Design and Implementation of a Hardware Accelerator IP for Post-Quantum Cryptography ML-DSA Compatible with the AXI-4 Interface

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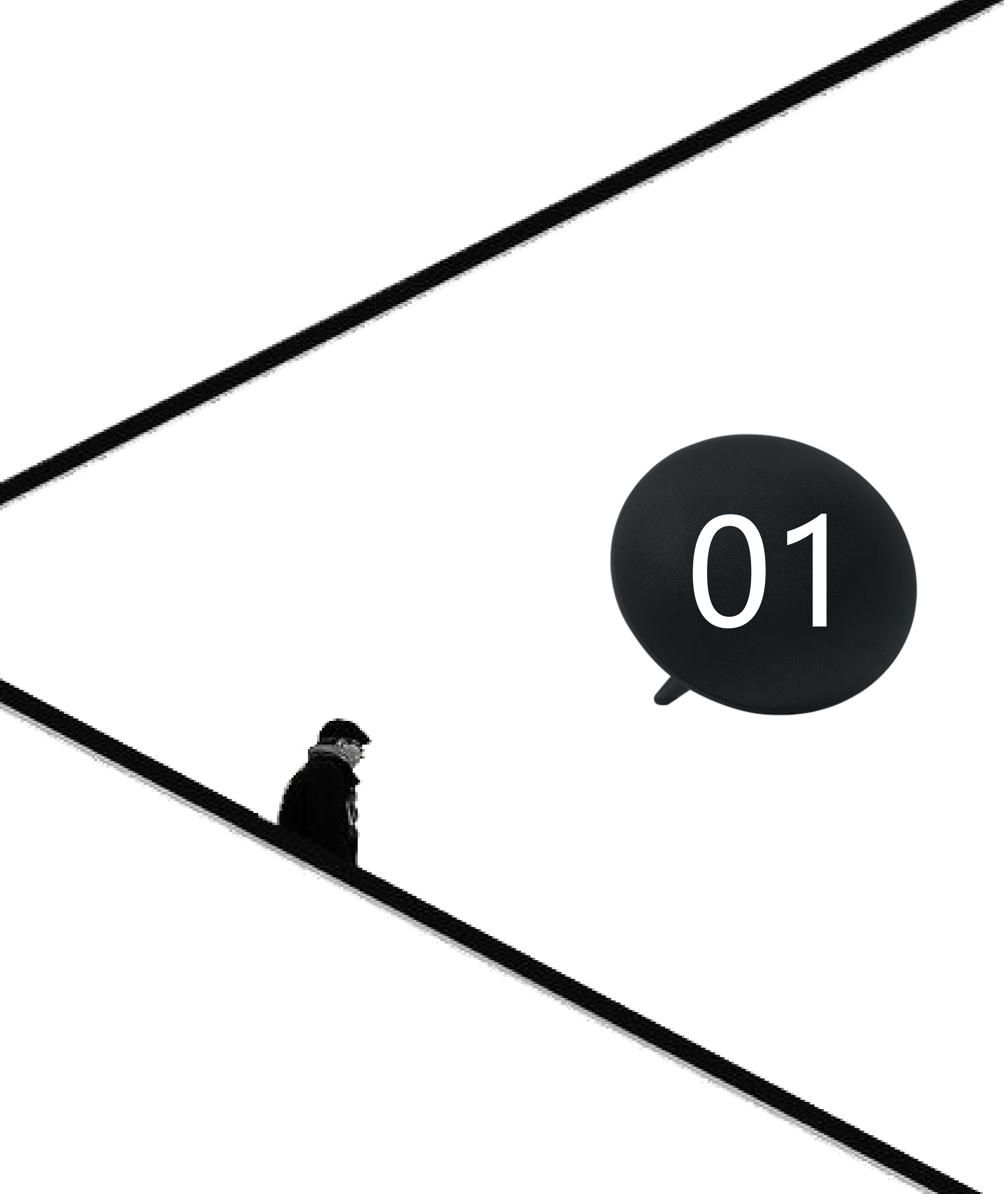


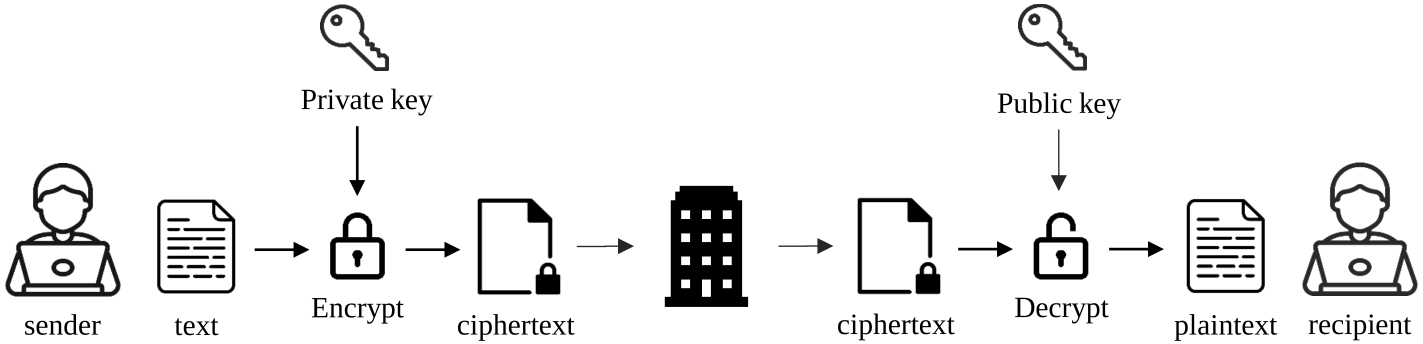
Introduction

Background

- ✓ Shor's algorithm, combined with a powerful quantum computer, will possibly break RSA and ECC.
- ✓ Initiated by NIST in 2016, the post-quantum cryptography standardization process, , it finalized the selection of **ML-DSA** as one of the encryption methods.
- ✓ Previously known as CRYSTAL-DILITHIUM

ML-DSA

- ✓ Defines method for generating digital signatures
- ✓ Based on the worst-case hardness of module lattice problems, it has potential resistance against both quantum and classical attacks.
- ✓ Advantages include fast arithmetic operations, efficient encryption, and compact signatures.
- ✓ Uses uniformly sampled high-entropy Gaussian-distributed secrets to generate random keys.
- ✓ The core security challenges of ML-DSA include MLWE problem and tMSIS problem



Fiat-Shamir with Aborts

1. Commitment：
   - The signer generates a random vector y ∈ R_q^{ℓ}
   - The commitment value is $w = Ay$
   - w is rounded to obtain w_1
2. Challenge：
   - The challenge c is generated by hashing w_1 and the message representative μ
3. Response：
   - The response $z = y + S_1 \cdot c$(where S_1 is part of the private key)
   - Use rejection sampling to check whether z meets specific coefficient bounds
4. Hint Calculation：
   - To enable the verifier to reconstruct w_1 from z and the compressed public value t_1
   - hint h ∈ R_q^{k}
5. Signature Composition：
   - The final signature consists of three parts: the rounded commitment w_1, the response z, and the hint h
6. Second Stage of Rejection Sampling：
   - To ensure the correctness of the signature, a second stage of rejection sampling must be performed

MLWE (module learning with errors)

Setup:

1. Modulus q=7.
2. Matrix A is of size 2×2, with elements selected randomly.
3. Secret vectors $s1$ and $s2$ are both of size 2×1.
4. Values for A, $s1$, $s2$:

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}, \quad s_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad s_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Calculation Steps:

1. Calculate $As1$:

$$As_1 = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 + 4 \cdot 3 \\ 1 \cdot 2 + 5 \cdot 3 \end{bmatrix} = \begin{bmatrix} 6 + 12 \\ 2 + 15 \end{bmatrix} = \begin{bmatrix} 18 \\ 17 \end{bmatrix}$$

2. Add the secret vector $s2$ to the result and take modulus q:

$$t = As_1 + s_2 = \begin{bmatrix} 18 \\ 17 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 19 \\ 21 \end{bmatrix}$$

$$t = \begin{bmatrix} 19 \mod 7 \\ 21 \mod 7 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

3. The public data is the matrix A and the result vector t:

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}, \quad t = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

MSIS (module shortest integer solution)

Setup:

1. Modulus $q = 7$
2. Matrix A is of size 3×2, with elements selected randomly.
3. Values for A:

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 5 \\ 6 & 2 \end{bmatrix}$$

Goal:

Find vectors z and u such that $Az + u = 0 \bmod q$.

Attempt to Solve:

1. Assume a vector z, u:

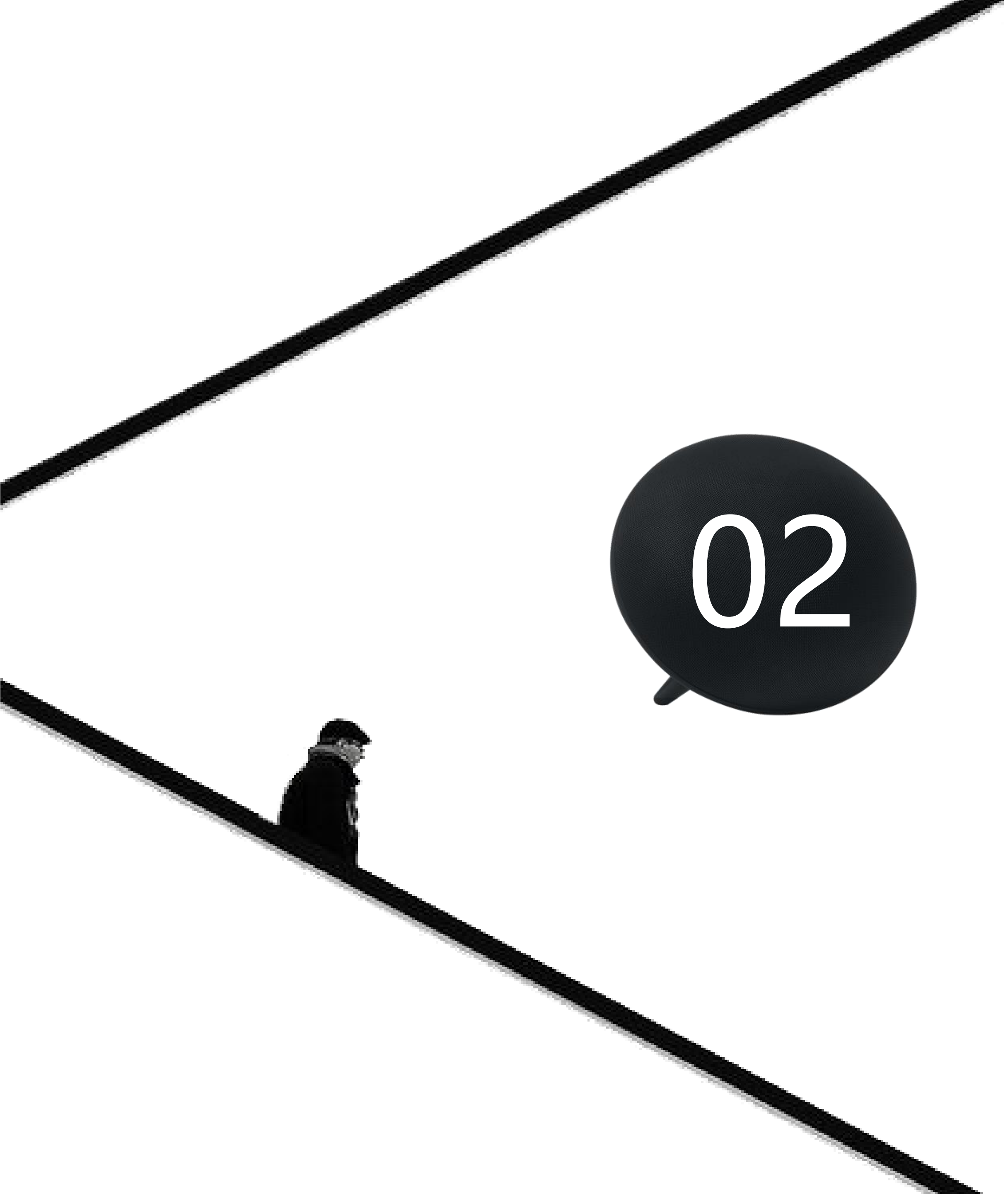
$$z = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad u = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}$$

2. Calculate $Az + u$ and take modulus q:

$$Az = \begin{bmatrix} 3 & 4 \\ 1 & 5 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 + 4 \cdot (-1) \\ 1 \cdot 2 + 5 \cdot (-1) \\ 6 \cdot 2 + 2 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 6 - 4 \\ 2 - 5 \\ 12 - 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 10 \end{bmatrix}$$

$$Az + u = \begin{bmatrix} 2 \\ -3 \\ 10 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 + (-1) \\ -3 + 3 \\ 10 + (-5) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

$$Az + u \mod 7 = \begin{bmatrix} 1 \mod 7 \\ 0 \mod 7 \\ 5 \mod 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

