01

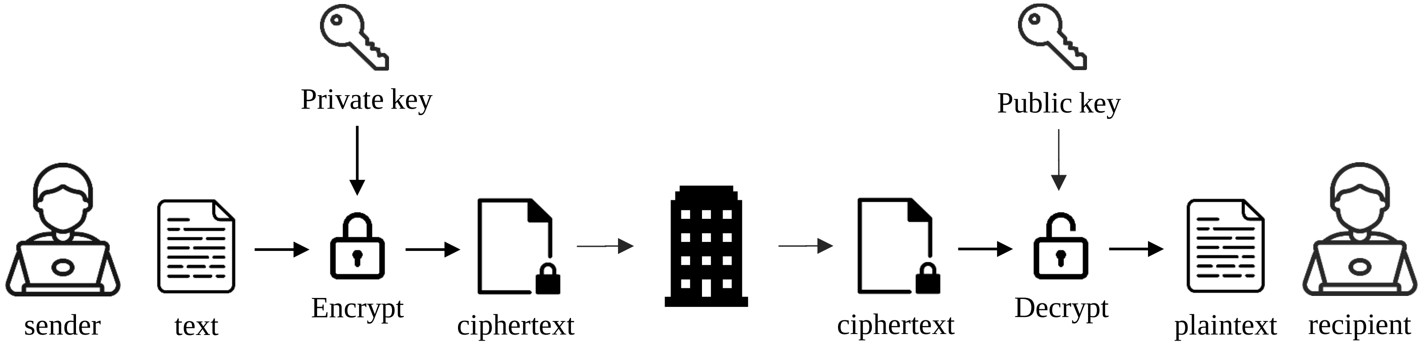
Introduction

Background

- ✓ Shor's algorithm, combined with a powerful quantum computer, will possibly break RSA and ECC.
- ✓ Initiated by NIST in 2016, the post-quantum cryptography standardization process, , it finalized the selection of **ML-DSA** as one of the encryption methods.
- ✓ Previously known as CRYSTAL-DILITHIUM

ML-DSA

- ✓ Defines method for generating digital signatures
- ✓ Based on the worst-case hardness of module lattice problems, it has potential resistance against both quantum and classical attacks.
- ✓ Advantages include fast arithmetic operations, efficient encryption, and compact signatures.
- ✓ Uses uniformly sampled high-entropy Gaussian-distributed secrets to generate random keys.
- ✓ The core security challenges of ML-DSA include MLWE problem and tMSIS problem



Fiat-Shamir with Aborts

1. Commitment：
   - The signer generates a random vector y ∈ R_q^{ℓ}
   - The commitment value is $w = Ay$
   - w is rounded to obtain w_1
2. Challenge：
   - The challenge c is generated by hashing w_1 and the message representative μ
3. Response：
   - The response $z = y + S_1 \cdot c$(where S_1 is part of the private key)
   - Use rejection sampling to check whether z meets specific coefficient bounds
4. Hint Calculation：
   - To enable the verifier to reconstruct w_1 from z and the compressed public value t_1
   - hint h ∈ R_q^{k}
5. Signature Composition：
   - The final signature consists of three parts: the rounded commitment w_1, the response z, and the hint h
6. Second Stage of Rejection Sampling：
   - To ensure the correctness of the signature, a second stage of rejection sampling must be performed

MLWE (module learning with errors)

Setup:

1. Modulus q=7.
2. Matrix A is of size 2×2, with elements selected randomly.
3. Secret vectors $s1$ and $s2$ are both of size 2×1.
4. Values for A, $s1$, $s2$:

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}, \quad s_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad s_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Calculation Steps:

1. Calculate $As1$:

$$As_1 = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 + 4 \cdot 3 \\ 1 \cdot 2 + 5 \cdot 3 \end{bmatrix} = \begin{bmatrix} 6 + 12 \\ 2 + 15 \end{bmatrix} = \begin{bmatrix} 18 \\ 17 \end{bmatrix}$$

2. Add the secret vector $s2$ to the result and take modulus q:

$$t = As_1 + s_2 = \begin{bmatrix} 18 \\ 17 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 19 \\ 21 \end{bmatrix}$$

$$t = \begin{bmatrix} 19 \mod 7 \\ 21 \mod 7 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

3. The public data is the matrix A and the result vector t:

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}, \quad t = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

MSIS (module shortest integer solution)

Setup:

1. Modulus q = 7
2. Matrix A is of size 3×2, with elements selected randomly.
3. Values for A:

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \\ 6 & 2 \end{bmatrix}$$

Goal:

Find vectors z and u such that $Az + u = 0 \bmod q$.

Attempt to Solve:

1. Assume a vector z, u:

$$z = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad u = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}$$

2. Calculate $Az + u$ and take modulus q:

$$Az = \begin{bmatrix} 3 & 4 \\ 1 & 5 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 + 4 \cdot (-1) \\ 1 \cdot 2 + 5 \cdot (-1) \\ 6 \cdot 2 + 2 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 6 - 4 \\ 2 - 5 \\ 12 - 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 10 \end{bmatrix}$$

$$Az + u = \begin{bmatrix} 2 \\ -3 \\ 10 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 + (-1) \\ -3 + 3 \\ 10 + (-5) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

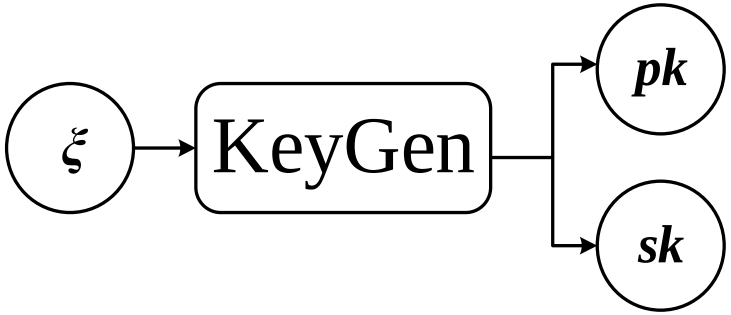
$$Az + u \mod 7 = \begin{bmatrix} 1 \mod 7 \\ 0 \mod 7 \\ 5 \mod 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

02

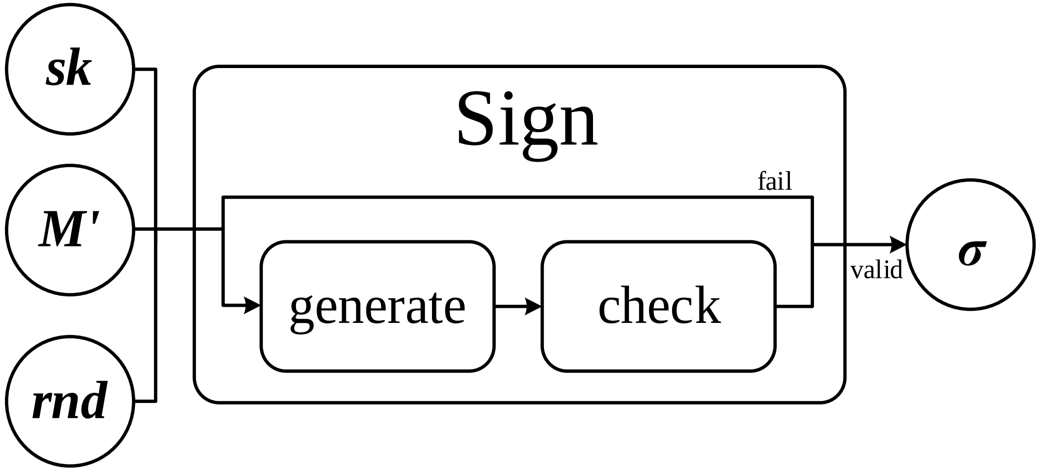
Algorithms

Algorithm

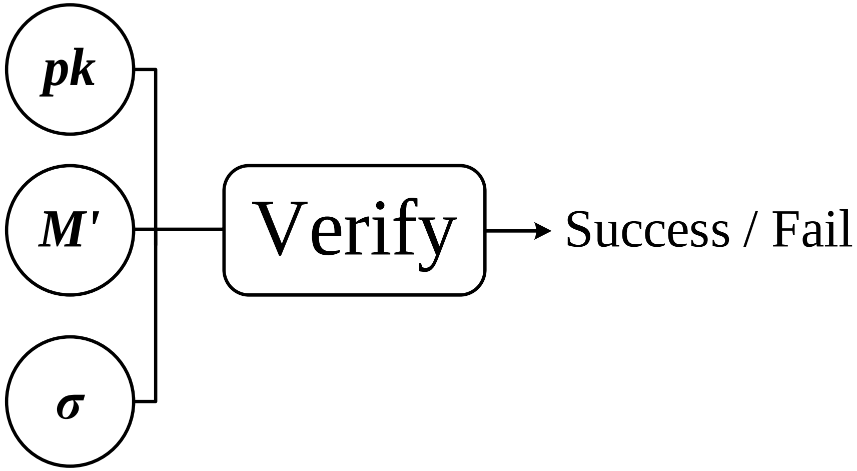
1. Key generation (KeyGen)



2. Signature generation (Sign)



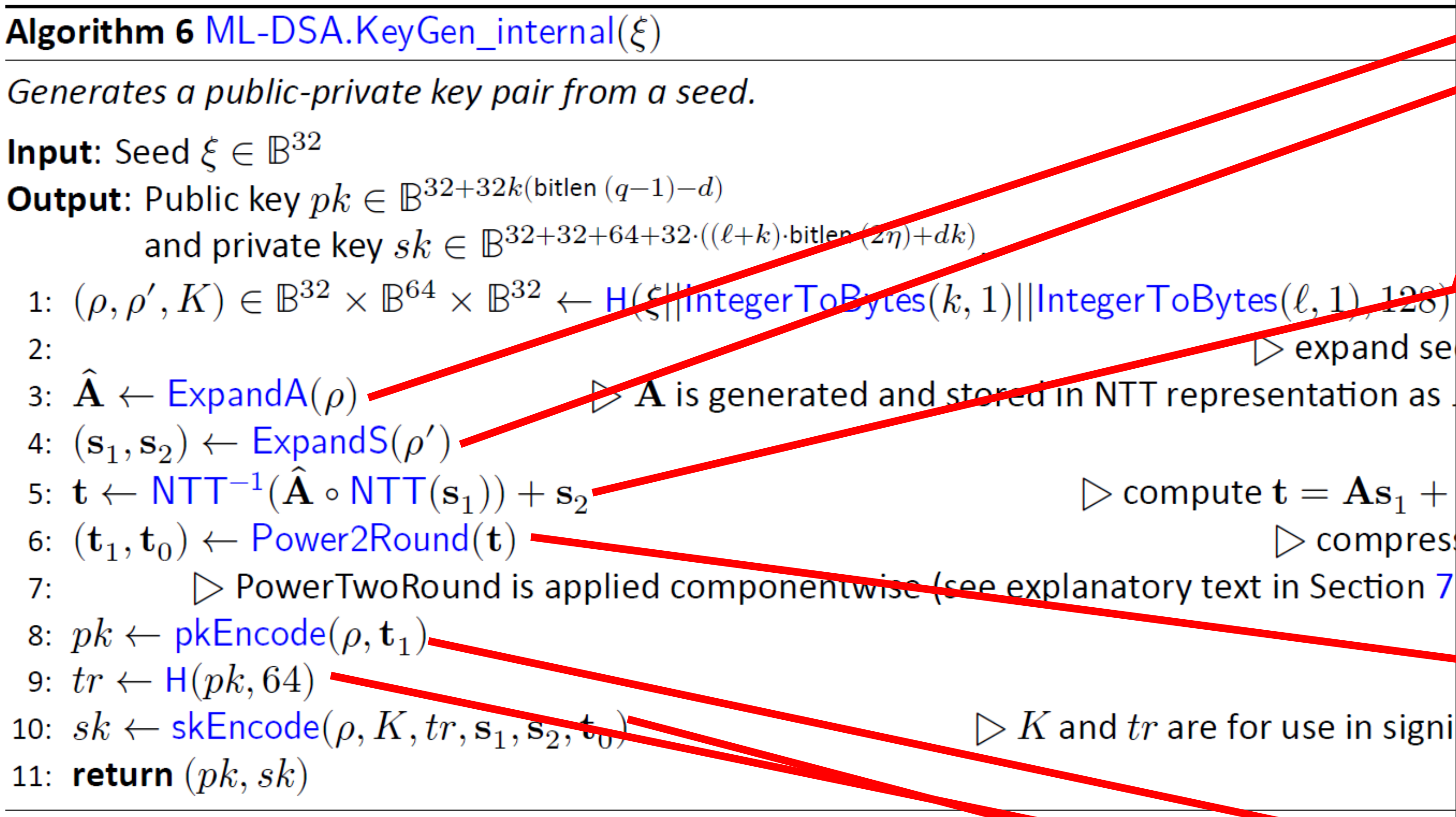
3. Signature verification (Verify)



