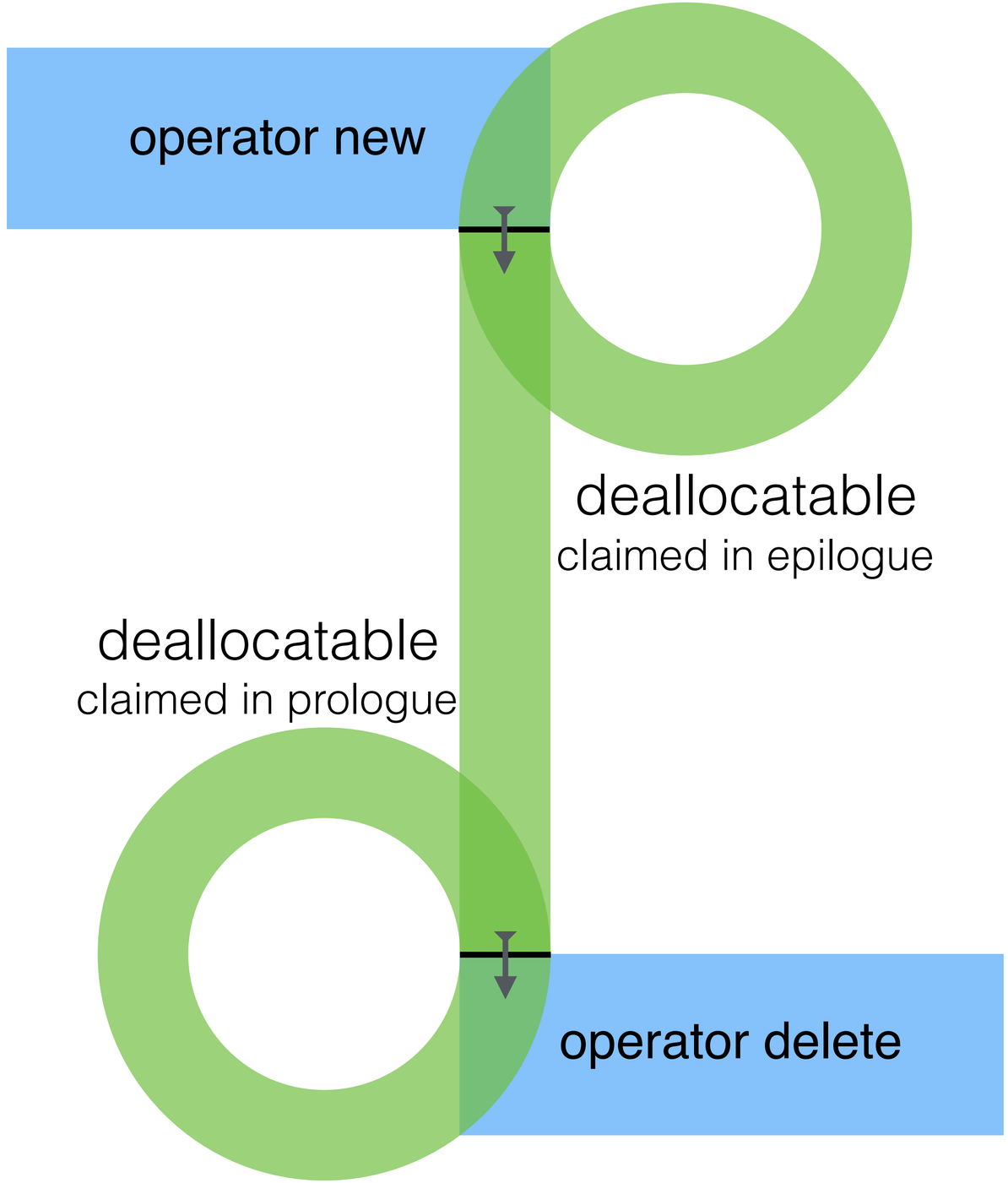

Implementation's point of view
Calling function's point of view
Proved by caller, posited by implementation
Proved by implementation, posited by caller