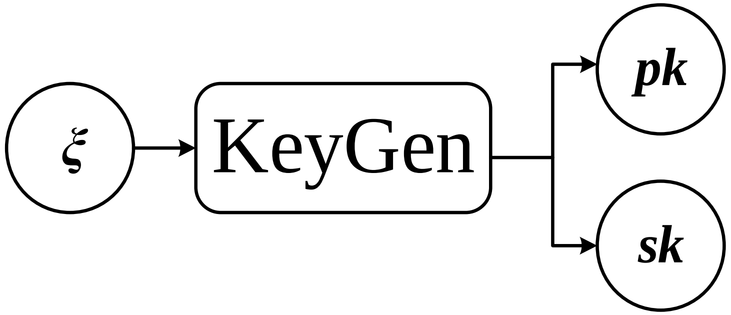


02

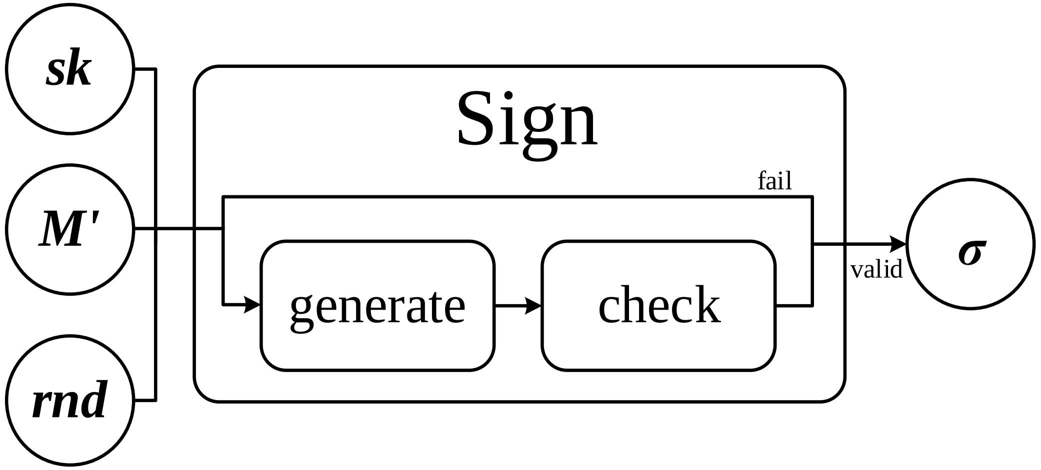
Algorithms

Algorithm

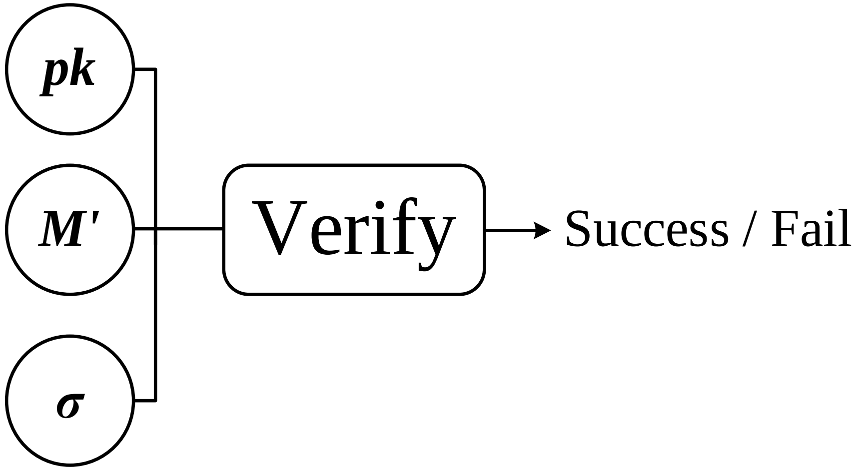
1. Key generation (KeyGen)



2. Signature generation (Sign)



3. Signature verification (Verify)



Symbols

- ***ξ*** : random seed
- ***p***k : public key
- ***sk*** : secret key
- ***M'*** : hash message
- ***rnd*** : random number
- ***σ*** : signature

Key Generation

Algorithm 6 ML-DSA.KeyGen_internal(ξ)

Generates a public-private key pair from a seed.

Input: Seed $\xi \in \mathbb{B}^{32}$

Output: Public key $pk \in \mathbb{B}^{32+32k(\text{bitlen }(q-1)-d)}$

and private key $sk \in \mathbb{B}^{32+32+64+32\cdot((\ell+k)\cdot\text{bitlen }(2\eta)+dk)}$

$(\rho, \rho', K) \in \mathbb{B}^{32} \times \mathbb{B}^{64} \times \mathbb{B}^{32} \leftarrow$ H$(\xi||$IntegerToBytes$(k, 1)||$IntegerToBytes$(\ell, 1), 128)$

$\triangleright$ expand se

$\hat{\mathbf{A}} \leftarrow$ ExpandA(ρ) $\triangleright$ $\mathbf{A}$ is generated and stored in NTT representation as

$(\mathbf{s}_1, \mathbf{s}_2) \leftarrow$ ExpandS(ρ')

$\mathbf{t} \leftarrow$ NTT$^{-1}(\hat{\mathbf{A}} \circ$ NTT$(\mathbf{s}_1)) + \mathbf{s}_2$ $\triangleright$ compute $\mathbf{t} = \mathbf{A}\mathbf{s}_1 +$

$(\mathbf{t}_1, \mathbf{t}_0) \leftarrow$ Power2Round$(\mathbf{t})$ $\triangleright$ compres

$\triangleright$ PowerTwoRound is applied componentwise (see explanatory text in Section 7

$pk \leftarrow$ pkEncode$(\rho, \mathbf{t}_1)$

$tr \leftarrow$ H$(pk, 64)$

$sk \leftarrow$ skEncode$(\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0)$ $\triangleright$ K and tr are for use in signi

return (pk, sk)

```python
def KeyGen(xi):
    H_xi = SHAKE_256(xi,1024)
    print(H_xi)
    p = H_xi[:32]
    p_prime = H_xi[32:96]
    K = H_xi[96:128]
    A_hat = ExpandA(p)
    s1, s2 = ExpandS(p_prime)
    s1Hat = [NTT(s) for s in s1]
    s1Hat = np.array(s1Hat)
    A_NTT_s1 = NTT_dot(A_hat,s1Hat)
    aHat_mul_s1Hat = [NTT_inv(s) for s in A_NTT_s1]
    t = []
    for i in range(ML_DSA["k"]):
        d = []
        for k in range(256):
            sum = aHat_mul_s1Hat[i][k] + s2[i][k]
            d.append(sum)
        t.append(d)
    t1 = []
    t0 = []
    for ti in range (ML_DSA["k"]):
        ta1 = []
        ta0 = []
        for tp in range(256):
            t1_temp,t0_temp = Power2Round(t[ti][tp])
            ta1.append(t1_temp)
            ta0.append(t0_temp)
        t1.append(ta1)
        t0.append(ta0)
    pk = pk_encode(p, t1)
    tr = SHAKE_256(pk,512)
    sk = sk_encode(p, K, tr, s1, s2, t0)
    return pk, sk
```

Key Generation

```
seed: 6CAE2E9C2CF64D2686C31C2118E0F24A47DD46DB85590910AAC9DF4C1B854E44

rho: C8BEADEDC6DBA5BF3BECA52C67CEAFB4F3EBF84190B2CFA6BCA132883129A28B

rhoPrime: 11779B16A7054953860C14796F63018C9EFD3957CC53A12AF727A5AFC64507445D9EA5E19B6403B3DD3ABAD9B1DAD1146E9C64410E372E7A6D9973F0D04D9632

k: B149C045A55EADA0C519069A8EE0602FBEDA8D2EDFEA09CAE01D542D47DCBA1E

aHat: [[[4518441, 4610216, 2805006, 6522567, 958931, 2266298, 7298857, 6160680, 4376220, 5886423, 2456656, 7246256, 4825911, 4337879, 2286865, 4
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```

Key Generation

```
s1: [[-2, -1, 1, 2, -2, -1, 2, 2, -1, 1, 1, 0, -2, 2, 0, 2, -2, 0, 2, 2, -1, 2, -1, 2, 0, -2, -2, 0, 0, -2, 2, -2, 2, -2, 1, 1
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s2: [[1, 1, -1, 0, 1, -1, 2, -2, 1, 0, 0, 1, -2, -1, 2, -1, 2, 1, 1, 0, 2, 1, -1, -1, 0, 2, -2, 0, 2, 2, 2, -2, -2, -2, -1, -2
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NTTInverse(aHat * s1Hat): [[4089385, 3243627, 2997576, 1860759, 7743501, 7853441, 1170077, 1195218, 7888106, 665458, 5751129,
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```

Key Generation

```
t: [[4089386, 3243628, 2997575, 1860759, 7743502, 7853440, 1170079, 1195216, 7888107, 665458, 5751129, 5154175, 7545299, 4808039, 4175100, 72
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t0: [[1578, -404, -697, 1175, 2062, -2688, -1377, -816, -789, 1906, 345, 1407, 467, -665, -2820, 3878, -2013, -413, -675, -103, -1522, -3652,
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t1: [[499, 396, 366, 227, 945, 959, 143, 146, 963, 81, 702, 629, 921, 587, 510, 885, 898, 534, 598, 982, 588, 490, 744, 917, 674, 991, 336, 7
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tr: 75A821E4FF2B52A3AB3DDD0C77C3A9F96FCC9BE360C2B75C97D7F9DEC97D1BDDE028D36C4FE18093AF6C5794AD19F9FA090C19A76F05A7F3B930B11792A13A7A
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sk: C8BEADEDC6DBA5BF3BECA52C67CEAFB4F3EBF84190B2CFA6BCA132883129A28BB149C045A55EADA0C519069A8EE0602FBEDA8D2EDFEA09CAE01D542D47DCBA1E75A821E4F
```

Signing

Algorithm 7 ML-DSA.Sign_internal(sk, M', rnd)

Deterministic algorithm to generate a signature for a formatted message M'.

Input: Private key $sk \in \mathbb{B}^{32+32+64+32\cdot((\ell+k)\cdot \text{bitlen}\,(2\eta)+dk)}$, formatted message $M' \in \{0,1\}^*$, and per message randomness or dummy variable $rnd \in \mathbb{B}^{32}$.
Output: Signature $\sigma \in \mathbb{B}^{\lambda/4+\ell\cdot 32\cdot(1+\text{bitlen}\,(\gamma_1-1))+\omega+k}$.

$(\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0) \leftarrow$ skDecode(sk)
$\hat{\mathbf{s}}_1 \leftarrow$ NTT$(\mathbf{s}_1)$
$\hat{\mathbf{s}}_2 \leftarrow$ NTT$(\mathbf{s}_2)$
$\hat{\mathbf{t}}_0 \leftarrow$ NTT$(\mathbf{t}_0)$
$\hat{\mathbf{A}} \leftarrow$ ExpandA(ρ) ▷ $\mathbf{A}$ is generated and stored in NTT representation as $\hat{\mathbf{A}}$
$\mu \leftarrow$ H(BytesToBits$(tr)||M', 64)$ ▷ message representative that may optionally be computed in a different cryptographic module
$\rho'' \leftarrow$ H$(K||rnd||\mu, 64)$ ▷ compute private random seed
$\kappa \leftarrow 0$ ▷ initialize counter κ
$(\mathbf{z}, \mathbf{h}) \leftarrow \perp$
while $(\mathbf{z}, \mathbf{h}) = \perp$ **do** ▷ rejection sampling loop
  $\mathbf{y} \in R_q^\ell \leftarrow$ ExpandMask(ρ'', κ)
  $\mathbf{w} \leftarrow$ NTT$^{-1}(\hat{\mathbf{A}} \circ$ NTT$(\mathbf{y}))$
  $\mathbf{w}_1 \leftarrow$ HighBits$(\mathbf{w})$ ▷ signer's commitment
  ▷ HighBits is applied componentwise (see explanatory text in Section 7.4)
  $\tilde{c} \leftarrow$ H$(\mu||$w1Encode$(\mathbf{w}_1), \lambda/4)$ ▷ commitment hash
  $c \in R_q \leftarrow$ SampleInBall$(\tilde{c})$ ▷ verifier's challenge
  $\hat{c} \leftarrow$ NTT(c)

```python
# 算法 2 ML-DSA.Sign(sk,M)
def Sign(sk,M,rnd):
    (p,K,tr,s1,s2,t0) = sk_decode(sk)
    s1_hat = [NTT(si) for si in s1]
    s2_hat = [NTT(si) for si in s2]
    t0_hat = [NTT(ti) for ti in t0]
    A_hat = ExpandA(p)
    u = tr + M
    u = SHAKE_256(u,512)
    p_prime = K + rnd + u
    p_prime = SHAKE_256(p_prime,512)
    ka = 0
    z = None
    h = None
    while z == None and h == None:
        y = ExpandMask(p_prime,ka)
        y_hat = [NTT(yi) for yi in y]
        w = NTT_dot(A_hat,y_hat)
        w = [NTT_inv(wi) for wi in w]
        w1 = [HighBits(w1i) for w1i in w]
        w1 = w1Encode(w1)
        c_tilde = u + w1
        c_tilde = SHAKE_256(c_tilde,2*ML_DSA["lamda"])
        c = SampleInBall(c_tilde)
        print(c)
        c_hat = NTT(c)
```

