國立臺灣科技大學

財務金融研究所碩士班

碩士學位論文

學號：M11218014

比特幣選擇權隱含風險中立機率密度之平滑尾部導取

Extracting Smooth Tails of Option Implied Risk-neutral Densities in the Bitcoin Market

研究生：王士誠

指導教授：薛博今 博士

中　華　民　國　一　一　四　年　五　月

摘要

本研究提出一種從比特幣選擇權價格中提取風險中性機率密度函數（Risk-neutral Density, RND）的方法。該方法透過單一點及其對應斜率配適 RND 的每一尾部，確保在接合點處具有連續性及平滑性。具體而言，實證 RND 的累積機率與斜率在接合點處均與廣義柏拉圖分配（Generalized Pareto Distribution, GPD）相等。這種配合方式確保了實證部分與估計部分之間的無縫銜接，從而維持整體分配的完整性。

為進一步驗證提出方法之有效性，本研究評估了 RND 的動差（即偏態和超額峰態）在預測比特幣報酬率方面的預測能力。利用 Deribit 平台 2020 年 1 月至 2024 年 4 月期間的比特幣選擇權交易資料，實證結果與多數先前文獻中發現相異，顯示偏態與日報酬率呈顯著負相關，表明投資人在比特幣市場中要求更高的風險溢酬以補償與負偏態相關的下行風險。此外，超額峰態與週報酬率呈負相關，意味著較高的尾部風險通常先於比特幣報酬率反轉出現。另外，從使用滾動視窗估計架構進行的樣本外驗證顯示，本研究提出的方法在預測日報酬率方面顯著優於 Birru 與 Figlewski (2012) 的方法，並在週報酬率預測上展現出些微優勢。本研究的發現為風險中性機率密度函數（RND）在加密貨幣市場中強化投資策略和改善風險管理的應用提供了寶貴的見解。

關鍵字：加密貨幣、比特幣、選擇權、隱含波動率、風險中立機率密度、廣義柏拉圖分布

Abstract

This study proposes a method for extracting risk-neutral density (RND) from Bitcoin options. Each tail of the empirical RND is fitted using one single point and its corresponding slope, ensuring continuity and smoothness at the junction point. Specifically, both the cumulative probability and the slope of the empirical RND equalize those of Generalized Pareto Distribution (GPD) at the joining point. This alignment guarantees a seamless transition between the empirical and parametric segments, thereby preserving the integrity of the overall distribution.

To further assess the effectiveness of the proposed method, this research evaluates the predictive power of the RND moments, namely skewness and excess kurtosis, for predicting Bitcoin returns. Utilizing Bitcoin options trading data from the Deribit platform covering the period from January 2020 to April 2024, consistent with most existing findings in the literature on index options, the empirical results show that both skewness and excess kurtosis exhibit a negative relationship with weekly returns. Nevertheless, the skewness and excess kurtosis become positively associated with daily returns, indicating that cryptocurrency investors may become less risk-averse and more inclined to seek tail risk as options close to maturity. Additionally, the proposed method significantly outperforms the method by Birru and Figlewski (2012) in predicting daily returns and demonstrates marginal superiority in weekly returns, as evidenced by out-of-sample validation using a rolling window estimation framework. The findings in this study offer valuable insights into the application of the RND for enhancing investment strategies and improving risk management in cryptocurrency markets.

Keywords: Cryptocurrency, Bitcoin, Option, Implied Volatility, Risk-neutral Density (RND), Generalized Pareto Distribution (GPD)

Acknowledgement

Contents

List of Figures

List of Tables

1. Introduction

1.1 Research Background and Motivation

Risk assessment has remained a core focus in the financial industry. In particular, with the rapid expansion of the cryptocurrency ecosystem, Bitcoin has witnessed explosive growth in its derivatives market (Akyildirim et al., 2020). According to The Block (*The Block*, 2025), Bitcoin futures and options trading volume exceeded $21 trillion in 2024, with options trading growing at 130% annually, far outpacing traditional derivatives markets. This reflects increasing demand for cryptocurrency risk management tools while providing researchers a unique perspective on price discovery in emerging markets (Zulfiqar & Gulzar, 2021).

Cryptocurrency markets differ significantly from traditional financial markets in several aspects such as 24/7 trading, extreme volatility, decentralized structure, and unique investors' composition. Bitcoin's historical annualized volatility frequently exceeds 100%, substantially higher than the 15-20% volatility of traditional stock indices (Liu & Tsyvinski, 2021). This extreme volatility makes risk management crucial while exhibiting specific challenges for interpreting option price information.

As a derivative financial instrument, options contain rich market information in their prices. Call options grant holders the right to purchase the underlying asset at a predetermined price at a specific future time while put options give holders the right to sell. Their non-linear payoff structure reflects market expectations of future trends and volatility risk assessments (Hull, 2021). Option prices reveal market consensus on risk expectations and provide insights into market microstructure and investors' behavior (Bakshi et al., 2003). While the Black-Scholes model (1973) provides theoretical foundation for option pricing, the normal distribution assumption fails to capture the fattailed distribution and negative skewness commonly observed in financial markets, particularly in high-volatility cryptocurrency markets (Chordia et al., 2021).

In the option pricing theory, the Black-Scholes model (1973) assumes geometric Brownian motion and applies no-arbitrage principles. Nevertheless, observed option prices often deviate from theoretical values, with implied volatility exhibiting systematic differences across strike prices, forming "volatility smile" or "volatility skew" (Rubinstein, 1994). This indicates that market expectations differ from the log-normal distribution assumed by Black-Scholes, particularly in the tail regions.

Extracting the risk-neutral density (RND) from option prices captures the market's complete expectation of future price distributions. According to Breeden and Litzenberger (1978), the RND can be derived by taking the second derivative of option prices with respect to strike prices. To extract the RND, implied volatility is converted from a finite number of observed market option prices through the Black-Scholes model. To obtain the continuum of implied volatility curve, Hagan and West (2006) indicated that quadratic splines can avoid the risk of overfitting while maintaining curve smoothness to ensure model robustness and reliability, particularly in the case of high market volatility. Haslip and Kaishev (2014) showed that when dealing with complex derivative financial products such as lookback options, quadratic splines combined with Fourier transforms can provide efficient and accurate pricing results, achieving a good balance between computational efficiency and precision, though with the risk of discontinuous first derivatives. Bliss and Panigirtzoglou (2004), Figlewski (2008) and Monteiro, Tütüncü, and Vicente (2008) introduced the interpolation of implied volatility using cubic spline functions, demonstrating high computational efficiency, relatively simple implementation and reasonable fitting results under most market conditions. Nevertheless, cubic splines only guarantee the continuity of the first derivative, while the second derivative may reveal discontinuous at certain nodes, resulting in non-smooth fitting when dealing with extreme volatility. To resolve the deficiency of smoothness in extracting RNDs, quartic spline functions with a single knot have been employed for implied volatility curve fitting (Figlewski, 2008; Birru & Figlewski, 2012; Reinke, 2020).

Due to limited option prices obtainable in the market, especially in deep out-of-the-money regions, RND extraction often lacks sufficient tail information. Previous research has developed two major streams of the RND tail-fitting methods: non-parametric and parametric approaches. Non-parametric methods make minimal assumptions about the underlying asset. Bondarenko (2000) proposed a non-parametric method for deriving the RND from option prices, indicating that daily RND changes correlate with index performance. Grith, Härdle and Schienle (2012) explored kernel smoothing and spline functions in the RND estimation, highlighting their flexibility in capturing complex distributional features such as skewness and multimodality. Monteiro and Santos (2022) addressed local constraint limitations in kernel-based estimation by imposing broader no-arbitrage constraints using the Heston model. Dong, Xu, and Cui (2024) introduced the Implied Willow Tree method, reconstructing complete risk-neutral processes directly from cross-maturity option data without preset parametric models.

Parametric methods commonly employ Extreme Value Theory (EVT) to extend the RND tails. Figlewski (2008) fitted each tail of the RND with Generalized Extreme Value Distribution (GEV) by imposing three continuity conditions: matching cumulative probabilities at the first joining point and ensuring equal density values at both the first and second joining points. Birru and Figlewski (2012) further replaced GEV with Generalized Pareto Distribution (GPD), discovering significant left skewness in the RND regardless of market volatility. Their findings confirmed that even during extreme market turbulence, the mean of RND remained close to the futures price, indicating effective no-arbitrage relationships. Other approaches include mixed distribution methods, which decompose the RND into core and tail components. Glatzer and Scheicher (2005) combined log-normal distributions for the core with GPD for the tails, demonstrating effectiveness in Eurozone bond markets. Markose and Alentorn (2011) parameterized the RND tails using Generalized Extreme Value Family to capture market expectations of extreme events. Monteiro et al. (2008) proposed density function extrapolation using cubic spline functions with non-negativity constraints and exponential functions for the tail extrapolation, though computationally simple but potentially limited in capturing complex tail features. Recent developments include novel parametric methods that do not rely on extreme value theory. Orosi (2015) established appropriate functional forms with parameter constraints to produce well-behaved RND estimates. Uberti (2023) developed a semi-parametric estimation method combining parametric stability with non-parametric flexibility. Y. Li, Nolte, and Pham (2024) introduced a Lognormal-Weibull mixture model offering improved performance when measuring skewness and analyzing multi-peak RNDs, demonstrating the continuous refinement in estimation methodology.

Beyond implied volatility, RND contains vastly useful information in higher-order moments, namely skewness and excess kurtosis, which have been broadly used to predict asset returns. Bali and Murray (2013) and Conrad, Dittmar, and Ghysels (2013) found a significant and negative relationship between risk-neutral skewness and future stock returns. Similar findings also appear in commodity futures (Fuertes et al., 2022), oil markets (Cortés et al., 2020), and foreign exchange (Chen et al., 2018). Most recently, Böök, Imbet, and Reinke (2025) demonstrated that volatility, skewness, and kurtosis derived from options data outperform historical return-based indicators in predicting stock risk premiums. While these relationships are established in the traditional financial markets, research in cryptocurrency markets remains limited.

This study extends the framework of Birru and Figlewski (2012) to derive the entire RND since GPD is particularly adept at capturing the behavior of the extreme events. Nonetheless, certain constraints are observed in the method by Birru and Figlewski (2012) (abbreviated as Birru-Figlewski method hereafter) when applied to cryptocurrency markets. Specifically, Birru-Figlewski method can produce discontinuities or kinks at the connection points. In addition, it increases computational complexity, potentially affecting estimation stability. To mitigate the limitations in Birru and Figlewski (2012), we propose a method which fits each tail of the RND using one single point with one slope to ensure the continuity with no kinks for both the cumulative distribution function and density function. The proposed method imposes two essential conditions. First, at the joining point, the cumulative probability of GPD must equal that of the empirical RND. Second, the slope of the GPD density function must match the slope of the empirical RND at the same point.

To further validate the effectiveness of the proposed method, this study examines the predictive power of the RND moments, such as skewness and excess kurtosis, for Bitcoin returns, in comparison with Birru-Figlewski method. Based on Bitcoin options trading data from the Deribit platform from January 2020 to April 2024, the proposed method significantly outperforms Birru-Figlewski method in terms of daily returns and slightly surpasses in weekly returns. Further, the robustness tests using a rolling window estimation (Campbell & Thompson, 2008) reaffirm the superior performance of the proposed method over Birru-Figlewski method.

In return prediction regression models, the proposed method indicate that the skewness has a positive effect on daily return, in contrast with the existing literature on index options (Bali & Murray, 2013; Conrad et al., 2013; Cujean & Hasler, 2017; Kim & Park, 2018; Y. Li et al., 2024). Moreover, excess kurtosis is positively associated with daily returns, inconsistent with the previous studies (Amaya et al., 2015; Mei et al., 2017)). Nevertheless, both the skewness and excess kurtosis become negatively associated with the weekly returns. This implies that cryptocurrency investors exhibit a reduced aversion to risk and are more likely to pursue tail risk as options close to expiration. As for the control variables in the prediction models, the significant predictive power of lagged returns supports the momentum effects in cryptocurrency markets (Liu & Tsyvinski, 2021). Also, the Crypto Fear and Greed Index shows a significant and positive relationship with future returns, highlighting that market sentiment serves as a contrarian indicator in cryptocurrency markets (M. He, Shen, Yaojie Zhang, and Yi Zhang (2023)).

This study contributes to the existing literature in several important ways. First, the proposed methodology ensures smooth continuity in the tail fitting of RNDs, eliminating kinks and enhancing computational efficiency relative to the Birru-Figlewski method. Second, it pioneers the application of RND moments to multi-horizon return prediction within cryptocurrency markets. The empirical findings offer meaningful insights and practical implications for researchers, practitioners, and policymakers aiming to enhance investment strategies and risk management practices in the evolving landscape of digital assets.

1.2 Thesis Structure

This dissertation comprises six chapters. Chapter 1 introduces the research background, motivation and critical findings. Chapter 2 reviews theoretical foundations of RND and relevant empirical studies. Chapter 3 describes data sources. Chapter 4 details the methodological framework, including the proposed tail-fitting method and empirical model specifications. Chapter 5 presents empirical results and method comparison. Chapter 6 makes the conclusion.

2. Literature Review

2.1 Fundamental Theory of the Risk-neutral Density

The risk-neutral density (RND), pioneered by Breeden and Litzenberger (1978), demonstrates that market expectations about future price distributions could be extracted from option prices. Shimko (1993) enhanced this methodology by converting option prices into implied volatility space for interpolation, leveraging the smoother volatility curves to improve RND estimation accuracy.

Christoffersen, Jacobs, and Chang (2013) established that risk-neutral skewness effectively predicts future return direction and magnitude, while Chang, Christoffersen, and Jacobs (2013) found higher risk-neutral kurtosis often precedes greater market volatility. For extreme market conditions, Birru and Figlewski (2012) documented significant RND shape transformations during the 2008 financial crisis, implementing the Generalized Pareto Distribution (GPD) for more accurate tail estimation. Jackwerth (2020) provided evidence that markets require time to fully incorporate major events, contributing valuable insights on market information efficiency.

2.2 Methods for Deriving the RND

Figlewski (2008) developed a comprehensive framework categorizing RND estimation into parametric and non-parametric approaches. McNeil and Frey (2000) combined GARCH models with Extreme Value Theory (EVT) for financial time series tail risk estimation, highlighting GPD's advantages in modeling extreme events.

Methodological advancements include Orosi's (2015) parametric estimation technique utilizing constrained functional forms, He, Peng, Zhang, and Zhao’s (2022) GPD application for tail estimation, and Uberti's (2023) semi-parametric approach combining parametric stability with non-parametric flexibility. Ammann and Feser (2019) developed robust estimation methods for risk-neutral moments that reduce bias under market noise and liquidity constraints, while Hayashi (2020) proposed a method for analyzing RNDs from volatility smiles that eliminates numerical approximation errors.

Recent innovation comes from Dong, Xu, and Cui (2024) with their Implied Willow Tree (IWT) method, which reconstructs complete risk-neutral processes from cross-maturity option data without preset parametric models, demonstrating effectiveness in pricing complex options and handling noisy data.

2.3 Moments of the RND

2.3.1 Higher-Order Moments

Higher-order moments, particularly skewness and kurtosis, play critical roles in financial research. Bali and Murray (2013) and Conrad et al. (2013) documented a significant and negative relationship between risk-neutral skewness and future stock returns, aligning with investor preference for negative skewness. Kim and Park (2018) confirmed this relationship persists after controlling for firm characteristics.

Mei, Liu, Ma, and Chen (2017) found realized skewness and kurtosis negatively affect future volatility, with skewness outperforming kurtosis in medium to long-term predictions. Fuertes, Liu, and Tang (2022) demonstrated the importance of risk-neutral skewness in commodity futures pricing, with strategies based on risk-neutral skewness values generating significant excess returns, particularly in contango markets.

Cortés, Mora-Valencia, and Perote (2020) showed log-semi-nonparametric distributions more accurately capture oil price RNDs than traditional log-normal distributions, with skewness and kurtosis containing valuable market expectation information. Recent work by Böök et al. (2025) derived robust conditional volatility, skewness, and kurtosis indicators from S&P 500 options that outperform historical return-based indicators in predicting equity risk premiums.

2.3.2 Tail Risk and Extreme Value Theory

Balkema and de Haan's (1974) threshold exceedance model established that samples exceeding sufficiently high thresholds converge to the Generalized Pareto Distribution, foundational for financial market tail risk estimation. Wang and Yen (2018) found option-implied tail risk indicators effectively predict underlying asset movements, particularly during recessions.

Chen, Hsieh, and Huang (2018) documented higher-order RND moments' explanatory power for crash risk and risk premiums, with skewness positively correlating with risk premiums and kurtosis with foreign exchange swap spreads. Lehnert (2022) challenged traditional views by showing short-selling in options markets creates a negative relationship between risk-neutral market skewness and returns.

Conrad et al. (2013) extended methods combining GARCH with mixed normal distributions to capture asymmetric volatility, while Neumann and Skiadopoulos (2013) found market expectations regarding volatility, skewness, and kurtosis exhibit significant predictability, especially during heightened market volatility.

2.4 Empirical Applications in Financial Markets

2.4.1 Traditional Financial Markets

Mohrschladt and Schneider (2021) revealed in-the-money options contain valuable market information through high-frequency trading data analysis. Li, Wu, H. Zhang, and

L. Zhang (2024) demonstrated risk-neutral skewness's predictive power for future stock returns, particularly during recessions, consistent with Cujean and Hasler's (2017) theoretical predictions.

Feng, He, and Zhang (2024) established strong associations between market sentiment and option-implied volatility during uncertainty periods, while Köse et al. (2024) found institutional investor behavior significantly impacts RND shapes. Amaya et al. (2015) documented realized skewness's explanatory power for cross-sectional stock returns, persisting after controlling for risk factors.

