# t-Distributed Stochastic Neighbor Embedding (t-SNE)

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# Introduction to t-SNE

- t-SNE is a nonlinear dimensionality reduction technique used for visualization.
- It maps high-dimensional data to a lower-dimensional space while preserving local structure.
- Often used to explore and cluster high-dimensional datasets.

# High-Dimensional Probability Distributions

- Given a dataset $X = \{x_1, x_2, \ldots, x_n\}$, we define pairwise similarities using a Gaussian distribution:

$$p_{j|i} = \frac{\exp(-\|x_i - x_j\|^2/2\sigma_i^2)}{\sum_{k \neq i} \exp(-\|x_i - x_k\|^2/2\sigma_i^2)} \tag{1}$$

- The joint probability distribution is then:

$$p_{ij} = \frac{p_{j|i} + p_{i|j}}{2n} \tag{2}$$

# Low-Dimensional Probability Distributions

- In the lower-dimensional space, we model similarities with a Student's t-distribution:

$$q_{ij} = \frac{(1 + \|y_i - y_j\|^2)^{-1}}{\sum_{k \neq l}(1 + \|y_k - y_l\|^2)^{-1}} \tag{3}$$

- This distribution allows better separation of clusters.

# KL Divergence Optimization

- The objective function minimizes the Kullback-Leibler (KL) divergence:

$$C = \sum_{i \neq j} p_{ij} \log \frac{p_{ij}}{q_{ij}} \tag{4}$$

- This is optimized using gradient descent to update the low-dimensional representation $Y$.

# Machine Learning Applications of t-SNE

- Visualizing high-dimensional datasets (e.g., MNIST, gene expression data).
- Discovering clusters and patterns in data.
- Understanding feature embeddings in deep learning.