Signing

```
message: DA5FBC7F78116BC8537E8E522DFBB6F68710FC36AE5AF0ACE5CFA3BA4FEBF6C86D966A44C2CF53CCC4FF5B9CC4E6403CFF3C23B4F292AAC01E35A21AB11F0707726E88048DD05887448B8

sk: 5AE5192442A0894AC775D84419BB7EDE9E8143A6E21CD709A47B58545EAEC7F59EAD08050900CBCF4093F9327CAD67B7600D9F8425084B6905BB6550F426F58B27B47C434BC194D466D83C14BF

rnd: 0000000000000000000000000000000000000000000000000000000000000000

rho: 5AE5192442A0894AC775D84419BB7EDE9E8143A6E21CD709A47B58545EAEC7F5

k: 9EAD08050900CBCF4093F9327CAD67B7600D9F8425084B6905BB6550F426F58B

tr: 27B47C434BC194D466D83C14BF5ABEA249E05287EC80D1B56BD0824E4EB84CE1CD4CCD0DBD1BDE59106AE10516E1150E841B03E759DFDD930F6C7BAC85753045

mu: A12A0CBDCEC8AEF5EC6AD85326CA82938BBEA6C371CA19083DC23BED8053940A544CA299C0CF8AF2DD7BDA21A995598DB01944BAEC75E78D9774652CB8CB3250

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Signing

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rhoPrime: A9A9F85C9C20C2B207E25FDC800A2E59F41FB874B7A5DB2CA80C3CD2F6FD1DD6D8F945C176A75CCDEE86D21C830C4164C2386D38968EC0F58ECAA2E6C5193BC7
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Signing

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[-29248, 56187, -92953, -5383, 29593, 10080, -55123, 90644, -91145, -64817, 2370, -92112, 22362, 55467, 61858, 104855, -3696, -32105, -94001, 98242, -33037, -85356, 11368
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NTT(y): [[3035491, 2809772, 6725498, 567655, 5447090, 6430497, 588219, 6597274, 7259775, 1642189, 5313552, 2601666, 2944974, 6295297, 3581905, 6874192, 210561, 5960622, 8
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aHat * NTT(y): [[6144420, 4965121, 7555555, 3905658, 6043149, 7189003, 8075652, 2203037, 2783350, 3674236, 2430017, 5393384, 4941164, 4296081, 6148283, 7660585, 2308491,
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w = NTTInverse(aHat * NTT(y)): [[4968407, 4718179, 462388, 1119871, 7817329, 3820487, 6327751, 947481, 6639094, 4489022, 8186388, 1912045, 4956263, 7739800, 7404478, 8379
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w1: [[26, 25, 2, 6, 41, 20, 33, 5, 35, 24, 43, 10, 26, 41, 39, 0, 34, 37, 1, 37, 20, 5, 15, 0, 42, 18, 29, 32, 31, 24, 10, 40, 18, 13, 17, 32, 11, 22, 40, 43, 22, 14, 24,
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w1Encode: 5A261829151623B62A5A7A0262199454F100AAD4811FA6A05213818B85AE96833959781E65B126E056866349665AD05020795E81443552C8308B9115CE7AA998A86EA8018CDAC43817B18D94E800E8C1

cTilde: 3202542EF1E239D32BE1BCE5AE4AC8052D578899D653E368E11BC11C5480BA06

c: [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 1,

cHat: [3919627, 1297980, 1398134, 6081972, 7171056, 414117, 5281780, 4348975, 7918931, 7431142, 1215943, 8115251, 2501891, 2979933, 1056051, 4629893, 4230848, 5517032, 50
```

Signing

$\langle\langle \mathbf{cs}_1 \rangle\rangle \leftarrow \mathsf{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_1)$

$\langle\langle \mathbf{cs}_2 \rangle\rangle \leftarrow \mathsf{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_2)$

$\mathbf{z} \leftarrow \mathbf{y} + \langle\langle \mathbf{cs}_1 \rangle\rangle$ ▷ signer's response

$\mathbf{r}_0 \leftarrow \mathsf{LowBits}(\mathbf{w} - \langle\langle \mathbf{cs}_2 \rangle\rangle)$

▷ LowBits is applied componentwise (see explanatory text in Section 7.4)

if $||\mathbf{z}||_\infty \geq \gamma_1 - \beta$ **or** $||\mathbf{r}_0||_\infty \geq \gamma_2 - \beta$ **then** $(\mathbf{z}, \mathbf{h}) \leftarrow \perp$ ▷ validity checks

else

$\langle\langle \mathbf{ct}_0 \rangle\rangle \leftarrow \mathsf{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{t}}_0)$

$\mathbf{h} \leftarrow \mathsf{MakeHint}(-\langle\langle \mathbf{ct}_0 \rangle\rangle, \mathbf{w} - \langle\langle \mathbf{cs}_2 \rangle\rangle + \langle\langle \mathbf{ct}_0 \rangle\rangle)$ ▷ Signer's hint

▷ MakeHint is applied componentwise (see explanatory text in Section 7.4)

if $||\langle\langle \mathbf{ct}_0 \rangle\rangle||_\infty \geq \gamma_2$ **or** the number of 1's in $\mathbf{h}$ is greater than ω, **then** $(\mathbf{z}, \mathbf{h}) \leftarrow \perp$

end if

end if

$\kappa \leftarrow \kappa + \ell$ ▷ increment counter

end while

$\sigma \leftarrow \mathsf{sigEncode}(\tilde{c}, \mathbf{z} \bmod^{\pm} q, \mathbf{h})$

return σ

```python
        cs1 = NTT_dot_l(s1_hat,c_hat)
        cs1 = [NTT_inv(csi) for csi in cs1]
        cs2 = NTT_dot_k(s2_hat,c_hat)
        cs2 = [NTT_inv(csi) for csi in cs2]
        z = array_plus_l(y,cs1)
        temp = array_minus_k(w,cs2)
        r0 = [LowBits(w1i) for w1i in temp]
        if (infinity_norm(z) >= ML_DSA["gamma_1"] - ML_DSA["beta"] or
            infinity_norm(r0) >= ML_DSA["gamma_2"] - ML_DSA["beta"]):
            z = None
            h = None
        else:
            ct0 = NTT_dot_k(t0_hat,c_hat)
            ct0 = [NTT_inv(cti) for cti in ct0]
            zero_array = [[0]*256] * ML_DSA["k"]
            w_minus_cs2 = array_minus_k(w,cs2)
            w_minus_cs2_pluse_ct0 = array_plus_k(w_minus_cs2,ct0)
            minus_ct0 = array_minus_k(zero_array,ct0)
            h,true_num = MakeHint(minus_ct0, w_minus_cs2_pluse_ct0)
            if (infinity_norm(c_tilde) >= ML_DSA["gamma_2"] or
                true_num > ML_DSA["omega"]):
                z = None
                h = None
        ka = ka + ML_DSA["l"]
    z_mod = []
    for i in range(ML_DSA["l"]):
        z_temp = []
        for j in range(256):
            z_temp.append(mod_pm(z[i][j]))
        z_mod.append(z_temp)
    Sigma = sigEncode(c_tilde,z_mod,h)
    return Sigma
```

Signing

```
cHat: [3919627, 1297980, 1398134, 6081972, 7171056, 414117, 5281780, 4348975, 7918931, 7431142, 1215943, 8115251, 2501891, 2979933, 1056051, 4629893, 4230848, 5517032, 50

cs1: [[2, 8380399, 8380416, 8380402, 8380407, 0, 1, 8380415, 6, 8380415, 9, 8380408, 0, 24, 7, 4, 5, 2, 8380406, 8380412, 6, 8, 3, 8380414, 8, 8380408, 8380414, 13, 83804
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cs2: [[7, 13, 8380415, 8380414, 8380406, 8380415, 5, 8380415, 8380415, 18, 8380405, 8380409, 8380415, 8380392, 5, 0, 2, 8380410, 8380407, 8, 4, 1, 11, 12, 8380406, 10, 83
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||z||: 130985, ||z|| check passed

r0: [[16336, -43434, 81462, -22910, 8316, 11209, 42434, -4837, -27144, -82132, -3552, 7413, 4201, -69199, -23623, -591, 50423, 27202, -43436, -94230, 1484, -49886, -81705
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||r0||95002, ||r0|| check passed

cHat * t0Hat: [[6685568, 3625903, 5012222, 7942920, 1902157, 4674810, 4210018, 6254152, 5621555, 4432127, 3161400, 2991916, 3326774, 463020, 6499212, 7545109, 1598396, 89
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Signing

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w - cs2 + ct0: [[4961016, 4702315, 465924, 1086137, 7810002, 3798349, 6339310, 972430, 6660148, 4493508, 8186370, 1911272, 4948125, 7727984, 7395494, 8379250, 6539211, 7063172, 143873, 69710
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h: [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
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||h||: 62, Hint check passed

||ct0||: 46263, ||ct0|| check passed

signature: 3202542EF1E239D32BE1BCE5AE4AC8052D578899D653E368E11BC11C5480BA06FED24E83A4361E358121DA338108794DDBF93ED0FDE9AD07C50F983BAAF01E985F9A6F380C6B14148AC829B67467CCF9F2A16D2594DB895BBE6
```

Verification

Algorithm 8 ML-DSA.Verify_internal(pk, M', σ)

Internal function to verify a signature σ *for a formatted message* M'.

Input: Public key $pk \in \mathbb{B}^{32+32k(\text{bitlen }(q-1)-d)}$ and message $M' \in \{0,1\}^*$

Input: Signature $\sigma \in \mathbb{B}^{\lambda/4+\ell\cdot 32\cdot(1+\text{bitlen }(\gamma_1-1))+\omega+k}$.

Output: Boolean

$(\rho, \mathbf{t}_1) \leftarrow$ pkDecode(pk)

$(\tilde{c}, \mathbf{z}, \mathbf{h}) \leftarrow$ sigDecode(σ) ▷ signer's commitment hash $\tilde{c}$,

if $\mathbf{h} = \bot$ **then return** false ▷ hint was

end if

$\hat{\mathbf{A}} \leftarrow$ ExpandA(ρ) ▷ $\mathbf{A}$ is generated and stored in NT

$tr \leftarrow$ H$(pk, 64)$

$\mu \leftarrow ($H$($BytesToBits$(tr)||M', 64))$ ▷ message representative computed in a different cryptographic module

$c \in R_q \leftarrow$ SampleInBall$(\tilde{c})$ ▷ compute ver

$\mathbf{w}'_{\text{Approx}} \leftarrow$ NTT$^{-1}(\hat{\mathbf{A}} \circ$ NTT$(\mathbf{z}) -$ NTT$(c) \circ$ NTT$(\mathbf{t}_1 \cdot 2^d))$ ▷ w

$\mathbf{w}'_1 \leftarrow$ UseHint$(\mathbf{h}, \mathbf{w}'_{\text{Approx}})$ ▷ reconstruction o

▷ UseHint is applied componentwise (see explanat

$\tilde{c}' \leftarrow$ H$(\mu||$w1Encode$(\mathbf{w}'_1), \lambda/4)$ ▷ hash

return $[[\ ||\mathbf{z}||_\infty < \gamma_1 - \beta]]$ and $[[\tilde{c} = \tilde{c}']]$

```python
def Ver(pk,M,signature):
    rho,t1 = pk_decode(pk)
    c_tilde,z,h = sigDecode(signature)
    if h == None:
        return False
    A_hat = ExpandA(rho)
    tr = SHAKE_256(pk,512)
    mu = SHAKE_256(tr + M,512)
    c = SampleInBall(c_tilde)
    z_hat = [NTT(zi) for zi in z]
    Ah_d_zh = NTT_dot(A_hat,z_hat)
    c_hat = NTT(c)
    t1_hat = [NTT(ti) for ti in t1]
    for i in range(ML_DSA["k"]):
        for j in range(256):
            t1_hat[i][j] = (t1_hat[i][j] * (2**ML_DSA["d"])) % ML_DSA["q"]
    ch_d_t12d = NTT_dot_k(t1_hat,c_hat)
    w_prime_approx = array_minus_k(Ah_d_zh,ch_d_t12d)
    w_prime_approx = [NTT_inv(wi) for wi in w_prime_approx]
    w1_prime = []
    for i in range(ML_DSA["k"]):
        w1_prime_temp = []
        for j in range(256):
            w1_prime_temp.append(UseHint(h[i][j],w_prime_approx[i][j]))
        w1_prime.append(w1_prime_temp)
    w1En = w1Encode(w1_prime)
    c_prime_tilde = SHAKE_256(mu + w1En,2 * ML_DSA["lamda"])
    return ((infinity_norm(z) < (ML_DSA["gamma_1"] - ML_DSA["beta"]))
            and (c_prime_tilde == c_tilde))
```