Symbols

- ξ : random seed
- ***p***k : public key
- ***sk*** : secret key
- ***M'*** : hash message
- ***rnd*** : random number
- $\boldsymbol{\sigma}$: signature

Key Generation

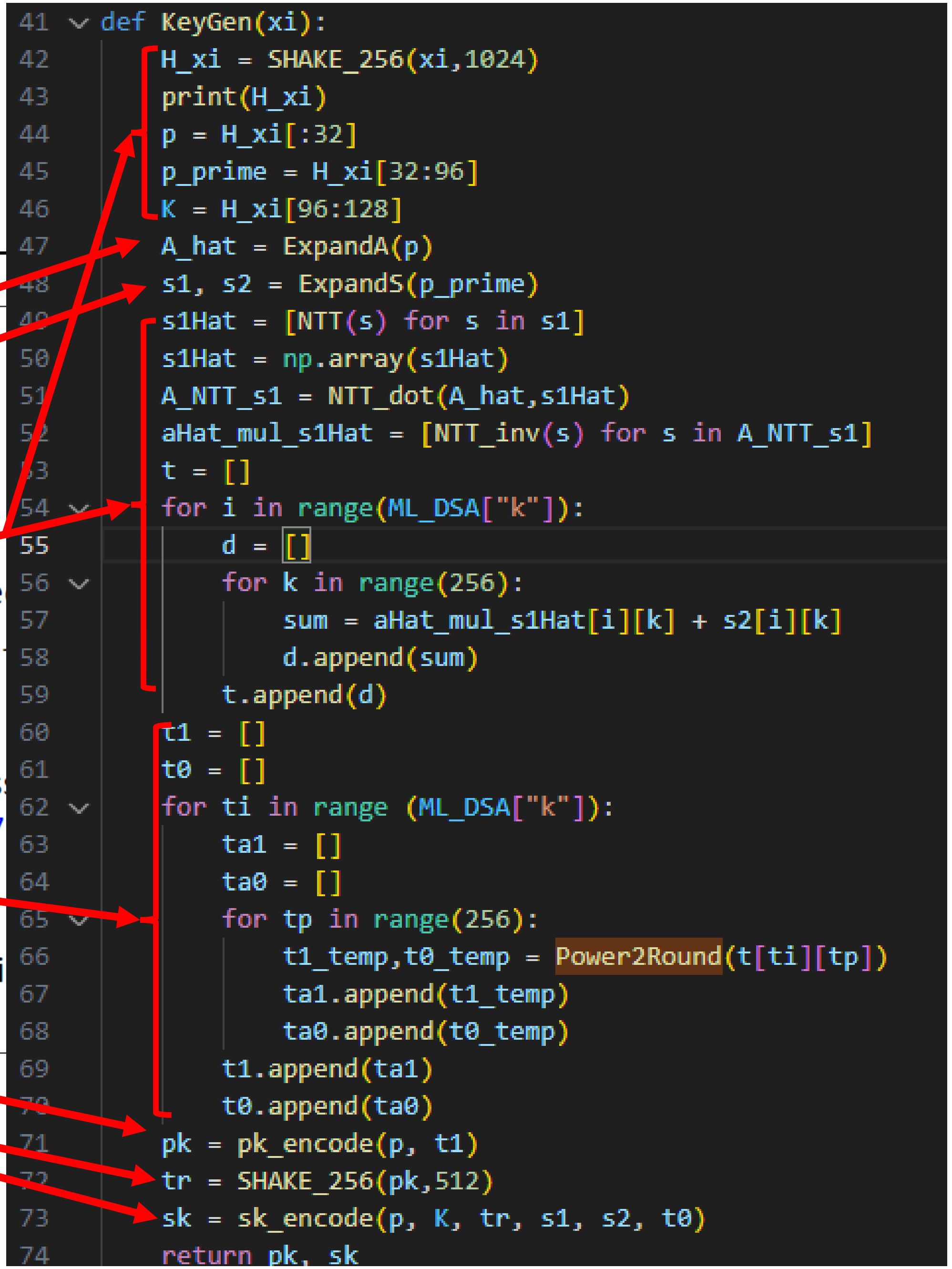


Algorithm 6 ML-DSA.KeyGen_internal(ξ)

Generates a public-private key pair from a seed.

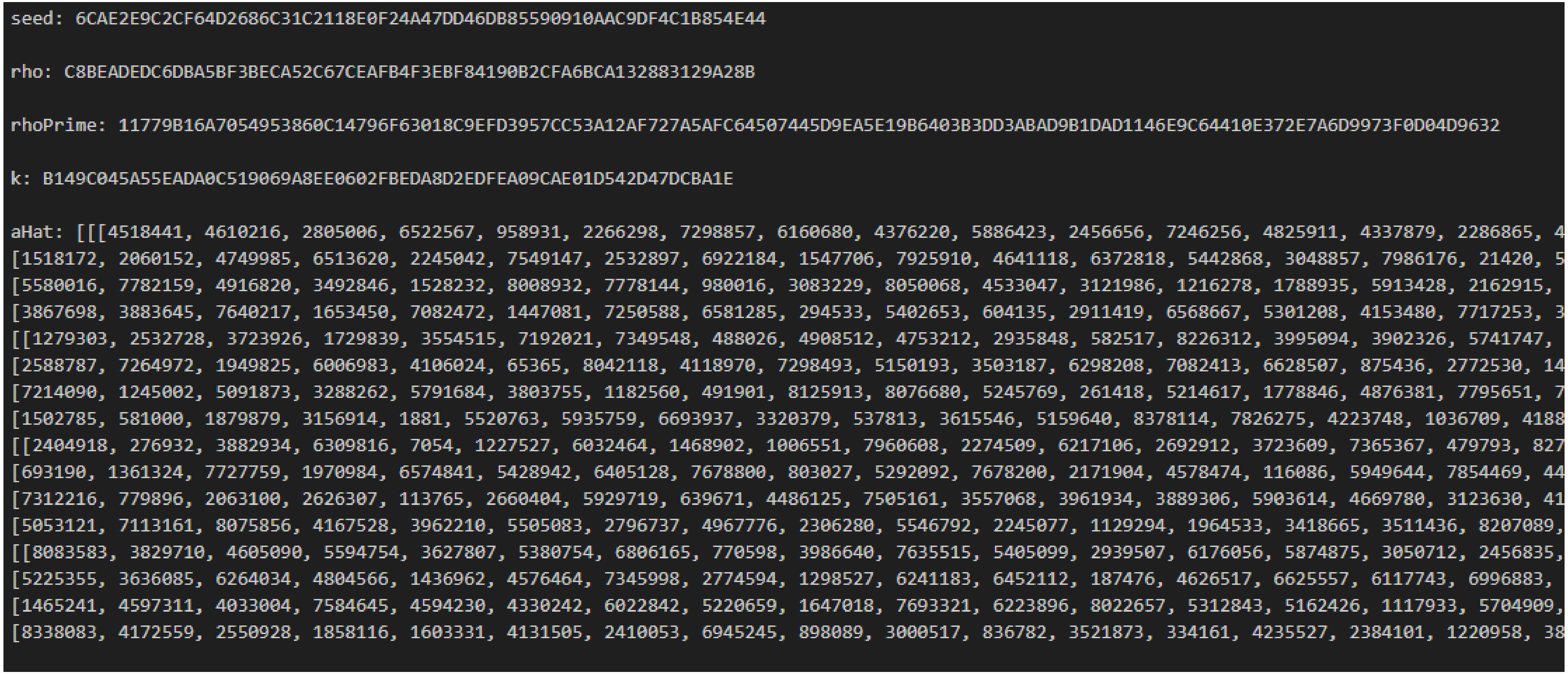
Input: Seed $\xi \in \mathbb{B}^{32}$
Output: Public key $pk \in \mathbb{B}^{32+32k(\text{bitlen }(q-1)-d)}$
and private key $sk \in \mathbb{B}^{32+32+64+32\cdot((\ell+k)\cdot\text{bitlen }(2\eta)+dk)}$

```
1: $(\rho, \rho', K) \in \mathbb{B}^{32} \times \mathbb{B}^{64} \times \mathbb{B}^{32} \leftarrow$ H($\xi$||IntegerToBytes($k, 1$)||IntegerToBytes($\ell, 1$), 128)
2:                                                      ▷ expand se
3: $\hat{\mathbf{A}} \leftarrow$ ExpandA($\rho$)          ▷ $\hat{\mathbf{A}}$ is generated and stored in NTT representation as
4: $(\mathbf{s}_1, \mathbf{s}_2) \leftarrow$ ExpandS($\rho'$)
5: $\mathbf{t} \leftarrow \text{NTT}^{-1}(\hat{\mathbf{A}} \circ \text{NTT}(\mathbf{s}_1)) + \mathbf{s}_2$          ▷ compute $\mathbf{t} = \mathbf{A}\mathbf{s}_1 +$
6: $(\mathbf{t}_1, \mathbf{t}_0) \leftarrow$ Power2Round($\mathbf{t}$)          ▷ compres
7:         ▷ PowerTwoRound is applied componentwise (see explanatory text in Section 7
8: $pk \leftarrow$ pkEncode($\rho, \mathbf{t}_1$)
9: $tr \leftarrow$ H($pk$, 64)
10: $sk \leftarrow$ skEncode($\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0$)          ▷ $K$ and $tr$ are for use in signi
11: return $(pk, sk)$
```



```python
def KeyGen(xi):
    H_xi = SHAKE_256(xi,1024)
    print(H_xi)
    p = H_xi[:32]
    p_prime = H_xi[32:96]
    K = H_xi[96:128]
    A_hat = ExpandA(p)
    s1, s2 = ExpandS(p_prime)
    s1Hat = [NTT(s) for s in s1]
    s1Hat = np.array(s1Hat)
    A_NTT_s1 = NTT_dot(A_hat,s1Hat)
    aHat_mul_s1Hat = [NTT_inv(s) for s in A_NTT_s1]
    t = []
    for i in range(ML_DSA["k"]):
        d = []
        for k in range(256):
            sum = aHat_mul_s1Hat[i][k] + s2[i][k]
            d.append(sum)
        t.append(d)
    t1 = []
    t0 = []
    for ti in range (ML_DSA["k"]):
        ta1 = []
        ta0 = []
        for tp in range(256):
            t1_temp,t0_temp = Power2Round(t[ti][tp])
            ta1.append(t1_temp)
            ta0.append(t0_temp)
        t1.append(ta1)
        t0.append(ta0)
    pk = pk_encode(p, t1)
    tr = SHAKE_256(pk,512)
    sk = sk_encode(p, K, tr, s1, s2, t0)
    return pk, sk
```

