

# Review of Model Reference Adaptive Control

Amritash Shekhar

Dept. of EEE

SRMIST

Chennai, India

amritashshekhar@gmail.com

Abhijeet Sharma

Dept. of EEE

SRMIST

Chennai, India

abhijeet5sharma5@gmail.com

**Abstract**—Adaptive control is a technique used for adjusting the parameters of a plant in real-time in order to maintain a desired level of performance when the parameters of the system are unknown and/or change with time. Model reference Adaptive Control (MRAC) offers an approach for the solution of problems related to Adaptive Control in real world. By creating a closed loop controller MRAC tries to compare the output of the plant with a standard reference response and various parameters of the plant change with this response. In this paper we had discussed about Adaptive Control, Model Reference Adaptive Control (MRAC), its historical perspective, its working, the rules that govern MRAC, its real-world applications and the present scenario.

**Keywords**—Adaptive Control, MRAC, Fuzzy Interference, closed loop systems, MIT Rule

## I. INTRODUCTION

According to Oxford dictionary “to adapt” means to adjust to a new situation. Standard controllers cannot be used in system where the parameters vary over time. Hence the need for adaptive controllers. An adaptive control system is like any other control system which has the ability to mutually adjust itself based on the inputs from the system in consideration of the system uncertainty. The parameters which can be adjusted are called adaptive parameters and the mechanism of adjustment, described by mathematical equations is called as the adaptive law. A compilation of methods for giving an ordered approach for adjusting the control systems in real time, so as to be able to give an optimal outcome when the various functions of the plant are amenable to change with time, is covered under adaptive control [1]. In this paper we will be discussing about Model Reference Adaptive control (MRAC) which is an approach to solve real world problems related to Adaptive Control. MRAC is used for making a closed loop controller which adjusts the variables of the system dynamically by comparing the output of the plant with a standard reference response [2]. Fig.1 depicts a simple block diagram of MRAC. It consists of an inner loop and an outer loop. The input signal is sent to the plant through a controller. The plant output is compared with a reference signal and the error signal is sent to the outer loop where an adjustment mechanism system dynamically alters the controller parameters.

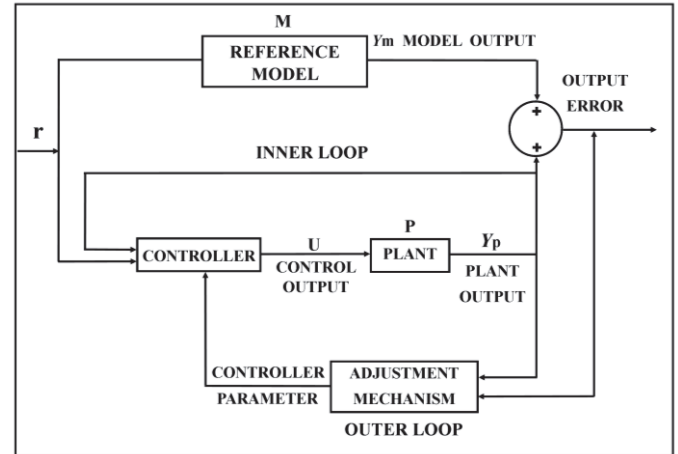


Fig. 1. Block Diagram of MRAC

Further, Model Reference Adaptive Control (MRAC) is classified into two types:

### 1) Direct Control type:

In this Control the system adjusts itself to the error signal which is described as the difference between the plant and the reference response. The controller parameter  $\theta$  of the controller C ( $\theta$ ) is updated in real time by adaptive law.

### 2) Indirect Control type:

In Indirect control the system adjusts itself by comparing the plant output to online standard reference. The value of parameter  $\theta$  is obtained by solving linear algebraic expressions that correlates  $\theta$  with online model of the plant for each time ( $t$ ) [3].

MRAC comprises of the following components:

1) Reference Model: It provides a reference against which the output is compared.

2) Controller: This is explained by a group of variables which are changeable and can be used to explain the control law.

3) Adjustment Mechanism: This is utilized to change the variables of the control system in order to match the system to an ideal model. Various mathematical approaches like Lyapunov theory MIT rule and theory of augmented error are tools to develop adjustment mechanisms [4-5].

MRAC is utilized in many industries ranging from aircraft manufacturing to biomedical field. Despite its widespread application, active research is underway on ways to utilize MRAC with techniques such as fuzzy logic and neural networks in order to improve the performance of controllers in real time.

We proceed as follows. In section 2 we will be discussing about the short history of adaptive control. In section 3 we will discussing about techniques of solving MRAC particularly MIT rule while, Section 4 covers important applications of MRAC. We conclude the paper by providing an insight into the challenges and modern trends of MRAC in section 5.