Verification

```
pk: 5B003CBFAF3E5166A85F8A45B9C1A4533FF216FB226CFEB83A81A20EE6E97E540FE2E3C6E44262A8C344330126E881551371383EA34EA2ADEDAD1185908B34905B09FC1E1304B

signature: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A1523FA43A15CC894A81285C4BD0E5063267D317BD1EA3E3A2F0AEA6BFAADF074926F5E522

message: DBAEDE95F7793725C9DB980AE6544EB2E2C4FC165C28A12B6EE675764F020C01C048BD0DC8064612E4B6858FB6871F71D104ECC4AA0FB27B9B79D1D95EF34E1072743826

rho: 5B003CBFAF3E5166A85F8A45B9C1A4533FF216FB226CFEB83A81A20EE6E97E54

t1: [[527, 248, 110, 915, 578, 536, 58, 275, 307, 384, 642, 519, 853, 68, 903, 248, 675, 147, 730, 950, 429, 324, 264, 558, 52, 740, 149, 1008, 7
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cTilde: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A152

z: [[-42047, -66894, 87908, 89595, 111780, 18060, -29472, 106675, -73405, 83736, -61194, -65178, -122794, 28095, -24402, 110453, -15, -102450, -2
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[-40359, -101301, 17776, -86009, -105279, 96534, -108073, -94116, -86484, -119434, 80204, -39970, -122636, 107842, 51703, 11226, 129241, -4976, 6

h: [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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Proper number of hints provided. Provided: 63, expected: <=80
||z||: 130971, ||z|| check passed
```

Verification

```
aHat: [[[2758750, 2429991, 2498833, 2173545, 8204600, 6778139, 1183134, 8086046, 462450, 3640781, 4260818, 97547, 7105425, 8135912, 7715776, 7341256, 5370857, 7268194, 27561
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trCandidate: A9CBDF7C927F677A561B66AB5BCD6E1F267279AFCCA1576595E6BA6E4D1B225C95123962C9523C7276AF324071DFC205F88140E81C44199FE92964744959CE99
muCandidate: BA55E8D45110BBFC1C9A23DDB43B93AC4DC6EB68780D5A1CE0FDF50895E1AB0E9F1EBCD34FE2E952B9C1BB8DE721E892A741C1C055190AB43563BF73479AB937

c: [0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, -1, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0

NTT(z): [[6959427, 6929453, 6113881, 4981143, 3545954, 6943164, 1647500, 2087619, 7374489, 5817057, 7760820, 6107494, 6733654, 3414907, 3084396, 6976092, 4524195, 1134448, 5
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aHat * NTT(z): [[8205148, 3682531, 6246060, 749942, 3179582, 6639911, 7079666, 4195088, 375904, 7358218, 7510083, 2890955, 2299967, 775515, 1123241, 4864387, 280980, 4284786
[198555, 3875383, 5473293, 1983473, 2018570, 7978287, 665150, 363524, 8164354, 6028711, 1364403, 4740059, 1911360, 7290277, 4965247, 4023333, 1408979, 7088204, 4279402, 8206
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NTT(t1): [[5436534, 311483, 2358223, 1202317, 7183104, 878370, 7569206, 7369810, 3782687, 2912922, 4980011, 6733999, 6136647, 8007920, 4194504, 7002838, 4687700, 7823600, 11
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```

Verification

```
NTT(t1) * 2^d: [[2550590, 4021968, 1701631, 2390889, 5080211, 5209254, 230169, 959452, 5370255, 3609825, 380156, 5015114, 5671058, 7356781, 1667068,
[315809, 5535877, 6043695, 91027, 8216044, 166520, 3359195, 5288668, 8281063, 1626068, 3577019, 8004444, 8155840, 533931, 1307082, 4312850, 7772849,
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NTT(c): [6740534, 4045296, 2602173, 4057786, 3828614, 2205945, 8167143, 6593076, 3152783, 8270939, 5589630, 286975, 2981496, 6078868, 5695322, 482104

NTT(c) * (NTT(t1) * 2^d): [[5324147, 7803716, 2455124, 465449, 6859752, 7840043, 3419480, 72744, 7270387, 7283536, 1228177, 1426655, 7968572, 3683039
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wPrimeApprox: [[7922246, 5265226, 5506507, 4646550, 2602037, 8056124, 4540058, 3683346, 947069, 5492323, 5072451, 2036902, 3275644, 360403, 1342966,
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w1Prime: [[42, 28, 29, 24, 14, 42, 24, 19, 5, 29, 27, 11, 17, 2, 7, 28, 40, 7, 12, 36, 2, 11, 11, 35, 6, 7, 43, 22, 13, 6, 19, 5, 34, 2, 31, 40, 38,
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w1EncodeTemp: 2AD7618E8A4D45B72D917070E8C190C2B28CC6B15A8D3115A2F0A1E6512111183D6700AD82A88D40E01D1156944E59880DB74C2BD4845A529194F170D2946CC1124A010