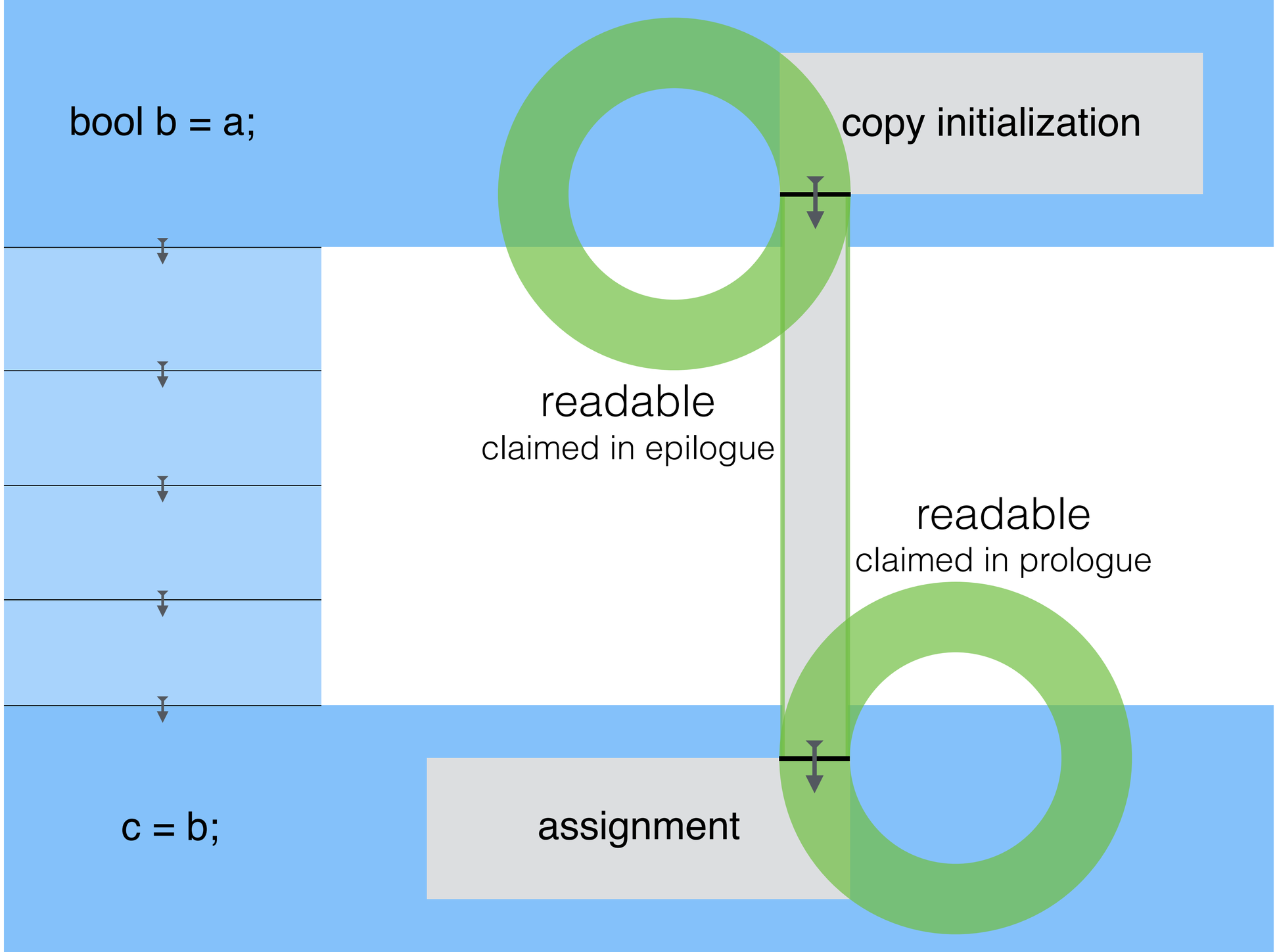
```
void *operator new( size_t s )
  {
   …
   implementation;
   …
   claim deallocatable( result, s );
   …
  }
```

```
void operator delete( void *p, size_t s )
  {
   …
   claim deallocatable( p, s );
   …
   implementation;
   …
  }
```

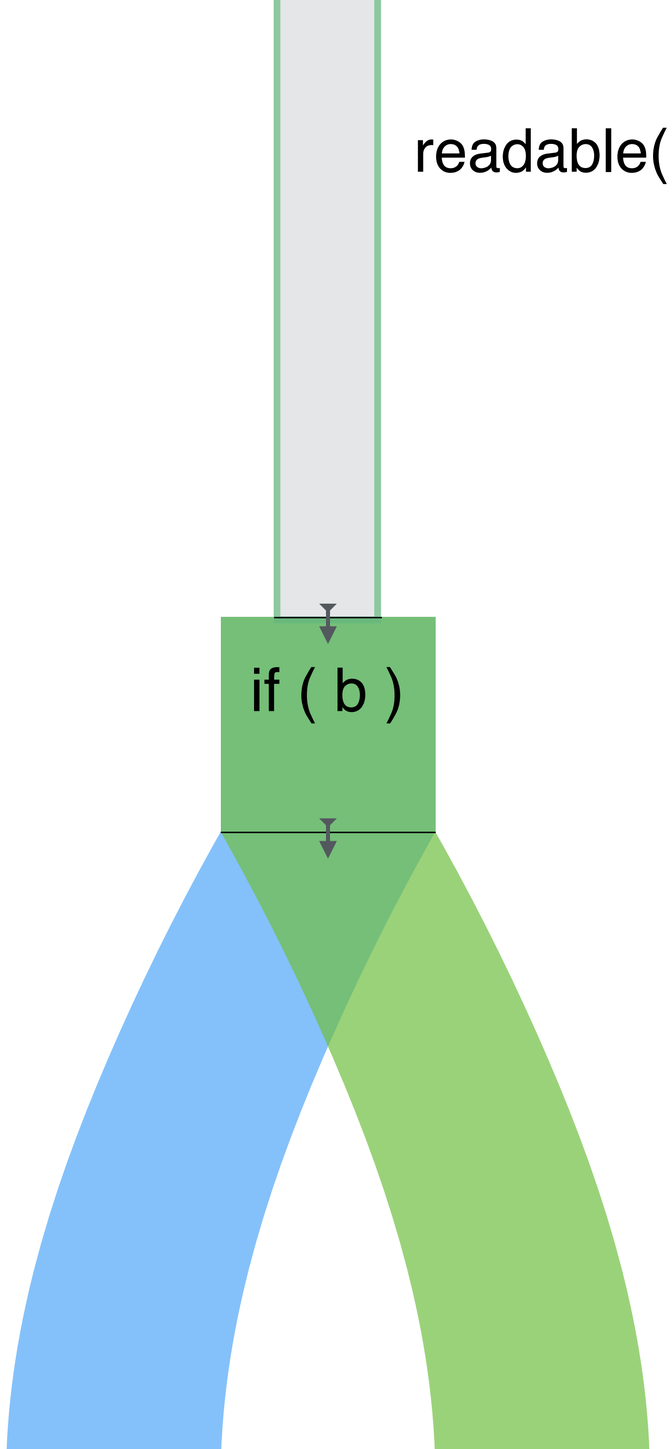

p = new T;
operator new
deallocatable
claimed in epilogue
deallocatable
claimed in prologue
delete p;
operator delete