## II. HISTORICAL PERSPECTIVE

As early as the 1950's the high performing aircraft auto pilots found the need to research the development of adaptive control systems. Since high-performance aircrafts are amenable to massive alteration in short periods of time, therefore ordinary controllers could not be used. Hence, the need for adaptive control was acknowledged and it subsequently increased. MRAC was first proposed by Whitaker in 1950's. MIT was one of the pioneers of MRAC and devised a rule named after it (to solve adaptive gain). 1960's was the most important period of development. State space model and Lyapunov stability criteria were introduced during this time. 1970's were the era of major innovations in the field of adaptive control systems and many MRAC models based on Lyapunov stability were designed. The success of 70's was quickly marred by practical problems on the applications and functioning of the adaptive control systems. From the 1990's and up to the present times, newer techniques such as fuzzy control, neural networks and burgeoning interest of famous agencies and establishments, like NASA has led to the development of more advanced adaptive controllers which can be applied into many diverse fields[6].

## III. MIT RULE

The MIT rule is the main approach to solve MRAC. To apply MRAC we need a closed loop system which can be adjusted by the parameter theta. Since we need an adjustment mechanism so that the error between the model and plant becomes zero, a tracking methodology can be evolved.

Let the error of the output(y) related to the closed loop system and output (ym) belonging to the model be:

$$e = y - y_m \quad (1)$$

Let the cost function be

$$J(\theta) = \frac{1}{2} e^2(\theta) \quad (2)$$

Which has to be minimized

Let the process of plant and model be:

$$G(s) = \frac{y}{u} \quad (3)$$

$$G_m(s) = \frac{y_m}{u_c} \quad (4)$$

The control law is represented as:

$$u(t) = f(u_c, y) \quad (5)$$

Error:

$$e_o = y - y_m \quad (6)$$

$$\frac{\partial e}{\partial \theta} = \frac{\partial y}{\partial \theta} \quad (7)$$

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta} \quad (8)$$

Where  $\gamma$  is tuning parameter and  $\frac{\partial e}{\partial \theta}$  is known as sensitivity derivative.

## IV. APPLICATIONS

Diabetes mellitus is among the top five illnesses affecting mankind. Type I diabetes or T1DM is a result of insufficient production of insulin, thereby increasing the blood glucose levels in the body. Prolonged glucose levels of above 150-160 mg/dl can cause serious health problems. For Type 1 diabetes, insulin must be taken every day in injection form, as oral Insulin will be destroyed by the digestive system. This human assisted Insulin injection through can be thought of as an open loop system. MRAC is employed on a simple controller to provide a closed loop system to control the glucose level of the patient by using an infusion pump. The insulin serves as an actuator of artificial pancreas. The meal data is logged in the simulation and the glucose level of the patient is compared with the standard data of a healthy adult. The pump then secretes the required amount of insulin. Further, the accuracy of the system is improved by applying fuzzy interference system [7]. In fig.2, block diagram of MRAC for glucose adjustment in diabetic subjects is depicted. The transfer function of the diabetic subject ( $Y_p$ ) is compared with that of a healthy model ( $Y_m$ ) and the error output is fed through the sensitivity derivative to the inner feedback loop comprising of PI controller, gain(X) and sensitivity derivative constant ( $\gamma$ )

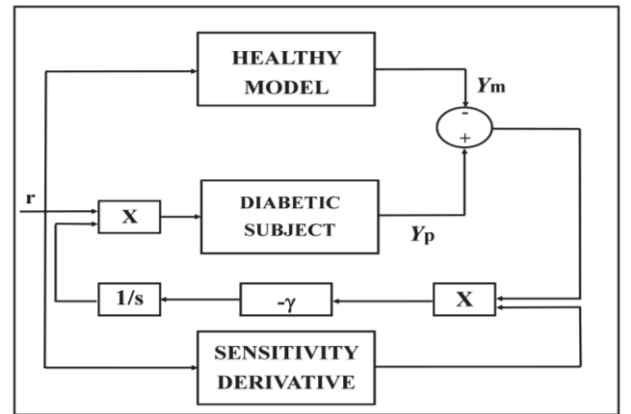


Fig. 2. Model Reference Adaptive Control of Glucose using MIT Rule

In the contemporary world need for energy has become ubiquitous, to meet the increasing requirements of energy renewable forms of energy are used for fuel cells.