cTilde: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A152

cTildePrime: E98901A3F79293983D935DCF3A4DC9BA8966F70CB2991E6E1E5942643D37A152

cTilde == cTildePrime, signature verified
```
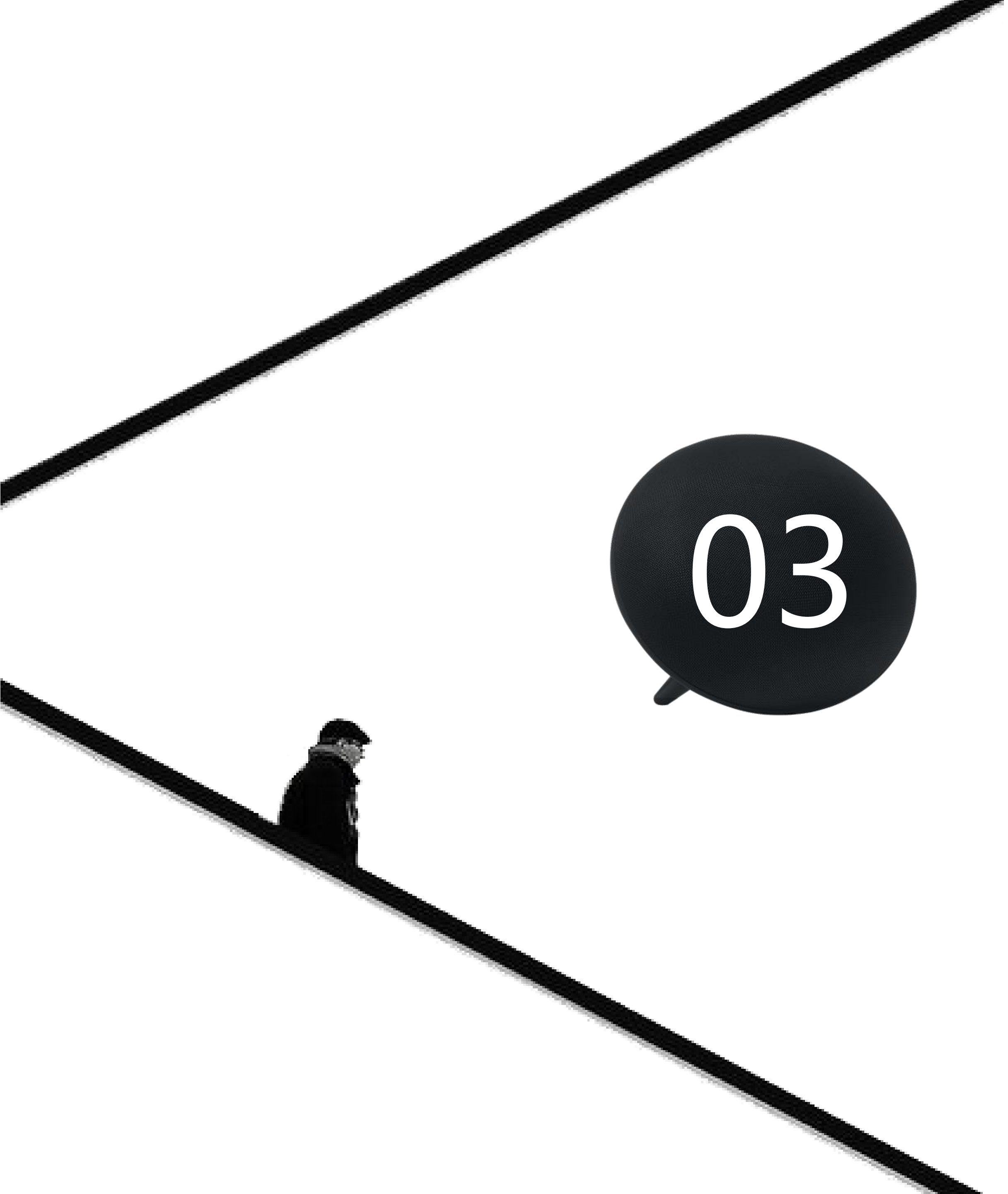
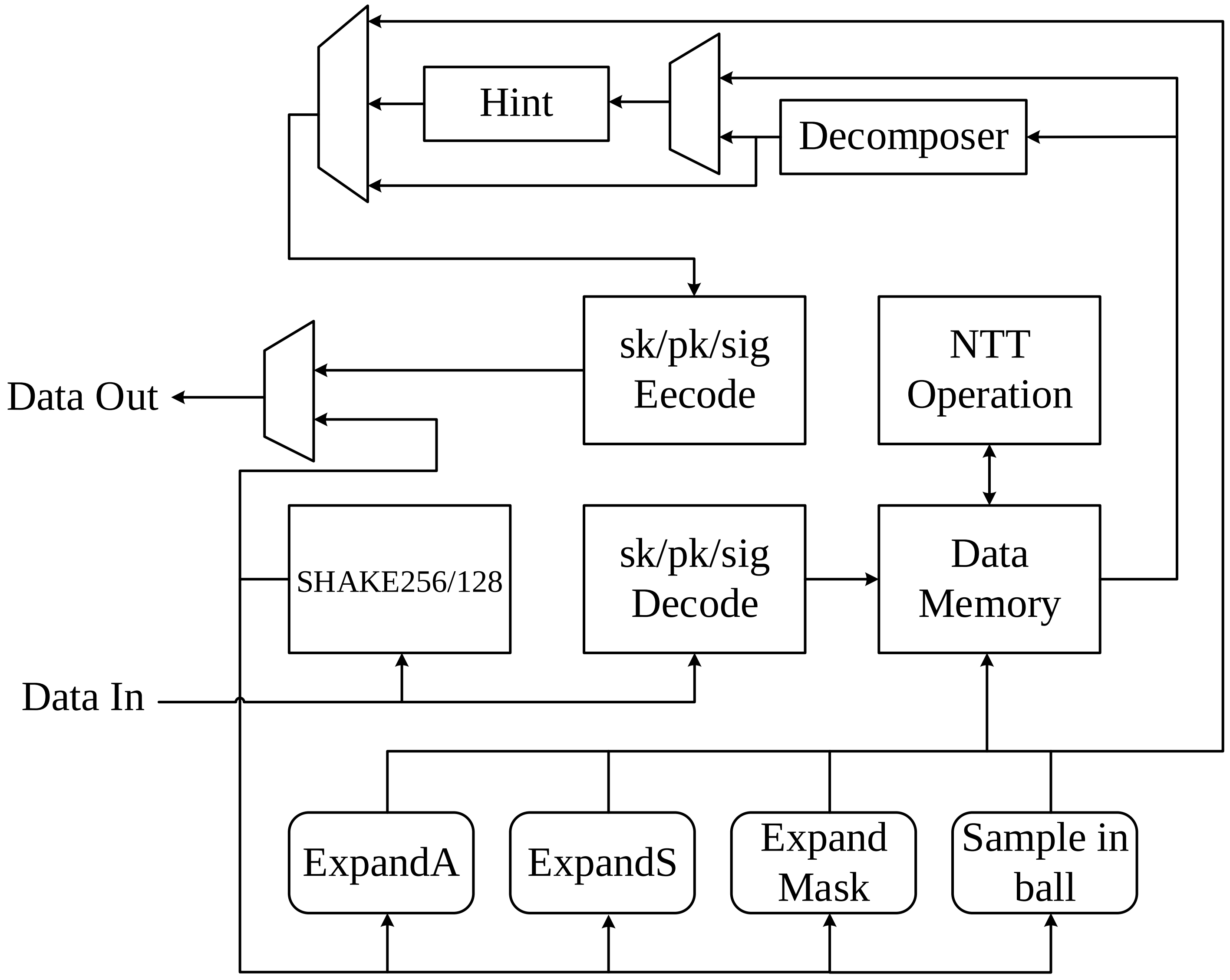
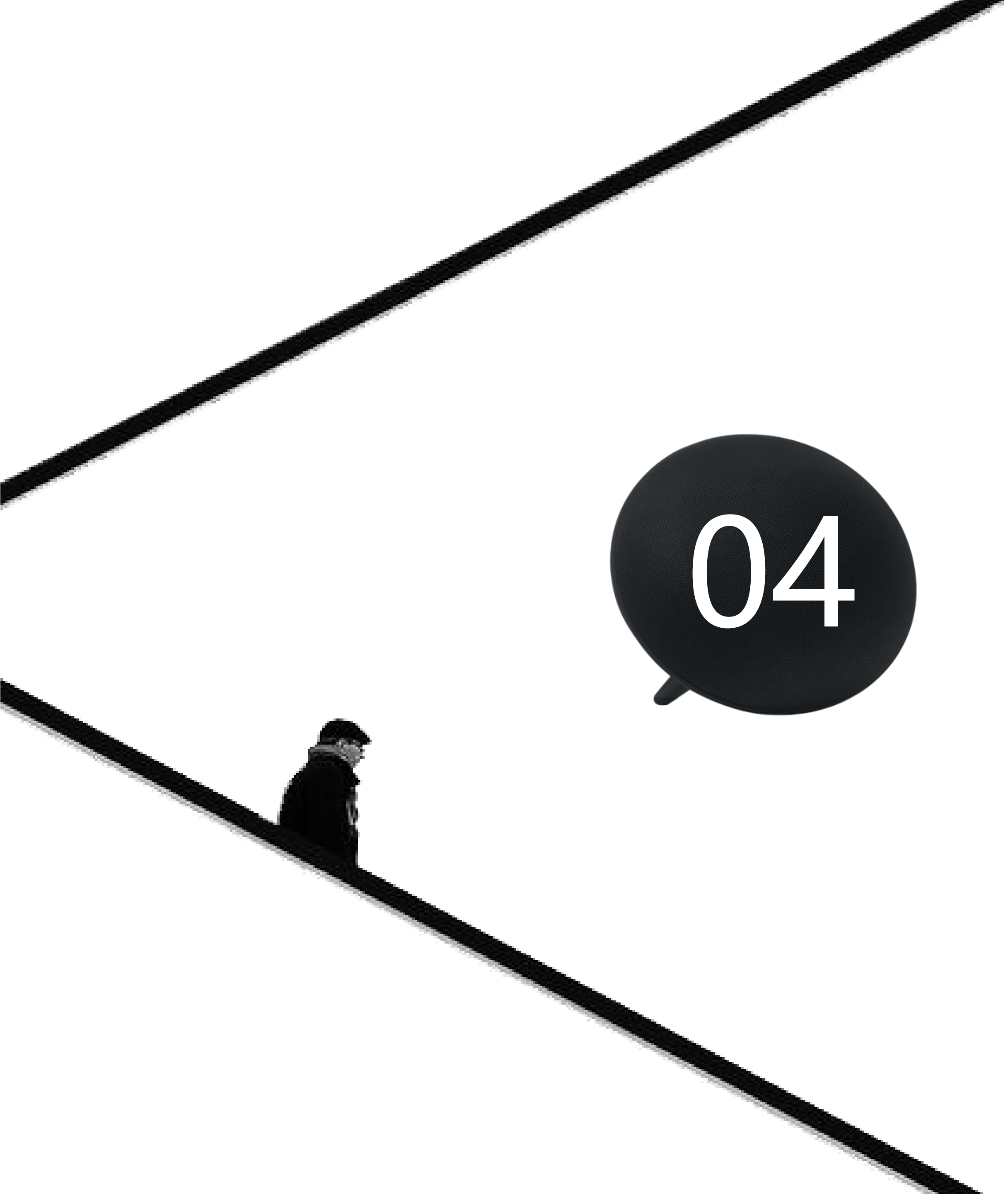



03

Architecture

Block Diagram





04

NTT

$$Y(x) = G(x) \cdot H(x) = \sum_{k=0}^{2(n-1)} y_k x^k$$

$$\boldsymbol{y}[k] = (\boldsymbol{g} * \boldsymbol{h})[k] = \sum_{i=0}^{k} \boldsymbol{g}[i]\boldsymbol{h}[k-i]$$

$$NWC(x) = Y(x) \bmod (x^n + 1)$$

$G(x)$ and $H(x)$ are defined in $Z_{7681}[4]/x^4 + 1$

$$G(x) = 1 + 2x + 3x^2 + 4x^3, \ and$$
$$H(x) = 5 + 6x + 7x^2 + 8x^3$$

or in vector notation:

$\boldsymbol{g} = [1, 2, 3, 4], \ and$

$\boldsymbol{h} = [5, 6, 7, 8].$

$$
\begin{array}{rrrrrrrr}
 & & & 1 & +\ 2x & +\ 3x^2 & +\ 4x^3 & \\
 & & & 5 & +\ 6x & +\ 7x^2 & +\ 8x^3 & \\
\hline
 & & & 8x^3 & +\ 16x^4 & +\ 24x^5 & +\ 32x^6 & \times \\
 & & 7x^2 & +\ 14x^3 & +\ 21x^4 & +\ 28x^5 & & \\
 & 6x & +\ 12x^2 & +\ 18x^3 & +\ 24x^4 & & & \\
5 & +\ 10x & +\ 15x^2 & +\ 20x^3 & & & & \\
\hline
5 & +\ 16x & +\ 34x^2 & +\ 60x^3 & +\ 61x^4 & +\ 52x^5 & +\ 32x^6 & +
\end{array}
$$

設計與實現基於AXI-4介面的後量子密法學ML-DSA之硬體加速器 – 未來規劃

$$
\begin{array}{r|l}
 & 32x^2 + 52x + 61 \\
\hline
x^4 + 1 & 32x^6 + 52x^5 + 61x^4 + 60x^3 + 34x^2 + 16x + 5 \\
 & 32x^6 + 0x^5 + 0x^4 + 0x^3 + 32x^2 \quad - \\
\cline{2-2}
 & 52x^5 + 61x^4 + 60x^3 + 2x^2 + 16x + 5 \\
 & 52x^5 + 0x^4 + 0x^3 + 0x^2 + 52x \quad - \\
\cline{2-2}
 & 61x^4 + 60x^3 + 2x^2 - 36x + 5 \\
 & 61x^4 + 0x^3 + 0x^2 + 0x + 61 \quad - \\
\cline{2-2}
 & 60x^3 + 2x^2 - 36x - 56
\end{array}
$$

Primitive 2n-th Root of Unity

Definition 3.1. *Let $\mathbb{Z}_q$ be an integer ring modulo q, and $n-1$ is the polynomial degree of $G(x)$ and $H(x)$. Such rings have a multiplicative identity (unity) of 1. Define ω as* **primitive n-th root of unity** *in $\mathbb{Z}_q$ if and only if:*

$$\omega^n \equiv 1 \mod q \tag{7}$$

and

$$\omega^k \not\equiv 1 \mod q \tag{8}$$

for $k < n$.

Definition 3.4. *Let $\mathbb{Z}_q$ be an integer ring modulo q, and $n-1$ is the polynomial degree of $G(x)$ and $H(x)$ and ω is its primitive n-th root of unity. Define ψ as the primitive $2n$-th root of unity if and only if:*

$$\psi^2 \equiv \omega \mod q \tag{13}$$

and

$$\psi^n \equiv -1 \mod q \tag{14}$$

NTT- Based Negative-Wrapped Convolution - NTT

$$\hat{\boldsymbol{a}}_j = \sum_{i=0}^{n-1} \psi^{2ij+i} a_i \mod q$$

Let $\boldsymbol{g} = [1, 2, 3, 4]$, $n = 4$ *and* $\psi = 1925$ *in the ring* $\mathbb{Z}_{7681}$.

$$\hat{\boldsymbol{g}} = \begin{bmatrix} \psi^{2(0\times0)+0} & \psi^{2(0\times1)+1} & \psi^{2(0\times2)+2} & \psi^{2(0\times3)+3} \\ \psi^{2(1\times0)+0} & \psi^{2(1\times1)+1} & \psi^{2(1\times2)+2} & \psi^{2(1\times3)+3} \\ \psi^{2(2\times0)+0} & \psi^{2(2\times1)+1} & \psi^{2(2\times2)+2} & \psi^{2(2\times3)+3} \\ \psi^{2(3\times0)+0} & \psi^{2(3\times1)+1} & \psi^{2(3\times2)+2} & \psi^{2(3\times3)+3} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad \hat{\boldsymbol{g}} = \begin{bmatrix} 1925^0 & 1925^1 & 1925^2 & 1925^3 \\ 1925^0 & 1925^3 & 1925^6 & 1925^1 \\ 1925^0 & 1925^5 & 1925^2 & 1925^7 \\ 1925^0 & 1925^7 & 1925^6 & 1925^5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\hat{\boldsymbol{g}} = \begin{bmatrix} \psi^0 & \psi^1 & \psi^2 & \psi^3 \\ \psi^0 & \psi^3 & \psi^6 & \psi^9 \\ \psi^0 & \psi^5 & \psi^{10} & \psi^{15} \\ \psi^0 & \psi^7 & \psi^{14} & \psi^{21} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} \psi^0 & \psi^1 & \psi^2 & \psi^3 \\ \psi^0 & \psi^3 & \psi^6 & \psi^1 \\ \psi^0 & \psi^5 & \psi^2 & \psi^7 \\ \psi^0 & \psi^7 & \psi^6 & \psi^5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad \hat{\boldsymbol{g}} = \begin{bmatrix} 1 & 1925 & 3383 & 6468 \\ 1 & 6468 & 4298 & 1925 \\ 1 & 5756 & 3383 & 1213 \\ 1 & 1213 & 4298 & 5756 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

Let $\boldsymbol{h} = [5, 6, 7, 8]$, $n = 4$ *and* $\psi = 1925$ *in the ring* $\mathbb{Z}_{7681}$.

$$\hat{\boldsymbol{h}} = \begin{bmatrix} 1 & 1925 & 3383 & 6468 \\ 1 & 6468 & 4298 & 1925 \\ 1 & 5756 & 3383 & 1213 \\ 1 & 1213 & 4298 & 5756 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 2489 \\ 7489 \\ 6478 \\ 6607 \end{bmatrix}$$

NTT- Based Negative-Wrapped Convolution - INTT

Note :

$$\psi^{-1} = 1213$$