Key Generation



```
seed: 6CAE2E9C2CF64D2686C31C2118E0F24A47DD46DB85590910AAC9DF4C1B854E44

rho: C8BEADEDC6DBA5BF3BECA52C67CEAFB4F3EBF84190B2CFA6BCA132883129A28B

rhoPrime: 11779B16A7054953860C14796F63018C9EFD3957CC53A12AF727A5AFC64507445D9EA5E19B6403B3DD3ABAD9B1DAD1146E9C64410E372E7A6D9973F0D04D9632

k: B149C045A55EADA0C519069A8EE0602FBEDA8D2EDFEA09CAE01D542D47DCBA1E

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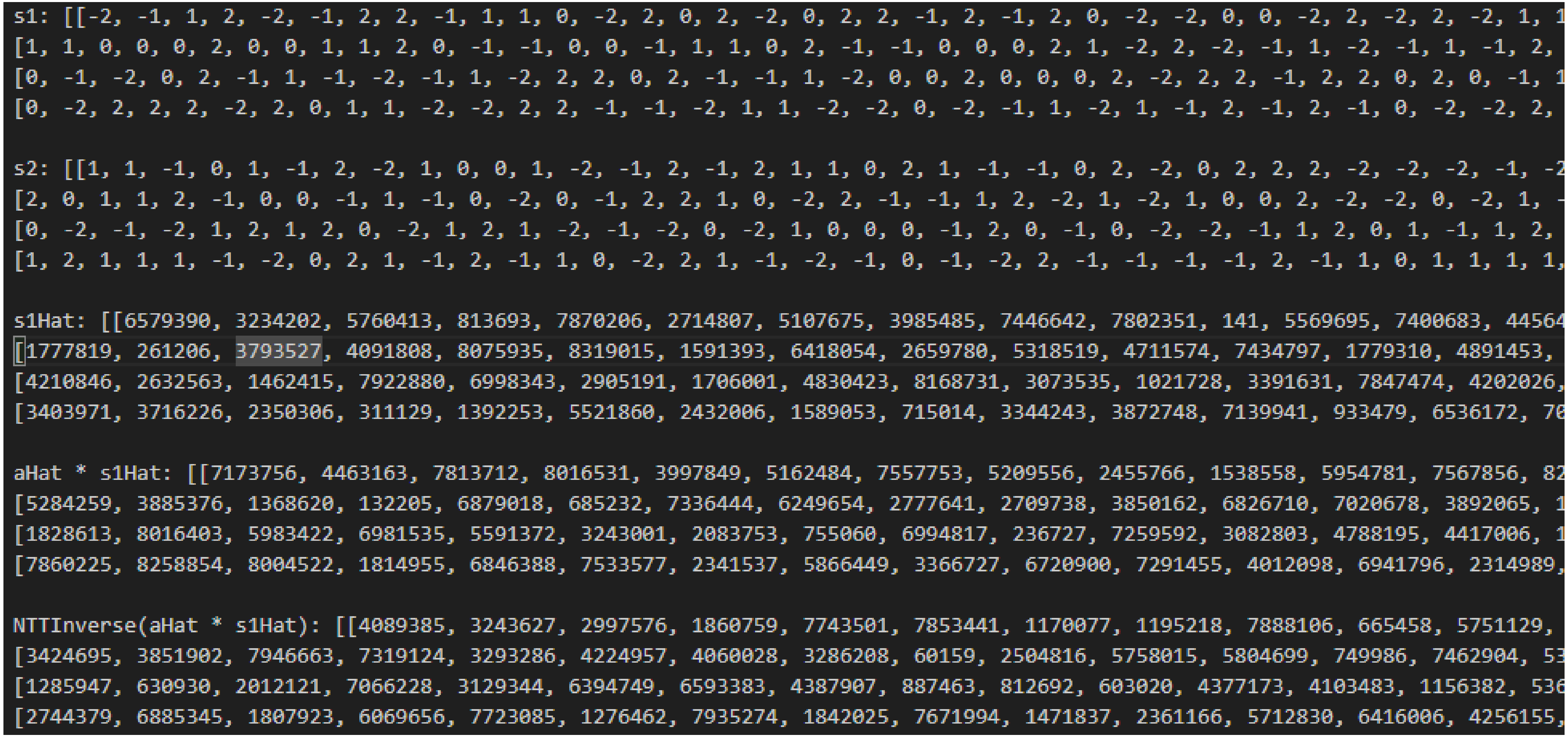
Key Generation



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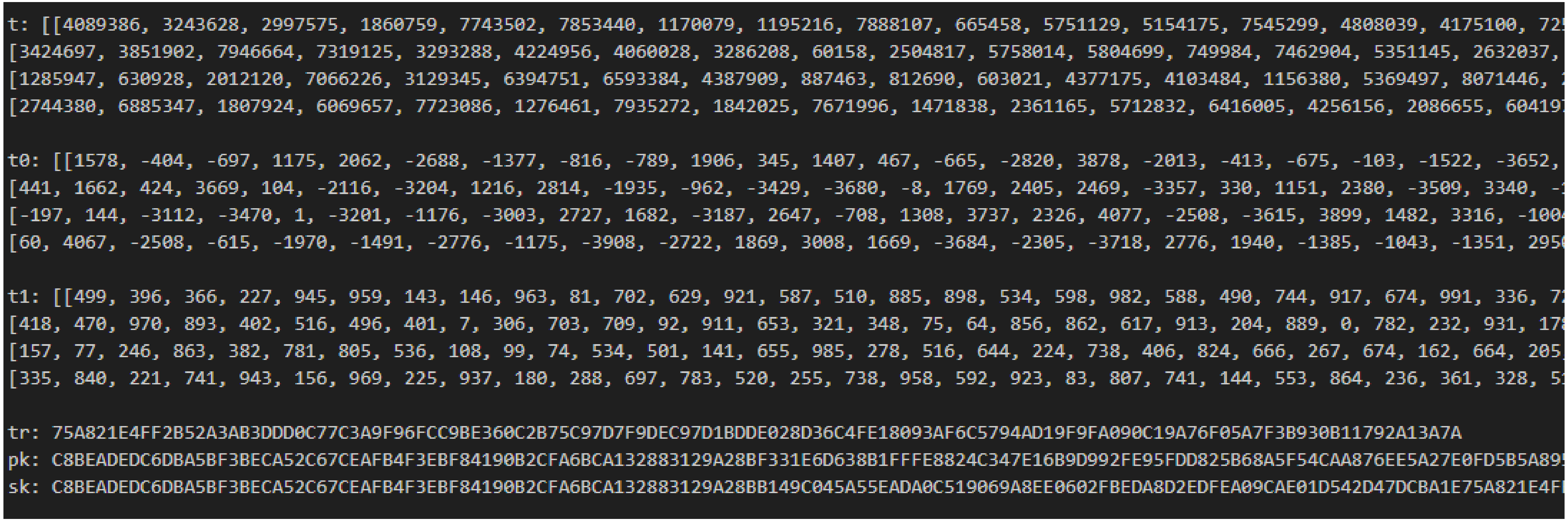
Key Generation



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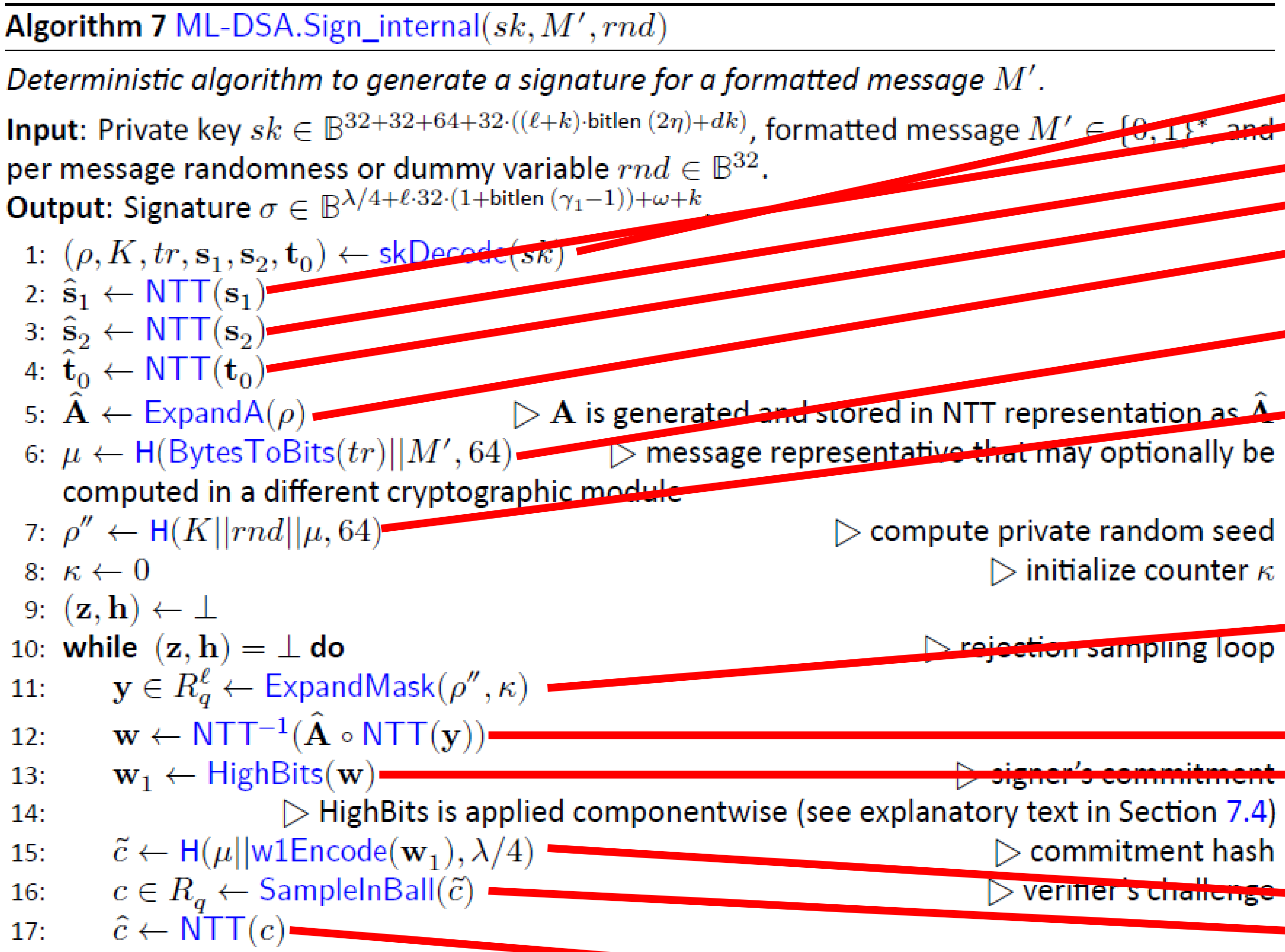
Signing



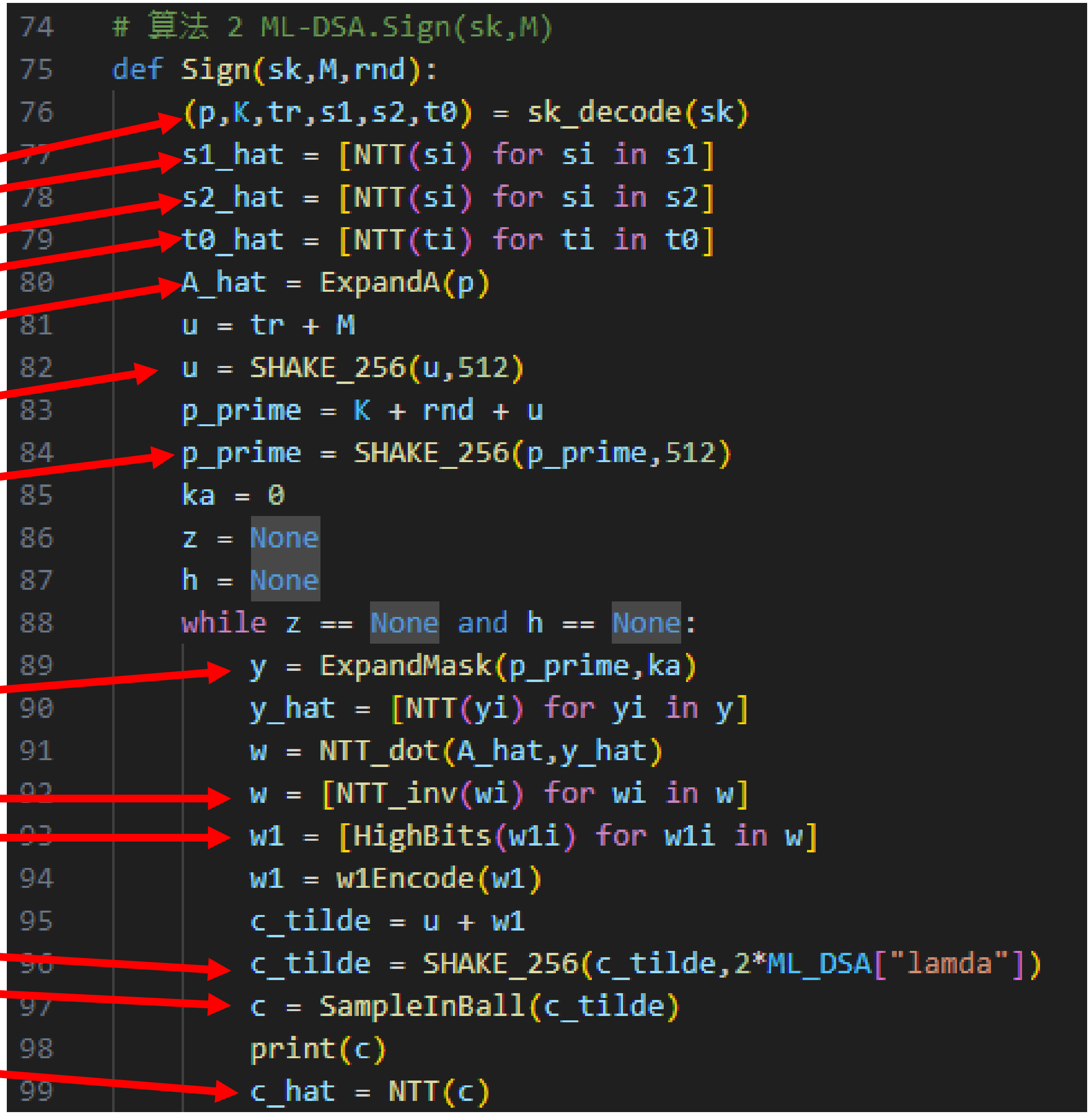
Algorithm 7 ML-DSA.Sign_internal(sk, M', rnd)

Deterministic algorithm to generate a signature for a formatted message M'.

Input: Private key $sk \in \mathbb{B}^{32+32+64+32\cdot((\ell+k)\cdot\text{bitlen}\,(2\eta)+dk)}$, formatted message $M' \in \{0,1\}^*$, and per message randomness or dummy variable $rnd \in \mathbb{B}^{32}$.

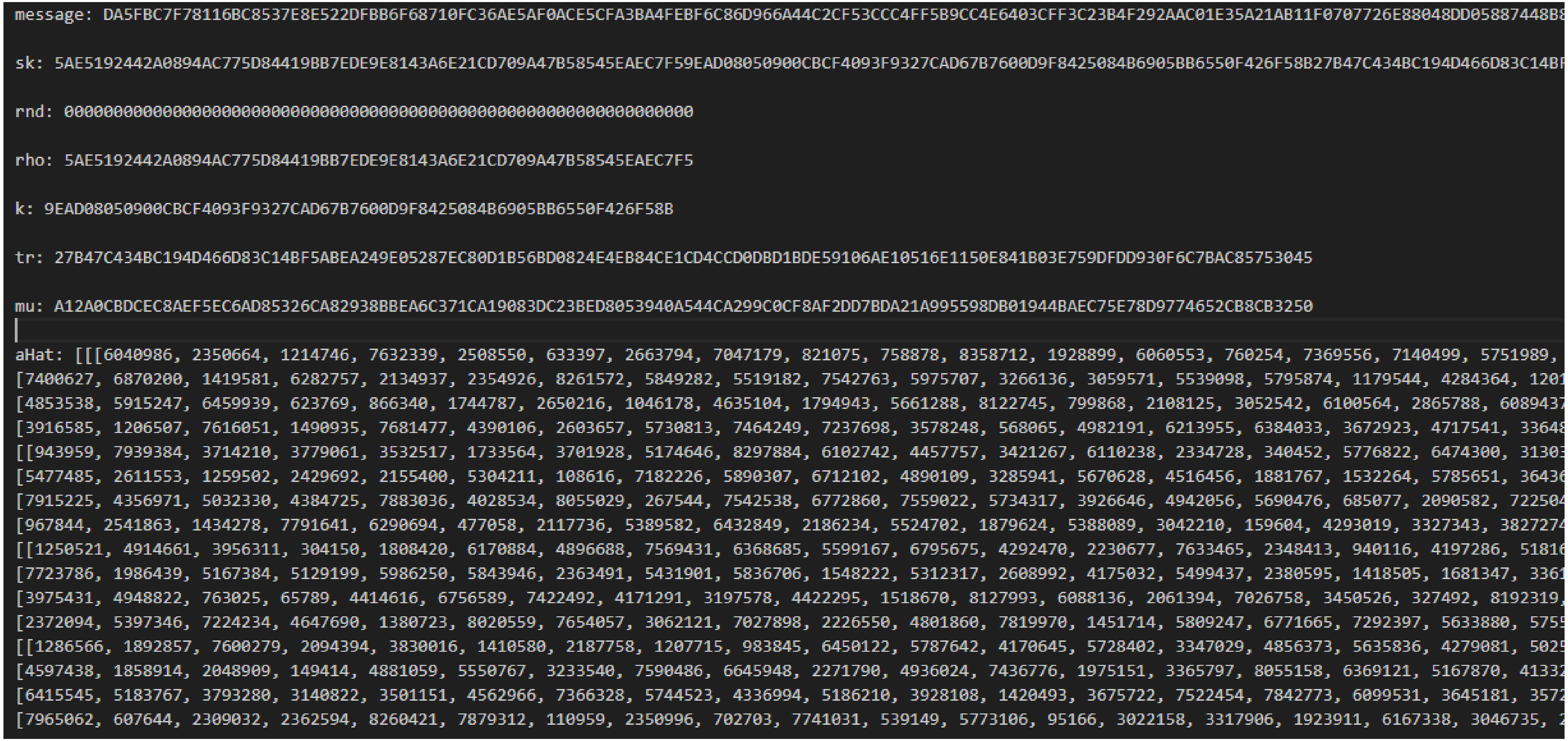
Output: Signature $\sigma \in \mathbb{B}^{\lambda/4+\ell\cdot 32\cdot(1+\text{bitlen}\,(\gamma_1-1))+\omega+k}$.