Heap implementation
neighborhood
(partial)

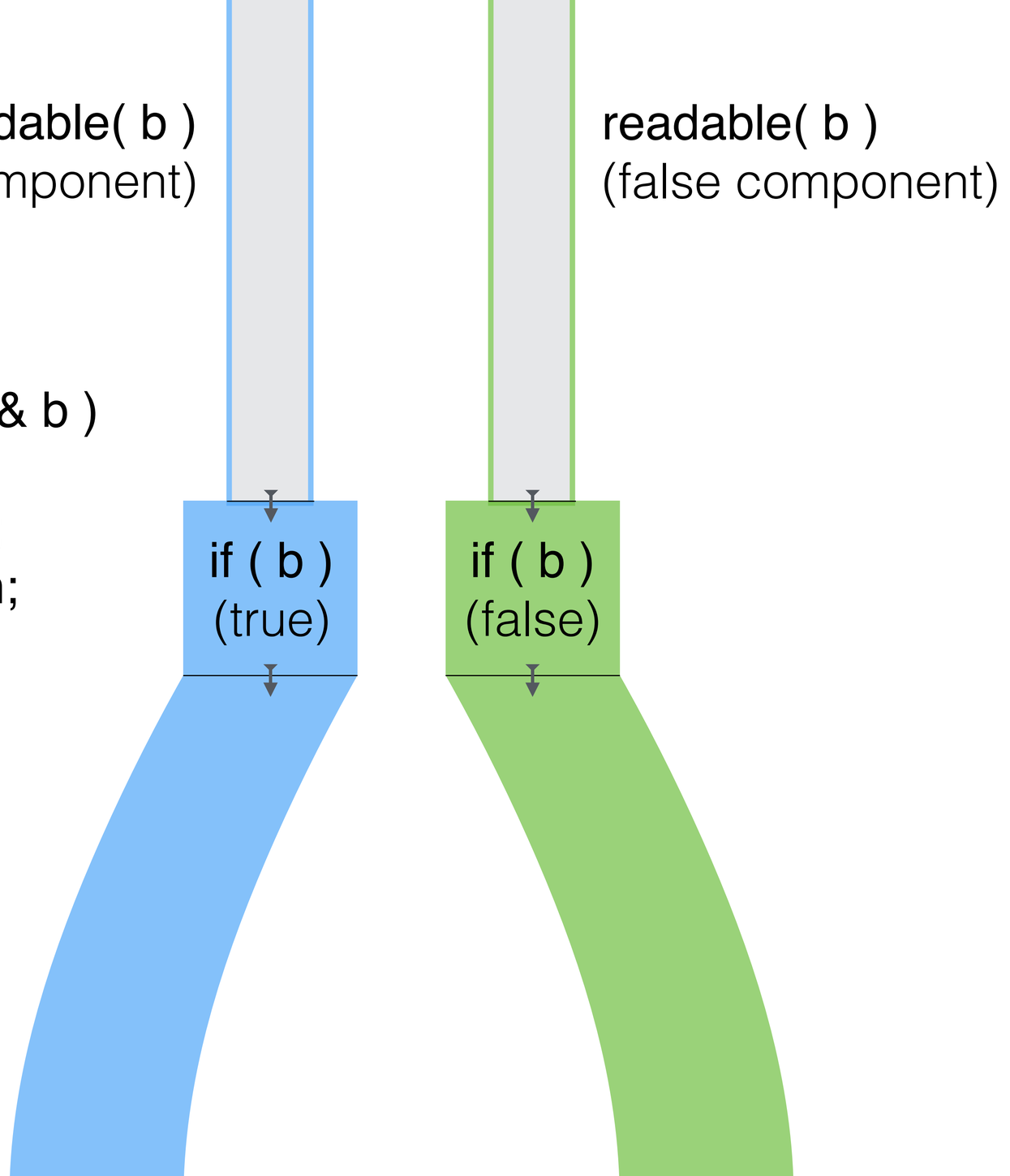



bool b = a;
copy initialization
readable
claimed in epilogue
readable
claimed in prologue
c = b;
assignment

```
void readable( const bool& b )
  {
   claim addressable( b );
   require implementation;
  }
```

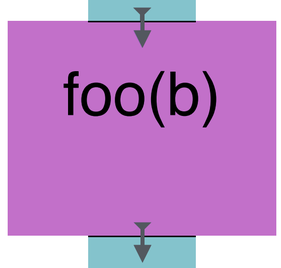


```
void readable( const bool& b )
  {
   claim addressable( b );
   require implementation;
  }
```