$$\boldsymbol{a}_i = n^{-1} \sum_{j=0}^{n-1} \psi^{-(2ij+j)} \hat{a}_j \mod q$$

$$Let\ NTT^{\psi}(\boldsymbol{g}) = \hat{\boldsymbol{g}} = [1467, 2807, 3471, 7621]\ and\ \psi = 1925\ in\ the\ ring\ \mathbb{Z}_{7681}.$$

$$\boldsymbol{g} = n^{-1} \begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & \psi^{-5} & \psi^{-7} \\ \psi^{-2} & \psi^{-6} & \psi^{-10} & \psi^{-14} \\ \psi^{-3} & \psi^{-9} & \psi^{-15} & \psi^{-21} \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$\boldsymbol{g} = 5761 \begin{bmatrix} 1213^0 & 1213^0 & 1213^0 & 1213^0 \\ 1213^1 & 1213^3 & 1213^5 & 1213^7 \\ 1213^2 & 1213^6 & 1213^2 & 1213^6 \\ 1213^3 & 1213^1 & 1213^7 & 1213^5 \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$\boldsymbol{g} = n^{-1} \begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & \psi^{-5} & \psi^{-7} \\ \psi^{-2} & \psi^{-6} & \psi^{-2} & \psi^{-6} \\ \psi^{-3} & \psi^{-1} & \psi^{-7} & \psi^{-5} \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$\boldsymbol{g} = 5761 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1213 & 5756 & 6468 & 1925 \\ 4298 & 3383 & 4298 & 3383 \\ 5756 & 1213 & 1925 & 6468 \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

NTT – INTT Example

$$INTT(\begin{bmatrix}1467\\2807\\3471\\7621\end{bmatrix}\circ\begin{bmatrix}2489\\7489\\6478\\6607\end{bmatrix}) = INTT(\begin{bmatrix}2888\\6407\\2851\\2992\end{bmatrix})$$

$$\begin{array}{r|l}
 & 32x^2+52x+61 \\ \hline
x^4+1 & 32x^6+52x^5+61x^4+60x^3+34x^2+16x+5 \\
 & 32x^6+0x^5+0x^4+0x^3+32x^2 \quad - \\ \hline
 & 52x^5+61x^4+60x^3+2x^2+16x+5 \\
 & 52x^5+0x^4+0x^3+0x^2+52x \quad - \\ \hline
 & 61x^4+60x^3+2x^2-36x+5 \\
 & 61x^4+0x^3+0x^2+0x+61 \quad - \\ \hline
 & 60x^3+2x^2-36x-56
\end{array}$$

$$= 5761\begin{bmatrix}1&1&1&1\\1213&5756&6468&1925\\4298&3383&4298&3383\\5756&1213&1925&6468\end{bmatrix}\begin{bmatrix}2888\\6407\\2851\\2992\end{bmatrix} = \begin{bmatrix}7625\\7645\\2\\60\end{bmatrix}$$

Cooley-Tukey(CT)Algorithm for Fast-NTT

$$\hat{\boldsymbol{a}}_j = \sum_{i=0}^{n-1} \psi^{2ij+i} a_i \mod q$$

$$= \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i} + \sum_{i=0}^{n/2-1} \psi^{4ij+2j+2i+1} a_{2i+1} \mod q$$

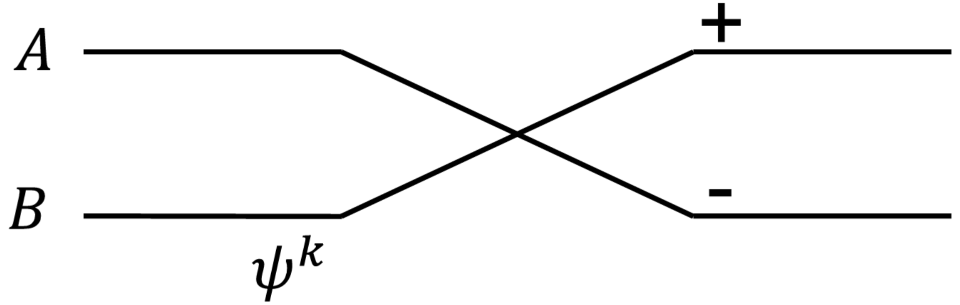
$$= \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i} + \psi^{2j+1} \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i+1} \mod q$$

$$\hat{\boldsymbol{a}}_{j+n/2} = \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i} - \psi^{2j+1} \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i+1} \mod q$$

Note :

periodicity : $\psi^{k+2n} = \psi^k$

symmetry : $\psi^{k+n} = -\psi^k$



Let $A_j = \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i}$ and $B_j = \sum_{i=0}^{n/2-1} \psi^{4ij+2i} a_{2i+1}$

$$\hat{\boldsymbol{a}}_j = A_j + \psi^{2j+1} B_j \mod q$$

$$\hat{\boldsymbol{a}}_{j+n/2} = A_j - \psi^{2j+1} B_j \mod q$$

Cooley-Tukey(CT)Algorithm for Fast-NTT

$$\hat{\boldsymbol{g}} = \begin{bmatrix} \psi^0 & \psi^1 & \psi^2 & \psi^3 \\ \psi^0 & \psi^3 & \psi^6 & \psi^9 \\ \psi^0 & \psi^5 & \psi^{10} & \psi^{15} \\ \psi^0 & \psi^7 & \psi^{14} & \psi^{21} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Based on the ψ periodicity:

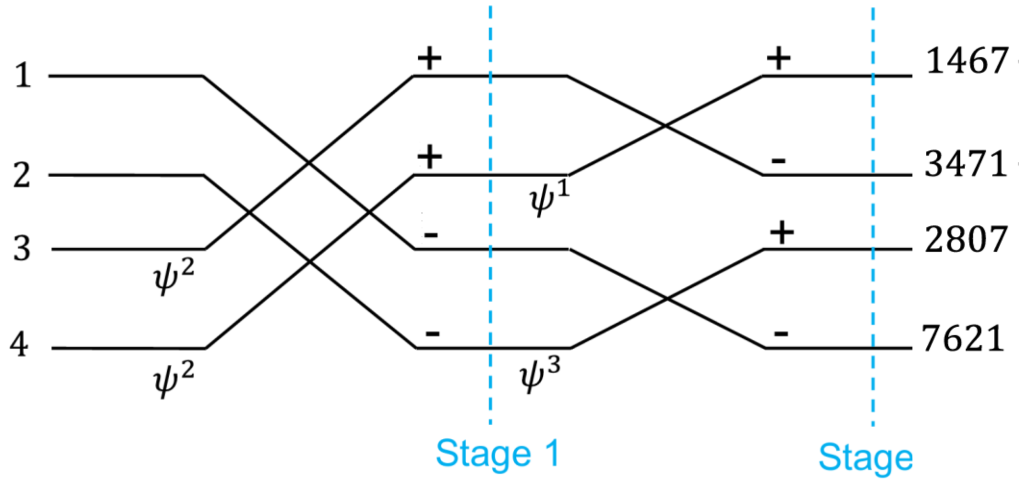
$$\hat{\boldsymbol{g}} = \begin{bmatrix} \psi^0 & \psi^1 & \psi^2 & \psi^3 \\ \psi^0 & \psi^3 & \psi^6 & \psi^1 \\ \psi^0 & \psi^5 & \psi^2 & \psi^7 \\ \psi^0 & \psi^7 & \psi^6 & \psi^5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Based on the ψ symmetry:

$$\hat{\boldsymbol{g}} = \begin{bmatrix} \psi^0 & \psi^1 & \psi^2 & \psi^3 \\ \psi^0 & \psi^3 & -\psi^2 & \psi^1 \\ \psi^0 & -\psi^1 & \psi^2 & -\psi^3 \\ \psi^0 & -\psi^3 & -\psi^2 & -\psi^1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\hat{g}_0 = 1\psi^0 + 2\psi^1 + 3\psi^2 + 4\psi^3$$
$$\hat{g}_1 = 1\psi^0 + 2\psi^3 - 3\psi^2 + 4\psi^1$$
$$\hat{g}_2 = 1\psi^0 - 2\psi^1 + 3\psi^2 - 4\psi^3$$
$$\hat{g}_3 = 1\psi^0 - 2\psi^3 - 3\psi^2 - 4\psi^1$$

$$\hat{g}_0 = \psi^0(1 + 3\psi^2) + \psi^1(2 + 4\psi^2)$$
$$\hat{g}_1 = \psi^0(1 - 3\psi^2) + \psi^3(2 - 4\psi^2)$$
$$\hat{g}_2 = \psi^0(1 + 3\psi^2) - \psi^1(2 + 4\psi^2)$$
$$\hat{g}_3 = \psi^0(1 - 3\psi^2) - \psi^3(2 - 4\psi^2)$$



Gentleman-Sande(GS)AlgorithmforFast-INTT

$$\boldsymbol{a}_i = \sum_{j=0}^{n-1} \psi^{-(2i+1)j} \hat{a}_j \mod q$$

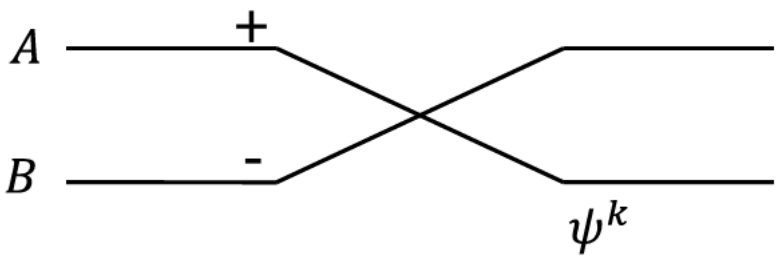