```
$(\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0) \leftarrow$ skDecode$(sk)$
$\hat{\mathbf{s}}_1 \leftarrow$ NTT$(\mathbf{s}_1)$
$\hat{\mathbf{s}}_2 \leftarrow$ NTT$(\mathbf{s}_2)$
$\hat{\mathbf{t}}_0 \leftarrow$ NTT$(\mathbf{t}_0)$
$\hat{\mathbf{A}} \leftarrow$ ExpandA$(\rho)$        ▷ $\mathbf{A}$ is generated and stored in NTT representation as $\hat{\mathbf{A}}$
$\mu \leftarrow$ H(BytesToBits$(tr)||M', 64)$        ▷ message representative that may optionally be computed in a different cryptographic module
$\rho'' \leftarrow$ H$(K||rnd||\mu, 64)$        ▷ compute private random seed
$\kappa \leftarrow 0$        ▷ initialize counter $\kappa$
$(\mathbf{z}, \mathbf{h}) \leftarrow \perp$
while $(\mathbf{z}, \mathbf{h}) = \perp$ do        ▷ rejection sampling loop
    $\mathbf{y} \in R_q^\ell \leftarrow$ ExpandMask$(\rho'', \kappa)$
    $\mathbf{w} \leftarrow$ NTT$^{-1}(\hat{\mathbf{A}} \circ$ NTT$(\mathbf{y}))$
    $\mathbf{w}_1 \leftarrow$ HighBits$(\mathbf{w})$        ▷ signer's commitment
        ▷ HighBits is applied componentwise (see explanatory text in Section 7.4)
    $\tilde{c} \leftarrow$ H$(\mu||$w1Encode$(\mathbf{w}_1), \lambda/4)$        ▷ commitment hash
    $c \in R_q \leftarrow$ SampleInBall$(\tilde{c})$        ▷ verifier's challenge
    $\hat{c} \leftarrow$ NTT$(c)$
```