Bali and Zhou (2016) identified significant associations between market uncertainty and expected returns during heightened macroeconomic uncertainty. Jondeau, Wang, Yan, and Zhang (2020) demonstrated individual stock skewness effectively predicts S&P 500 index futures returns, persisting after controlling for liquidity risk and economic cycles.

2.4.2 Cryptocurrency Market

Zulfiqar and Gulzar (2021) noted cryptocurrency exchange options provide diverse hedging instruments, while Baur and Smales (2022) found leveraged fund traders maintain key roles and net short positions in Bitcoin futures markets, accurately predicting major market fluctuations.

López-Cabarcos, Pérez-Pico, Piñeiro-Chousa, and Šević (2021) established social media sentiment indicators' predictive power for short-term Bitcoin price movements. Chordia, Lin, and Xiang (2021) documented significant left skewness and excess kurtosis in Bitcoin options' RND, while Akyildirim, Corbet, Lucey, Sensoy, and Yarovaya (2020) found cryptocurrency volatility increases with investor fear sentiment, correlating with traditional market volatility indicators.

Liu and Tsyvinski (2021) identified significant momentum effects in cryptocurrency markets, with Li, Urquhart, Wang, and Zhang (2021) confirming particularly strong MAX momentum effects. Liu and Chen (2024) documented market capitalization-dependent skewness patterns, with asymmetric risk negatively correlating with future returns. Liu, Li, Nekhili, and Sultan (2023) employed machine learning to confirm lagged returns' strong predictive power for cryptocurrency returns.

3. Data

3.1 Data

This study utilizes historical trading data from Deribit exchange (*Deribit*, 2025), the dominant platform in the global cryptocurrency options market. The dataset spans January 2020 to April 2024, encompassing daily trading volume, closing prices, implied volatility, spot prices, futures prices, and related trading metrics.

According to The Block (*The Block*, 2025), Deribit commands over 80% market share in open interest among major trading platforms (Deribit, OKX, and Binance), attributable to its established operational history and strategic market development (as shown in Figure 3-1). Founded in 2016 in the Netherlands, Deribit pioneered professional cryptocurrency options trading, with its name reflecting the fusion of "Derivatives" and "Bitcoin."

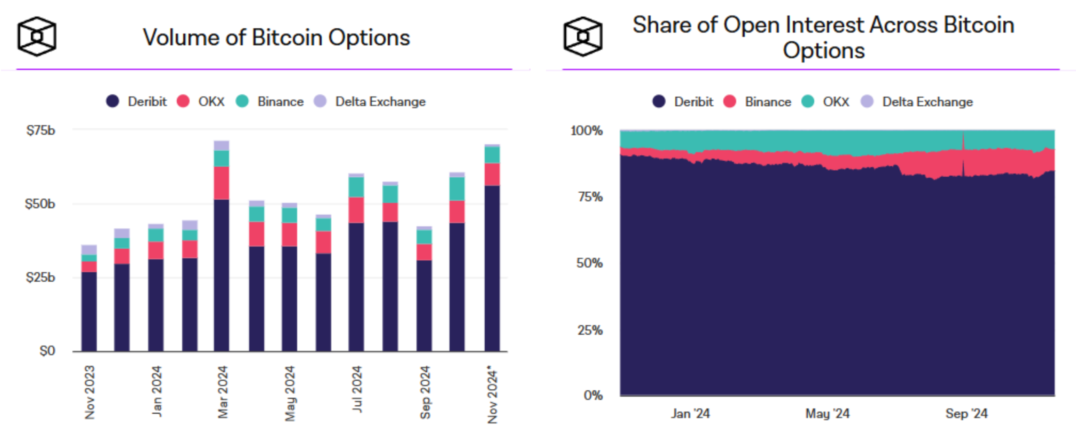

Figure 3-1: Statistics on Bitcoin Options Market Trading Volume and Open Interest
(Data Source: The Block Official Website)

Deribit's Bitcoin European cash-settled options operate on a continuous 24/7 trading mechanism with uniform expiration at 08:00 UTC (*Deribit Options*, 2025). The exchange offers a comprehensive product matrix including short-term contracts (1-day, 2-day, 3-day), medium-term contracts (1-week, 2-week, 3-week), month-end expiration contracts

(January, February, April, May, July, August, October, November), and quarterly expiration contracts (March, June, September, December). This diversified product architecture satisfies varied investor requirements while enhancing market liquidity and price discovery efficiency, with particularly significant market participation growth following the October 2020 introduction of daily and weekly expiration options.

3.2 Overview of Bitcoin Options Trading Market

Transaction counts (Figure 3-2) and trading volumes (Figure 3-3) have grown substantially since October 2020, coinciding with Deribit's product diversification strategy that introduced daily and weekly expiration options. Market activity surged again in late 2023, with call option volume reaching historic highs in February 2024 amid rising Bitcoin prices, reflecting market optimism. The increasing trading volume since H2 2023 indicates improved market liquidity and depth, enhancing price discovery efficiency and attracting institutional participation.

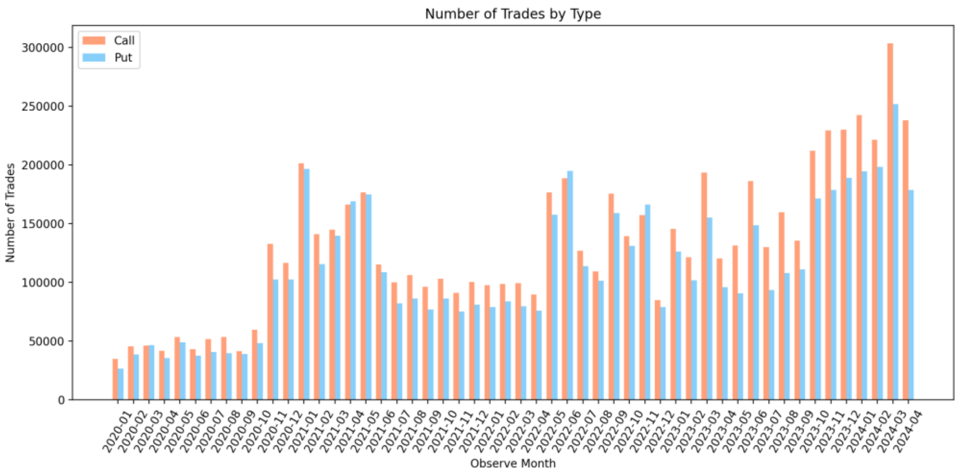

Figure 3-2: Monthly Transaction Counts of Bitcoin Call and Put Options from January 2020 to April 2024

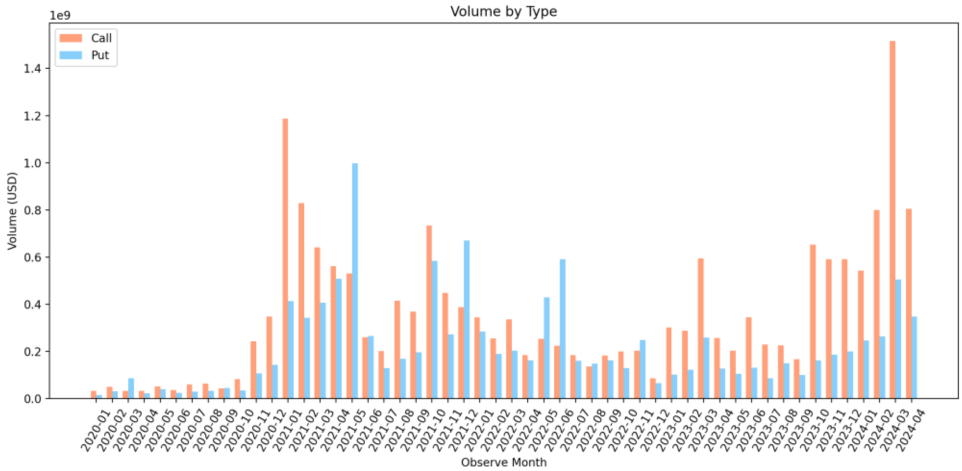

Figure 3-3: Monthly Trading Volume of Bitcoin Call and Put Options from January 2020 to April 2024

Trading activity heat maps reveal distinct patterns. Call options trading (Figure 3-4) concentrates around at-the-money positions (Moneyness ratio $\left(\frac{StrikePrice(K)}{SpotPrice(S)}\right)$ 0.9-1.1), with slightly out-of-the-money options (1.0-1.1) recording the highest volume ($2,072M), indicating investor preference for leveraged positions. Short-term call options (31-90 days) maintain a substantial volume across various moneyness levels, while extremely short-term options (<14 days) show significant at-the-money activity, reflecting speculative demand. Long-term (>180 days) call options exhibit limited trading except for anomalous volume ($392M) in deep out-of-the-money positions (>2.0), likely reflecting institutional hedging strategies.

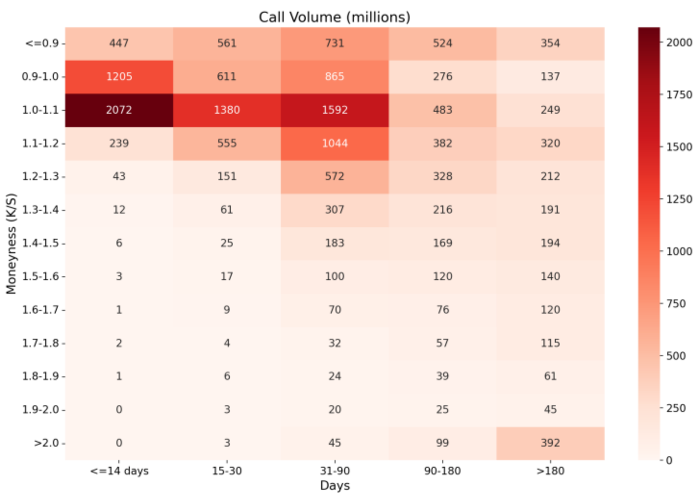

Figure 3-4: Heat Map of Total Bitcoin Call Options Trading Volume from January 2020 to April 2024

Put options (Figure 3-5) demonstrate different characteristics, with trading heavily concentrated near at-the-money (0.9-1.0) positions, peaking at $1,676M. Deep out-of-the-money puts (≤0.7) show limited activity, suggesting minimal demand for protection against significant price declines. Short-term puts are most active near at-the-money, while medium-term puts (90-180 days) show substantial volume ($821M) in deep in-the-money positions (>1.1). Long-term puts (>180 days) exhibit relatively uniform volume distribution across moneyness levels.

Figure 3-5: Heat Map of Total Bitcoin Put Options Trading Volume from January 2020 to April 2024

The market demonstrates considerable depth with trading concentrated around at-the-money positions and preference for short-term strategies. Higher call option volumes relative to puts reflect bullish market sentiment, while anomalous trading patterns in specific segments likely represent specialized institutional strategies. Despite its significant growth, the market remains characterized by predominantly short-term speculative trading behavior.

4. Research Methodology

4.1 Deriving the RND

In the following text, symbols C, P, S, K, and T represent as follows: C is call option price, P is put option price, S is underlying asset current price, K is strike price, r is risk-free rate, T is days to option expiration. This research also uses $f(K)$ to represent the Risk-neutral Density Function (RND) and $F(K) = \int_{-\infty}^{K} f(z)dz$ to represent the Risk-neutral Cumulative Function.

The call option price is the expected payoff before its expiration day T, discounted back to the present value. Under risk-neutral conditions, this expected price can be calculated based on risk-neutral probability and discounted using the risk-free rate, as follows:

$$C = \int_{K}^{\infty} e^{-rT} (S_T - K) f(S_T) dS_T \tag{1}$$

Next, by taking the first partial derivative of call option price with respect to strike price, we can derive the risk-neutral distribution function $F(K)$, as follows:

$$\begin{aligned}
\frac{\partial C}{\partial K} &= \frac{\partial}{\partial K}\left[\int_{K}^{\infty} e^{-rT} (S_T - K) f(S_T) dS_T\right] \\
&= e^{-rT}\left[-(K-K)f(K) + \int_{K}^{\infty} -f(S_T)dS_T\right] \\
&= -e^{-rT}\int_{K}^{\infty} f(S_T)dS_T \\
&= -e^{-rT}[1 - F(K)]
\end{aligned}$$

Rearranging terms, we obtain the risk-neutral distribution function $F(K)$:

$$F(K) = e^{rT}\frac{\partial C}{\partial K} + 1 \tag{2}$$

Then, by taking another partial derivative of equation (2) with respect to strike price, we can derive the RND at strike price:

$$f(K) = \frac{\partial}{\partial K}\left[e^{rT}\frac{\partial C}{\partial K} + 1\right] = e^{rT}\frac{\partial^2 C}{\partial K^2} \tag{3}$$

In actual options trading markets, since strike prices are in discrete form, we can use observed option prices and apply Finite Difference Methods (FDM) to obtain approximate solutions to equations (2) and (3). Assuming that at time T to expiration, there are N options with different strike prices in the market, where K_1 represents the lowest strike price and K_n represents the highest strike price. We use three options with strike prices K_{n-1}, K_n and K_{n+1} to calculate the approximate value centered at K_n, as follows:

$$F(K_n) \approx e^{rT}\left[\frac{C_{n+1} - C_{n-1}}{K_{n+1} - K_{n-1}}\right] + 1 \tag{4}$$

$$f(K_n) \approx e^{rT}\frac{C_{n+1} - 2C_n + C_{n-1}}{(\Delta K)^2} \tag{5}$$

Equations (1) to (5) explain how to theoretically derive the RND between strike prices K_2 and K_{n-1} from a set of call option prices.

In this research, ΔK is a fixed constant value used to construct artificially spaced option prices to fill gaps between discrete strike prices in the market. This approach addresses the problem of sparse or uneven trading data and ensures consistent spacing between strike prices, facilitating numerical calculations through finite difference methods and improving the accuracy of estimation.

4.2 Deriving the RND Using Bitcoin Options

4.2.1 Applying the Black-Scholes Model

In traditional financial markets, option pricing models (such as the Black-Scholes model) typically use risk-free interest rates as parameters, which are generally represented by the yields of low-risk assets such as government bonds. Nevertheless, in cryptocurrency markets such as Bitcoin, the applicability of risk-free interest rates is limited and therefore not widely used. This is because the Bitcoin market lacks a unified risk-free asset. Due to the decentralized nature of cryptocurrency markets, there are no widely accepted risk-free assets such as government bonds, making it difficult to determine a single universal risk-free rate, thus limiting its applicability in this market.

Additionally, Bitcoin price volatility is far higher than traditional assets. This highly volatile has a more significant impact on option prices than risk-free interest rates, causing traders to focus more on changes in implied volatility rather than risk-free rates. Furthermore, in cryptocurrency markets, the interest rate environment may be influenced by exchange rules and market supply and demand, not necessarily related to traditional risk-free rates, making traditional interest rate indicators difficult to reflect the actual situation in cryptocurrency markets. Moreover, the cost of holding Bitcoin differs from the cost of holding traditional currencies or assets, including security aspects and technological risks, which are difficult to quantify through risk-free interest rates, further limiting the applicability of risk-free rates in Bitcoin option pricing.

This research uses Bitcoin option trading prices from the Deribit exchange, which has adopted a more suitable model (priced in Bitcoin) to compute option prices, adapting to the cryptocurrency market and meeting trading market needs. To meet research requirements, this study observes the traditional Black-Scholes model (Equation (6)) and compares it with the calculation formula provided by Deribit exchange (Equation (7)). It can be seen that multiplying the Deribit exchange quote $C_{Deribit}$ by the Bitcoin spot price S_0 yields the Bitcoin option price denominated in US dollars.

$$C_{BS} = S_0 \times N(d_1) - Ke^{-rT} \times N(d_2)$$

$$\Rightarrow C_{BS} = S_0 \times \left[N(d_1) - \frac{Ke^{-rT}}{S_0} \times N(d_2) \right] \; and \; F = S_0 e^{rT}$$

$$\Rightarrow \frac{C_{BS}}{S_0} = N(d_1) - \frac{K}{F} \times N(d_2) \tag{6}$$

$$C_{Deribit} = N(d_1) - \frac{K}{F} \times N(d_2) \tag{7}$$

, where $d_1 = \frac{\ln\left(\frac{F}{K}\right) + \left(\frac{\sigma^2}{2}\right) \times T}{\sigma \times \sqrt{T}}$, $d_2 = d_1 - \sigma \times \sqrt{T}$, C_{BS} is the Black-Scholes call option price (denominated in USD), $C_{Deribit}$ is the Deribit exchange Bitcoin call option price (denominated in Bitcoin), S_0 is the Bitcoin spot price, $N(x)$ is the cumulative distribution function (CDF) of the normal distribution, K is the strike price, r is the risk-free interest rate, T is the option's time to expiration, F is the Bitcoin futures price, ln is the natural logarithm, σ is the annualized standard deviation.

4.2.2 Calculating Bitcoin Option Implied Volatility

In the Bitcoin options market, traders are predominantly risk-seeking and tend to operate with out-of-the-money (OTM) options, primarily due to their lower cost, high leverage effect, and particularly strong sensitivity to volatility. For buyers, given the extremely high volatility of the Bitcoin market itself, these options are highly attractive to speculators and high-risk-preferring investors. Despite the higher probability of these options expiring worthless, traders are still willing to take on such risks. For sellers, since Bitcoin price volatility is significantly higher than traditional financial markets, the premium levels for OTM options are usually higher, further enhancing the incentives for seller participation, making OTM options a core tool for many sellers to create stable cash flow. In summary, OTM options have better trading volume and liquidity, and their prices can more efficiently reflect market sentiment. This research uses OTM option trading data to compute implied volatility; nevertheless, to avoid anomalies caused by unreasonable trades in deeply out-of-the-money areas, options data with strike prices below $10 is excluded.

Shimko (1993) proposed converting market option prices to implied volatility (IV) before interpolation, as implied volatility curves are typically smoother and more continuous than price data, making them suitable for interpolation and smoothing processes. The interpolated curves are then converted back to call option prices to extract the RND. This method does not rely on option prices conforming to Black-Scholes model assumptions, but rather uses the Black-Scholes formula merely as a calculation tool to convert data into a form more suitable for smoothing.