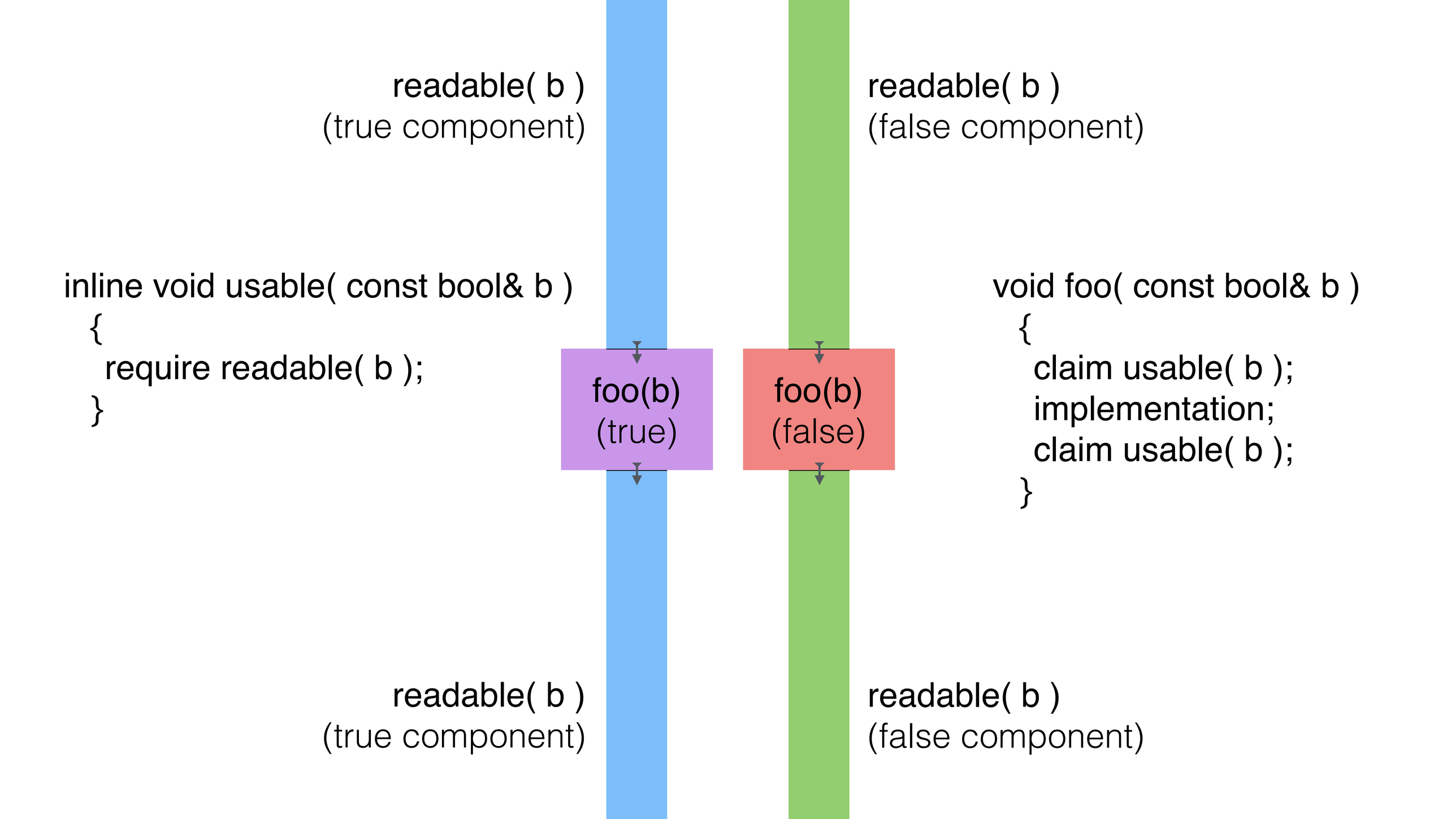
readable(b)

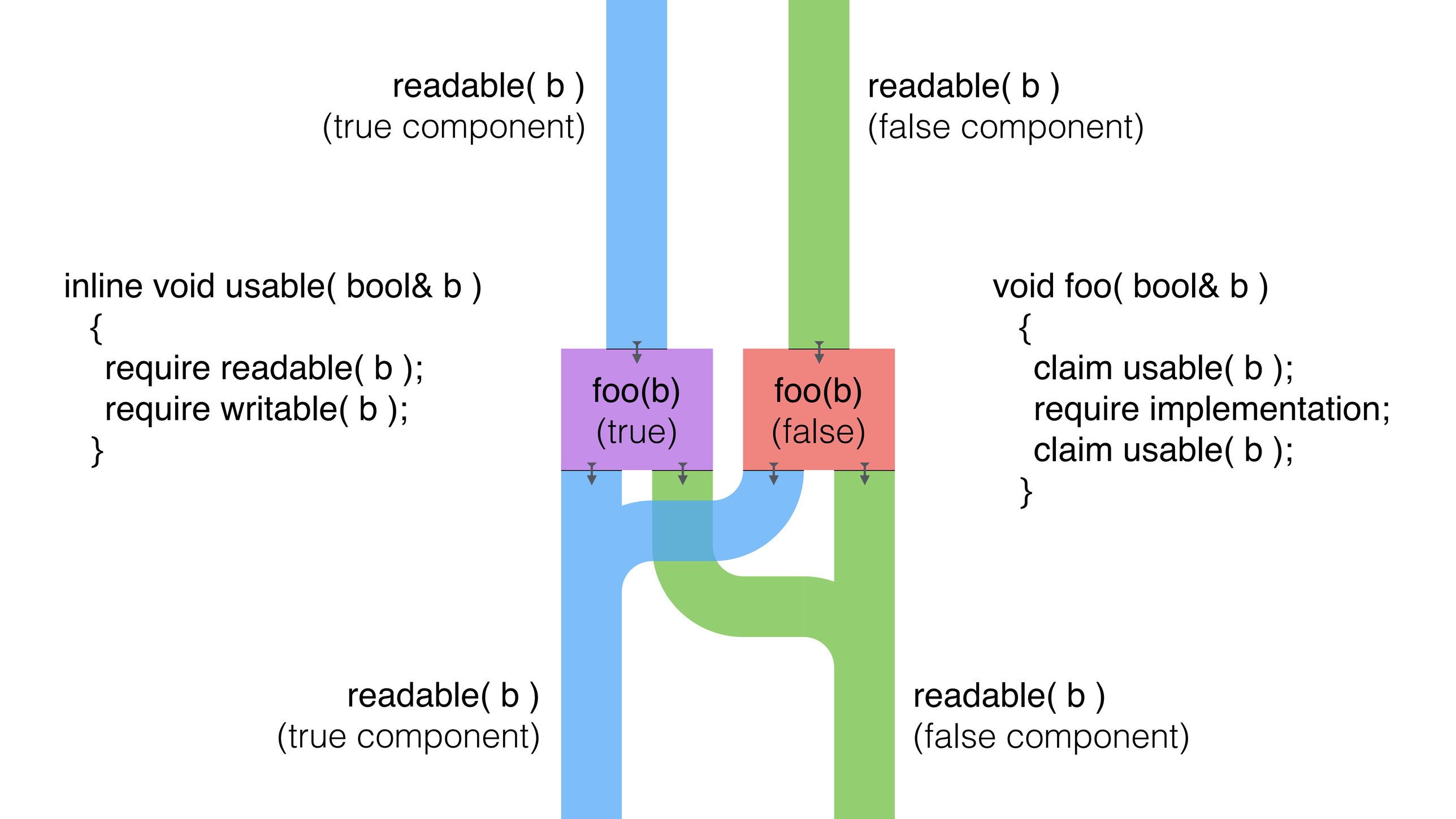
```
inline void usable( const bool& b )
  {
   require readable( b );
  }
```