```python
# 算法 2 ML-DSA.Sign(sk,M)
def Sign(sk,M,rnd):
    (p,K,tr,s1,s2,t0) = sk_decode(sk)
    s1_hat = [NTT(si) for si in s1]
    s2_hat = [NTT(si) for si in s2]
    t0_hat = [NTT(ti) for ti in t0]
    A_hat = ExpandA(p)
    u = tr + M
    u = SHAKE_256(u,512)
    p_prime = K + rnd + u
    p_prime = SHAKE_256(p_prime,512)
    ka = 0
    z = None
    h = None
    while z == None and h == None:
        y = ExpandMask(p_prime,ka)
        y_hat = [NTT(yi) for yi in y]
        w = NTT_dot(A_hat,y_hat)
        w = [NTT_inv(wi) for wi in w]
        w1 = [HighBits(w1i) for w1i in w]
        w1 = w1Encode(w1)
        c_tilde = u + w1
        c_tilde = SHAKE_256(c_tilde,2*ML_DSA["lamda"])
        c = SampleInBall(c_tilde)
        print(c)
        c_hat = NTT(c)
```

Signing



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sk: 5AE5192442A0894AC775D84419BB7EDE9E8143A6E21CD709A47B58545EAEC7F59EAD08050900CBCF4093F9327CAD67B7600D9F8425084B6905BB6550F426F58B27B47C434BC194D466D83C14BF

rnd: 0000000000000000000000000000000000000000000000000000000000000000

rho: 5AE5192442A0894AC775D84419BB7EDE9E8143A6E21CD709A47B58545EAEC7F5

k: 9EAD08050900CBCF4093F9327CAD67B7600D9F8425084B6905BB6550F426F58B

tr: 27B47C434BC194D466D83C14BF5ABEA249E05287EC80D1B56BD0824E4EB84CE1CD4CCD0DBD1BDE59106AE10516E1150E841B03E759DFDD930F6C7BAC85753045

mu: A12A0CBDCEC8AEF5EC6AD85326CA82938BBEA6C371CA19083DC23BED8053940A544CA299C0CF8AF2DD7BDA21A995598DB01944BAEC75E78D9774652CB8CB3250

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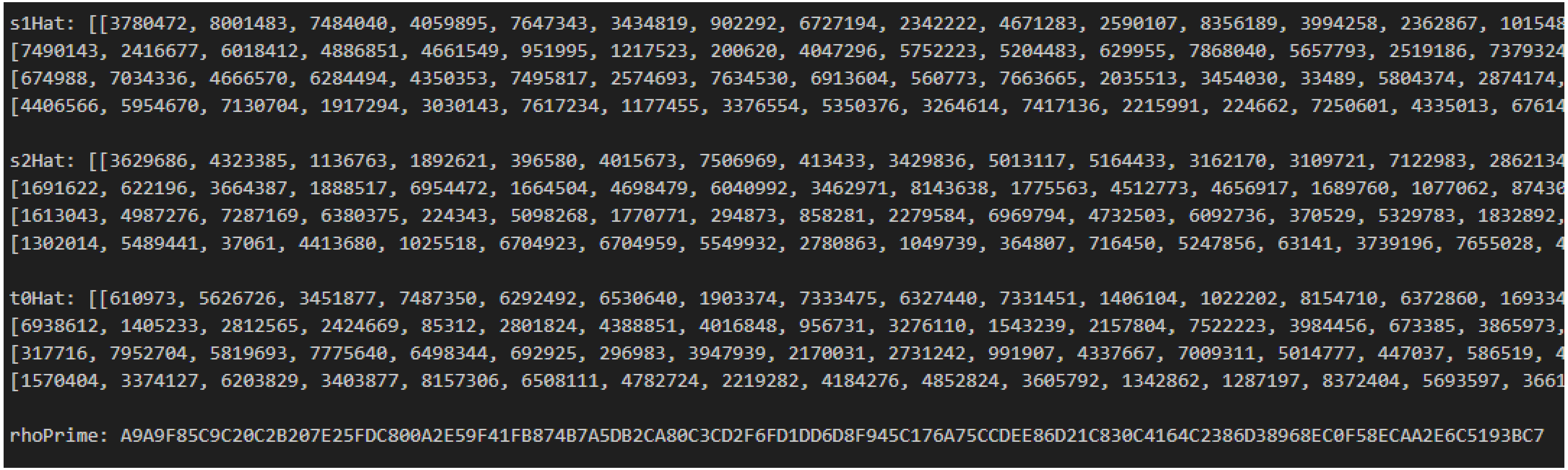
Signing



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rhoPrime: A9A9F85C9C20C2B207E25FDC800A2E59F41FB874B7A5DB2CA80C3CD2F6FD1DD6D8F945C176A75CCDEE86D21C830C4164C2386D38968EC0F58ECAA2E6C5193BC7
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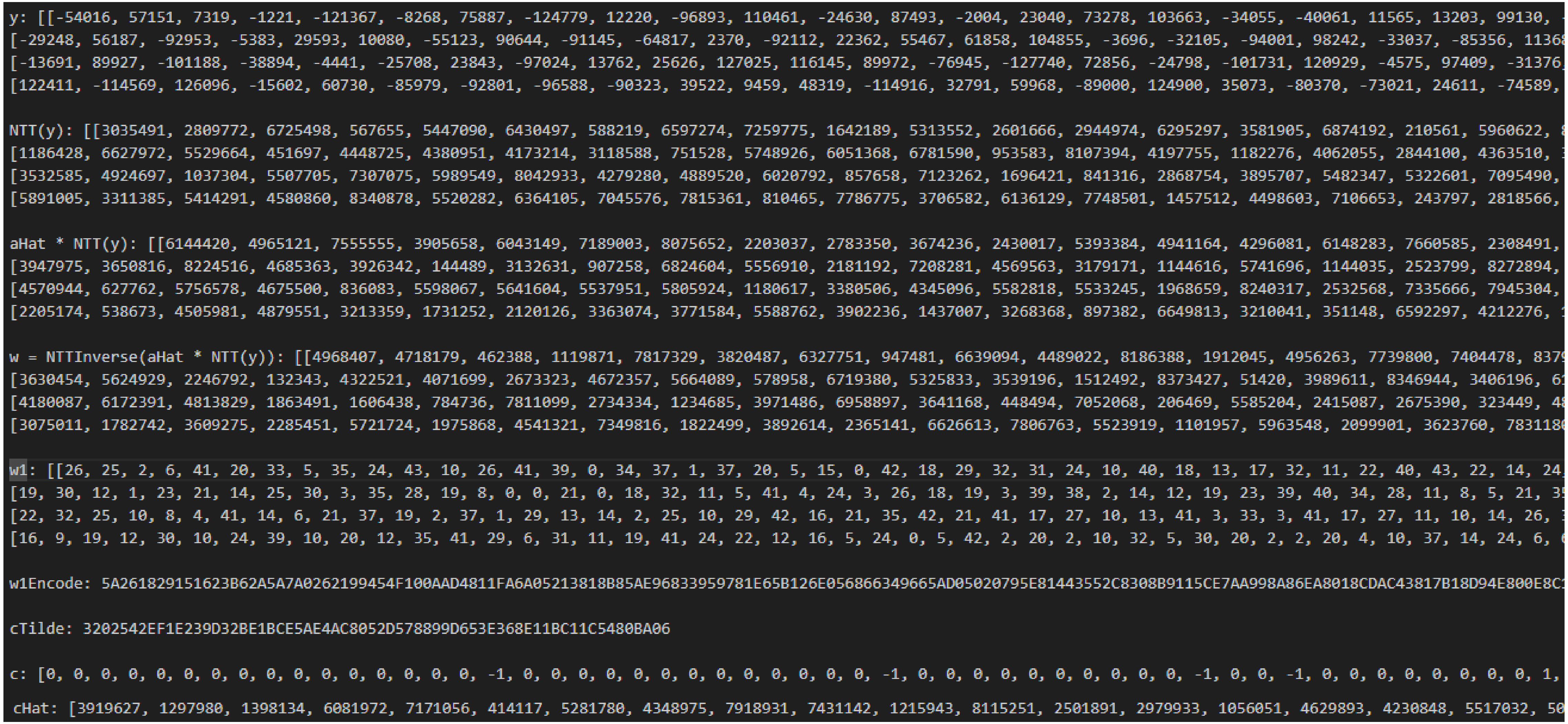
Signing



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c: [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 1,
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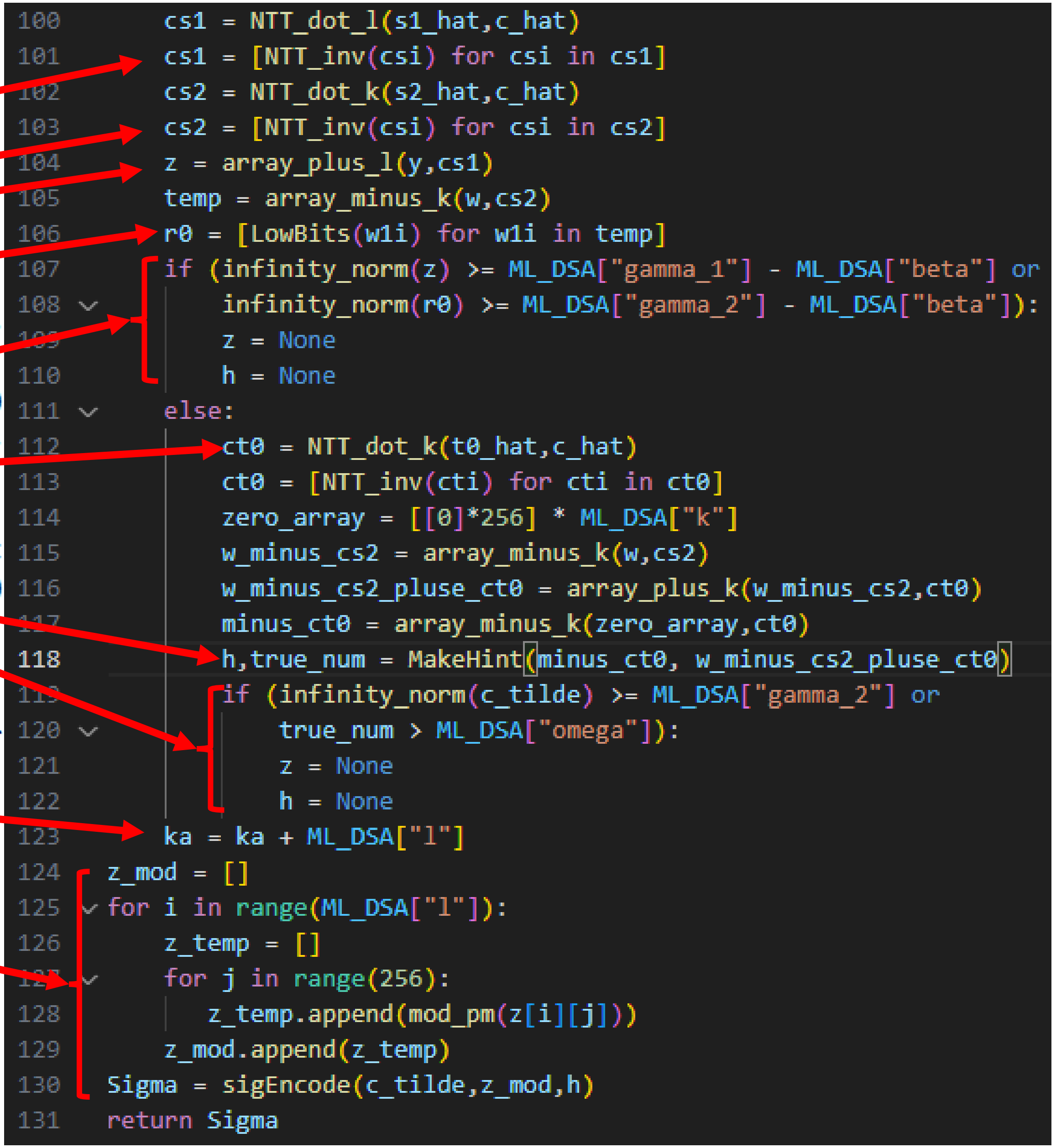
```

