```
.    .    .    Bits(J + M ) = Bits(J) + 1
.    .    ...  FIN
.    .    M = M * 2
.    ...  FIN
...  FIN
```

A1.13 THE ICOSAHEDRON

Geodesic dome constructions provide a useful way to partition the sphere (hence the three-dimensional directions) into relatively uniform patches. The resulting polyhedra look like those of Fig. A1.13.

The icosahedron has 12 vertices, 20 faces, and 30 edges. Let its center be at the origin of Cartesian coordinates and let each vertex be a unit distance from the center. Define

$$t\text{, the golden ratio} = \frac{1+\sqrt{5}}{2}$$

$$a = \frac{\sqrt{t}}{5^{1/4}}$$

$$b = \frac{1}{(\sqrt{t}\ 5^{1/4})}$$

$$c = a + 2b = \frac{1}{b}$$

$$d = a + b = \frac{t^{3/2}}{5^{1/4}}$$

$$A = \text{angle subtended by edge at origin} = \arccos\left(\frac{\sqrt{5}}{5}\right)$$

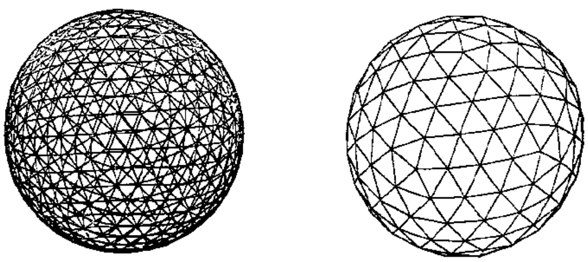

Fig. A1.13 Multifaceted polyhedra from the icosahedron.

Then

angle between radius and an edge $= b = \arccos(b)$
edge length $= 2b$
distance from origin to center of edge $= a$
distance from origin to center of face $= \frac{ta}{\sqrt{3}}$

The 12 vertices may be placed at

$$(\ 0,\ \pm a,\ \pm b)$$
$$(\pm b,\ 0,\ \pm a)$$
$$(\pm a,\ \pm b,\ 0)$$

Then midpoints of the 20 faces are given by

$$\tfrac{1}{3}(\pm d,\ \pm d,\ \pm d)$$
$$\tfrac{1}{3}(\ 0,\ \pm a,\ \pm c)$$
$$\tfrac{1}{3}(\pm c,\ 0,\ \pm a)$$
$$\tfrac{1}{3}(\pm a,\ \pm c,\ 0)$$

To subdivide icosahedral faces further, several methods suggest themselves, the simplest being to divide each edge into n equal lengths and then construct n^2 congruent equilateral triangles on each face, pushing them out to the radius of the sphere for their final position. (There are better methods than this if more uniform face sizes are desired.)

A1.14 ROOT FINDING

Since polynomials of fifth and higher degree are not soluble in closed form, numerical (approximate) solutions are useful for them as well as for nonpolynomial functions. The Newton–Raphson method produces successive approximations to a real root of a differentiable function of one variable.

$$x^{i+1} = x^i - \frac{f(x^i)}{f'(x^i)}$$

Here x^i is the ith approximation to the root, and $f(x^i)$ and $f'(x^i)$ are the function and its derivative evaluated at x^i. The new approximation to the root is x^{i+1}. The successive generation of approximations can stop when they converge to a single value. The convergence to a root is governed by the choice of initial approximation to the root and by the behavior of the function in the vicinity of the root. For instance, several roots close together can cause problems.

The one-dimensional form of this method extends in a natural way to solving systems of simultaneous nonlinear equations. Given n functions F_i, each of n parameters, the problem is to find the set of parameters that drives all the functions to zero. Write the parameter vector $\mathbf{x}$.