Photovoltaic cells have DC voltage which varies in magnitude so as to make regulated supply of DC/DC converter supply which is controlled using PWM pulse and by laws of control system. The problem in this system is that it is nonlinear and it has non-minimum phase, therefore it becomes difficult to form an ideal control system. For the improvement of transient response in the system a Simple Adaptive control (SAC) approach is described as follows. If we have to apply the SAC on a plant then a need to have a mandatory positive real (ASPR) transfer functions. A system can be called an ASPR if it gives static output feedback so that closed loop transfer function is positive and real (SPR). Boost converter is not ASPR, to make it ASPR a parallel feed forward compensator (PFC) is used. By applying this technique we can use SAC algorithm for making an optimal controller for the control system. First a model of DC/DC boost converter is made, and transfer function is obtained. Thereafter the design of PI controller, feed-forward and model reference are employed. After this by applying MIT rule a design of MRAC for DC/DC boost converter is done [8]. To increase the robustness of the system a fractional order parameter law is used. By using fractional order parameter law the design of reference model, fractional PI and fractional feed-forward is done and then simulation is done to see the system response [9]

Fig.(3) depicts block diagram of MRAC for DC-DC boost converter. The input is fed to the controller where a part of it is sent to the fractional feed forward loop and the remaining signal is sent to the plant via PI controller. The two signals are compared and the resultant error signal is fed to outer feedback loop which alters the controller parameter.

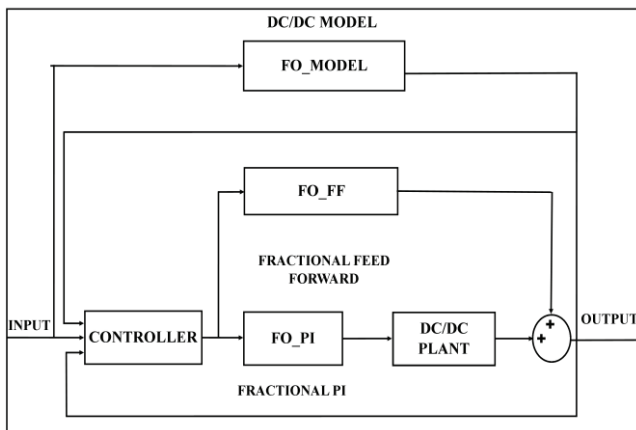


Fig. 3. Block Diagram of DC-DC Boost Converter using MRAC

For developing smart a structure designing is important therefore; multiple strategies for development of control systems and their utilization needs to be generated. For the modelling of structural behaviours of piezoelectric smart structures, finite element model having high number of degree of freedom is utilized. The control system can be direct or indirect control methods. Direct method is superior to the indirect method since it uses a lesser amount of calculations and labor. Direct MRAC techniques are an effort to form a common control system so that it will not need identification and observation in the control loop and simultaneously will maintain or improvise the outcome and quality of the control system in relation to a fixed gain control system or can replace the fixed gain control system where they do not have an application. Non-linear model reference adaptive control law (MRAC) is also applied for

the suppression of vibratory movements in smart piezoelectric mechanical structures [10].

In this section use of MRAC in micro- grid, a subset of power grid is discussed. Distributed Generation (DG) technology is used to meet the demands of power to small scale areas. DG provides advantages over conventional systems, such as higher efficiency and less carbon-dioxide emission, hence causes less pollution. Using standard controllers for micro-grid has several drawbacks such as slow transient response, deviations in grid frequency and unbalanced harmonic current sharing. Hence, an adaptive MRAC controller with MIT rule employing speed droops for active power sharing is utilized. It improves the grid frequency and current sharing. In this case, PV cells connected to boost converters with Maximum power point capability (MPPT) are utilized. Hence, the converter doubles as a three phase inverter to use Phase Locked Loop (PLL) for finding the frequency in order to be able to work for AC signals in the grid. This method is designed to help in the speed control of the rotors of synchronous generators for active power sharing in autonomous micro-grid. This suggested method will have a better response in comparison to PI [11]. Fig (4) depicts the block diagram of MRAC for a proposed microgrid. It consists of 3 distributed generation (DG) units and 3 different types of loads connected to the main grid through isolation switches.