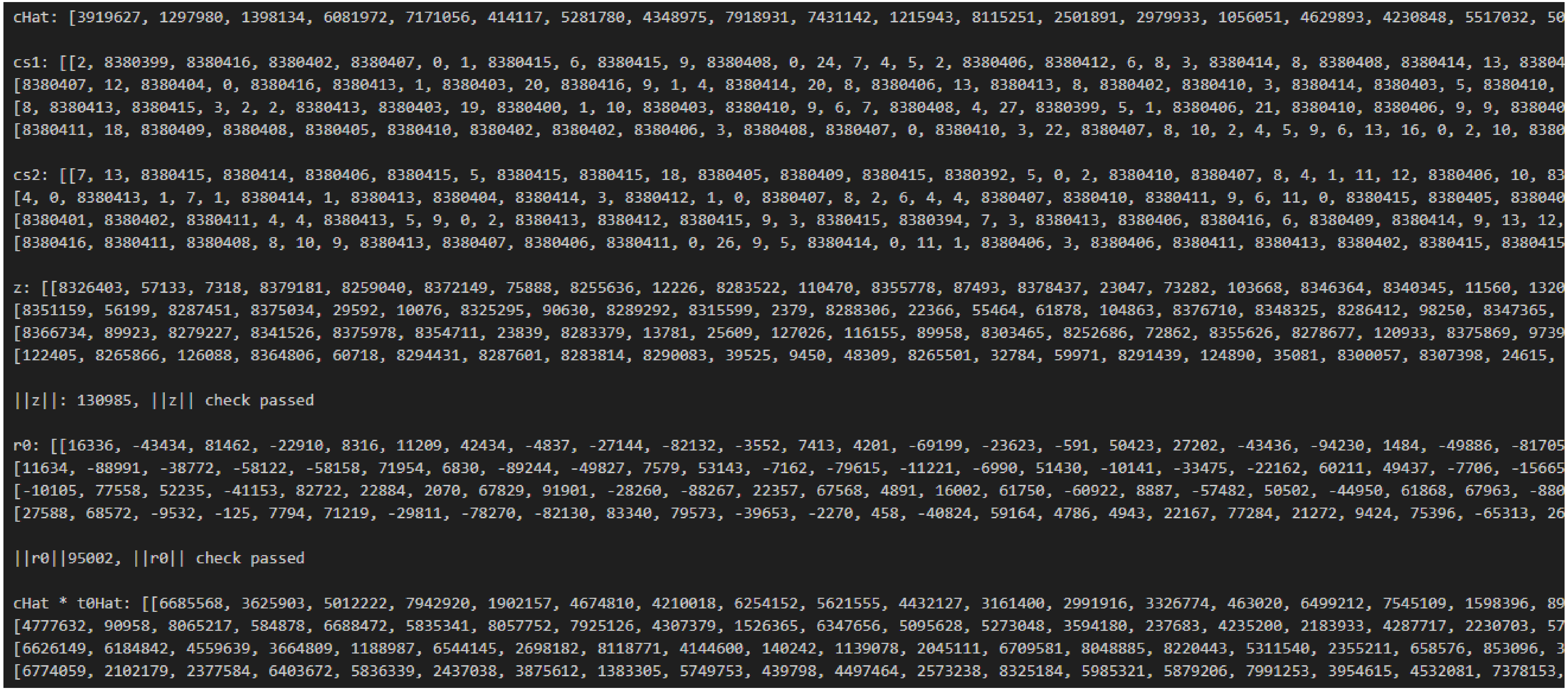
Signing

```
    $\langle\langle c\mathbf{s}_1\rangle\rangle \leftarrow \mathsf{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_1)$
    $\langle\langle c\mathbf{s}_2\rangle\rangle \leftarrow \mathsf{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_2)$
    $\mathbf{z} \leftarrow \mathbf{y} + \langle\langle c\mathbf{s}_1\rangle\rangle$                                ▷ signer's response
    $\mathbf{r}_0 \leftarrow \mathsf{LowBits}(\mathbf{w} - \langle\langle c\mathbf{s}_2\rangle\rangle)$
                ▷ LowBits is applied componentwise (see explanatory text in Section 7.4)
    if $||\mathbf{z}||_\infty \geq \gamma_1 - \beta$ or $||\mathbf{r}_0||_\infty \geq \gamma_2 - \beta$ then $(\mathbf{z}, \mathbf{h}) \leftarrow \bot$        ▷ validity checks
    else
        $\langle\langle c\mathbf{t}_0\rangle\rangle \leftarrow \mathsf{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{t}}_0)$
        $\mathbf{h} \leftarrow \mathsf{MakeHint}(-\langle\langle c\mathbf{t}_0\rangle\rangle, \mathbf{w} - \langle\langle c\mathbf{s}_2\rangle\rangle + \langle\langle c\mathbf{t}_0\rangle\rangle)$        ▷ Signer's hint
                ▷ MakeHint is applied componentwise (see explanatory text in Section 7.4)
        if $||\langle\langle c\mathbf{t}_0\rangle\rangle||_\infty \geq \gamma_2$ or the number of 1's in $\mathbf{h}$ is greater than $\omega$, then $(\mathbf{z}, \mathbf{h}) \leftarrow \bot$
        end if
    end if
    $\kappa \leftarrow \kappa + \ell$                                ▷ increment counter
end while
$\sigma \leftarrow \mathsf{sigEncode}(\tilde{c}, \mathbf{z} \bmod^{\pm} q, \mathbf{h})$
return $\sigma$
```



```
        cs1 = NTT_dot_l(s1_hat,c_hat)
        cs1 = [NTT_inv(csi) for csi in cs1]
        cs2 = NTT_dot_k(s2_hat,c_hat)
        cs2 = [NTT_inv(csi) for csi in cs2]
        z = array_plus_l(y,cs1)
        temp = array_minus_k(w,cs2)
        r0 = [LowBits(w1i) for w1i in temp]
        if (infinity_norm(z) >= ML_DSA["gamma_1"] - ML_DSA["beta"] or
            infinity_norm(r0) >= ML_DSA["gamma_2"] - ML_DSA["beta"]):
            z = None
            h = None
        else:
            ct0 = NTT_dot_k(t0_hat,c_hat)
            ct0 = [NTT_inv(cti) for cti in ct0]
            zero_array = [[0]*256] * ML_DSA["k"]
            w_minus_cs2 = array_minus_k(w,cs2)
            w_minus_cs2_pluse_ct0 = array_plus_k(w_minus_cs2,ct0)
            minus_ct0 = array_minus_k(zero_array,ct0)
            h,true_num = MakeHint(minus_ct0, w_minus_cs2_pluse_ct0)
            if (infinity_norm(c_tilde) >= ML_DSA["gamma_2"] or
                true_num > ML_DSA["omega"]):
                z = None
                h = None
        ka = ka + ML_DSA["l"]
    z_mod = []
    for i in range(ML_DSA["l"]):
        z_temp = []
        for j in range(256):
           z_temp.append(mod_pm(z[i][j]))
        z_mod.append(z_temp)
    Sigma = sigEncode(c_tilde,z_mod,h)
    return Sigma
```

Signing



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||z||: 130985, ||z|| check passed

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||r0||95002, ||r0|| check passed

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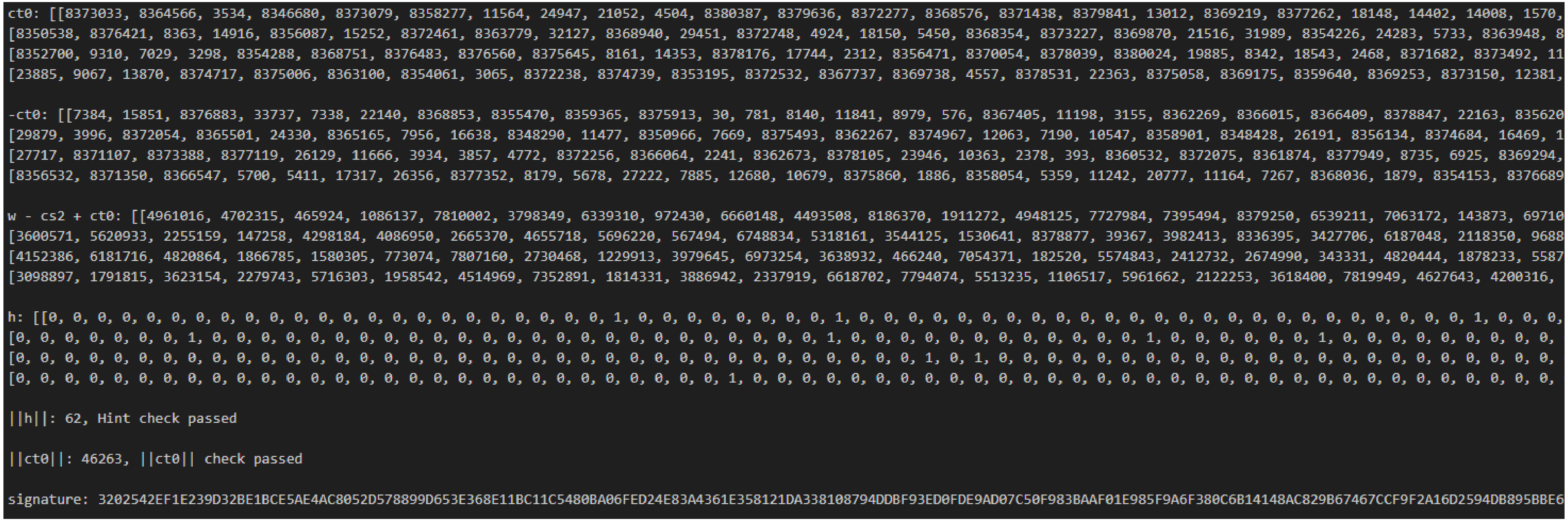
Signing