Figlewski (2008) proposed a method aimed at resolving abnormal fluctuations in implied volatility data near at-the-money options, particularly the jump phenomenon in call and put option prices near the at-the-money point. Such jumps may lead to non-smooth implied volatility curves, thereby affecting the stability of the RND extraction. Following this method, if a strike price is between 0.9 and 1.1 times the futures price, the average of call and put implied volatilities is taken as the data point. The formula is as follows:

$$IV_{mix}(K) = 0.5 \times IV_{call}(K) + 0.5 \times IV_{put}(K)$$

$$K \in [0.9 \times F, 1.1 \times F]$$

As shown in Figure 4-1 through Figure 4-3, this approach effectively reduces fluctuation amplitude near the at-the-money point while maintaining OTM option data outside this range.

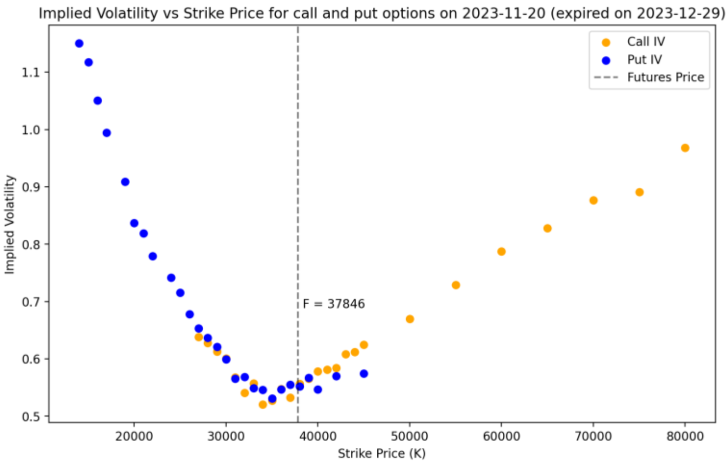

Figure 4-1: Bitcoin Option Implied Volatility Distribution on November 20, 2023 (Expiring on December 29, 2023)

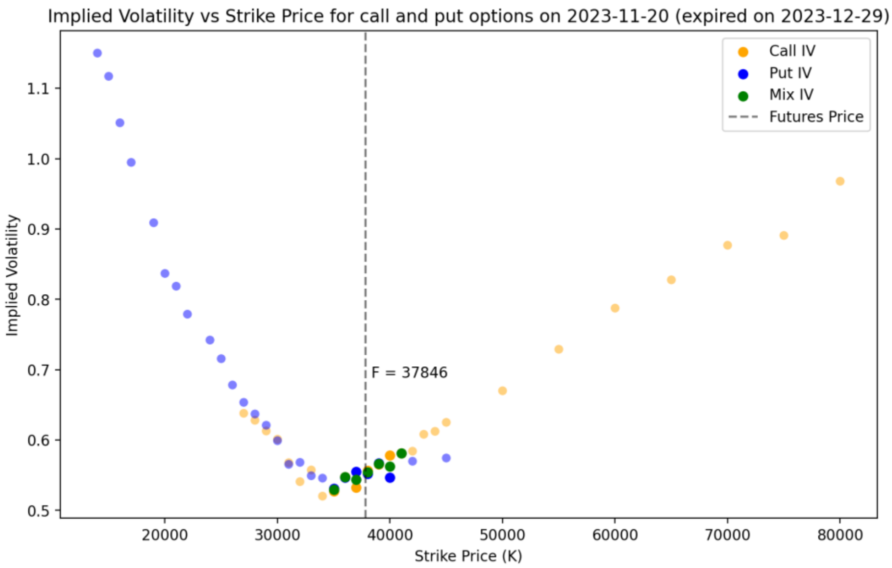

Figure 4-2: Bitcoin Option Implied Volatility Distribution on November 20, 2023 (Expiring on December 29, 2023)

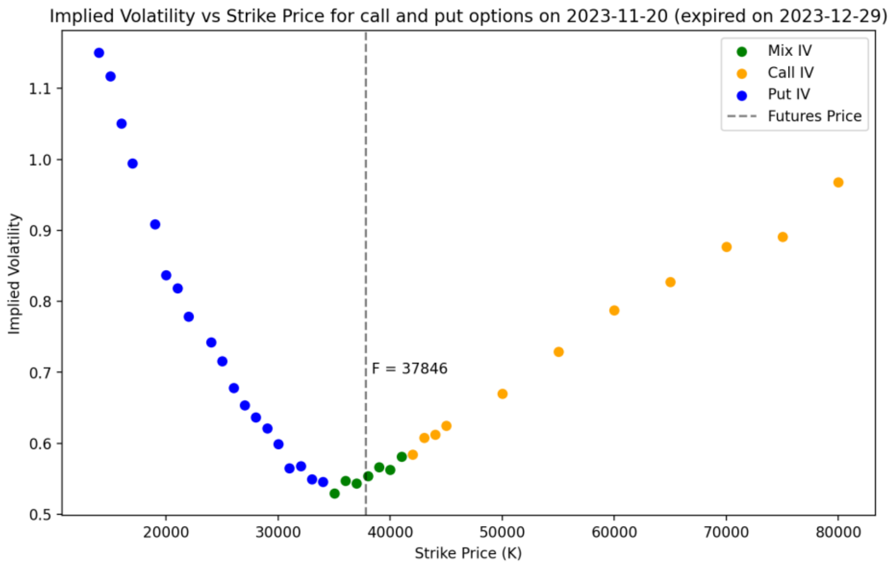

Figure 4-3: Bitcoin Option Implied Volatility Distribution on November 20, 2023 (Expiring on December 29, 2023)

4.2.3 Fitting Bitcoin Option Implied Volatility Curve

To more precisely fit the processed implied volatility data, this research adopts a 4th-order spline function with a single knot for curve fitting. The knot is placed at the futures price, a design that allows greater flexibility at this key position while maintaining the overall continuity of the curve. Using a 4th-order spline function ensures that the fitted curve has third-order continuous differentiability (C^3 continuity), effectively capturing subtle changes in the implied volatility curve while avoiding over-fitting problems.

The mathematical representation of the 4th-order spline function is as follows:

$$S(x) = \begin{cases} \sum_{i=0}^{4} a_i\,(x - x_0)^i, & x < k \\ \sum_{i=0}^{4} b_i\,(x - x_0)^i, & x \geq k \end{cases}$$

, where k is the knot position (i.e., the futures price), x_0 is the reference point, and a_i and b_i are coefficients to be determined. At the knot, the function must satisfy the following continuity conditions:

$$\begin{cases} \sum_{i=0}^{4} a_i(k - x_0)^i = \sum_{i=0}^{4} b_i(k - x_0)^i \\ \sum_{i=1}^{4} i\,a_i(k - x_0)^{i-1} = \sum_{i=1}^{4} i\,b_i(k - x_0)^{i-1} \\ \sum_{i=2}^{4} i(i-1)a_i(k - x_0)^{i-2} = \sum_{i=2}^{4} i(i-1)b_i(k - x_0)^{i-2} \\ \sum_{i=3}^{4} i(i-1)(i-2)a_i(k - x_0)^{i-3} = \sum_{i=3}^{4} i(i-1)(i-2)b_i(k - x_0)^{i-3} \end{cases}$$

These conditions ensure the continuity of the function value and its first, second, and third derivatives at the knot.

Setting the sole knot at the futures price has important economic significance, as this position typically corresponds to at-the-money options. This knot placement divides the curve into two segments, corresponding to areas above and below the futures price, allowing the fitted curve to more accurately reflect volatility near the at-the-money point. This segmented fitting method is particularly suitable for handling the asymmetric features that may appear in option implied volatility before and after the at-the-money position.

At the implementation level, this research uses the LSQUnivariateSpline method from Python's SciPy package for curve fitting. This method employs the least squares approach for parameter estimation, effectively handling non-uniformly distributed data points and achieving segmented fitting through specified internal knots. Through the combination of the least squared method and knot placement, LSQUnivariateSpline provides a flexible and stable mathematical tool capable of fitting smooth and accurate implied volatility curves in non-uniformly distributed data (as shown in Figure 4-4), providing a solid foundation for subsequent RND extractions. The least squares method formula is as follows:

$$\min_{\{a_i\},\{b_i\}} \sum_{j=1}^{n} [y_j - S(x_j)]^2$$

, where y_j is the actual observed value, $S(x_j)$ is the observed value of the spline function at x_j, $\{a_i\},\{b_i\}$ is the set of parameters to be estimated for the spline function, and n is the total number of data points.

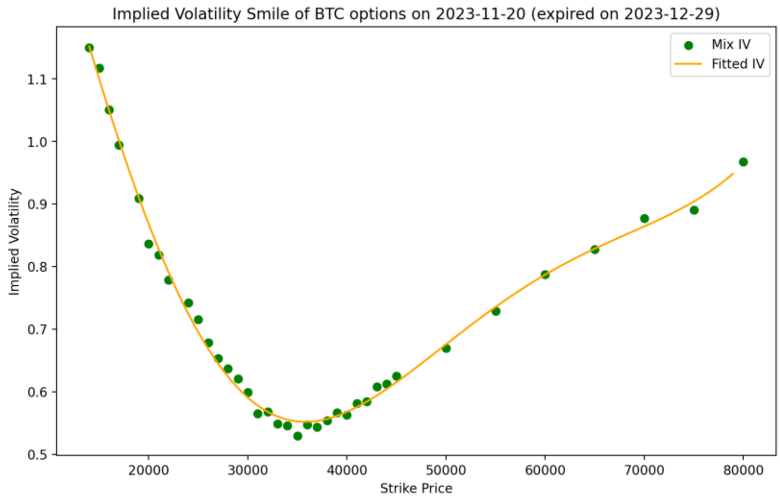

Figure 4-4: Bitcoin Option Implied Volatility Fitted Curve on November 20, 2023 (Expiring on December 29, 2023)

4.2.4 Extracting the Empirical RND from Bitcoin Options

After completing the fitting of the implied volatility curve, the derivation of the risk-neutral probability density follows. First, this research uses the fitted implied volatility curve, combined with the pricing model adopted by the Deribit exchange (Formula (7)), to calculate theoretical call option prices at different strike prices, as shown in Figure 4-5.

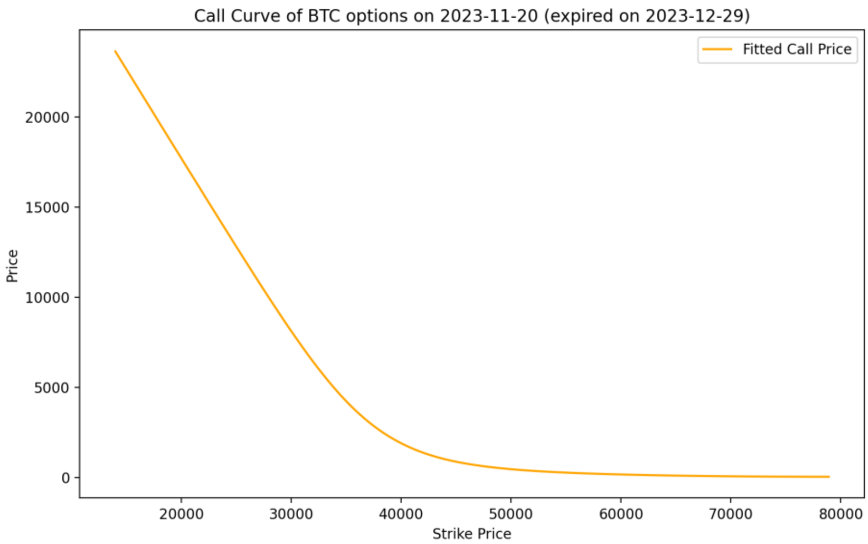

Figure 4-5: Bitcoin Option Theoretical Call Option Prices on November 20, 2023 (Expiring on December 29, 2023)

After obtaining the theoretical call option prices, this research employs the finite difference method (central difference method) for discrete data differentiation to derive the empirical RND. Compared to forward or backward differences, the central difference method can effectively reduce truncation errors (Formulas (4) and (5)). To ensure the stability and accuracy of numerical calculations, this research adopts an equidistant partitioning approach in setting the price spacing, with ΔK set to 0.1. Choosing a smaller price spacing not only provides more refined density estimation, but the equidistant partitioning approach also helps improve the stability of numerical differentiation calculations.

After completing extracting the empirical RND, to ensure the reliability and reasonableness of the data, this research focuses on the empirical CDF for data validation. In terms of data integrity validation, we first ensure that empirical CDF value does not contain missing values, with observations containing missing values excluded from the analysis scope. Furthermore, to maintain theoretical consistency of extraction, this research further restricts these two probability values to be strictly between 0 and 1, while excluding boundary values equal to 0 or 1, to avoid extreme cases affecting subsequent analysis.

$$\begin{cases} F(K)\ exists\ (non-missing) \\ F_R(K)\ exists\ (non-missing) \\ 0 < F(K) < 1 \\ 0 < F_R(K) < 1 \end{cases}$$

, where $F(K)$ is the cumulative distribution function, and $F_R(K) = 1 - F(K)$ 。

The validation criteria are based on three aspects of consideration. First, from the perspective of theoretical consistency, these criteria ensure that the estimation conforms to the basic properties of CDF in probability theory, while also satisfying the basic requirements of PDF. Second, in terms of numerical stability, this filtering approach can avoid computational problems in subsequent analyses due to extreme or abnormal values, effectively improving the reliability of the overall estimation. Finally, from a practical application perspective, removing abnormal values that might lead to misinterpretation better ensures that results accurately reflect market participants' true price expectations.

The derive empirical RND and its CDF are shown in Figure 4-6 and Figure 4-7:

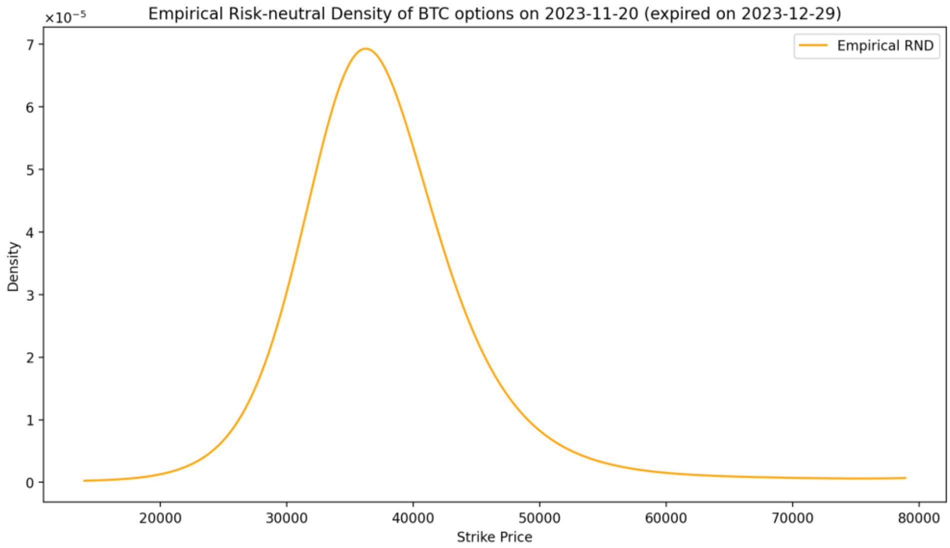

Figure 4-6: Bitcoin Option Empirical RND on November 20, 2023 (Expiring on December 29, 2023)

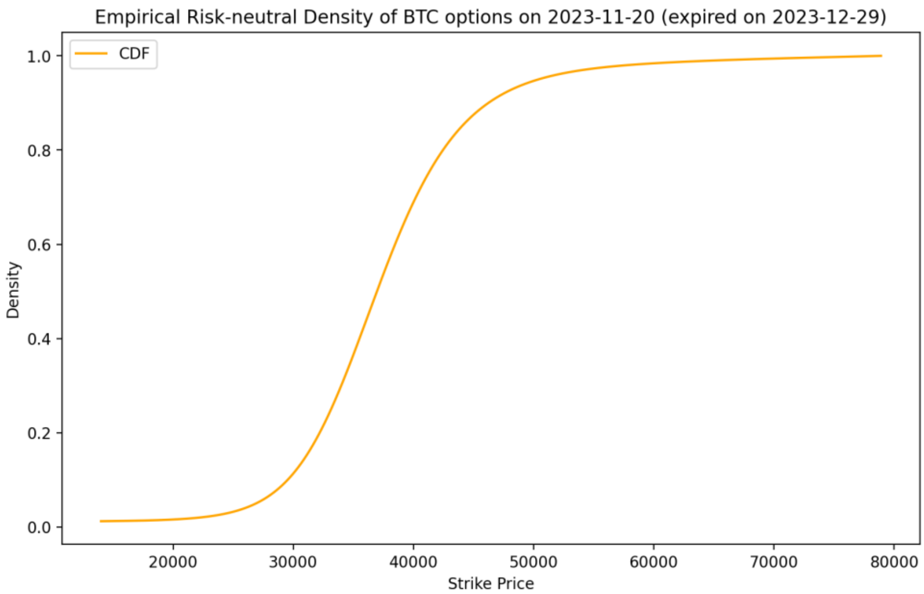

Figure 4-7: CDF of Bitcoin Option Empirical RND on November 20, 2023 (Expiring on December 29, 2023)

4.3 Fitting the Tails of the Empirical RND

4.3.1 Fitting Method with GEV

The Empirical RND extracted from market option prices can only cover the range of effective trading strike prices. To fully describe market expectations, the tails of the empirical RND need to be extended. Figlewski (2008) proposed using the Generalized Extreme Value Distribution (GEV) to fit the tails of the empirical RND. This method requires setting three conditions for each tail to ensure the continuity of tail fitting:

$$\text{Right tail conditions: } \begin{cases} F_{GEVR}\big(K(\alpha_{1R})\big) = \alpha_{1R} \\ f_{GEVR}\big(K(\alpha_{1R})\big) = f_{EMP}\big(K(\alpha_{1R})\big) \\ f_{GEVR}\big(K(\alpha_{2R})\big) = f_{EMP}\big(K(\alpha_{2R})\big) \end{cases}$$

$$\text{Left tail conditions: } \begin{cases} F_{GEVL}\big(-K(\alpha_{1L})\big) = 1 - \alpha_{1L} \\ f_{GEVR}\big(-K(\alpha_{1L})\big) = f_{EMP}\big(K(\alpha_{1L})\big) \\ f_{GEVR}\big(-K(\alpha_{2L})\big) = f_{EMP}\big(K(\alpha_{2L})\big) \end{cases}$$

, where F_{GEVR} and f_{GEVR} are the CDF and PDF of the right tail GEV, respectively, F_{GEVL} is the CDF of the left tail GEV, f_{EMP} is the RND function derived in this research, and $K(\alpha)$ is the strike price corresponding to the α quantile of the Empirical RND. The fitting conditions from Figlewski (2008) can be summarized as:

(1) At the first joining point, the CDF of the GEV tail and the CDF of the empirical RND must be equal

(2) At the first joining point, the GEV density function value and the density function value of the empirical RND must be equal

(3) At the second joining point, the GEV density function value and the density function value of the empirical RND must be equal

4.3.2 GPD Distribution Theory

This research adopts the Generalized Pareto Distribution (GPD) for tail fitting, based on Balkema and Haan's (1974) proof that observations exceeding a high threshold asymptotically converge to GPD. GPD requires only two parameters (scale σ and shape ξ), enhancing computational efficiency and reducing overfitting risk compared to GEV's three parameters. Research by Hosking and Wallis (1987), McNeil and Frey (2000), and Birru and Figlewski (2012) confirmed GPD's advantages in financial applications.