$$= \left[\sum_{j=0}^{\frac{n}{2}-1} \psi^{-(2i+1)j} \hat{a}_j + \sum_{j=0}^{\frac{n}{2}-1} \psi^{-(2i+1)(j+\frac{n}{2})} \hat{a}_{(j+\frac{n}{2})} \right] \mod q$$

$$= \psi^{-i} \left[\sum_{j=0}^{\frac{n}{2}-1} \psi^{-2ij} \hat{a}_j + \sum_{j=0}^{\frac{n}{2}-1} \psi^{-2i(j+\frac{n}{2})} \hat{a}_{(j+\frac{n}{2})} \right] \mod q$$

$$\boldsymbol{a}_{2i} = \psi^{-2i} \left[\sum_{j=0}^{\frac{n}{2}-1} \psi^{-4ij} \hat{a}_j + \sum_{j=0}^{\frac{n}{2}-1} \psi^{-4i(j+\frac{n}{2})} \hat{a}_{(j+\frac{n}{2})} \right] \mod q$$

$$\boldsymbol{a}_{2i} = \psi^{-2i} \sum_{j=0}^{\frac{n}{2}-1} \left[\hat{a}_j + \hat{a}_{(j+\frac{n}{2})} \right] \psi^{-4ij} \mod q$$

$$\boldsymbol{a}_{2i+1} = \psi^{-2i} \sum_{j=0}^{\frac{n}{2}-1} \left[\hat{a}_j - \hat{a}_{(j+\frac{n}{2})} \right] \psi^{-4ij} \mod q$$



Let $A_i = \sum_{j=0}^{\frac{n}{2}-1} \hat{a}_j \psi^{-4ij}$ and $B_i = \sum_{j=0}^{\frac{n}{2}-1} \hat{a}_{j+\frac{n}{2}} \psi^{-4ij}$

$$\boldsymbol{a}_{2i} = (A_i + B_i)\psi^{-2i} \mod q$$

$$\boldsymbol{a}_{2i+1} = (A_i - B_i)\psi^{-2i} \mod q$$

Gentleman-Sande(GS)AlgorithmforFast-INTT

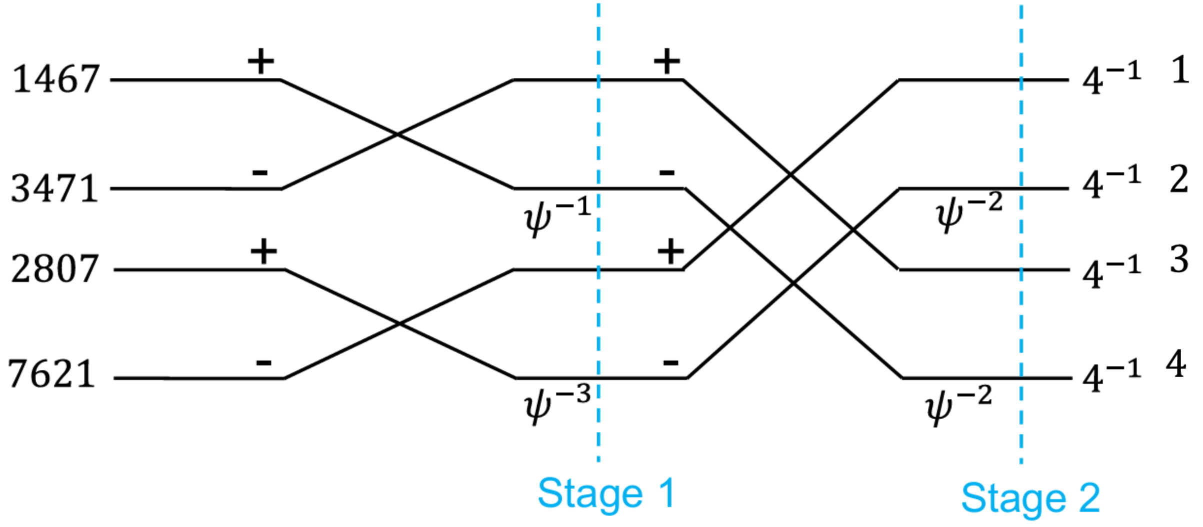
$$\boldsymbol{g} = n^{-1}\begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & \psi^{-5} & \psi^{-7} \\ \psi^{-2} & \psi^{-6} & \psi^{-10} & \psi^{-14} \\ \psi^{-3} & \psi^{-9} & \psi^{-15} & \psi^{-21} \end{bmatrix}\begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$\boldsymbol{g} = n^{-1}\begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & \psi^{-5} & \psi^{-7} \\ \psi^{-2} & \psi^{-6} & \psi^{-2} & \psi^{-6} \\ \psi^{-3} & \psi^{-1} & \psi^{-7} & \psi^{-5} \end{bmatrix}\begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$\boldsymbol{g} = n^{-1}\begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & -\psi^{-1} & -\psi^{-3} \\ \psi^{-2} & -\psi^{-2} & \psi^{-2} & -\psi^{-2} \\ \psi^{-3} & \psi^{-1} & -\psi^{-3} & -\psi^{-1} \end{bmatrix}\begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$g_0 = [1467\psi^{-0} + 2807\psi^{-0} + 3471\psi^{-0} + 7621\psi^{-0}]n^{-1}$$
$$g_1 = [1467\psi^{-1} + 2807\psi^{-3} - 3471\psi^{-1} - 7621\psi^{-3}]n^{-1}$$
$$g_2 = [1467\psi^{-2} - 2807\psi^{-2} + 3471\psi^{-2} - 7621\psi^{-2}]n^{-1}$$
$$g_3 = [1467\psi^{-3} + 2807\psi^{-1} - 3471\psi^{-3} - 7621\psi^{-1}]n^{-1}$$

$$g_0 = [(1467 + 3471)\psi^{-0} + (2807 + 7621)\psi^{-0}]\psi^{-0}n^{-1}$$
$$g_1 = [(1467 - 3471)\psi^{-1} + (2807 - 7621)\psi^{-3}]\psi^{-0}n^{-1}$$
$$g_2 = [(1467 + 3471)\psi^{-0} - (2807 + 7621)\psi^{-0}]\psi^{-2}n^{-1}$$
$$g_3 = [(1467 - 3471)\psi^{-1} - (2807 - 7621)\psi^{-3}]\psi^{-2}n^{-1}$$



Gentleman-Sande(GS)AlgorithmforFast-INTT

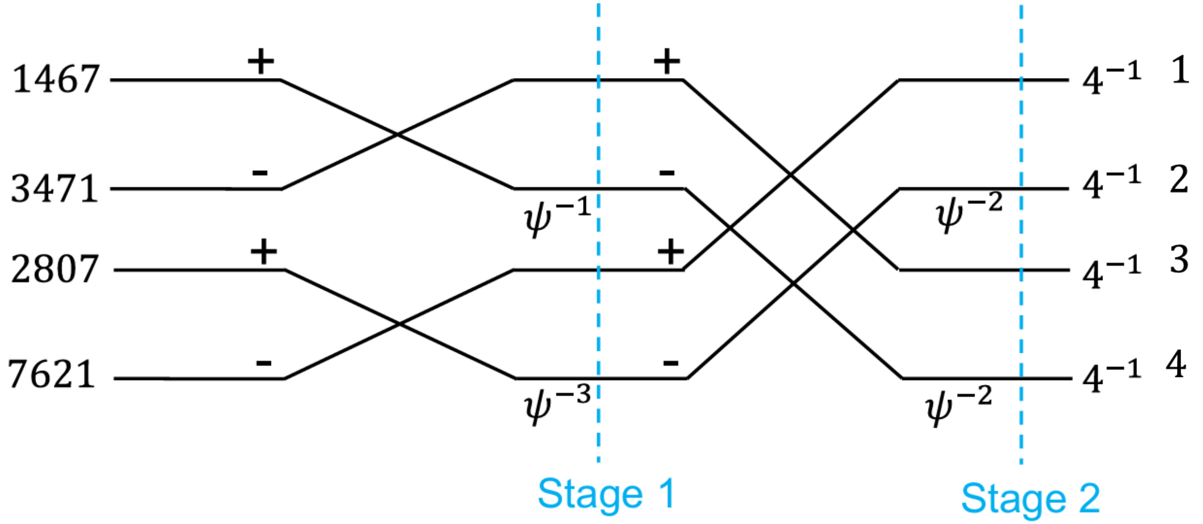
$$\boldsymbol{g} = n^{-1}\begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & \psi^{-5} & \psi^{-7} \\ \psi^{-2} & \psi^{-6} & \psi^{-10} & \psi^{-14} \\ \psi^{-3} & \psi^{-9} & \psi^{-15} & \psi^{-21} \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$\boldsymbol{g} = n^{-1}\begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & \psi^{-5} & \psi^{-7} \\ \psi^{-2} & \psi^{-6} & \psi^{-2} & \psi^{-6} \\ \psi^{-3} & \psi^{-1} & \psi^{-7} & \psi^{-5} \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

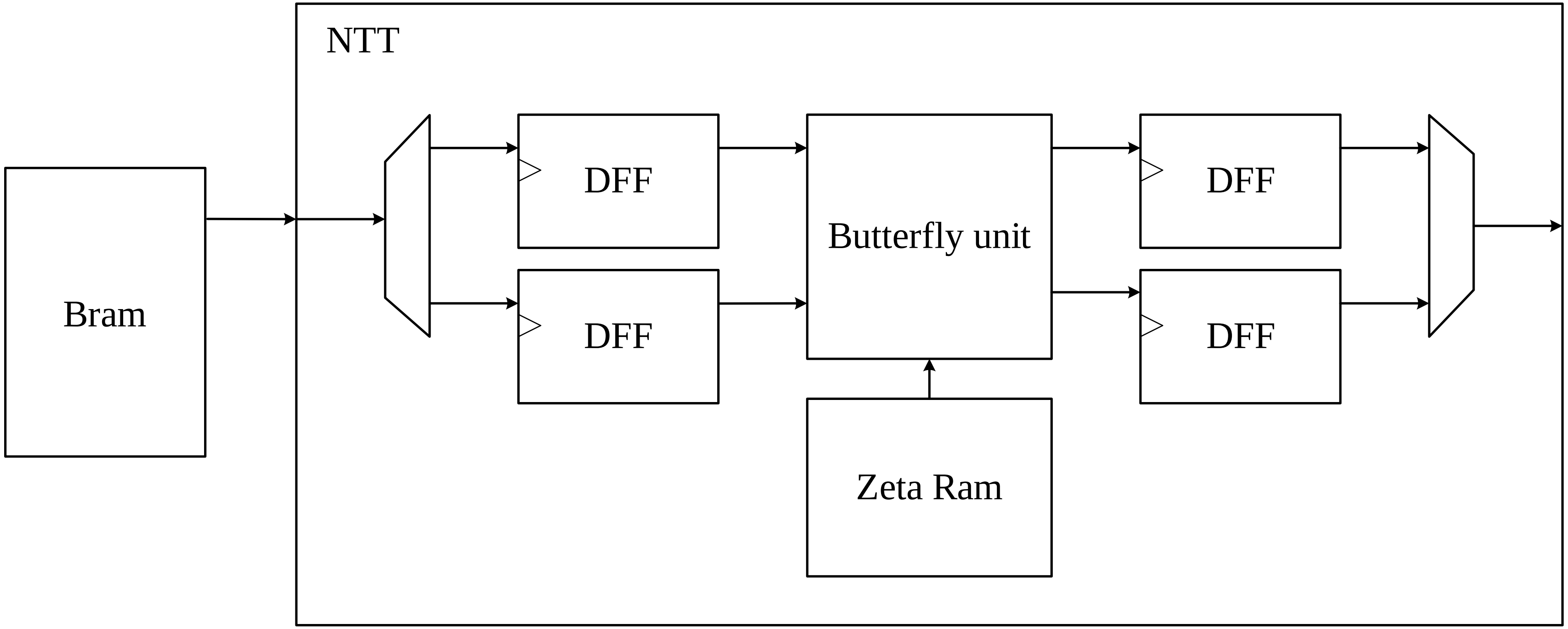
$$\boldsymbol{g} = n^{-1}\begin{bmatrix} \psi^{-0} & \psi^{-0} & \psi^{-0} & \psi^{-0} \\ \psi^{-1} & \psi^{-3} & -\psi^{-1} & -\psi^{-3} \\ \psi^{-2} & -\psi^{-2} & \psi^{-2} & -\psi^{-2} \\ \psi^{-3} & \psi^{-1} & -\psi^{-3} & -\psi^{-1} \end{bmatrix} \begin{bmatrix} 1467 \\ 2807 \\ 3471 \\ 7621 \end{bmatrix}$$

$$g_0 = [1467\psi^{-0} + 2807\psi^{-0} + 3471\psi^{-0} + 7621\psi^{-0}]n^{-1}$$
$$g_1 = [1467\psi^{-1} + 2807\psi^{-3} - 3471\psi^{-1} - 7621\psi^{-3}]n^{-1}$$
$$g_2 = [1467\psi^{-2} - 2807\psi^{-2} + 3471\psi^{-2} - 7621\psi^{-2}]n^{-1}$$
$$g_3 = [1467\psi^{-3} + 2807\psi^{-1} - 3471\psi^{-3} - 7621\psi^{-1}]n^{-1}$$

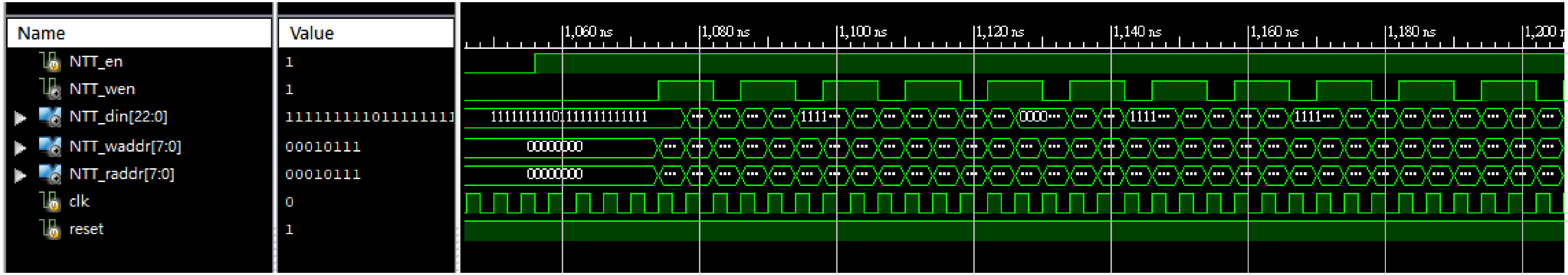
$$g_0 = [(1467 + 3471)\psi^{-0} + (2807 + 7621)\psi^{-0}]\psi^{-0}n^{-1}$$
$$g_1 = [(1467 - 3471)\psi^{-1} + (2807 - 7621)\psi^{-3}]\psi^{-0}n^{-1}$$
$$g_2 = [(1467 + 3471)\psi^{-0} - (2807 + 7621)\psi^{-0}]\psi^{-2}n^{-1}$$
$$g_3 = [(1467 - 3471)\psi^{-1} - (2807 - 7621)\psi^{-3}]\psi^{-2}n^{-1}$$

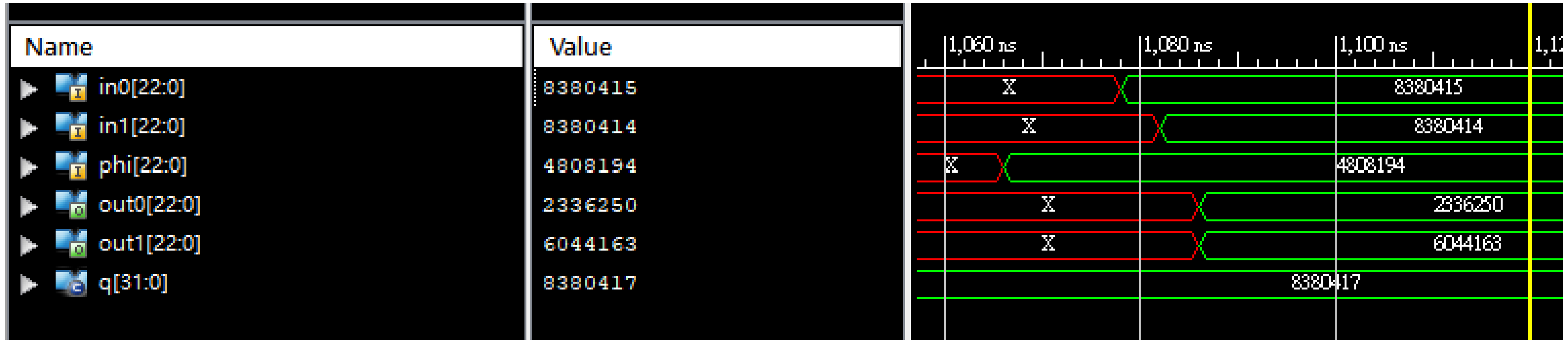


NTT Block Diagram



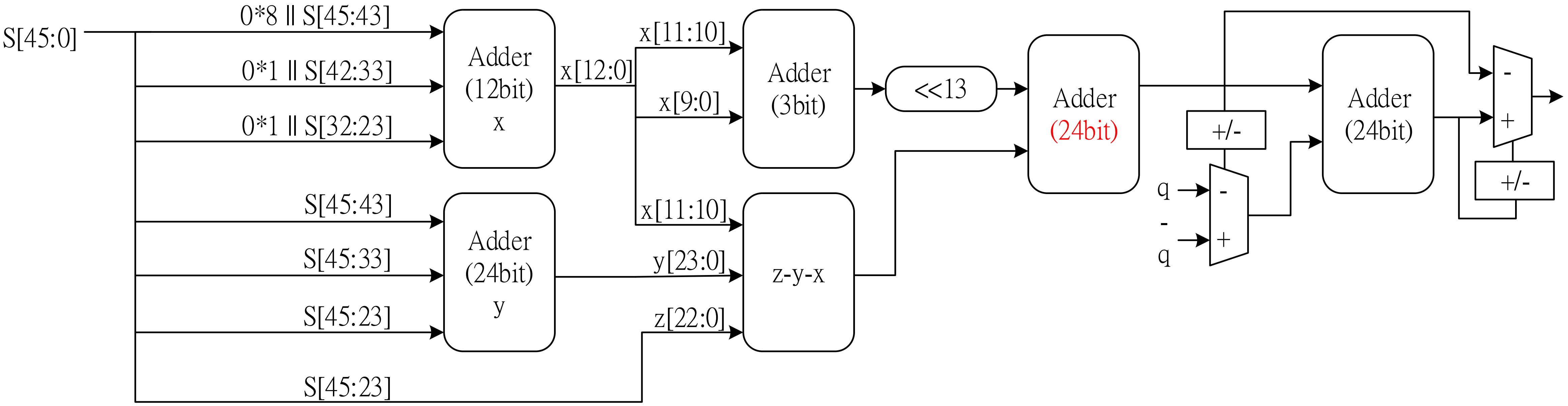
NTT RTL simulation

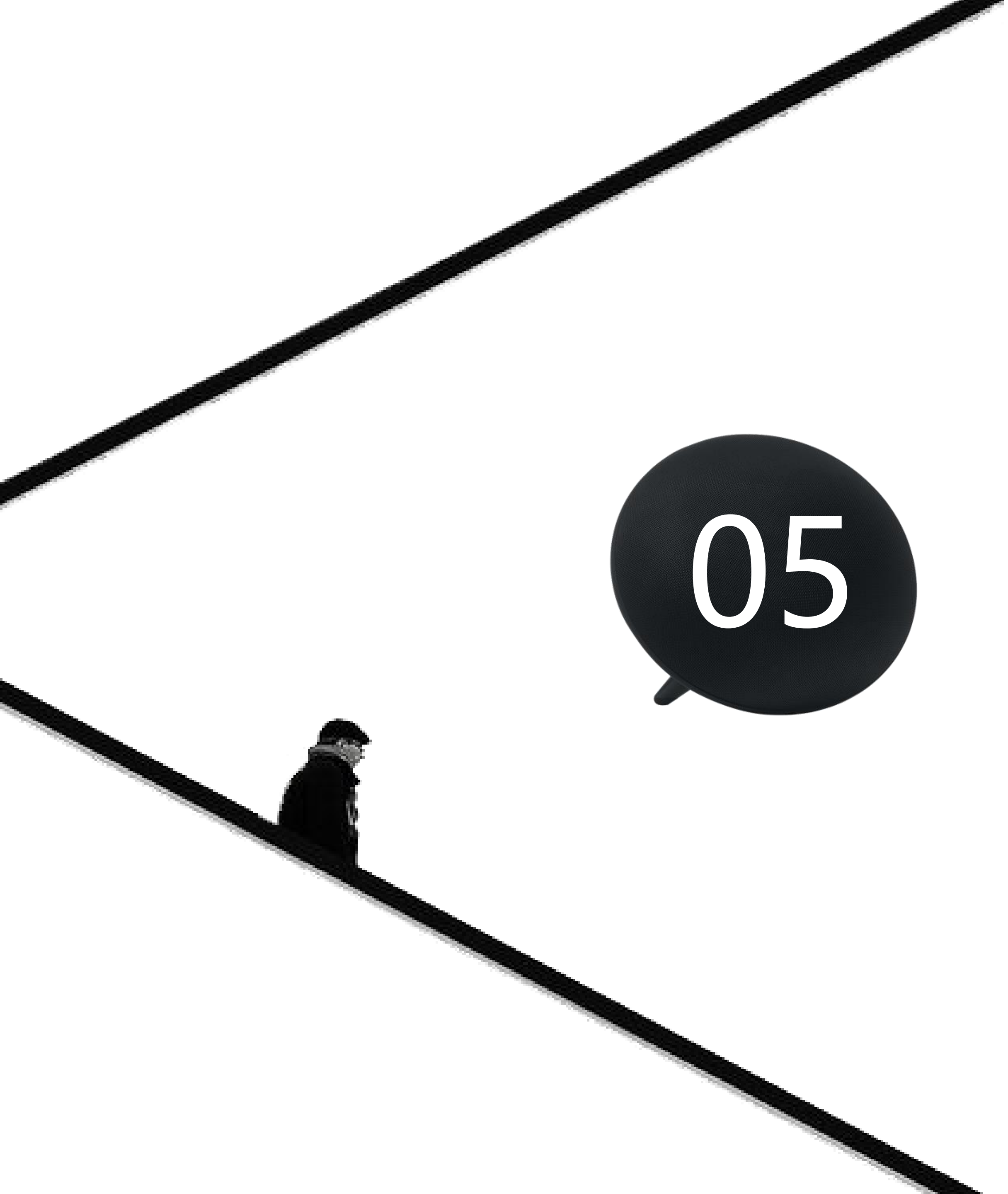