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w - cs2 + ct0: [[4961016, 4702315, 465924, 1086137, 7810002, 3798349, 6339310, 972430, 6660148, 4493508, 8186370, 1911272, 4948125, 7727984, 7395494, 8379250, 6539211, 7063172, 143873, 69710
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||h||: 62, Hint check passed

||ct0||: 46263, ||ct0|| check passed

signature: 3202542EF1E239D32BE1BCE5AE4AC8052D578899D653E368E11BC11C5480BA06FED24E83A4361E358121DA338108794DDBF93ED0FDE9AD07C50F983BAAF01E985F9A6F380C6B14148AC829B67467CCF9F2A16D2594DB895BBE6
```
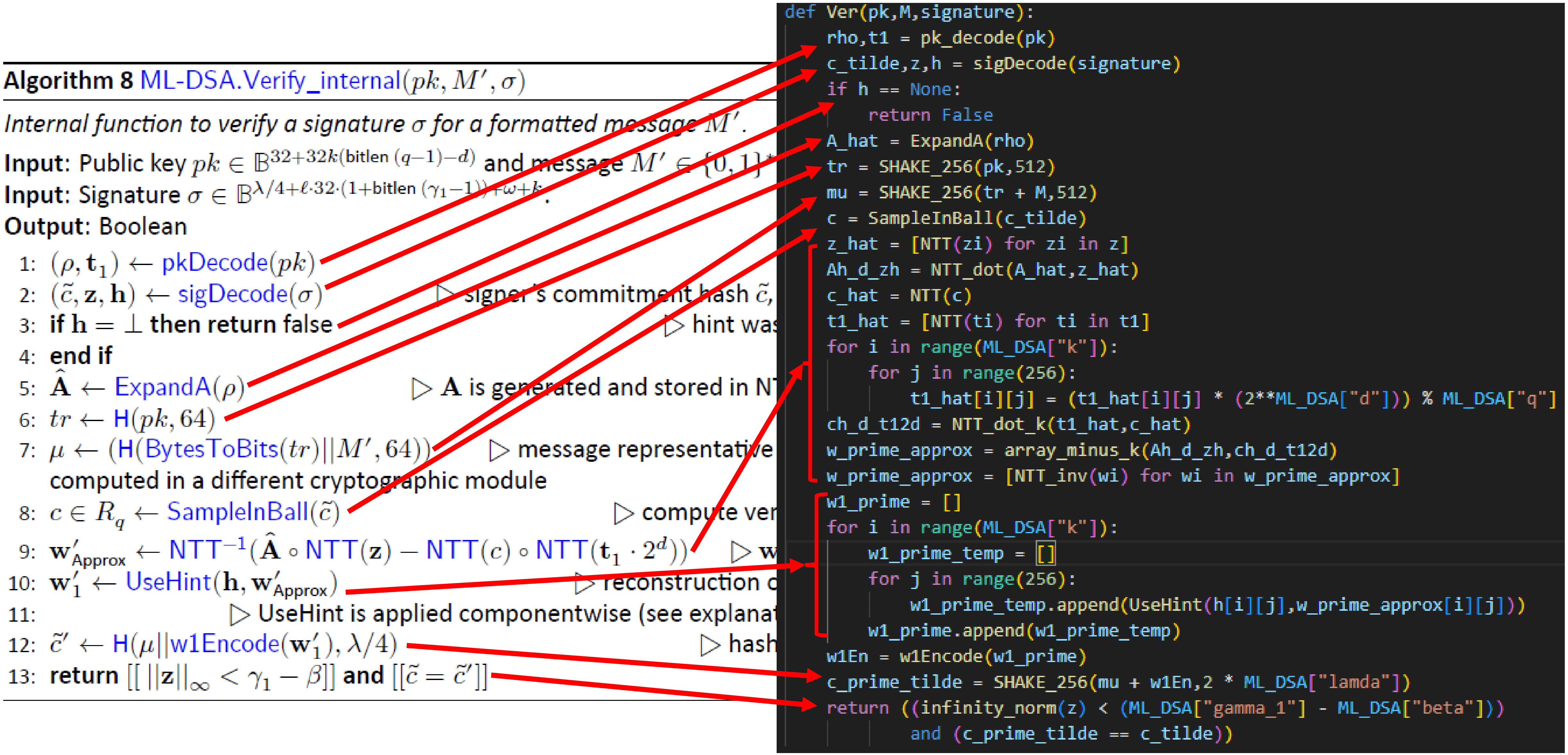

Verification



Algorithm 8 ML-DSA.Verify_internal(pk, M', σ)

Internal function to verify a signature σ for a formatted message M'.

Input: Public key $pk \in \mathbb{B}^{32+32k(\text{bitlen}\,(q-1)-d)}$ and message $M' \in \{0,1\}^*$

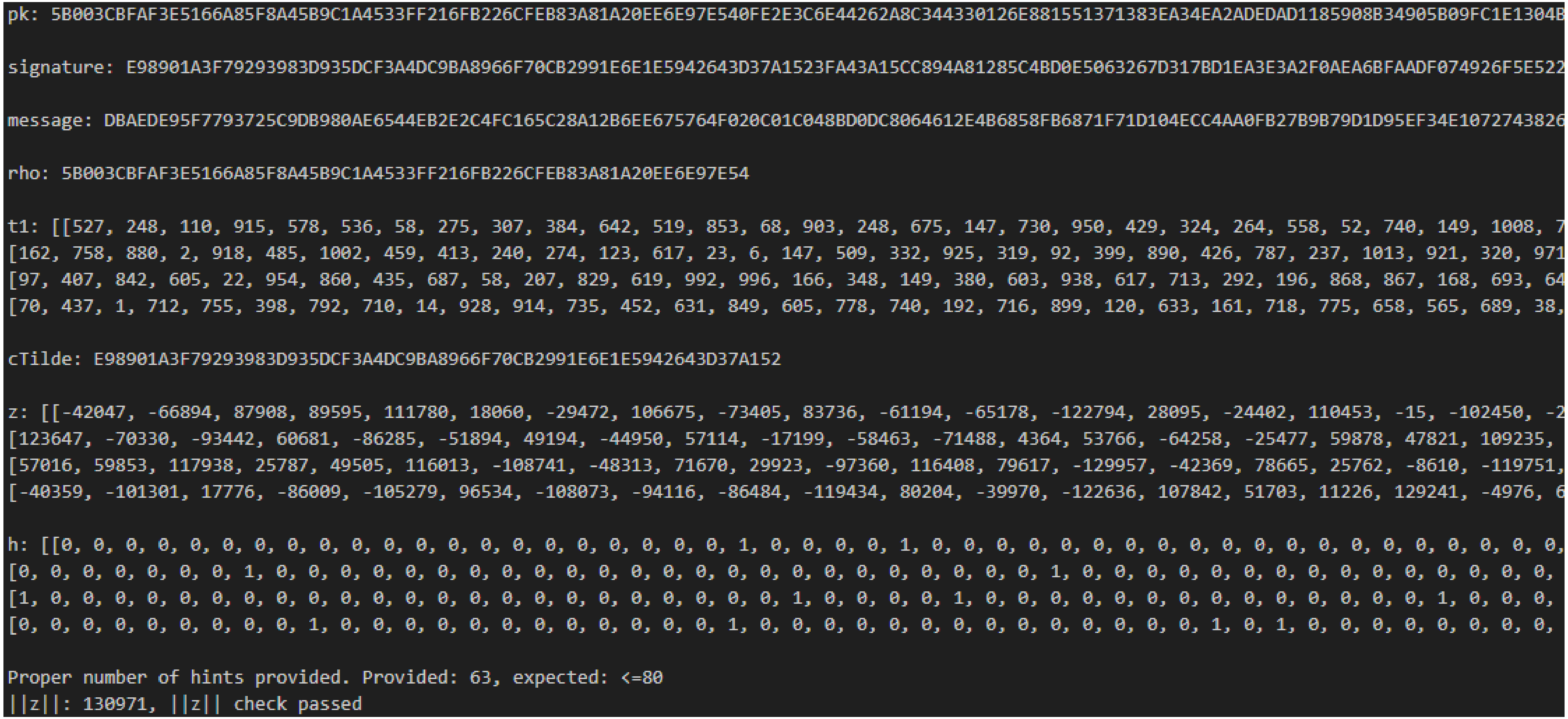
Input: Signature $\sigma \in \mathbb{B}^{\lambda/4+\ell\cdot 32\cdot(1+\text{bitlen}\,(\gamma_1-1))+\omega+k}$.

Output: Boolean

```
$(\rho, \mathbf{t}_1) \leftarrow$ pkDecode$(pk)$
$(\tilde{c}, \mathbf{z}, \mathbf{h}) \leftarrow$ sigDecode$(\sigma)$        ▷ signer's commitment hash $\tilde{c}$,
if $\mathbf{h} = \bot$ then return false        ▷ hint was
end if
$\hat{\mathbf{A}} \leftarrow$ ExpandA$(\rho)$        ▷ $\mathbf{A}$ is generated and stored in NT
$tr \leftarrow$ H$(pk, 64)$
$\mu \leftarrow ($H(BytesToBits$(tr)||M', 64))$        ▷ message representative
computed in a different cryptographic module
$c \in R_q \leftarrow$ SampleInBall$(\tilde{c})$        ▷ compute ver
$\mathbf{w}'_{\text{Approx}} \leftarrow$ NTT$^{-1}(\hat{\mathbf{A}} \circ$ NTT$(\mathbf{z}) -$ NTT$(c) \circ$ NTT$(\mathbf{t}_1 \cdot 2^d))$        ▷ w
$\mathbf{w}'_1 \leftarrow$ UseHint$(\mathbf{h}, \mathbf{w}'_{\text{Approx}})$        ▷ reconstruction c
        ▷ UseHint is applied componentwise (see explanat
$\tilde{c}' \leftarrow$ H$(\mu||$w1Encode$(\mathbf{w}'_1), \lambda/4)$        ▷ hash
return $[[\ ||\mathbf{z}||_\infty < \gamma_1 - \beta]]$ and $[[\tilde{c} = \tilde{c}']]$
```