The mathematical expression of the GPD's CDF is as follows:

$$F_{GPD}(x;\sigma,\xi) = \begin{cases} 1-(1+\xi\dfrac{x}{\sigma})^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1-exp\,(-\dfrac{x}{\sigma}), & \xi = 0 \end{cases}$$

The mathematical expression of the GPD's PDF is as follows:

$$f_{GPD}(x;\sigma,\xi) = \begin{cases} \dfrac{1}{\sigma}(1+\xi\dfrac{x}{\sigma})^{-\frac{1}{\xi}-1}, & \xi \neq 0 \\ \dfrac{1}{\sigma}exp\,(-\dfrac{x}{\sigma}), & \xi = 0 \end{cases}$$

, where $\sigma > 0$ is the scale parameter, used to control the degree of dispersion of the distribution. A larger σ value indicates a greater variability. The shape parameter ξ determines the type and tail of the distribution:

(1) When $\xi > 0$: The distribution is a Pareto Distribution with a heavy tail; the distribution has infinite support, with domain $[0,\infty)$; the tail decays more slowly.

(2) When $\xi = 0$: The distribution degenerates to an Exponential-type Distribution with a fixed decay rate; it is the simplest continuous memoryless distribution; the tail decays at a moderate speed.

(3) When $\xi < 0$: The distribution is defined with finite upper bound on the interval $\left[0, -\frac{\sigma}{\xi}\right)$; it is less commonly used in financial market applications because asset returns typically do not have a clear upper limit

4.3.3 Fitting Method with GPD

Birru and Figlewski (2012) proposed a fitting method for GPD tails, using two joining points to compare the density function values of GPD and the empirical RND. The tail fitting conditions are set as follows:

(1) At the first joining point, the GPD density function value and the empirical RND density function value must be equal

(2) At the second joining point, the GPD density function value and the empirical RND density function value must be equal

The mathematical expressions are as follows:

Right tail conditions:

$$\begin{cases} f_{GPD}(K(\alpha_{1R})) = f_{EMP}(K(\alpha_{1R})) & (PDF\ condition) \\ f_{GPD}(K(\alpha_{2R})) = f_{EMP}(K(\alpha_{2R})) & (PDF\ condition) \end{cases}$$

Left tail conditions:

$$\begin{cases} f_{GPD}(K(\alpha_{1L})) = f_{EMP}(K(\alpha_{1L})) & (PDF\ condition) \\ f_{GPD}(K(\alpha_{2L})) = f_{EMP}(K(\alpha_{2L})) & (PDF\ condition) \end{cases}$$

The scale parameter and shape parameter of GPD are solved by minimizing the following objective functions:

Right tail parameter minimization objective function:

$$\min_{\sigma,\xi}\{[f_{GPD}(K(\alpha_{1R})) - f_{EMP}(K(\alpha_{1R}))]^2 + [f_{GPD}(K(\alpha_{2R})) - f_{EMP}(K(\alpha_{2R}))]^2\}$$

Left tail parameter minimization objective function:

$$\min_{\sigma,\xi}\{[f_{GPD}(K(\alpha_{1L}))-f_{EMP}(K(\alpha_{1L}))]^2+[f_{GPD}(K(\alpha_{2L}))-f_{EMP}(K(\alpha_{2L}))]^2\}$$

, where α_{1L} is preset to 0.05; α_{2L} is preset to 0.02; α_{1R} is preset to 0.95; α_{2R} is preset to 0.98. The empirical RND curve and its CDF completed using Birru-Figlewski method are shown in Figure 4-8 and Figure 4-9.

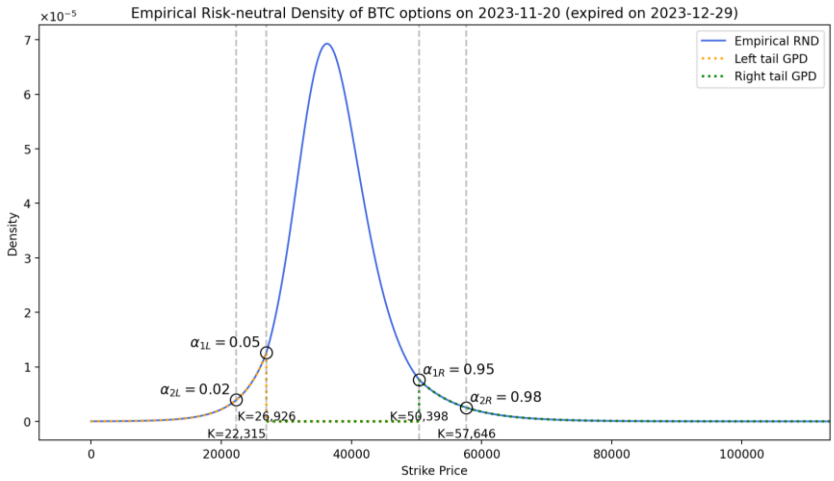

Figure 4-8: Bitcoin Option Empirical RND and GPD Tail Fitting on November 20, 2023 (Birru-Figlewski method)

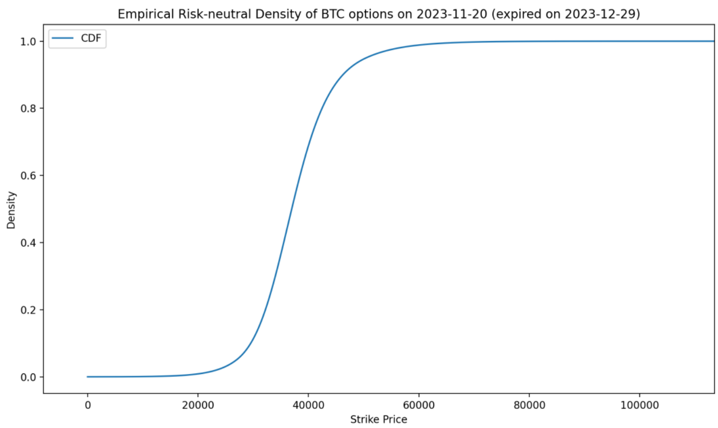

Figure 4-9: CDF of Bitcoin Option RND on November 20, 2023 (Birru-Figlewski method)

4.3.4 Fitting Method with GPD Using the Proposed Method

This research proposes fitting the empirical RND using the proposed method with GPD. The proposed method not only considers the fitting of CDF values but also adds continuity conditions for the slope of the density function. The main advantage of this method is that it can simultaneously ensure the continuity and smoothness of the density function while simplifying the fitting process and improving computational efficiency. The tail fitting conditions set in this research are as follows:

(1) At the joining point, the CDF of GPD and the CDF of the empirical RND must be equal.

(2) At the joining point, the slope of the GPD density function and the slope of the empirical RND density function must be equal.

The mathematical expressions are as follows:

Right tail conditions:

$$\begin{cases} F_{GPD}(K(\alpha_{1R})) = \alpha_{1R} & (CDF\ condition) \\ \dfrac{f_{GPD}(K(\alpha_{1R}) + \Delta x) - f_{GPD}(K(\alpha_{1R}))}{\Delta x} = \dfrac{f_{EMP}(K(\alpha_{1R}) + \Delta x) - f_{EMP}(K(\alpha_{1R}))}{\Delta x} & (Slope\ condition) \end{cases}$$

Left tail conditions:

$$\begin{cases} F_{GPD}(-K(\alpha_{1L})) = \alpha_{1L} & (CDF\ condition) \\ \dfrac{f_{GPD}(-K(\alpha_{1L}) + \Delta x) - f_{GPD}(-K(\alpha_{1L}))}{\Delta x} = \dfrac{f_{EMP}(K(\alpha_{1L}) + \Delta x) - f_{EMP}(K(\alpha_{1L}))}{\Delta x} & (Slope\ condition) \end{cases}$$

The scale parameter and shape parameter of GPD are solved by minimizing the following objective functions:

Right tail parameter minimization objective function:

$$\min_{\sigma,\xi}\left\{\left[F_{GPD}\big(K(\alpha_{1R})\big)-\alpha_{1R}\right]^2 + \left[\frac{f_{GPD}(K(\alpha_{1R})+\Delta x)-f_{GPD}(K(\alpha_{1R}))}{\Delta x} - \frac{f_{EMP}(K(\alpha_{1R})+\Delta x)-f_{EMP}(K(\alpha_{1R}))}{\Delta x}\right]^2\right\}$$

Left tail parameter minimization objective function:

$$\min_{\sigma,\xi}\left\{[F_{GPD}(-K(\alpha_{1L}))-\alpha_{1L}]^2 + \left[\frac{f_{GPD}(-K(\alpha_{1L})+\Delta x)-f_{GPD}(-K(\alpha_{1L}))}{\Delta x} - \frac{f_{EMP}(K(\alpha_{1L})+\Delta x)-f_{EMP}(K(\alpha_{1L}))}{\Delta x}\right]^2\right\}$$

, where Δx is a fixed constant value used to construct artificially spaced option prices, preset to 0.1; α_{1L} is preset to 0.05; α_{1R} is preset to 0.95.

This research performs fitting separately for the left and right tails. For the left tail, we select the point with a cumulative probability of 5% as the joining point; for the right tail, we select the point with a cumulative probability of 95% as the joining point. This design ensures the smoothness of tail fitting while maintaining the continuity of the overall distribution. To minimize fitting errors, this research adopts the least squares method for parameter estimation, solving for the scale parameter and shape parameter of GPD through numerical optimization methods. The completed RND curve and its CDF are shown in Figures 4-10 and 4-11.

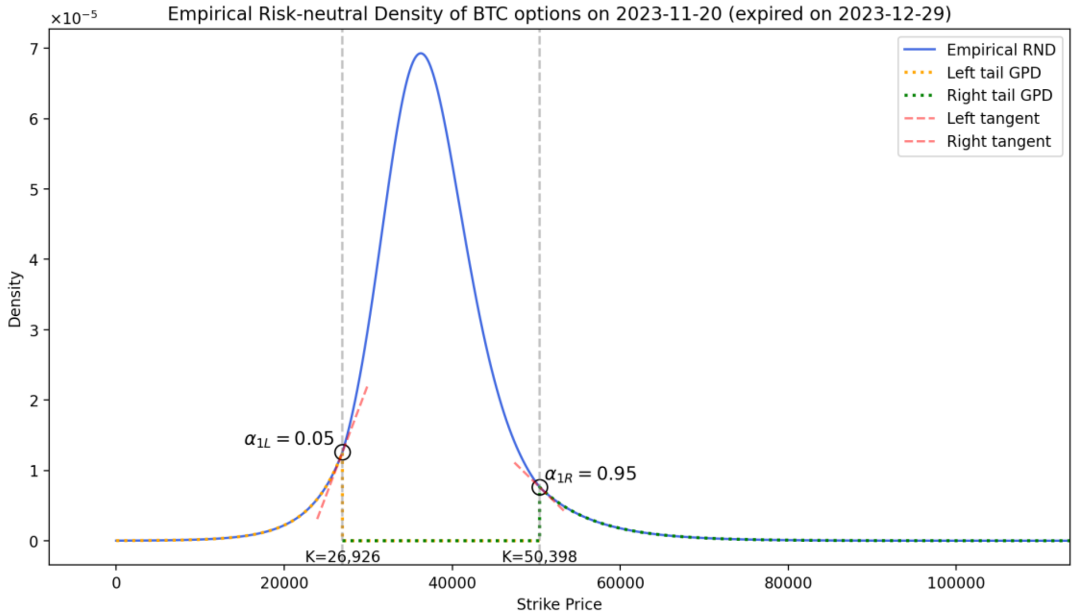

Figure 4-10: Bitcoin Option Empirical RND and GPD Tail Fitting on November 20, 2023 (The Proposed Method)

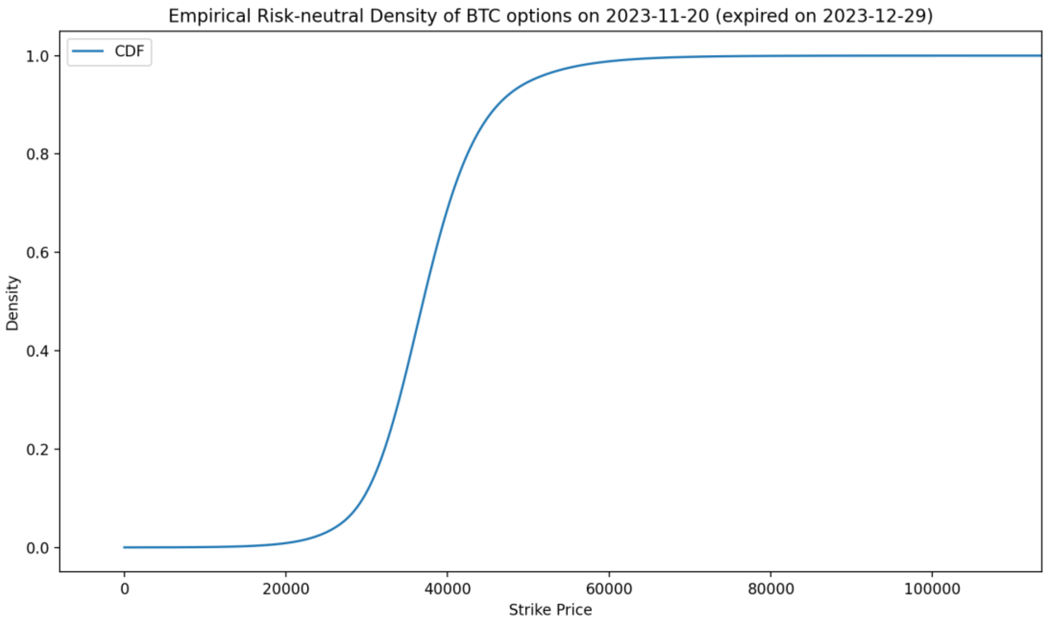

Figure 4-11: CDF of Bitcoin Option RND on November 20, 2023 (The Proposed Method)

4.4 Deriving the Moments of the RND

4.4.1 Definition and Implications of Moments

Moments are important statistical measures for describing the characteristics of probability distributions and can be divided into raw moments and central moments. For discrete strike prices, the nth-order raw moment is defined as:

$$m'_n = E[K^n] = \sum_{i=1}^{J} K_i^n f(K_i) \Delta K$$

And the nth-order central moment is defined as:

$$m_n = E[(K - \bar{K})^n] = \sum_{i=1}^{J} (K_i - \bar{K})^n f(K_i) \Delta K$$

, where $f(K_i)$ is the RND; ΔK is the price interval, set to 0.1; $\bar{K}$ is the expected value, i.e., the first-order raw moment m'_1; J is the total number of RND observations.

4.4.2 Moments of the RND

Recent research shows that option-implied moments not only effectively describe market expectations but also possess significant predictive power. Chang et al. (2013) found that risk-neutral skewness can effectively predict stock returns, especially during periods of high market volatility; while Neumann and Skiadopoulos (2013) pointed out that changes in risk-neutral kurtosis often lead market trends, providing important signals for investment decisions. This research computes the following four main moments:

1. Mean

The mean is the first-order raw moment, reflecting the overall market expectation for the future price level of the underlying asset. Bali and Murray (2013) pointed out that under risk-neutral pricing theory, the mean of the RND should equal the forward price discounted by the risk-free rate, providing an important arbitrage constraint. The formula is as follows:

$$\bar{K} = m_1' = E[K] = \sum_{i=1}^{J} K_i f(K_i)\,\Delta K$$

2. Standard Deviation

The standard deviation is the square root of the second-order central moment, measuring the degree of price dispersion. Christoffersen et al. (2013) found that option-implied standard deviation has stronger predictive power than historical volatility, especially in emerging markets. Standard deviation reflects market expectations for price volatility, with higher values indicating greater uncertainty among market participants regarding future price movements. The formula is as follows:

$$\sigma = \sqrt{m_2} = \sqrt{E[(K - \bar{K})^2]} = \sqrt{\sum_{i=1}^{J} (K_i - \bar{K})^2 f(K_i) \Delta K}$$

3. Skewness

Skewness is the standardized third-order central moment, describing the asymmetry of the distribution:

$$Skewness = \frac{m_3}{\sigma^3} = \frac{E[(K - \bar{K})^3]}{\sigma^3} = \frac{\sum_{i=1}^{J} (K_i - \bar{K})^3 f(K_i) \Delta K}{\sigma^3}$$

The skewness coefficient has important implications in financial markets. When positive skewness is observed, it indicates that the price distribution has a longer right tail, implying that market participants expect a higher probability of significant upward movements, reflecting overall optimistic market sentiment. Conversely, negative skewness indicates that the price distribution has a longer left tail, representing a greater perceived downside risk in the market, usually reflecting higher hedging demand among market participants. Research by Conrad et al. (2013) showed that risk-neutral skewness not only reflects market sentiment but also contains investors' expectations for extreme events. Li et al. (2024) showed that the dynamic changes in the skewness coefficient can often serve as a leading indicator of market sentiment shifts, with its changing trends providing important reference value for investment decisions.