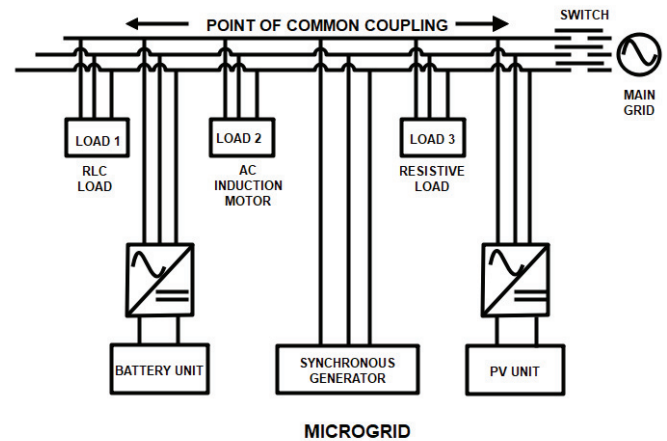


Fig. 4. Block Diagram of MRAC for Micro grid

In this section speed control of DC motor is discussed. These motors require low inertia and high starting torque. DC motors are appropriate for hassle free, high performance and varied uses of speed alterations because they can be easily employed for automotive and aircraft application. Normal feedback controllers like PID cannot vary according to the variation in plant parameters, so an approach to MRAC design using MIT rule is provided for speed control of the DC motors. Through simulations one can conclude that, when the value of adaptation increase or when the amplitude of reference signal is huge, the adaptive system becomes oscillatory; therefore to overcome this problem the MIT rule is used with normalized algorithm and it is termed as modified MIT rule. First MRAC was designed by using MIT rule to derive a law for adjusting the parameter  $\theta$ . Thereafter modified MIT rule is applied for the adjustment feed forward gain. Modeling of DC motor is

done after this and a transfer function is obtained .For simulation purpose 5 different cases are considered. By taking different adaption gain and different input signals the simulation is carried out and results were compared for MIT and modified MIT rule, and normalized algorithm was found to be less sensitive and hence modified MIT rule is a better approach [12].

Teleoperation is the operation of a machine or system from a distance. Teleoperation system has applications in medical areas, sub aquatic investigations and operations in inaccessible or dangerous places. With the increase in reliable telecommunication lines the potential use of teleoperation system has increased. Time delay of transfer line is an important parameter that increases the poor transparency and instability of teleoperation system. So a novel method has been evolved to overcome this handicap and to ensure stability and good performance of the teleoperation system. Also it allows for significant time delays in transmission lines. A new algorithm is proposed for this purpose which can estimate time delays and output of the plant when white and color noise is present. A model reference adaptive controller (MRAC) for master and slave site is designed. First system modeling is done to find transfer function and other parameters. Then time delay in the presence of white noise is done. Then to compensate the delay in MRAC plant prediction is performed and equations are formed. Then, MRAC is designed for general plant and is further extended to a master and slave site. Simulations results indicated better tracking performance and superiority to the closely-related previous method[13].Fig.(5) depicts the block diagram of slave site of a Teleoperation system wherein the slave site system is connected to master site through a communication channel.

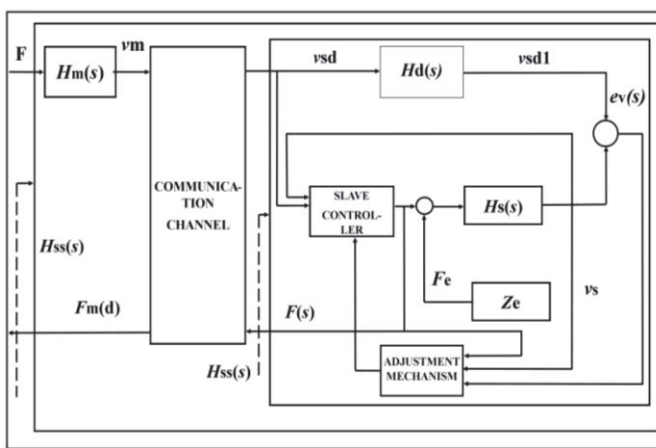


Fig. 5. Teleoperation System with Slave Site

## V. RECENT DEVELOPMENTS AND CHALLENGES

MRAC using fuzzy interference and Variable structure MRAC (VS-MRAC) has been developed which have better performance than standard MRAC. MIT Rule has been modified and new algorithms are in developmental process. At present, MRAC are being developed using Machine Learning models, prototypes of MRAC using Neural Networks which have better robustness are being studied.

MRAC embedded with embedded systems are being used in electronic industry and IOT interfaced MRAC are also in developing stage [14]. Despite years of technological developments Model Reference Adaptive Control still has shortcomings like; MRAC does not work effectively for Non-linear systems. Sensitivity, Controllability, observeability, stability and robustness are few drawbacks of MRAC. Many open problems still need to solve like nonlinear regression, transient performance, under actuated systems [16-17].

## VI. CONCLUSION

Since its inception in the 1950's to the present day, MRAC has undergone significant changes in its design and performance. MRAC was developed as a control system which has applications in a range of industries particularly in the aviation industry. This paper discusses about MRAC, its historical perspective and MIT rule. Some applications of MRAC have been discussed in brief. Although extensive research has been done on MRAC yet its widespread utility is still not present due to controversies and certain drawbacks[18-19].The future looks promising for MRAC as modified MRAC like fuzzy MRAC and Variable structure MRAC (VS-MRAC) have been designed which are better performing. Recently, prototypes of MRAC using Neural Networks and embedded system MRAC are being studied[20]. This paper is a short review of Model Reference Adaptive Control.

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