```python
def Ver(pk,M,signature):
    rho,t1 = pk_decode(pk)
    c_tilde,z,h = sigDecode(signature)
    if h == None:
        return False
    A_hat = ExpandA(rho)
    tr = SHAKE_256(pk,512)
    mu = SHAKE_256(tr + M,512)
    c = SampleInBall(c_tilde)
    z_hat = [NTT(zi) for zi in z]
    Ah_d_zh = NTT_dot(A_hat,z_hat)
    c_hat = NTT(c)
    t1_hat = [NTT(ti) for ti in t1]
    for i in range(ML_DSA["k"]):
        for j in range(256):
            t1_hat[i][j] = (t1_hat[i][j] * (2**ML_DSA["d"])) % ML_DSA["q"]
    ch_d_t12d = NTT_dot_k(t1_hat,c_hat)
    w_prime_approx = array_minus_k(Ah_d_zh,ch_d_t12d)
    w_prime_approx = [NTT_inv(wi) for wi in w_prime_approx]
    w1_prime = []
    for i in range(ML_DSA["k"]):
        w1_prime_temp = []
        for j in range(256):
            w1_prime_temp.append(UseHint(h[i][j],w_prime_approx[i][j]))
        w1_prime.append(w1_prime_temp)
    w1En = w1Encode(w1_prime)
    c_prime_tilde = SHAKE_256(mu + w1En,2 * ML_DSA["lamda"])
    return ((infinity_norm(z) < (ML_DSA["gamma_1"] - ML_DSA["beta"]))
            and (c_prime_tilde == c_tilde))
```

Verification



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signature: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A1523FA43A15CC894A81285C4BD0E5063267D317BD1EA3E3A2F0AEA6BFAADF074926F5E522

message: DBAEDE95F7793725C9DB980AE6544EB2E2C4FC165C28A12B6EE675764F020C01C048BD0DC8064612E4B6858FB6871F71D104ECC4AA0FB27B9B79D1D95EF34E1072743826

rho: 5B003CBFAF3E5166A85F8A45B9C1A4533FF216FB226CFEB83A81A20EE6E97E54

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Proper number of hints provided. Provided: 63, expected: <=80
||z||: 130971, ||z|| check passed
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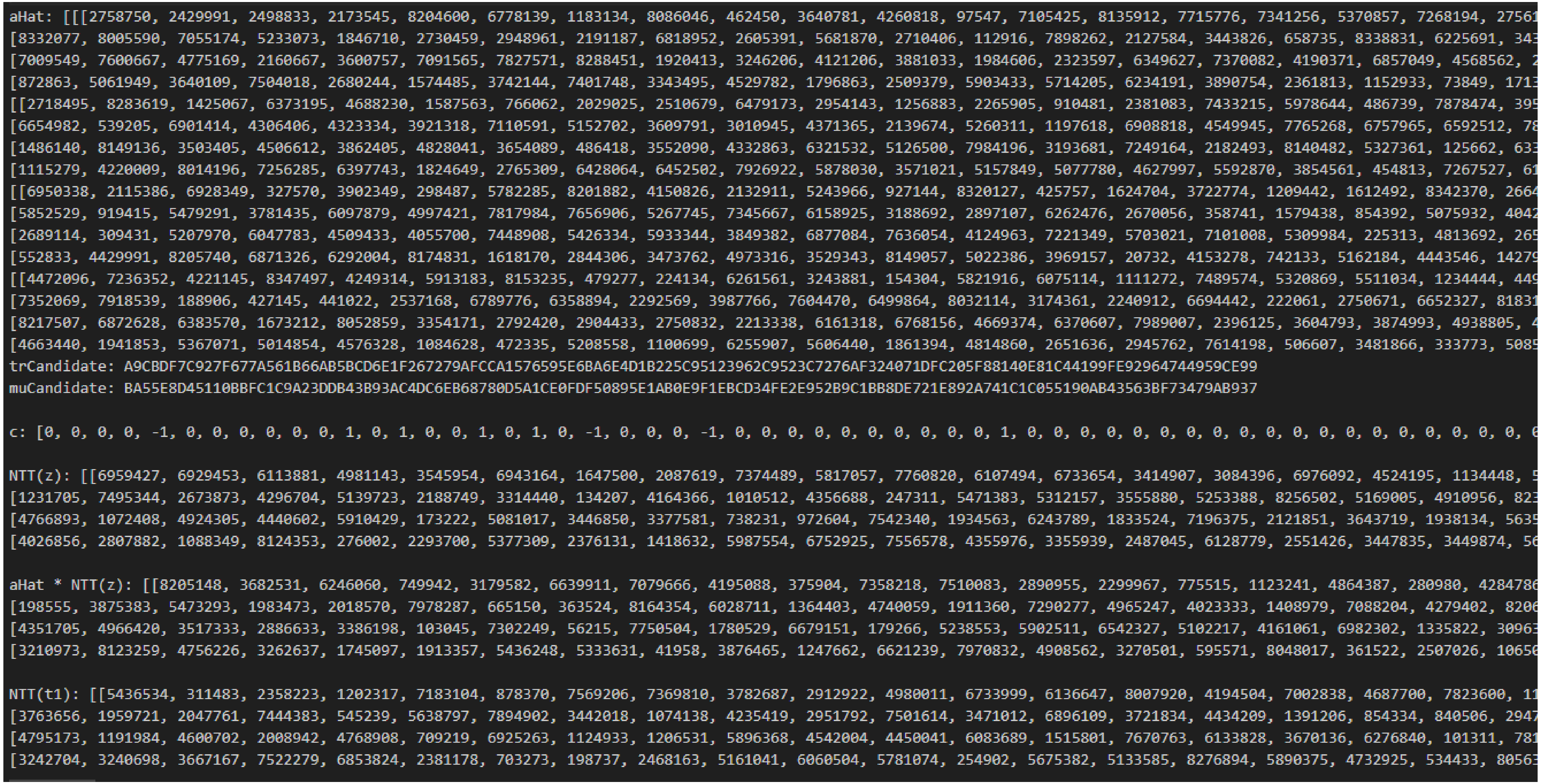
Verification



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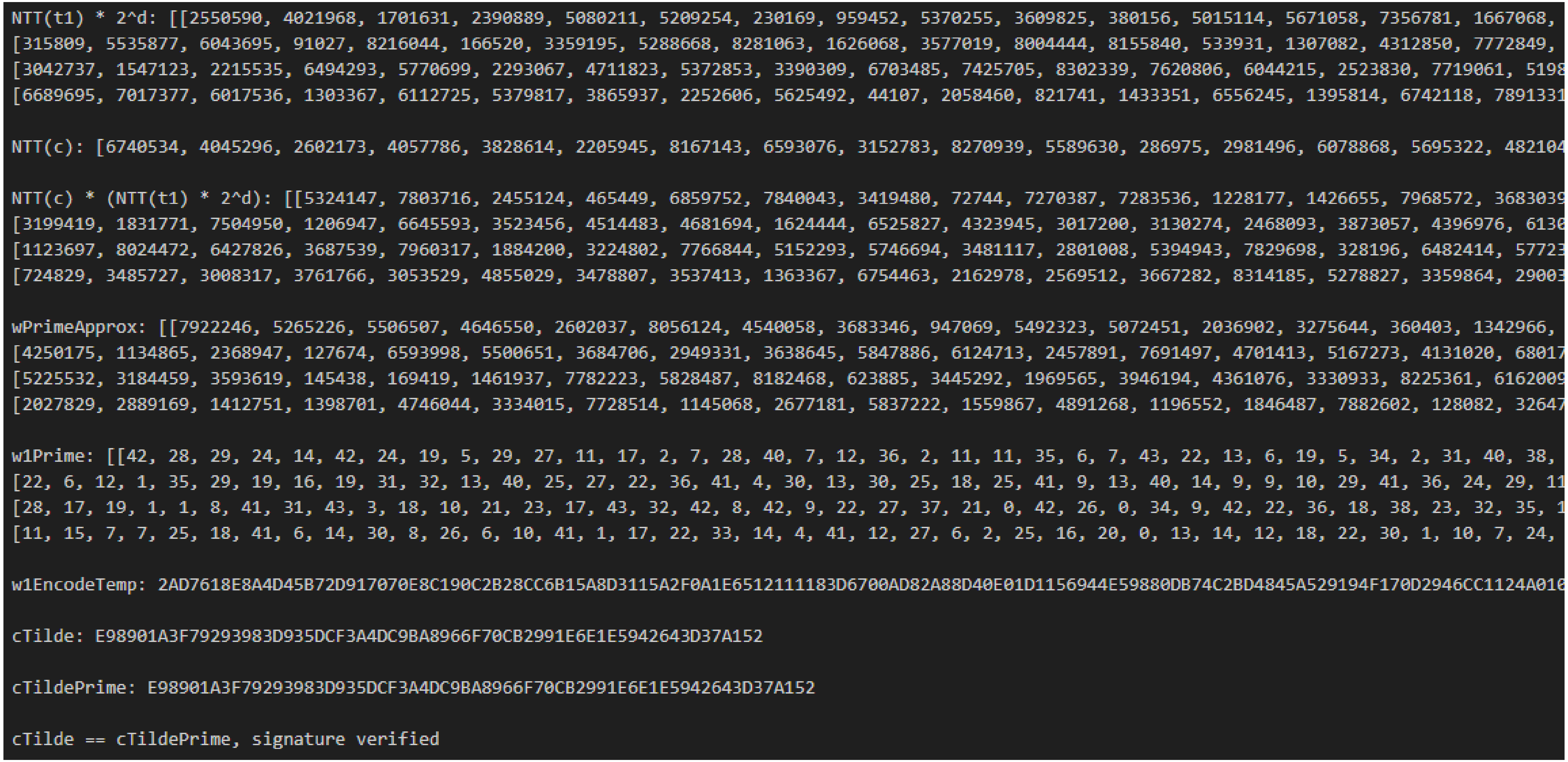
Verification



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NTT(c): [6740534, 4045296, 2602173, 4057786, 3828614, 2205945, 8167143, 6593076, 3152783, 8270939, 5589630, 286975, 2981496, 6078868, 5695322, 482104

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cTilde: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A152

cTildePrime: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A152

cTilde == cTildePrime, signature verified
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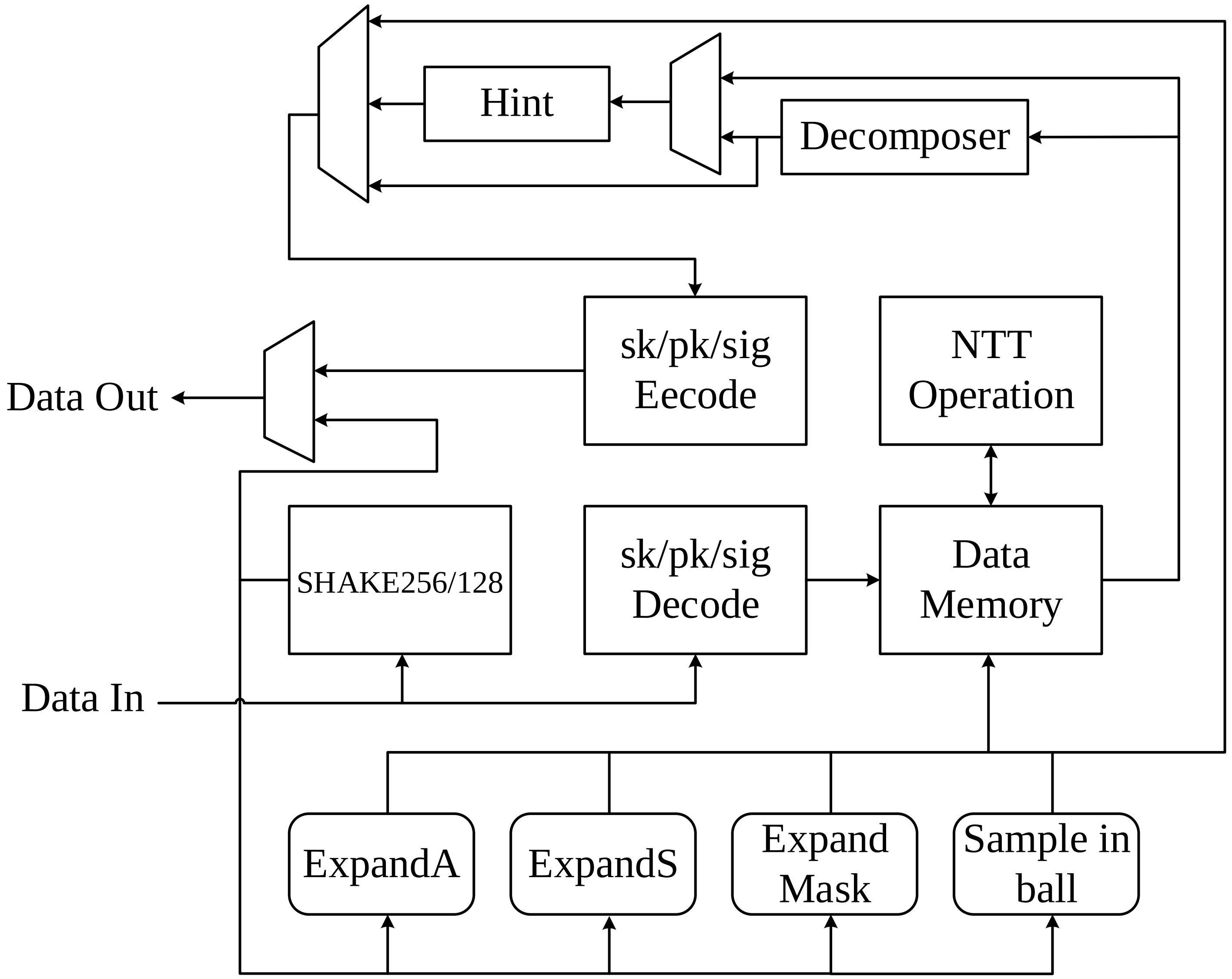
03

Architecture

Block Diagram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Expand
Mask
Sample in
ball