4. Excess Kurtosis

Excess Kurtosis is the standardized fourth-order central moment minus 3 (the kurtosis value of the normal distribution), used to describe the tail of the distribution. It is calculated as:

$$Excess\ Kurtosis = \frac{m_4}{\sigma^4} - 3 = \frac{E[(K-\bar{K})^4]}{\sigma^4} - 3 = \frac{\sum_{i=1}^{J}(K_i-\bar{K})^4 f(K_i)\Delta K}{\sigma^4} - 3$$

In practical applications, excess kurtosis is an important indicator for assessing extreme market risk. Amaya et al. (2015) pointed out that excess kurtosis can effectively capture extreme market risk, with its predictive power being particularly significant during financial crises. When positive excess kurtosis is observed, it indicates that the distribution has more pronounced fat tails compared to the normal distribution, meaning that the probability of extreme events occurring is higher than expected under a normal distribution.

4.4.3 Market Implications of the Moments

These moments provide rich market information beyond mathematical description, including price expectations, risk assessment, and market sentiment indicators. This research employs regression analysis to empirically test their predictive power for spot returns.

4.5 Regression Analysis

4.5.1 Theoretical Background for Regression Models

To explore the predictive power of Bitcoin option-implied RND for underlying asset price movements, this research adopts multiple regression analysis for testing. The dependent variable (Y) is set as the logarithmic return of Bitcoin, while the independent variables (X) gradually incorporate moments of the RND such as Mean, Standard Deviation, Skewness, Excess Kurtosis, Median, as well as market sentiment indicators such as the Cryptocurrency Fear and Greed Index and the Chicago Board Options Exchange Volatility Index (VIX), along with historical returns from the previous 1 to 4 periods, to construct the most explanatory predictive model.

Bali and Zhou (2016) demonstrated that moments of the RND, particularly skewness and kurtosis, effectively predict cross-sectional asset return variations, reflecting market participants' risk preferences and containing crucial pricing information. Amaya et al. (2015) further established that the RND excess kurtosis specifically excels in predicting extreme market risk, a finding particularly relevant for high-volatility markets such as cryptocurrencies.

López-Cabarcos et al. (2021) examined relationships between Bitcoin volatility, stock market performance, and investor sentiment, indicating that during market stability periods, VIX returns and investor sentiment significantly impact Bitcoin volatility. Akyildirim et al. (2020), utilizing high-frequency data, identified a time-varying positive correlation between cryptocurrencies and market panic indicators (VIX, VSTOXX), which intensifies during periods of financial market stress. M. He et al. (2023) employed the daily updated Cryptocurrency Fear and Greed Index as a predictor, demonstrating significant in-sample and out-of-sample predictive power for individual cryptocurrency and market index returns over one-day to one-week horizons.

Liu and Tsyvinski (2021) identified significant momentum effects in cryptocurrency markets, with Bitcoin's current period returns substantially predicting returns for the subsequent 1-6 days. Y. Li et al. (2021) documented a positive MAX momentum effect, where cryptocurrencies exhibiting higher extreme daily returns tend to generate superior future returns. Liu et al. (2023), applying machine learning methodologies to cryptocurrency return prediction, determined that previous 1-day returns possess the strongest predictive power, exceeding the combined effect of all other variables.

4.5.2 Regression Model Specification

Campbell and Thompson (2008) introduced "economic significance threshold," showing that by gradually introducing predictive variables and setting strict statistical significance standards, one can effectively distinguish variables with substantial predictive power. In Gu, Kelly, and Xiu (2020), a "staged variable introduction framework" was proposed specifically for high-dimensional data, which can alleviate overfitting problems compared to models that introduce all variables at once.

This research adopts a multi-level regression analysis, gradually expanding from univariate to four-variable models, to systematically explore the predictive power of various RND moments for future returns. The following details the design of regression models at each level:

1. Univariate Regression Model

The univariate regression model is mainly used to test the explanatory power of individual variables for future returns. Its basic form is:

$$R_t = \beta_0 + \beta_1 Variable_{i,t-1} + \varepsilon_t, i \in \{1, 2, \ldots, 11\}$$

, where $R_t = ln(\frac{Close_t}{Close_{t-1}})$ is the return of Bitcoin price in the next period (T). If the option sample expires in 7 days, then the return is calculated using the closing price from the current day to the closing price 7 days later (expiration date).

$Variable_i$ is sequentially replaced with the following variables for univariate regression analysis: Mean, Standard Deviation (Std), Skewness, Excess Kurtosis, Median, Cryptocurrency Fear and Greed Index, Chicago Board Options Exchange Volatility Index (VIX), T-1 Return, T-2 Return, T-3 Return, T-4 Return.

2. Bivariate Regression Model

Considering the importance of Skewness in option pricing theory, this research designs bivariate models with Skewness as a fixed factor. The model is set as follows:

$$R_t = \beta_0 + \beta_1 Skewness_{t-1} + \beta_2 Variable_{i,t-1} + \varepsilon_t, i \in \{1, 2, \ldots, 10\}$$

$Variable_i$ is sequentially replaced with the following variables, paired with Skewness for bivariate regression analysis: Mean, Standard Deviation (Std), Excess Kurtosis, Median, Cryptocurrency Fear and Greed Index, Chicago Board Options Exchange Volatility Index (VIX), T-1 Return, T-2 Return, T-3 Return, T-4 Return.

3. Three-Variable Regression Model

The three-variable model further incorporates Excess Kurtosis as a fixed factor, forming:

$$R_t = \beta_0 + \beta_1 Skewness_{t-1} + \beta_2 ExcessKurtosis_{t-1} + \beta_3 Variable_{i,t-1} + \varepsilon_t, i \in \{1, 2, \dots, 9\}$$

$Variable_i$ is sequentially replaced with the following variables, paired with Skewness and Excess Kurtosis for three-variable regression analysis: Mean, Standard Deviation (Std), Median, Cryptocurrency Fear and Greed Index, Chicago Board Options Exchange Volatility Index (VIX), T-1 Return, T-2 Return, T-3 Return, T-4 Return.

4. Four-Variable Regression Model

Building on the three-variable model, the four-variable model adds the Standard Deviation (Std) variable. The complete model is as follows:

$$R_t = \beta_0 + \beta_1 Skewness_{t-1} + \beta_2 ExcessKurtosis_{t-1} + \beta_3 Std_{t-1} + \beta_4 Variable_{i,t-1} + \varepsilon_t,$$

$$i \in 1,2, \dots, 8$$

$Variable_i$ is sequentially replaced with the following variables, paired with Skewness, Excess Kurtosis, and Standard Deviation for four-variable regression analysis: Mean, Median, Cryptocurrency Fear and Greed Index, Chicago Board Options Exchange Volatility Index (VIX), T-1 Return, T-2 Return, T-3 Return, T-4 Return.

4.5.3 Validation of Model Effectiveness

The proposed method, compared to Birru-Figlewski method, has the advantage of computational efficiency in practical applications. To objectively validate the effectiveness of this method, this research adopts three quantitative indicators for evaluation: Mean Squared Error (MSE), Out-of-sample R-squared (R_{OS}^2) (Campbell & Thompson, 2008; Welch & Goyal, 2008), and computational efficiency, to comprehensively compare the accuracy and practicality of predictive models constructed by the two methods.

1. Mean Squared Error (MSE) Calculation

Mean Squared Error is a commonly used indicator for evaluating the accuracy of predictive models (Orosi, 2015). Its calculation formula is as follows:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

, where y_i is the actual observed value (i.e., the actual Bitcoin return), $\hat{y}_i$ is the model's predicted value, and n is the sample size. A smaller MSE value indicates smaller prediction errors and higher prediction accuracy.

2. Out-of-sample R-squared (R_{OS}^2) Calculation

Referring to Campbell and Thompson (2008) and Welch and Goyal (2008), this research adopts out-of-sample R-squared (R_{OS}^2) to evaluate the predictive power of the model. R_{OS}^2 measures the improvement in prediction of the predictive model relative to the historical average benchmark model. Its calculation formula is as follows:

$$R_{OS}^2 = 1 - \frac{\sum_{t=s_0+1}^{T}(R_{t+1} - \hat{R}_{t+1})^2}{\sum_{t=s_0+1}^{T}(R_{t+1} - \bar{R}_{t+1})^2}$$

, where R_{t+1} is the actual return, $\hat{R}_{t+1}$ is the predicted value from the predictive model, $\bar{R}_{t+1}$ is the historical average return (benchmark model), s_0 is the initial in-sample period length, and T is the total sample size. When R_{OS}^2 is greater than zero, it indicates that the predictive model outperforms the historical average benchmark model;

the larger the R_{OS}^2 value, the more significant the improvement in prediction.

This research adopts a rolling window approach for out-of-sample prediction, with the initial in-sample period set to 80% of the total sample size, and gradually advancing forward for prediction. By comparing the R_{OS}^2 values of the proposed method and Birru-Figlewski method, the difference in out-of-sample predictive power between the two methods can be objectively evaluated.

3. Computational Efficiency Comparison

In addition to prediction accuracy, this research also values the practical applicability of the method, especially its computational efficiency when processing large amounts of data. To objectively evaluate the computational performance of the two methods, this research selects ten representative option expiration dates, derives the 7-day risk-neutral probability density for each expiration date, and uses both the proposed method and Birru-Figlewski method to fit the tails with the Generalized Pareto Distribution (GPD). To ensure the reliability and stability of the results, this research performs ten repeated computations for each method, records their execution times, and computes the average, thereby comprehensively evaluating the differences in computational efficiency between the two methods in practical applications.

5. Empirical Results

This chapter presents empirical comparisons between the proposed method and Birru-Figlewski method for RND tail fitting, analyzing fitting characteristics, computational efficiency, and predictive performance across different option expiration horizons.

Bitcoin's 24-hour trading environment, substantial liquidity, and efficient price discovery mechanism make it ideal for studying RND moments. The market's rapid information processing and technology-oriented trader base enable short-term options to effectively capture real-time risk assessments.

5.1 Analysis of Fitting Effects

5.1.1 Comparison between the Proposed Method and Birru-Figlewski method

Comparing samples from July 10, 2022 (expiring July 11, 2022) and September 27, 2023 (expiring September 28, 2023), the proposed method demonstrates better stability, with smoother and more continuous fitting curves (Figure 5-1 and Figure 5-2).

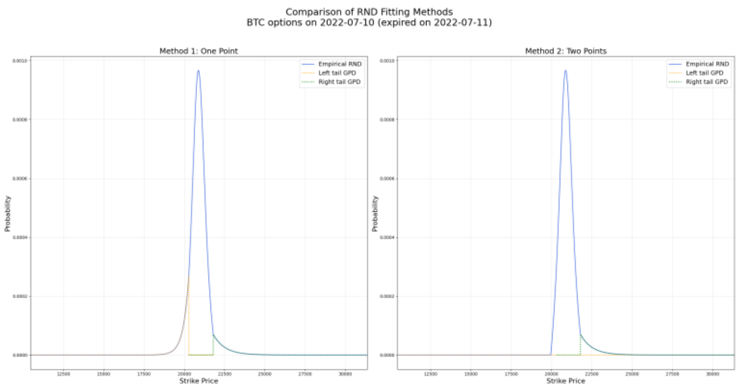

Figure 5-1: Comparison of Bitcoin Option GPD Tail Fitting on July 10, 2022
(Left: The proposed method; Right: Birru-Figlewski method)

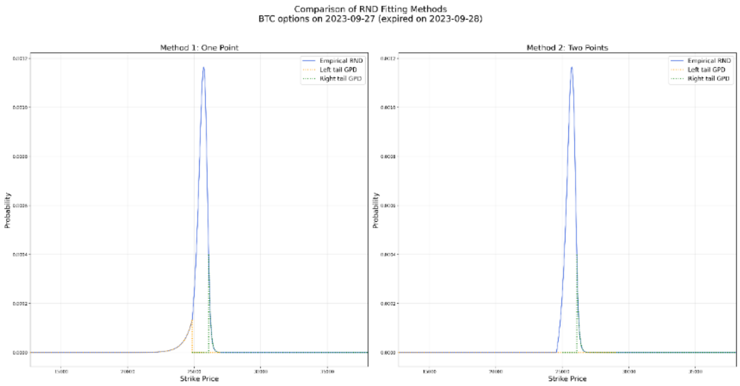

Figure 5-2: Comparison of Bitcoin Option GPD Tail Fitting on September 27, 2023

(Left: The proposed method; Right: Birru-Figlewski method)

The proposed method (left panels) reveals robust RND curve fitting across both samples, maintaining continuity at joining points and producing gradually decreasing tails that conform to probability density function properties. This demonstrates superior robustness in handling extreme values.

Conversely, while the Birru-Figlewski method (right panels) generally produces reasonable fitting curves, it occasionally exhibits fitting failures or discontinuities under certain market conditions. This limitation stems from simultaneous continuity requirements at two joining points, which becomes problematic during extreme market fluctuations or highly skewed price distributions.

The proposed method offers enhanced computational efficiency and reduced fitting failures, particularly valuable in large sample analyses.

5.1.2 Comparison of Computational Efficiency

Testing both methods across 10 option expiration dates (September-December 2023) using identical hardware configurations revealed significant performance differences (Table 5-1). The proposed method demonstrated superior performance, averaging 309.86 seconds execution time compared to Birru-Figlewski method 347.95 seconds, representing a 10.95% reduction in computation time.

This efficiency advantage stems from fundamental algorithmic differences. While Birru-Figlewski method requires simultaneous satisfaction of continuity conditions at two joining points, introducing additional optimization constraints, the proposed method addresses only a single joining point, enabling more direct optimization with faster parameter convergence. The proposed method also exhibits greater execution time stability (lower standard deviation), indicating more consistent computational performance.

Table 5-1: Comparison of Computational Efficiency for Bitcoin Option GPD Tail Fitting
(Left: The proposed method; Right: Birru-Figlewski method)

Execution Time:	2025/2/5 00:48		
Execution Conditions:	Each time generates 10 weekly return RNDs with GPD distribution tail fitting.		
Option Expiration Dates:	2023/9/22, 2023/9/29, 2023/10/13, 2023/10/20, 2023/10/27, 2023/11/10, 2023/11/17, 2023/11/24, 2023/12/15, 2023/12/22		
The proposed method		Birru-Figlewski method	
1st Execution Time (sec)	297.58	1st Execution Time (sec)	346.49
2nd Execution Time (sec)	300.23	2nd Execution Time (sec)	346.95
3rd Execution Time (sec)	313.52	3rd Execution Time (sec)	347.78
4th Execution Time (sec)	313.51	4th Execution Time (sec)	347.68
5th Execution Time (sec)	312.25	5th Execution Time (sec)	347.81
6th Execution Time (sec)	311.37	6th Execution Time (sec)	347.94
7th Execution Time (sec)	313.52	7th Execution Time (sec)	348.39
8th Execution Time (sec)	311.86	8th Execution Time (sec)	349.10
9th Execution Time (sec)	312.36	9th Execution Time (sec)	348.59
10th Execution Time	312.42	10th Execution Time	348.80
Shortest Execution Time (sec)	297.58	Shortest Execution Time (sec)	346.49
Longest Execution Time (sec)	313.52	Longest Execution Time (sec)	349.10
Average Execution Time (sec)	309.86	Average Execution Time (sec)	347.95

5.2 Regression Analysis with 1 Day to Expiration

5.2.1 Fitting Tails with GPDs Based on the Proposed Method

This section utilizes options expiring daily from January 10, 2021, to April 30, 2024, deriving the RNDs from observation dates one day prior to expiration using the proposed method. Table 5-2 presents descriptive statistics for the variables, comprising 824 observations.

Table 5-2: Descriptive Statistics of the RND Moments and Bitcoin Returns for Options with 1 Day to Expiration (The proposed method)

	Count	Mean	Std	Min	25%	Median	75%	Max
T Return (Y)	824	-0.0004	0.0336	-0.1670	-0.0157	-0.0003	0.0155	0.1353
Mean	824	36100.6584	13956.0790	278.8385	25143.0730	34347.8549	46114.0725	72614.4638
Std	824	1792.3028	1423.7678	249.9084	825.4532	1468.7422	2295.5535	12224.7221
Skewness	824	0.1108	1.5802	-6.0025	-0.7608	0.2488	1.0899	9.1946
Excess Kurtosis	824	3.6683	8.2169	-15.7904	-0.2814	1.6635	4.0938	82.1349
Median	824	37000.7834	14051.2206	7860.0000	25800.8750	35768.0000	46933.4000	72597.4000
Fear and Greed Index	824	46.9563	22.5256	6.0000	26.0000	49.0000	68.2500	95.0000
VIX	824	20.1261	5.2924	12.0700	16.2500	19.2600	23.0500	37.2100
T-1 Return	824	-0.0001	0.0337	-0.1670	-0.0155	-0.0002	0.0160	0.1353
T-2 Return	824	0.0000	0.0338	-0.1670	-0.0152	-0.0002	0.0161	0.1353
T-3 Return	824	-0.0001	0.0338	-0.1670	-0.0155	-0.0002	0.0161	0.1353
T-4 Return	824	-0.0003	0.0338	-0.1670	-0.0158	-0.0004	0.0160	0.1353

The univariate regression results presented in Table 5-3 demonstrate that Mean, Excess Kurtosis, Median, and T-4 Return achieve statistical significance at the 5%, 10%, 5%, and 10% levels, respectively. Mean and median exhibit negative relationships with subsequent returns, while Excess Kurtosis and T-4 return display a positive relationship. Notably, market sentiment indicators such as the Cryptocurrency Fear and Greed Index and the Volatility Index (VIX) demonstrate negligible predictive capacity, suggesting that technical indicators may possess greater value than sentiment metrics in this context.