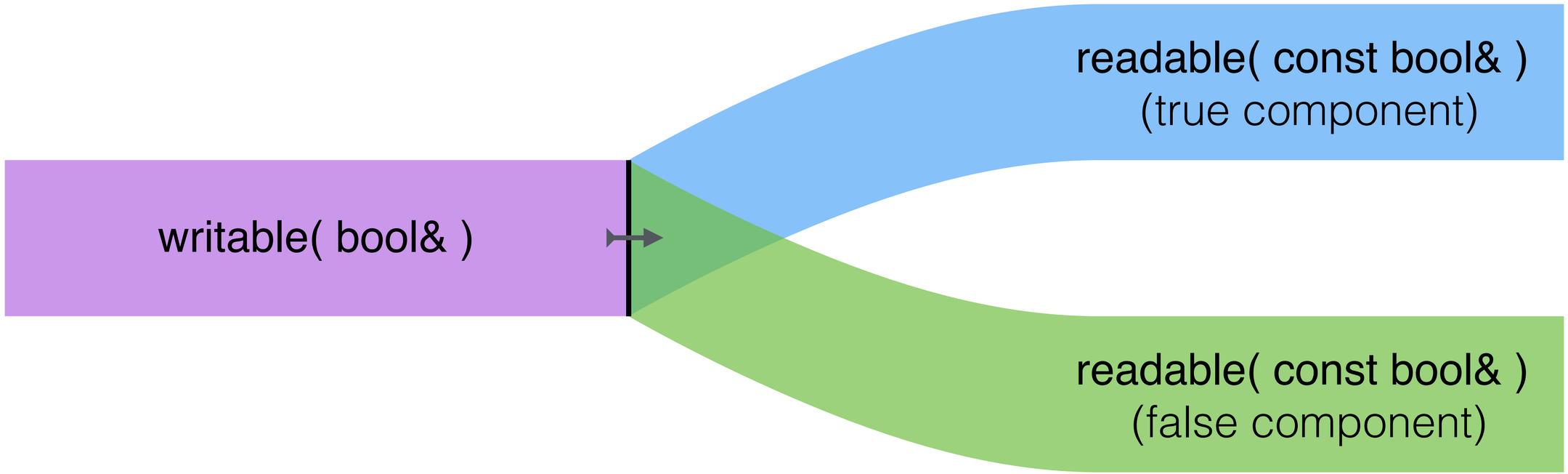



```
void foo( const bool& b )
  {
   claim usable( b );
   implementation;
   claim usable( b );
  }
```

readable(b)