```
zeta = 4808194
t = 3572223
w_hat[j] = 8380415
w_hat[j + length] = 8380414
w_hat[j] = 2336250
w_hat[j + length] = 6044163
```

Modular Rduction

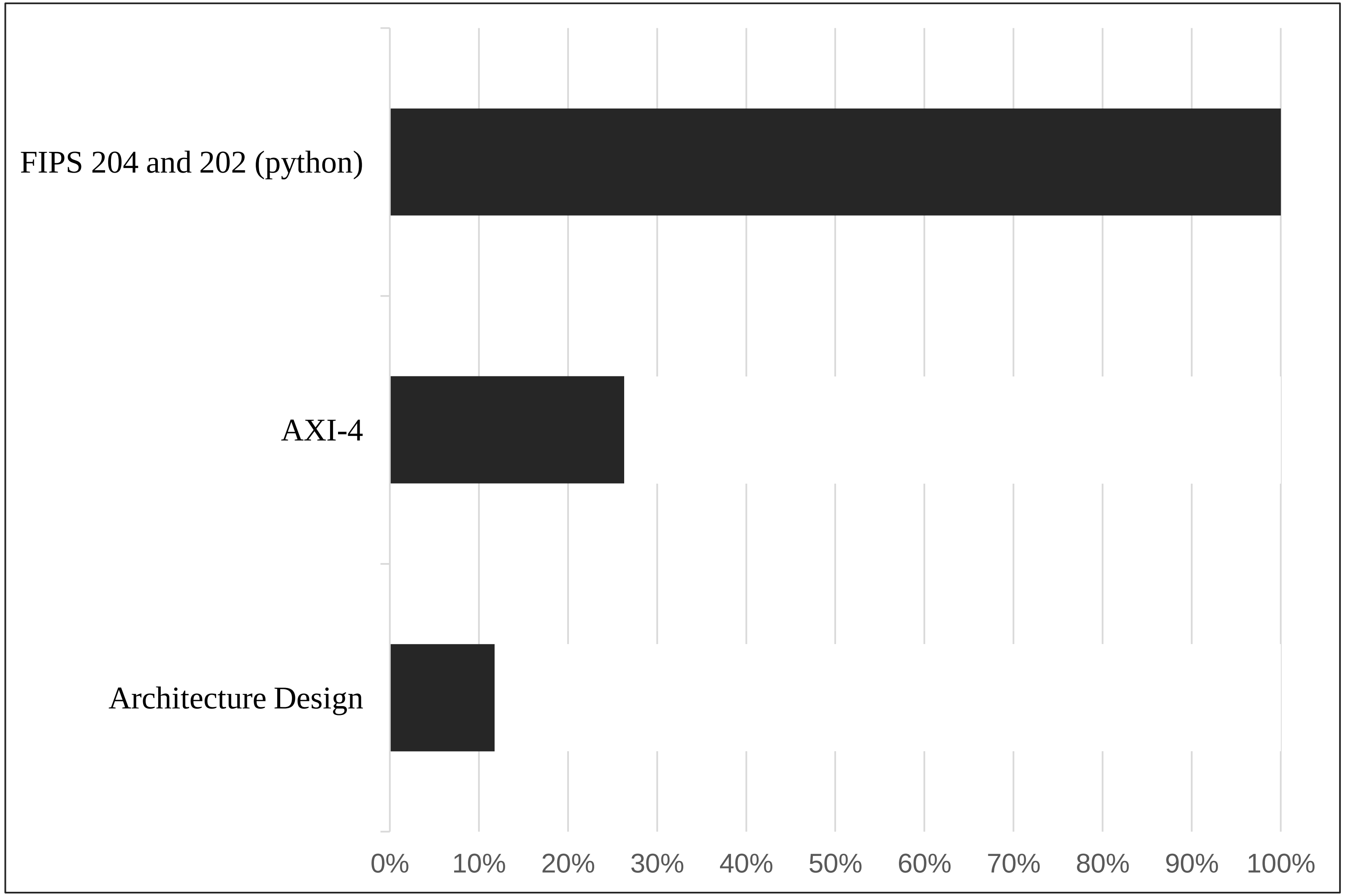


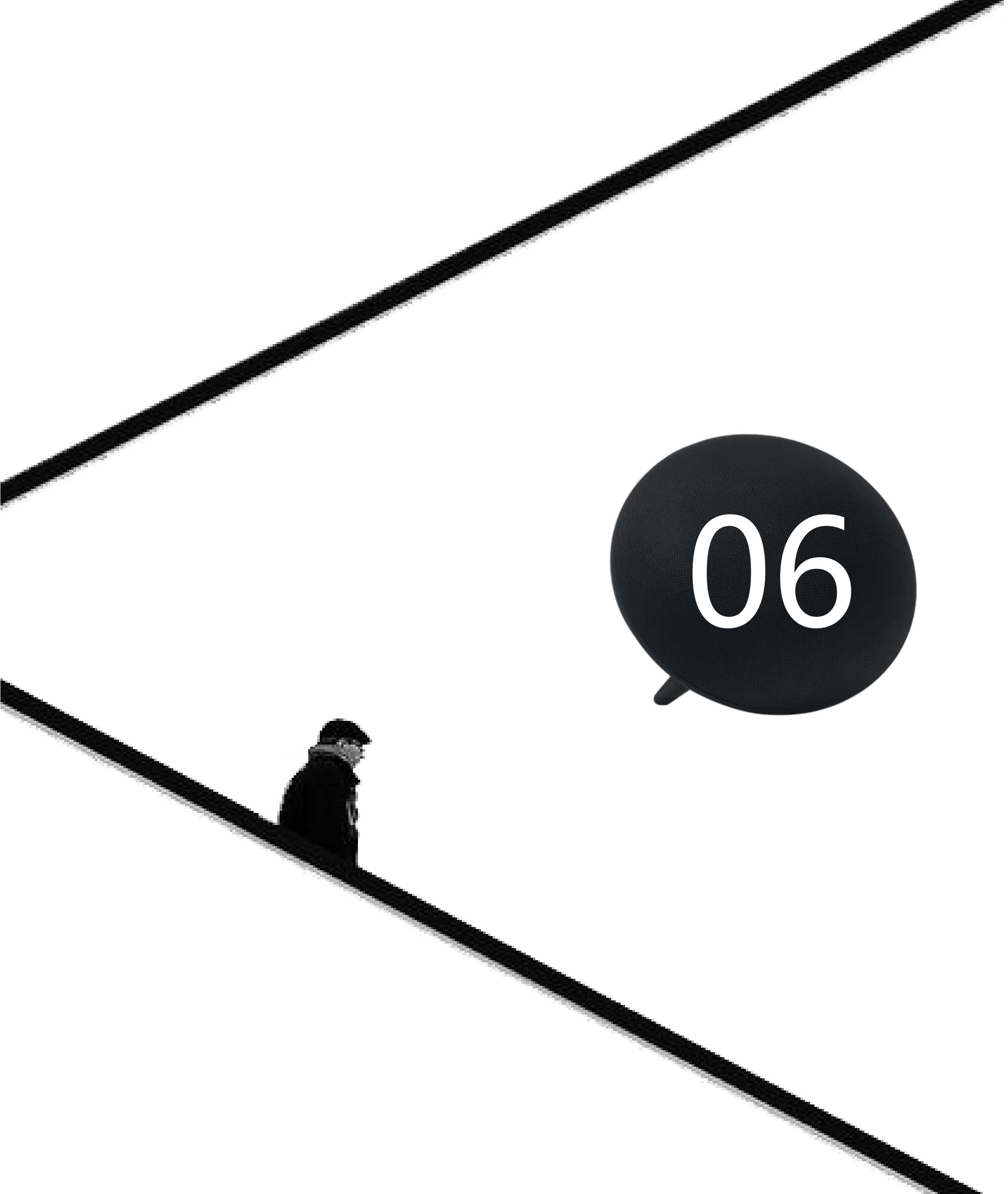


05

Current progress

Current progress





06

References

References

[1] “Module-Lattice-Based digital signature standard,” National Institute of Standards and Technology, Gaithersburg, MD, Aug. 2023. Accessed: Aug. 05, 2024. [Online]. Available: http://dx.doi.org/10.6028/nist.fips.204.ipd

[2] 蔡秉邕， Dilithium數位簽章系統的安全性估計，碩士論文，國立清華大學，2022。

[3] 黃彥霖，容錯學習密碼學的加密演算法與其實作，碩士論文，國立交通大學，2019。

[4] A. Satriawan, R. Mareta, and H. Lee, *A Complete Beginner Guide to the Number Theoretic Transform* (*NTT*), Dept. of Electrical and Computer Engineering, Inha University, Incheon, South Korea.

[5] K. Denisse Ortega L., and Luis J. Dominguez Perez, “Implementing crystal-dilithium on FRDM-K64,” in *Proceedings of the 2021 IEEE 12th Annual Ubiquitous Computing*, Electronics & Mobile Communication Conference (UEMCON), pp. 0178–0183, New York, NY, USA, Dec. 2021, Available: http://dx.doi.org/10.1109/uemcon53757.2021.9666622

[6] T.-H. Nguyen, B. Kieu-Do-Nguyen, C.-K. Pham, and T.-T. Hoang, “High-speed NTT accelerator for crystal-kyber and crystal-dilithium,” *IEEE Access*, vol. 12, pp. 34918–34930, Feb. 2024, Available: https://doi.org/10.1109/access.2024.3371581