Table 5-3: Univariate Regression Results for Options with 1 Day to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared
Mean	-0.0782	0.0247	**	0.0061
Std	-0.0223	0.5228		0.0005
Skewness	0.0190	0.5866		0.0004
Excess Kurtosis	0.0577	0.0978	*	0.0033
Median	-0.0733	0.0353	**	0.0054
Fear and Greed Index	-0.0070	0.8403		0.0000
VIX	0.0033	0.9246		0.0000
T-1 Return	-0.0356	0.3074		0.0013
T-2 Return	0.0296	0.3955		0.0009
T-3 Return	0.0109	0.7553		0.0001
T-4 Return	0.0612	0.0793	*	0.0037

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Table 5-4 presents bivariate regression results with skewness as an independent variable. The inclusion of Mean, Excess Kurtosis, Median, and T-4 Return variables provides stable predictive capability with enhanced explanatory power (higher R-squared values), indicating that RND median and short-term momentum effects contain significant predictive information for Bitcoin returns. Three-variable and four-variable regression analyses are documented in Appendix Table 1 and Appendix Table 2.

Table 5-4: Bivariate Regression Results for Options with 1 Day to Expiration (The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	R-squared
Mean	-0.0771	0.0281	**	0.0100	0.7744		0.0062
Std	-0.0238	0.4964		0.0207	0.5540		0.0009
Excess Kurtosis	0.0620	0.0788	*	0.0283	0.4216		0.0041
Median	-0.0722	0.0391	**	0.0126	0.7184		0.0055
Fear and Greed Index	-0.0075	0.8297		0.0192	0.5832		0.0004
VIX	0.0016	0.9631		0.0188	0.5911		0.0004
T-1 Return	-0.0337	0.3374		0.0147	0.6758		0.0015
T-2 Return	0.0301	0.3889		0.0196	0.5737		0.0013
T-3 Return	0.0099	0.7774		0.0184	0.5978		0.0005
T-4 Return	0.0613	0.0790	*	0.0193	0.5802		0.0041

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Following multi-level regression analysis, we adopt a three-variable regression model (Model 1, Table 5-5) incorporating Skewness, Excess Kurtosis, and T-4 return. The results indicate that Excess Kurtosis and T-4 return positively influence Bitcoin return prediction, both at the significant level of 10% while Skewness also has a positive influence yet has a p value of 0.4141. The model yields an R-squared value of 0.0080 and a Mean Squared Error (MSE) of 0.9908.

We further construct an alternative model (Model 2, Table 5-6) with Excess Kurtosis as an independent variable, incorporating Median and T-4 Return as control variables. The results reveal that Excess Kurtosis has a significant and positive relationship (0.0551, p = 0.1029) at the significant level of 10.29%, Median a significant and negative relationship (-0.0720, p = 0.0385) at the significant level of 5%, while T-4 return demonstrates a significant and positive relationship (0.0634, p=0.0677) at the significant level of 10%. This model achieves an R-squared value of 0.0123 and an MSE of 0.9865, indicating marginally superior predictive capacity compared to Model 1. Nevertheless, none of the added variables are statistically significant (Appendix Table 3 and Appendix Table 4), suggesting that this model has already demonstrated relatively stable predictive ability.

Table 5-5: Model 1 for Options with 1 Day to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared	MSE
Skewness	0.0287	0.4141		0.0080	0.9908
Excess Kurtosis	0.0628	0.0748	*		
T-4 Return	0.0620	0.0749	*		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Table 5-6: Model 2 for Options with 1 Day to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared	MSE
Excess Kurtosis	0.0541	0.1199		0.0123	0.9865
Median	-0.0716	0.0399	**		
T-4 Return	0.0635	0.0677	*		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

5.2.2 Fitting Tails with GPDs Based on Birru-Figlewski method

This section employs identical options data spanning January 10, 2021, to April 30, 2024, and constructs RND functions using the Birru-Figlewski method. Table 5-7 presents descriptive statistics for 824 observations.

Table 5-7: Descriptive Statistics of the RND Moments and Bitcoin Returns for Options with 1 Day to Expiration (Birru-Figlewski method)

	Count	Mean	Std	Min	25%	Median	75%	Max
T Return (Y)	824	-0.0004	0.0336	-0.1670	-0.0157	-0.0003	0.0155	0.1353
Mean	824	36063.4368	13913.2243	771.3413	25127.8634	34173.5420	46118.1417	72614.4638
Std	824	1737.0639	1175.6058	179.8905	857.0913	1503.7339	2281.9034	10729.4925
Skewness	824	0.3270	1.6594	-5.9903	-0.5630	0.4926	1.1707	16.9925
Excess Kurtosis	824	3.1693	8.0741	-46.5680	-0.7257	1.3681	3.6538	61.3908
Median	824	37023.8734	14018.8714	15890.6000	25841.0750	35768.0000	46933.4000	72597.4000
Fear and Greed Index	824	46.9854	22.5166	6.0000	26.0000	49.0000	68.2500	95.0000
VIX	824	20.1163	5.2975	12.0700	16.2350	19.2300	23.0500	37.2100
T-1 Return	824	-0.0001	0.0337	-0.1670	-0.0155	-0.0003	0.0160	0.1353
T-2 Return	824	0.0000	0.0338	-0.1670	-0.0152	-0.0002	0.0161	0.1353
T-3 Return	824	-0.0001	0.0338	-0.1670	-0.0155	-0.0002	0.0161	0.1353
T-4 Return	824	-0.0003	0.0338	-0.1670	-0.0158	-0.0004	0.0160	0.1353

Univariate regression results (Table 5-8) reveal that Mean, Median, and T-4 Return achieve statistical significance, similar to the proposed method. However, in bivariate regression analysis with skewness as an independent variable (Table 5-9), Skewness becomes insignificant across all model specifications. The results of the three-variable and four-variable regression analyses are shown in Appendix Table 5 and Appendix Table 6.

Table 5-8: Univariate Regression Results for Options with 1 Day to Expiration (Birru-Figlewski method)

	Coefficient	p value	Significance	R-squared
Mean	-0.0741	0.0334	**	0.0055
Std	-0.0177	0.6119		0.0003
Skewness	-0.0289	0.4079		0.0008
Excess Kurtosis	0.0372	0.2863		0.0014
Median	-0.0735	0.0348	**	0.0054
Fear and Greed Index	-0.0071	0.8392		0.0001
VIX	0.0034	0.9231		0.0000
T-1 Return	-0.0356	0.3075		0.0013
T-2 Return	0.0296	0.3957		0.0009
T-3 Return	0.0109	0.7557		0.0001
T-4 Return	0.0612	0.0793	*	0.0037

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Table 5-9: Bivariate Regression Results for Options with 1 Day to Expiration (Birru-Figlewski method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	R-squared
Mean	-0.0779	0.0262	**	-0.0368	0.2930		0.0068
Std	-0.0143	0.6842		-0.0271	0.4414		0.0010
Excess Kurtosis	0.0321	0.3708		-0.0212	0.5543		0.0018
Median	-0.0761	0.0294	**	-0.0345	0.3224		0.0066
Fear and Greed Index	-0.0057	0.8712		-0.0286	0.4133		0.0009
VIX	0.0063	0.8584		-0.0295	0.4005		0.0009
T-1 Return	-0.0423	0.2330		-0.0366	0.3018		0.0026
T-2 Return	0.0281	0.4219		-0.0273	0.4352		0.0016
T-3 Return	0.0106	0.7611		-0.0288	0.4096		0.0009
T-4 Return	0.0599	0.0862	*	-0.0259	0.4580		0.0044

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

For comparative purposes, we select Skewness, Excess Kurtosis, and T-4 Return as explanatory variables for Model 1 (Table 5-10). The results indicate that neither Skewness nor Excess Kurtosis achieves significance, with only T-4 Return significant at the significant level of 10%. The model yields an R-squared value of merely 0.0054 and an

MSE of 0.9934, demonstrating unstable predictive capacity.

An alternative model (Model 2, Table 5-11) with Excess Kurtosis as an independent variable, incorporating Median and T-4 Return as control variables shows that Median is significant at the significant level of 5% (-0.0735, p = 0.0348) and T-4 return at the significant level of 10% (0.0625, p = 0.0721), achieving an R-squared value of 0.0105 and an MSE of 0.9883.

Table 5-10: Model 1 for Options with 1 Day to Expiration (Birru-Figlewski method)

	Coefficient	p value	Significance	R-squared	MSE
Skewness	-0.0183	0.6112		0.0054	0.9934
Excess Kurtosis	0.0321	0.3703			
T-4 Return	0.0599	0.0863	*		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Table 5-11: Model 2 for Options with 1 Day to Expiration (Birru-Figlewski method)

	Coefficient	p value	Significance	R-squared	MSE
Excess Kurtosis	0.0331	0.3421		0.0105	0.9883
Median	-0.0735	0.0348	**		
T-4 Return	0.0626	0.0722	*		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

5.2.3 Comparison

In the three-variable model (Table 5-12), the proposed method demonstrates that Excess Kurtosis and T-4 Return both achieve at the significant level of 10%, while Skewness exhibits a positive coefficient with the p value of 0.41. In contrast, the Birru-Figlewski method yields significance only for T-4 Return.

The proposed method demonstrates superior explanatory power in Model 1 (R-squared = 0.0080 vs. 0.0054) and lower MSE (0.9908 vs. 0.9934). Out-of-sample R-squared values further confirm this advantage (0.0004 vs. 0.0002). In Model 2, where

Median replaces Skewness, the proposed method's superiority becomes even more evident (R-squared = 0.0123 vs. 0.0105; MSE = 0.9865 vs. 0.9883; Out-of-sample R-squared = 0.0007 vs. 0.0003).

Table 5-12: Comparison of Regression Results for Options with 1 Day to Expiration
(Left: The proposed method; Right: Birru-Figlewski method)

	The proposed method						**Birru-Figlewski method**					
	Coef.	p value	Sig.	R-squared	MSE	R^2_{OS}	Coef.	p value	Sig.	R-squared	MSE	R^2_{OS}
Model 1												
Skewness	0.0287	0.4141		0.0080	0.9908	0.0004	-0.0183	0.6112		0.0054	0.9934	0.0002
Excess Kurtosis	0.0628	0.0748	*				0.0321	0.3703				
T-4 Return	0.0620	0.0749	*				0.0599	0.0863	*			
Model 2												
Excess Kurtosis	0.0541	0.1199		0.0123	0.9865	0.0007	0.0331	0.3421		0.0105	0.9883	0.0003
Median	-0.0716	0.0399	**				-0.0735	0.0348	**			
T-4 Return	0.0635	0.0677	*				0.0626	0.0722	*			

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

5.3 Regression Analysis with 7 Days to Expiration

5.3.1 Fitting Tails with GPDs Based on the Proposed Method

This section uses options expiring daily from January 15, 2021, to April 19, 2024, deriving the RND from the observation date seven days before expiration, and constructs complete the RND functions using the proposed method. Moments such as mean, standard deviation, skewness, and kurtosis are then calculated as explanatory variables. Using the next period's Bitcoin spot return as the explained variable, multi-level regression analysis is conducted to observe whether the RND has predictive effects. The descriptive statistics of the variables are shown in Table 5-13, with a total of 119 samples. The means of Skewness and Excess Kurtosis are both greater than 0, and there are relatively few extreme values.

Table 5-13: Descriptive Statistics of the RND Moments and Bitcoin Returns for Options with 7 Days to Expiration (The proposed method)

	Count	Mean	Std	Min	25%	Median	75%	Max
T Return (Y)	119	-0.0070	0.0978	-0.3516	-0.0511	-0.0071	0.0423	0.3071
Mean	119	36529.4634	13781.4125	16591.1342	26070.6956	35496.4638	45453.7462	69393.9514
Std	119	3599.4222	2428.7791	710.3376	1727.0200	2868.8427	5085.2385	16566.9193
Skewness	119	0.0719	0.6877	-1.4832	-0.2484	0.0372	0.2935	4.2156
Excess Kurtosis	119	2.1573	2.9048	0.4791	1.2927	1.6335	1.9781	26.4692
Median	119	36497.0748	13738.8186	16678.0000	26066.9000	35379.1000	45497.1000	69137.5000
Fear and Greed Index	119	46.5378	22.4287	9.0000	25.0000	48.0000	69.0000	93.0000
VIX	119	20.0029	5.1284	12.2800	16.2950	18.8100	22.8100	32.0200
T-1 Return	119	-0.0060	0.0980	-0.3516	-0.0480	-0.0065	0.0463	0.3071
T-2 Return	119	-0.0061	0.0980	-0.3516	-0.0480	-0.0065	0.0463	0.3071
T-3 Return	119	-0.0055	0.0977	-0.3516	-0.0439	-0.0065	0.0463	0.3071
T-4 Return	119	-0.0055	0.0977	-0.3516	-0.0439	-0.0065	0.0463	0.3071

Following the regression analysis set in Section 4.5, the univariate regression results are shown in Table 5-14. It can be observed that Mean, Std, and Median are significant, while the individual predictive ability of market sentiment indicators such as the Cryptocurrency Fear and Greed Index and Volatility Index (VIX) remains extremely low.

Table 5-14: Univariate Regression Results for Options with 7 Days to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared
Mean	-0.1559	0.0904	*	0.0243
Std	-0.1547	0.0931	*	0.0239
Skewness	-0.0607	0.5122		0.0037
Excess Kurtosis	-0.1145	0.2148		0.0131
Median	-0.1553	0.0917	*	0.0241
Fear and Greed Index	0.0277	0.7648		0.0008
VIX	-0.0505	0.5858		0.0025
T-1 Return	-0.0398	0.6677		0.0016
T-2 Return	0.0126	0.8920		0.0002
T-3 Return	0.0043	0.9626		0.0000
T-4 Return	-0.0849	0.3587		0.0072

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

In conducting multiple regression analyses with two, three, and four variables, we found that most explanatory variables did not exhibit significant predictive effects, as documented in Appendix Table 7 to Appendix Table 9. This phenomenon indicates that merely increasing the number of variables cannot effectively enhance the model's predictive capability and may instead lead to overfitting problems.

After iteratively testing various variable combinations, this research discovered that when predicting Bitcoin weekly returns, the pairing of Excess Kurtosis and Median demonstrated superior predictive performance. Building on this foundation, we further incorporated market sentiment indicators by adding the Cryptocurrency Fear and Greed Index to the model, which exhibited significant predictive power. Through careful selection of variable combinations, rather than indiscriminately increasing the number of variables, this study ultimately identified a prediction model with both statistical significance and economic meaning. The regression results are presented in Table 5-15 and Table 5-16.

In Model 1, all three variables demonstrate statistical significance, with Excess Kurtosis and Fear and Greed Index at the significant level of 10%, and Median significant at the significant level of 5%. The model achieves an R-squared value of 0.0666 and MSE of 0.9256. Additionally, we constructed a second model replacing Excess Kurtosis with Skewness, as shown in Model 2. In Model 2, Skewness demonstrates a negative relationship with returns at the significant level of 12% level, while Median and Fear and Greed Index remain at the significant level of 5%. This model yields a slightly lower R-squared of 0.0616 and higher MSE of 0.9305 compared to Model 1, suggesting that Excess Kurtosis provides better predictive power than Skewness for weekly Bitcoin returns.

Table 5-15: Model 1 for Options with 7 Days to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared	MSE
Excess Kurtosis	-0.1620	0.0874	*	0.0666	0.9256
Median	-0.2706	0.0144	**		
Fear and Greed Index	0.2171	0.0561	*		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Table 5-16: Model 2 for Options with 7 Days to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared	MSE
Skewness	-0.1659	0.1284		0.0616	0.9305
Median	-0.2838	0.0120	**		
Fear and Greed Index	0.2693	0.0388	**		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

5.3.2 Fitting Tails with GPDs Based on Birru-Figlewski method

This section employs options contracts expiring daily between January 15, 2021, and April 19, 2024, using observations 7 days prior to expiration to derive the RND. We construct the complete RND function using Birru-Figlewski method, then calculate statistics including mean, standard deviation, skewness, and kurtosis as explanatory variables. Using subsequent Bitcoin spot returns as the dependent variable, we conduct multi-level regression analyses to examine whether the RND possesses predictive power. The descriptive statistics of the variables are presented in Table 5-17, with a total sample size of 119. Both Skewness and Excess Kurtosis have means greater than 0, and the descriptive statistics are highly similar to those of the proposed method, indicating that in weekly returns, the two methods do not exhibit substantial differences.

Table 5-17: Descriptive Statistics of the RND Moments and Bitcoin Returns for Options with 7 Days to Expiration (Birru-Figlewski method)

	Count	Mean	Std	Min	25%	Median	75%	Max
T Return (Y)	119	-0.0070	0.0978	-0.3516	-0.0511	-0.0071	0.0423	0.3071
Mean	119	36529.5600	13781.3082	16592.5497	26071.3029	35496.4638	45453.7462	69393.9514
Std	119	3598.8321	2429.3086	708.7932	1725.0112	2868.8427	5085.2385	16566.9193
Skewness	119	0.0723	0.6879	-1.4832	-0.2469	0.0372	0.2912	4.2156
Excess Kurtosis	119	2.1404	2.9086	0.4327	1.2819	1.6078	1.9556	26.4692
Median	119	36497.0748	13738.8186	16678.0000	26066.9000	35379.1000	45497.1000	69137.5000
Fear and Greed Index	119	46.5378	22.4287	9.0000	25.0000	48.0000	69.0000	93.0000
VIX	119	20.0029	5.1284	12.2800	16.2950	18.8100	22.8100	32.0200
T-1 Return	119	-0.0060	0.0980	-0.3516	-0.0480	-0.0065	0.0463	0.3071
T-2 Return	119	-0.0061	0.0980	-0.3516	-0.0480	-0.0065	0.0463	0.3071
T-3 Return	119	-0.0055	0.0977	-0.3516	-0.0439	-0.0065	0.0463	0.3071
T-4 Return	119	-0.0055	0.0977	-0.3516	-0.0439	-0.0065	0.0463	0.3071

Following the regression analysis specified in Section 4.5, the univariate regression results are presented in Table 5-18. We observe that Mean, Standard Deviation, and Median exhibit statistical significance.