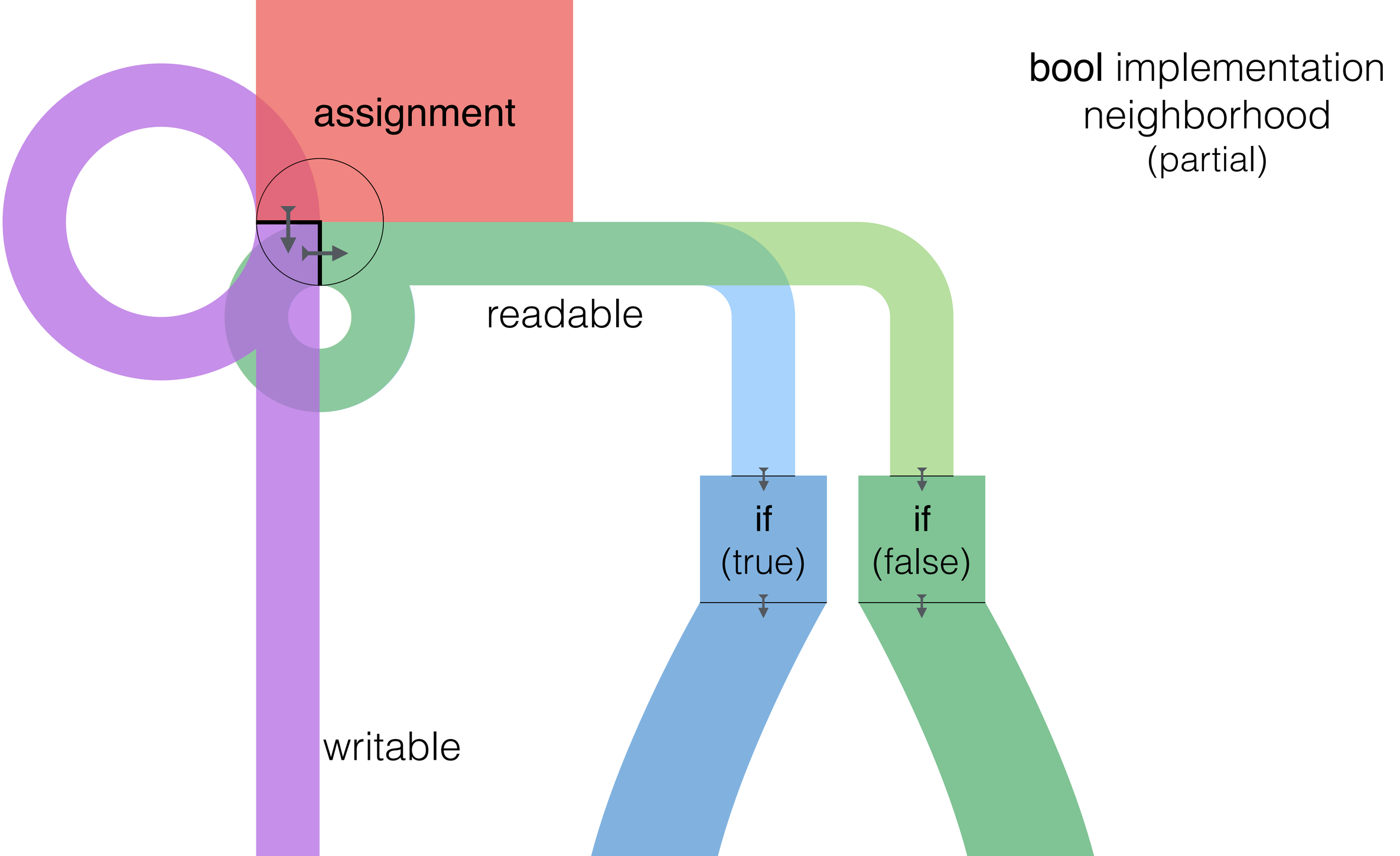

readable(b)
(true component)
readable(b)
(false component)
inline void usable(const bool& b)
{
require readable(b);
}
foo(b)
(true)
foo(b)
(false)
void foo(const bool& b)
{
claim usable(b);
implementation;
claim usable(b);
}
readable(b)
(true component)
readable(b)
(false component)


readable(b)
(true component)
readable(b)
(false component)
inline void usable(bool& b)
{
require readable(b);
require writable(b);
}
foo(b)
(true)
foo(b)
(false)
void foo(bool& b)
{
claim usable(b);
require implementation;
claim usable(b);
}
readable(b)
(true component)
readable(b)
(false component)




```
void writable( bool& b )
  {
   claim addressable( b );
   require implementation;
   require readable( b );
  }
```

```
void readable( const bool& b )
  {
   claim addressable( b );
   require implementation;
  }
```



bool implementation
neighborhood
(partial)