Table 5-18: Univariate Regression Results for Options with 7 Days to Expiration (The proposed method)

	Coefficient	p value	Significance	R-squared
Mean	-0.1559	0.0904	*	0.0243
Std	-0.1547	0.0930	*	0.0239
Skewness	-0.0608	0.5115		0.0037
Excess Kurtosis	-0.1157	0.2100		0.0134
Median	-0.1553	0.0917	*	0.0241
Fear and Greed Index	0.0277	0.7648		0.0008
VIX	-0.0505	0.5858		0.0025
T-1 Return	-0.0398	0.6677		0.0016
T-2 Return	0.0126	0.8920		0.0002
T-3 Return	0.0043	0.9626		0.0000
T-4 Return	-0.0849	0.3587		0.0072

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Birru-Figlewski method, when conducting multiple regression analyses with two, three, and four variables, encounters the same issues as the proposed method, with most explanatory variables lacking significant predictive effects. These results are documented in Appendix Table 10 to Appendix Table 12.

To facilitate comparison with the regression model constructed using the proposed method, this section also selects Excess Kurtosis, Median, and the Cryptocurrency Fear and Greed Index as explanatory variables for Model 1, and Skewness, Median, and the Cryptocurrency Fear and Greed Index for Model 2. The model results are presented in Table 5-19, with variable significance and model predictive capability closely resembling those of the proposed method model.

Table 5-19: Model 1 for Options with 7 Days to Expiration (Birru-Figlewski method)

	Coefficient	p value	Significance	R-squared	MSE
Excess Kurtosis	-0.1620	0.0874	*	0.0666	0.9256
Median	-0.2706	0.0144	**		
Fear and Greed Index	0.2171	0.0561	*		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Table 5-20: Model 2 for Options with 7 Days to Expiration (Birru-Figlewski method)

	Coefficient	p value	Significance	R-squared	MSE
Skewness	-0.1662	0.1277		0.0617	0.9304
Median	-0.2840	0.0119	**		
Fear and Greed Index	0.2696	0.0386	**		

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

5.3.3 Comparison

For 7-day expiration options, both methods identify statistically significant variable combinations with remarkably similar RND moments. Both methods select Excess Kurtosis, Median, and the Cryptocurrency Fear and Greed Index as optimal predictors, with identical explanatory power (R-squared = 0.0666) and prediction error (MSE = 0.9256), as summarized in Table 5-21

The significant predictive power of Excess Kurtosis aligns with Amaya et al. (2015), who noted that excess kurtosis effectively captures extreme market risks. The Cryptocurrency Fear and Greed Index's significant predictive power in weekly returns is consistent with He et al. (2023) and López-Cabarcos et al. (2021). The negative relationship between Skewness and returns aligns with the existing literature (Bali & Murray, 2013; Conrad et al., 2013; Cujean & Hasler, 2017; Kim & Park, 2018; Y. Li et al., 2024).

Out-of-sample R-squared values are positive and substantial for both methods (0.3342 for the proposed method vs. 0.3335 for Birru-Figlewski method), outperforming historical average benchmark models.

Table 5-21: Comparison of Regression Results for Options with 7 Days to Expiration
(Left: The proposed method; Right: Birru-Figlewski method)

	The proposed method						**Birru-Figlewski method**					
	Coef.	p value	Sig.	R-squared	MSE	R^2_{OS}	Coef.	p value	Sig.	R-squared	MSE	R^2_{OS}
Model 1												
Excess Kurtosis	-0.1620	0.0874	*	0.0666	0.9256	0.3342	-0.1620	0.0874	*	0.0666	0.9256	0.3335
Median	-0.2706	0.0144	**				-0.2706	0.0144	**			
Fear and Greed Index	0.2171	0.0561	*				0.2171	0.0561	*			
Model 2												
Skewness	-0.1659	0.1284		0.0616	0.9305	0.1712	-0.1662	0.1277		0.0617	0.9304	0.1704
Median	-0.2838	0.0120	**				-0.2840	0.0119	**			
Fear and Greed Index	0.2693	0.0388	**				0.2696	0.0386	**			

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

5.4 Summary

In daily return prediction, the proposed method statistically outperforms Birru-Figlewski method in variable significance and explanatory power. In the three-variable model, the proposed method yields statistically significant coefficients for both Excess Kurtosis and T-4 Return at the significant level of 10% while Birru-Figlewski method produces significance only for T-4 Return. In weekly return prediction, the performances of the two methods are comparable, possibly reflecting diminished impact of tail fitting methods over longer horizons.

Out-of-sample prediction results demonstrate positive R-squared values for both methods across both timeframes, with the proposed method generally yielding higher values. Regarding computational efficiency, the proposed method achieves a 10.95 % reduction in execution time and exhibits lower variance in runtime.

Different moments of risk-neutral probability density functions reveal varying predictive capabilities across timeframes. In daily return prediction, Excess Kurtosis and historical returns demonstrate stronger predictive power, with Excess Kurtosis showing a positive relationship with returns that contradicts findings in traditional markets. In weekly return prediction, excess kurtosis and market sentiment indicators play more important roles.

Overall, the proposed method improves daily return prediction accuracy and runtime efficiency, providing potentially valuable benefits for large-scale or time-sensitive applications.

6. Conclusions

6.1 Summary

This study proposes a method to fit each tail of the empirical RND using one single point and its corresponding slope in Bitcoin options markets. The proposed method ensures no kinks at the connection points and reduces computational complexity, achieving greater efficiency compared to the method by Birru and Figlewski (2012).

Empirical analysis using Deribit trading data from January 2021 to April 2024 demonstrates distinct predictive patterns across time horizons. In return prediction models, the results show what both skewness and excess kurtosis have positive effects on daily returns, in contrast with the existing literature (Amaya et al., 2015; Bali & Murray, 2013; Conrad et al., 2013; Cujean & Hasler, 2017; Kim & Park, 2018; Y. Li et al., 2024) while the skewness and excess kurtosis are negatively associated with weekly returns. This may suggest that cryptocurrency investors become less risk-averse and more inclined to seek tail risk as options approach maturity. The results reinforce that option-implied RND moments contain valuable forward-looking information, particularly in cryptocurrency markets characterized by extreme volatility. Out-of-sample validation using a rolling window framework (Campbell & Thompson, 2008) shows that the proposed method significantly outperforms Birru-Figlewski method in daily return prediction and slightly surpasses in weekly prediction. The findings in this study further highlight the effectiveness of RNDs in refining investment strategies and risk management in cryptocurrency markets."

6.2 Recommendations for Future Research

While this study offers insightful findings, several areas can be further explored. Subsequent research could extend to other cryptocurrencies and traditional financial markets to evaluate the comparative performance of the proposed method versus Birru-Figlewski method across diverse market structures. For instance, examining Ethereum options or analyzing stock index options markets would provide valuable insights into the cross-market predictive capabilities of RND moments. Moreover, this study emphasizes options price-implied information. Future research could incorporate market microstructure factors (e.g., trading volume, bid-ask spread) to assess their relationship with cryptocurrency returns. Examining how these factors interact with RND moments may reveal whether microstructure effects complement or subsume the predictive power of option-implied information, potentially enhancing return prediction models in cryptocurrency markets.

References

Akyildirim, E., Corbet, S., Lucey, B., Sensoy, A., & Yarovaya, L. (2020). The relationship between implied volatility and cryptocurrency returns. *Finance Research Letters*, *33*, 101212. https://doi.org/10.1016/j.frl.2019.06.010

Amaya, D., Christoffersen, P., Jacobs, K., & Vasquez, A. (2015). Does realized skewness predict the cross-section of equity returns? *Journal of Financial Economics*, *118*(1), 135–167. https://doi.org/10.1016/j.jfineco.2015.02.009

Ammann, M., & Feser, A. (2019). Robust estimation of risk-neutral moments. *Journal of Futures Markets*, *39*(9), 1137–1166. https://doi.org/10.1002/fut.22020

Bakshi, G., Kapadia, N., & Madan, D. (2003). Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *The Review of Financial Studies*, *16*(1), 101–143. https://doi.org/10.1093/rfs/16.1.0101

Bali, T. G., & Murray, S. (2013). Does Risk-Neutral Skewness Predict the Cross Section of Equity Option Portfolio Returns? *The Journal of Financial and Quantitative Analysis*, *48*(4), 1145–1171.

Bali, T. G., & Zhou, H. (2016). Risk, Uncertainty, and Expected Returns. *The Journal of Financial and Quantitative Analysis*, *51*(3), 707–735.

Balkema, A. A., & Haan, L. de. (1974). Residual Life Time at Great Age. *The Annals of Probability*, *2*(5), 792–804. https://doi.org/10.1214/aop/1176996548

Baur, D. G., & Smales, L. A. (2022). Trading behavior in bitcoin futures: Following the “smart money.” *Journal of Futures Markets*, *42*(7), 1304–1323. https://doi.org/10.1002/fut.22332

Birru, J., & Figlewski, S. (2012). Anatomy of a meltdown: The risk neutral density for the S&P 500 in the fall of 2008. *Journal of Financial Markets*, *15*(2), 151–180. https://doi.org/10.1016/j.finmar.2011.09.001

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, *81*(3), 637–654.

Bliss, R. R., & Panigirtzoglou, N. (2004). Option-Implied Risk Aversion Estimates. *The Journal of Finance*, *59*(1), 407–446. https://doi.org/10.1111/j.1540-6261.2004.00637.x

Bondarenko, O. (2000). *Recovering Risk-Neutral Densities: A New Nonparametric Approach* (SSRN Scholarly Paper No. 246063). Social Science Research Network. https://doi.org/10.2139/ssrn.246063

Böök, A., Imbet, J. F., Reinke, M., & Sala, C. (2025). The Forecasting Power of Short-Term Options. *The Journal of Derivatives*, *32*(3), 80–116. https://doi.org/10.3905/jod.2025.1.221

Breeden, D. T., & Litzenberger, R. H. (1978). Prices of State-Contingent Claims Implicit in Option Prices. *The Journal of Business*, *51*(4), 621–651.

Campbell, J. Y., & Thompson, S. B. (2008). Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average? *The Review of Financial Studies*, *21*(4), 1509–1531. https://doi.org/10.1093/rfs/hhm055

Chang, B. Y., Christoffersen, P., & Jacobs, K. (2013). Market skewness risk and the cross section of stock returns. *Journal of Financial Economics*, *107*(1), 46–68. https://doi.org/10.1016/j.jfineco.2012.07.002

Chen, R.-R., Hsieh, P., & Huang, J. (2018). Crash risk and risk neutral densities. *Journal of Empirical Finance*, *47*, 162–189. https://doi.org/10.1016/j.jempfin.2018.03.006

Chordia, T., Lin, T.-C., & Xiang, V. (2021). Risk-Neutral Skewness, Informed Trading, and the Cross Section of Stock Returns. *Journal of Financial and Quantitative Analysis*, *56*(5), 1713–1737. https://doi.org/10.1017/S0022109020000551

Christoffersen, P., Jacobs, K., & Chang, B. Y. (2013). Chapter 10—Forecasting with Option-Implied Information. In G. Elliott & A. Timmermann (Eds.), *Handbook of Economic Forecasting* (Vol. 2, pp. 581–656). Elsevier. https://doi.org/10.1016/B978-0-444-53683-9.00010-4

Conrad, J., Dittmar, R. F., & Ghysels, E. (2013). Ex Ante Skewness and Expected Stock Returns. *The Journal of Finance*, *68*(1), 85–124. https://doi.org/10.1111/j.1540-6261.2012.01795.x

Cortés, L. M., Mora-Valencia, A., & Perote, J. (2020). Retrieving the implicit risk neutral density of WTI options with a semi-nonparametric approach. *The North American Journal of Economics and Finance*, *54*, 100862. https://doi.org/10.1016/j.najef.2018.10.010

Cujean, J., & Hasler, M. (2017). Why Does Return Predictability Concentrate in Bad Times? *The Journal of Finance*, *72*(6), 2717–2758. https://doi.org/10.1111/jofi.12544

Deribit. (2025). https://www.deribit.com/

Deribit Options. (2025). https://www.deribit.com/

Dong, B., Xu, W., & Cui, Z. (2024). Implied Willow Tree. *The Journal of Derivatives*, *31*(4), 44–74. https://doi.org/10.3905/jod.2024.1.200

Feng, Y., He, M., & Zhang, Y. (2024). Market Skewness and Stock Return Predictability: New Evidence from China. *Emerging Markets Finance and Trade*, *60*(2), 233–244. https://doi.org/10.1080/1540496X.2023.2217327

Figlewski, S. (2008). *Estimating the Implied Risk Neutral Density for the U.S. Market Portfolio* (SSRN Scholarly Paper No. 1256783). Social Science Research Network. https://papers.ssrn.com/abstract=1256783

Fuertes, A.-M., Liu, Z., & Tang, W. (2022). Risk-neutral skewness and commodity futures pricing. *Journal of Futures Markets*, *42*(4), 751–785. https://doi.org/10.1002/fut.22308

Glatzer, E., & Scheicher, M. (2005). What moves the tail? The determinants of the option-implied probability density function of the DAX index. *Journal of Futures Markets*, *25*(6), 515–536. https://doi.org/10.1002/fut.20157

Grith, M., Härdle, W. K., & Schienle, M. (2012). Nonparametric Estimation of Risk-Neutral Densities. In J.-C. Duan, W. K. Härdle, & J. E. Gentle (Eds.), *Handbook of Computational Finance* (pp. 277–305). Springer. https://doi.org/10.1007/978-3-642-17254-0_11

Gu, S., Kelly, B., & Xiu, D. (2020). Empirical Asset Pricing via Machine Learning. *The Review of Financial Studies*, *33*(5), 2223–2273. https://doi.org/10.1093/rfs/hhaa009

Hagan, P. S., & and West, G. (2006). Interpolation Methods for Curve Construction. *Applied Mathematical Finance*, *13*(2), 89–129. https://doi.org/10.1080/13504860500396032

Haslip, G. G., & and Kaishev, V. K. (2014). Lookback option pricing using the Fourier transform B-spline method. *Quantitative Finance*, *14*(5), 789–803. https://doi.org/10.1080/14697688.2014.882010

Hayashi, F. (2020). Analytically Deriving Risk-Neutral Densities from Volatility Smiles in Delta. *The Journal of Derivatives*, *27*(4), 6–12. https://doi.org/10.3905/jod.2020.1.099

He, M., Shen, L., Zhang, Y., & Zhang, Y. (2023). Predicting cryptocurrency returns for real-world investments: A daily updated and accessible predictor. *Finance Research Letters*, *58*, 104406. https://doi.org/10.1016/j.frl.2023.104406

He, Y., Peng, L., Zhang, D., & Zhao, Z. (2022). Risk Analysis via Generalized Pareto Distributions. *Journal of Business & Economic Statistics*, *40*(2), 852–867. https://doi.org/10.1080/07350015.2021.1874390

Hosking, J. R. M., & Wallis, J. R. (1987). Parameter and Quantile Estimation for the Generalized Pareto Distribution. *Technometrics*, *29*(3), 339–349. https://doi.org/10.2307/1269343

Hull, J. (2021). *Options, Futures, and Other Derivatives: Global Edition*. Pearson Deutschland. https://elibrary.pearson.de/book/99.150005/9781292410623

Jackwerth, J. (2020). What Do Index Options Teach Us About COVID-19? *The Review of Asset Pricing Studies*, *10*(4), 618–634. https://doi.org/10.1093/rapstu/raaa012

Jondeau, E., Wang, X., Yan, Z., & Zhang, Q. (2020). Skewness and index futures return. *Journal of Futures Markets*, *40*(11), 1648–1664. https://doi.org/10.1002/fut.22112

Kim, T. S., & Park, H. (2018). Is stock return predictability of option-implied skewness affected by the market state? *Journal of Futures Markets*, *38*(9), 1024–1042. https://doi.org/10.1002/fut.21921

Köse, N., Yildirim, H., Ünal, E., & Lin, B. (2024). The Bitcoin price and Bitcoin price uncertainty: Evidence of Bitcoin price volatility. *Journal of Futures Markets*, *44*(4), 673–695. https://doi.org/10.1002/fut.22487

Lehnert, T. (2022). Is Risk-Neutral Skewness an Indicator of Downside Risk? Evidence from Tail Risk Taking of Hedge Funds. *The Journal of Derivatives*, *29*(3), 65–84. https://doi.org/10.3905/jod.2022.1.148

Li, X., Wu, Z., Zhang, H., & Zhang, L. (2024). Risk-neutral skewness and stock market returns: A time-series analysis. *The North American Journal of Economics and Finance*, *70*, 102040. https://doi.org/10.1016/j.najef.2023.102040

Li, Y., Nolte, I., & Pham, M. C. (2024). Parametric risk-neutral density estimation via finite lognormal-Weibull mixtures. *Journal of Econometrics*, *241*(2), 105748. https://doi.org/10.1016/j.jeconom.2024.105748

Li, Y., Urquhart, A., Wang, P., & Zhang, W. (2021). MAX momentum in cryptocurrency markets. *International Review of Financial Analysis*, *77*, 101829. https://doi.org/10.1016/j.irfa.2021.101829

Liu, Y., & Chen, Y. (2024). Skewness risk and the cross-section of cryptocurrency returns. *International Review of Financial Analysis*, *96*, 103626. https://doi.org/10.1016/j.irfa.2024.103626

Liu, Y., Li, Z., Nekhili, R., & Sultan, J. (2023). Forecasting cryptocurrency returns with machine learning. *Research in International Business and Finance*, *64*, 101905. https://doi.org/10.1016/j.ribaf.2023.101905

Liu, Y., & Tsyvinski, A. (2021). Risks and Returns of Cryptocurrency. *The Review of Financial Studies*, *34*(6), 2689–2727. https://doi.org/10.1093/rfs/hhaa113

López-Cabarcos, M. Á., Pérez-Pico, A. M., Piñeiro-Chousa, J., & Šević, A. (2021). Bitcoin volatility, stock market and investor sentiment. Are they connected? *Finance Research Letters*, *38*, 101399. https://doi.org/10.1016/j.frl.2019.101399

Markose, S., & Alentorn, A. (2011). The Generalized Extreme Value Distribution, Implied Tail Index, and Option Pricing. *The Journal of Derivatives*, *18*(3), 35–60. https://doi.org/10.3905/jod.2011.18.3.035

McNeil, A. J., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. *Journal of Empirical Finance*, *7*(3), 271–300. https://doi.org/10.1016/S0927-5398(00)00012-8

Mei, D., Liu, J., Ma, F., & Chen, W. (2017). Forecasting stock market volatility: Do realized skewness and kurtosis help? *Physica A: Statistical Mechanics and Its Applications*, *481*, 153–159. https://doi.org/10.1016/j.physa.2017.04.020

Mohrschladt, H., & Schneider, J. C. (2021). Option-implied skewness: Insights from ITM-options. *Journal of Economic Dynamics and Control*, *131*, 104227. https://doi.org/10.1016/j.jedc.2021.104227

Monteiro, A. M., & Santos, A. A. F. (2022). Option prices for risk-neutral density estimation using nonparametric methods through big data and large-scale problems. *Journal of Futures Markets*, *42*(1), 152–171. https://doi.org/10.1002/fut.22258

Monteiro, A. M., Tütüncü, R. H., & Vicente, L. N. (2008). Recovering risk-neutral probability density functions from options prices using cubic splines and ensuring nonnegativity. *European Journal of Operational Research*, *187*(2), 525–542. https://doi.org/10.1016/j.ejor.2007.02.041

Neumann, M., & Skiadopoulos, G. (2013). Predictable Dynamics in Higher-Order Risk-Neutral Moments: Evidence from the S&P 500 Options. *The Journal of Financial and Quantitative Analysis*, *48*(3), 947–977.