Category of measurements

exemplar functor

Category of things to measure

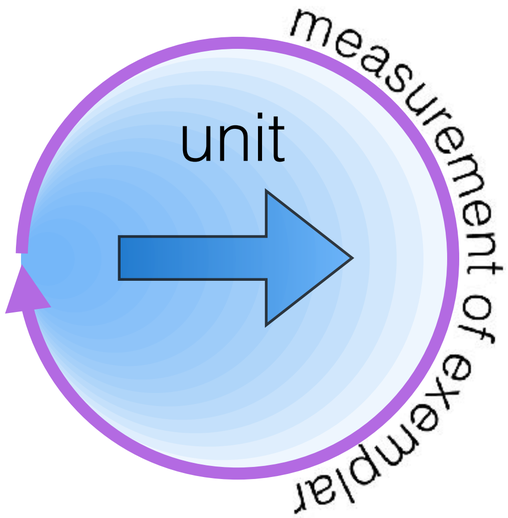
Sets
&functions

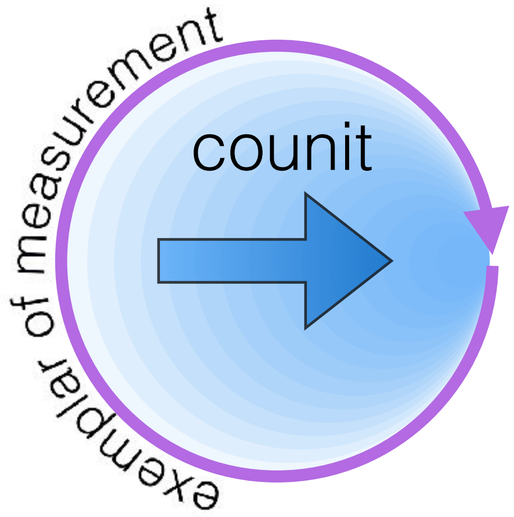
discrete space

set of points

Topological spaces
&continuous functions

measurement functor





Sets
&functions

discrete space

set of points

Topological spaces
&continuous functions

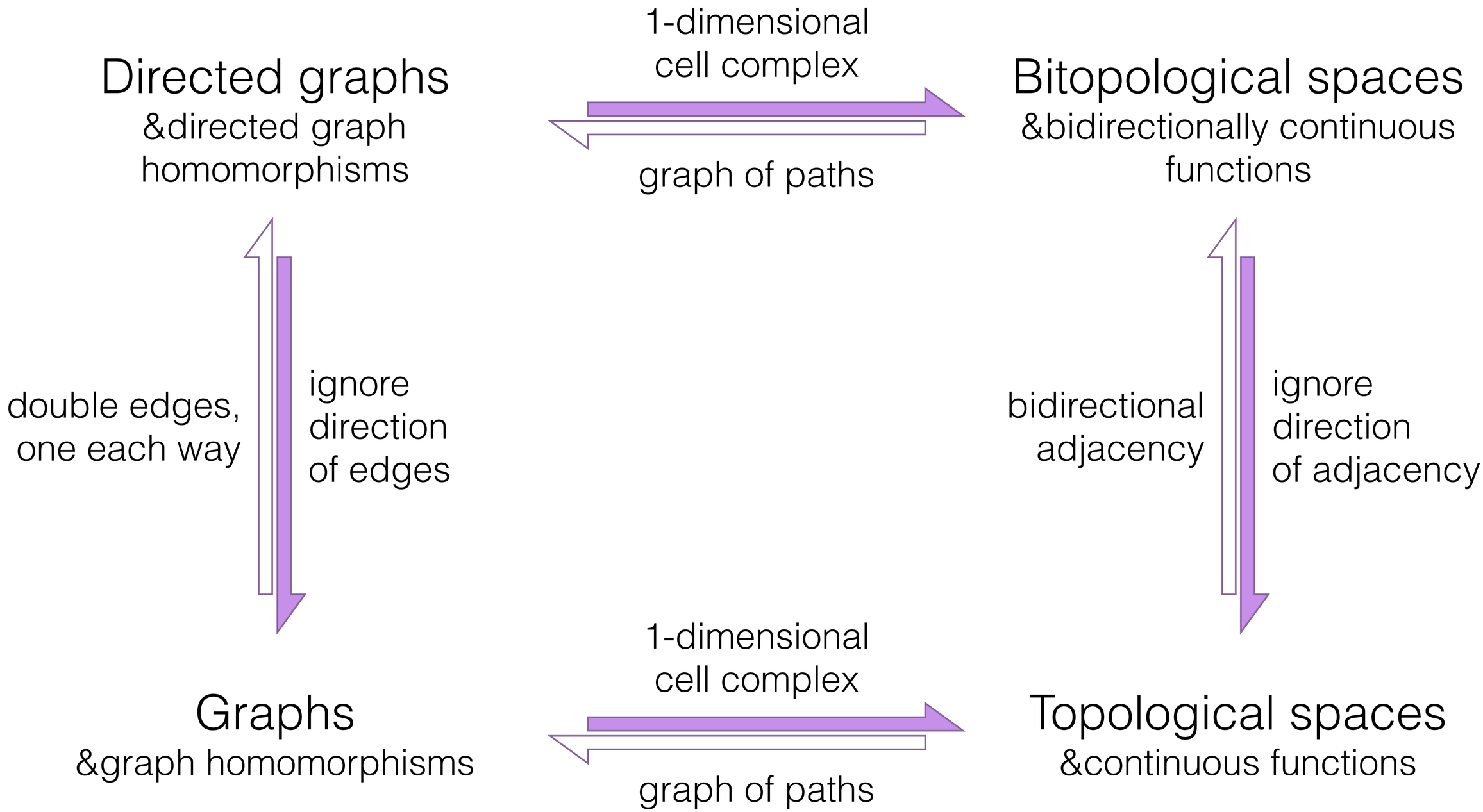
forget
edges

edgeless
graph

1-dimensional
cell complex

graph of paths

Graphs
&graph homomorphisms

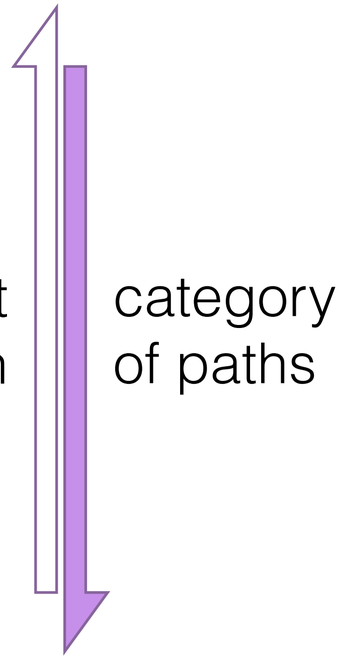

Directed graphs
&directed graph homomorphisms
1-dimensional cell complex
graph of paths
Bitopological spaces
&bidirectionally continuous functions
double edges, one each way
ignore direction of edges
bidirectional adjacency
ignore direction of adjacency
Graphs
&graph homomorphisms
1-dimensional cell complex
graph of paths
Topological spaces
&continuous functions