Orosi, G. (2015). Estimating Option-Implied Risk-Neutral Densities: A Novel Parametric Approach. *The Journal of Derivatives*, *23*(1), 41–61. https://doi.org/10.3905/jod.2015.23.1.041

Reinke, M. (2020). Risk-Neutral Density Estimation: Looking at the Tails. *The Journal of Derivatives*, *27*(3), 99–125. https://doi.org/10.3905/jod.2019.1.090

Rubinstein, M. (1994). Implied Binomial Trees. *The Journal of Finance*, *49*(3), 771–818. https://doi.org/10.2307/2329207

Shimko, D. (1993). Bounds of probability. *Risk*, *6*(4), 33–37.

The Block. (2025). The Block. https://www.theblock.co/data/crypto-markets/options

Uberti, P. (2023). A theoretical generalization of the Markowitz model incorporating skewness and kurtosis. *Quantitative Finance*, *23*(5), 877–886. https://doi.org/10.1080/14697688.2023.2176250

Wang, Y.-H., & Yen, K.-C. (2018). The information content of option-implied tail risk on the future returns of the underlying asset. *Journal of Futures Markets*, *38*(4), 493–510. https://doi.org/10.1002/fut.21887

Welch, I., & Goyal, A. (2008). A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. *The Review of Financial Studies*, *21*(4), 1455–1508. https://doi.org/10.1093/rfs/hhm014

Zulfiqar, N., & Gulzar, S. (2021). Implied volatility estimation of bitcoin options and the stylized facts of option pricing. *Financial Innovation*, *7*(1), 67. https://doi.org/10.1186/s40854-021-00280-y

Appendix

Appendix Table 1: Three-Variable Regression Results for Options with 1 Day to Expiration (The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	R-squared
Mean	-0.0736	0.0361	**	0.0191	0.5895		0.0575	0.1030		0.0094
Std	-0.0235	0.5018		0.0300	0.3958		0.0619	0.0795	*	0.0047
Median	-0.0680	0.0525	*	0.0216	0.5420		0.0569	0.1073		0.0087
Fear and Greed Index	-0.0038	0.9138		0.0284	0.4209		0.0618	0.0807	*	0.0041
VIX	-0.0035	0.9211		0.0287	0.4185		0.0623	0.0786	*	0.0041
T-1 Return	-0.0286	0.4166		0.0243	0.4941		0.0595	0.0928	*	0.0049
T-2 Return	0.0300	0.3896		0.0290	0.4110		0.0620	0.0791	*	0.0050
T-3 Return	0.0131	0.7073		0.0277	0.4322		0.0627	0.0761	*	0.0043
T-4 Return	0.0620	0.0749	*	0.0287	0.4141		0.0628	0.0748	*	0.0080

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 2: Four-Variable Regression Results for Options with 1 Day to Expiration (The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	Std_Coef	Std_p	Std_Sig	R-squared
Mean	-0.0755	0.0467	**	0.0185	0.6045		0.0574	0.1038		0.0051	0.8926		0.0095
Median	-0.0727	0.0663	*	0.0204	0.5683		0.0566	0.1096		0.0101	0.7972		0.0088
Fear and Greed Index	0.0015	0.9658		0.0300	0.3961		0.0620	0.0798	*	-0.0238	0.5068		0.0047
VIX	-0.0051	0.8840		0.0306	0.3902		0.0623	0.0787	*	-0.0238	0.4966		0.0047
T-1 Return	-0.0294	0.4042		0.0260	0.4665		0.0593	0.0940	*	-0.0244	0.4852		0.0055
T-2 Return	0.0291	0.4037		0.0306	0.3872		0.0618	0.0798	*	-0.0224	0.5226		0.0055
T-3 Return	0.0111	0.7522		0.0294	0.4059		0.0625	0.0773	*	-0.0225	0.5222		0.0048
T-4 Return	0.0611	0.0797	*	0.0302	0.3918		0.0627	0.0755	*	-0.0207	0.5526		0.0084

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 3: Four-Variable Regression Results Based on the Model 1 (Daily Return The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	T-4 Return_Coef	T-4 Return_p	T-4 Return_Sig	R-squared
Mean	-0.0752	0.0320	**	0.0194	0.5845		0.0582	0.0984	*	0.0639	0.0659	*	0.0135
Std	-0.0207	0.5526		0.0302	0.3918		0.0627	0.0755	*	0.0611	0.0797	*	0.0084
Excess Kurtosis	-0.0694	0.0473	**	0.0218	0.5361		0.0576	0.1026		0.0636	0.0674	*	0.0127
Fear and Greed Index	-0.0127	0.7182		0.0290	0.4110		0.0620	0.0790	*	0.0638	0.0698	*	0.0081
VIX	0.0012	0.9723		0.0286	0.4188		0.0627	0.0764	*	0.0621	0.0755	*	0.0080
T-1 Return	-0.0290	0.4102		0.0247	0.4870		0.0603	0.0883	*	0.0622	0.0742	*	0.0088
T-2 Return	0.0283	0.4166		0.0294	0.4044		0.0627	0.0750	*	0.0613	0.0788	*	0.0088
T-3 Return	0.0155	0.6571		0.0280	0.4264		0.0636	0.0716	*	0.0626	0.0725	*	0.0082

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 4: Four-Variable Regression Results Based on the Model 2 (Daily Return The proposed method)

	Coef	p value	Sig	Kurtosis_Coef	Kurtosis _p	Kurtosis _Sig	Median_Coef	Median_p	Median_Sig	T-4 Return_Coef	T-4 Return_p	T-4 Return_Sig	R-squared
Mean	-0.3367	0.1683		0.0601	0.0865	*	0.2620	0.2840		0.0645	0.0635	*	0.0145
Std	0.0176	0.6530		0.0539	0.1214		-0.0796	0.0420	**	0.0645	0.0642	*	0.0125
Skewness	0.0218	0.5361		0.0576	0.1026		-0.0694	0.0473	**	0.0636	0.0674	*	0.0127
Fear and Greed Index	0.0413	0.3304		0.0553	0.1122		-0.0945	0.0247	**	0.0583	0.0974	*	0.0134
VIX	-0.0243	0.5153		0.0552	0.1131		-0.0801	0.0315	**	0.0619	0.0759	*	0.0128
T-1 Return	-0.0306	0.3799		0.0522	0.1346		-0.0708	0.0422	**	0.0637	0.0670	*	0.0132
T-2 Return	0.0289	0.4053		0.0540	0.1212		-0.0721	0.0386	**	0.0627	0.0713	*	0.0131
T-3 Return	0.0181	0.6028		0.0552	0.1136		-0.0719	0.0391	**	0.0642	0.0650	*	0.0126

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 5: Three-Variable Regression Results for Options with 1 Day to Expiration (Birru-Figlewski method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	R-squared
Mean	-0.0764	0.0293	**	-0.0300	0.4058		0.0281	0.4337		0.0076
Std	-0.0155	0.6590		-0.0191	0.5973		0.0327	0.3625		0.0020
Median	-0.0744	0.0337	**	-0.0280	0.4369		0.0271	0.4502		0.0073
Fear and Greed Index	-0.0038	0.9140		-0.0211	0.5572		0.0319	0.3755		0.0018
VIX	0.0040	0.9099		-0.0217	0.5485		0.0318	0.3769		0.0018
T-1 Return	-0.0397	0.2659		-0.0294	0.4222		0.0283	0.4322		0.0033
T-2 Return	0.0282	0.4205		-0.0196	0.5857		0.0322	0.3698		0.0026
T-3 Return	0.0113	0.7457		-0.0211	0.5575		0.0324	0.3673		0.0019
T-4 Return	0.0599	0.0863	*	-0.0183	0.6112		0.0321	0.3703		0.0054

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 6: Four-Variable Regression Results for Options with 1 Day to Expiration (Birru-Figlewski method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	Std_Coef	Std_p	Std_Sig	R-squared
Mean	-0.0912	0.0240	**	-0.0357	0.3334		0.0261	0.4676		0.0298	0.4608		0.0082
Median	-0.0966	0.0231	**	-0.0353	0.3383		0.0241	0.5044		0.0394	0.3554		0.0083
Fear and Greed Index	0.0004	0.9909		-0.0191	0.5975		0.0328	0.3636		-0.0156	0.6688		0.0020
VIX	0.0031	0.9310		-0.0195	0.5929		0.0325	0.3676		-0.0153	0.6636		0.0021
T-1 Return	-0.0401	0.2615		-0.0273	0.4603		0.0289	0.4229		-0.0165	0.6397		0.0036
T-2 Return	0.0279	0.4244		-0.0176	0.6280		0.0328	0.3616		-0.0151	0.6675		0.0028
T-3 Return	0.0099	0.7788		-0.0191	0.5975		0.0329	0.3600		-0.0145	0.6816		0.0021
T-4 Return	0.0597	0.0877	*	-0.0163	0.6528		0.0327	0.3625		-0.0146	0.6775		0.0056

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 7: Two-Variable Regression Results for Options with 7 Days to Expiration (The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	R-squared
Mean	-0.1505	0.1066		-0.0401	0.6654		0.0259
Std	-0.1636	0.1212		0.0185	0.8601		0.0242
Excess Kurtosis	-0.1390	0.2782		0.0353	0.7822		0.0138
Median	-0.1502	0.1062		-0.0429	0.6430		0.0259
Fear and Greed Index	0.0803	0.4581		-0.1021	0.3462		0.0084
VIX	-0.0667	0.4830		-0.0750	0.4298		0.0079
T-1 Return	-0.0249	0.7963		-0.0538	0.5779		0.0043
T-2 Return	0.0248	0.7932		-0.0653	0.4901		0.0043
T-3 Return	0.0115	0.9021		-0.0620	0.5077		0.0038
T-4 Return	-0.0789	0.3982		-0.0515	0.5811		0.0098

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 8: Three-Variable Regression Results for Options with 7 Days to Expiration (The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	R-squared
Mean	-0.1591	0.0884	*	0.0685	0.5935		-0.1555	0.2228		0.0385
Std	-0.1410	0.2082		0.0644	0.6194		-0.0822	0.5435		0.0273
Median	-0.1591	0.0876	*	0.0660	0.6065		-0.1560	0.2212		0.0386
Fear and Greed Index	0.0621	0.5716		-0.0054	0.9709		-0.1264	0.3322		0.0165
VIX	-0.0355	0.7259		0.0163	0.9069		-0.1225	0.3708		0.0148
T-1 Return	-0.0750	0.4713		0.0820	0.5680		-0.1766	0.2032		0.0182
T-2 Return	0.0150	0.8744		0.0312	0.8120		-0.1370	0.2893		0.0140
T-3 Return	-0.0017	0.9859		0.0357	0.7840		-0.1393	0.2833		0.0138
T-4 Return	-0.0908	0.3330		0.0552	0.6700		-0.1524	0.2374		0.0218

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 9: Four-Variable Regression Results for Options with 7 Days to Expiration (The proposed method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	Std_Coef	Std_p	Std_Sig	R-squared
Mean	-0.2019	0.2372		0.0648	0.6165		-0.1846	0.2507		0.0612	0.7640		0.0392
Median	-0.2031	0.2339		0.0615	0.6348		-0.1861	0.2477		0.0631	0.7574		0.0394
Fear and Greed Index	0.1350	0.2543		-0.0131	0.9283		-0.0336	0.8126		-0.1941	0.1101		0.0384
VIX	-0.0413	0.6830		0.0427	0.7614		-0.0622	0.6664		-0.1431	0.2038		0.0288
T-1 Return	-0.0743	0.4741		0.1105	0.4462		-0.1196	0.4106		-0.1406	0.2105		0.0317
T-2 Return	0.0145	0.8779		0.0603	0.6497		-0.0803	0.5561		-0.1410	0.2103		0.0275
T-3 Return	-0.0007	0.9940		0.0645	0.6251		-0.0823	0.5478		-0.1410	0.2102		0.0273
T-4 Return	-0.0904	0.3338		0.0841	0.5219		-0.0957	0.4819		-0.1407	0.2094		0.0353

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 10: Two-Variable Regression Results for Options with 7 Days to Expiration (Birru-Figlewski method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	R-squared
Mean	-0.1505	0.1066		-0.0401	0.6654		0.0259
Std	-0.1636	0.1212		0.0185	0.8601		0.0242
Excess Kurtosis	-0.1390	0.2782		0.0353	0.7822		0.0138
Median	-0.1502	0.1062		-0.0429	0.6430		0.0259
Fear and Greed Index	0.0803	0.4581		-0.1021	0.3462		0.0084
VIX	-0.0667	0.4830		-0.0750	0.4298		0.0079
T-1 Return	-0.0249	0.7963		-0.0538	0.5779		0.0043
T-2 Return	0.0248	0.7932		-0.0653	0.4901		0.0043
T-3 Return	0.0115	0.9021		-0.0620	0.5077		0.0038
T-4 Return	-0.0789	0.3982		-0.0515	0.5811		0.0098

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 11: Three-Variable Regression Results for Options with 7 Days to Expiration (Birru-Figlewski method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	R-squared
Mean	-0.1591	0.0884	*	0.0685	0.5935		-0.1555	0.2228		0.0385
Std	-0.1410	0.2082		0.0644	0.6194		-0.0822	0.5435		0.0273
Median	-0.1591	0.0876	*	0.0660	0.6065		-0.1560	0.2212		0.0386
Fear and Greed Index	0.0621	0.5716		-0.0054	0.9709		-0.1264	0.3322		0.0165
VIX	-0.0355	0.7259		0.0163	0.9069		-0.1225	0.3708		0.0148
T-1 Return	-0.0750	0.4713		0.0820	0.5680		-0.1766	0.2032		0.0182
T-2 Return	0.0150	0.8744		0.0312	0.8120		-0.1370	0.2893		0.0140
T-3 Return	-0.0017	0.9859		0.0357	0.7840		-0.1393	0.2833		0.0138
T-4 Return	-0.0908	0.3330		0.0552	0.6700		-0.1524	0.2374		0.0218

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level

Appendix Table 12: Four-Variable Regression Results for Options with 7 Days to Expiration (Birru-Figlewski method)

	Coef	p value	Sig	Skewness_Coef	Skewness_p	Skewness_Sig	Kurtosis_Coef	Kurtosis_p	Kurtosis_Sig	Std_Coef	Std_p	Std_Sig	R-squared
Mean	-0.2019	0.2372		0.0648	0.6165		-0.1846	0.2507		0.0612	0.7640		0.0392
Median	-0.2031	0.2339		0.0615	0.6348		-0.1861	0.2477		0.0631	0.7574		0.0394
Fear and Greed Index	0.1350	0.2543		-0.0131	0.9283		-0.0336	0.8126		-0.1941	0.1101		0.0384
VIX	-0.0413	0.6830		0.0427	0.7614		-0.0622	0.6664		-0.1431	0.2038		0.0288
T-1 Return	-0.0743	0.4741		0.1105	0.4462		-0.1196	0.4106		-0.1406	0.2105		0.0317
T-2 Return	0.0145	0.8779		0.0603	0.6497		-0.0803	0.5561		-0.1410	0.2103		0.0275
T-3 Return	-0.0007	0.9940		0.0645	0.6251		-0.0823	0.5478		-0.1410	0.2102		0.0273
T-4 Return	-0.0904	0.3338		0.0841	0.5219		-0.0957	0.4819		-0.1407	0.2094		0.0353

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level; *** indicates significance at the 1% level