Directed graphs
&directed graph
homomorphisms

1-dimensional
cell complex

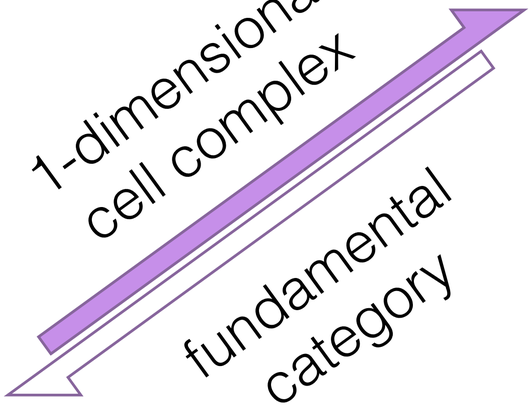
graph of paths

Bitopological spaces
&bidirectionally continuous
functions



forget
composition

category
of paths



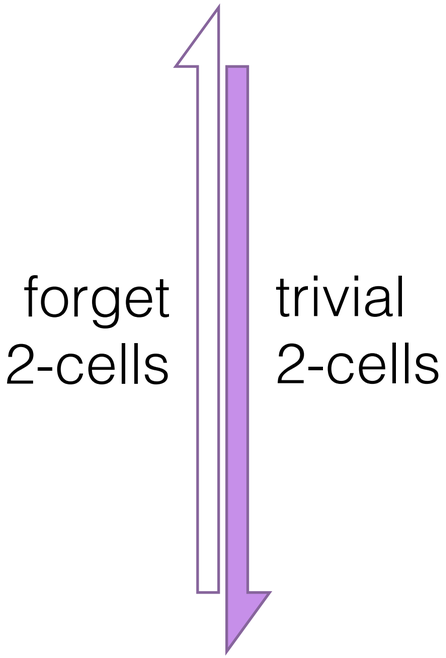
Categories
&functors

Categories
&functors

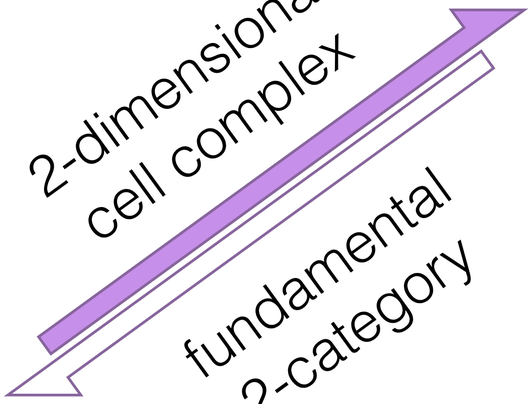
1-dimensional
cell complex

fundamental
category

Bitopological spaces
&bidirectionally continuous
functions



2-dimensional
cell complex

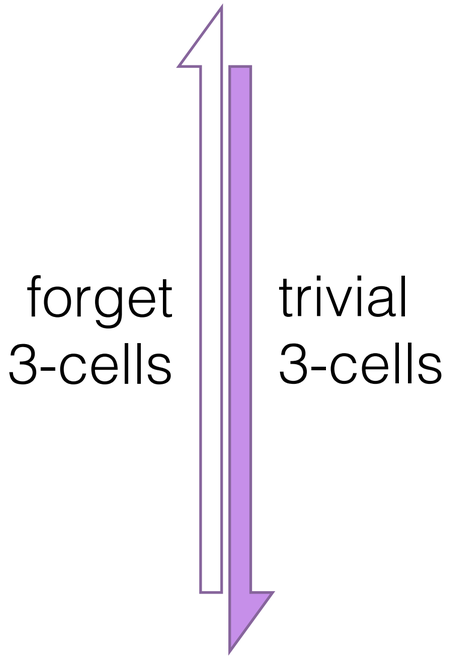


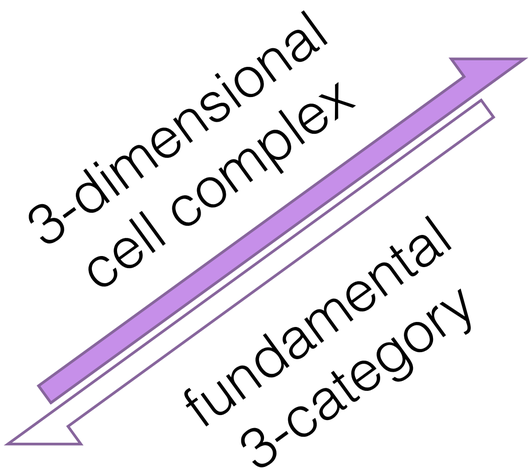
fundamental
2-category

2-Categories
&2-functors

2-Categories
&2-functors

Bitopological spaces
&bidirectionally continuous
functions

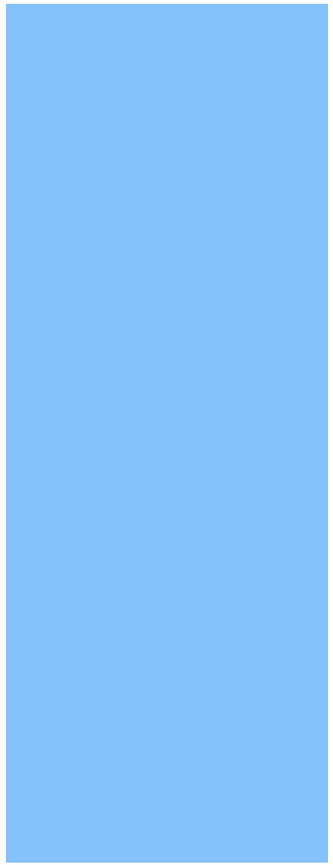




3-Categories
&3-functors

What is a morphism between computer programs?
(written to the same interfaces, above and below)

Thesis: a morphism between computer programs is a morphism between their bitopological spaces.
(that commutes with the inclusion maps from the interfaces)



All of these neighborhoods are composed of *nothing but edges*.

It’s edges all the way down.

And it’s edges all the way up.

Questions?