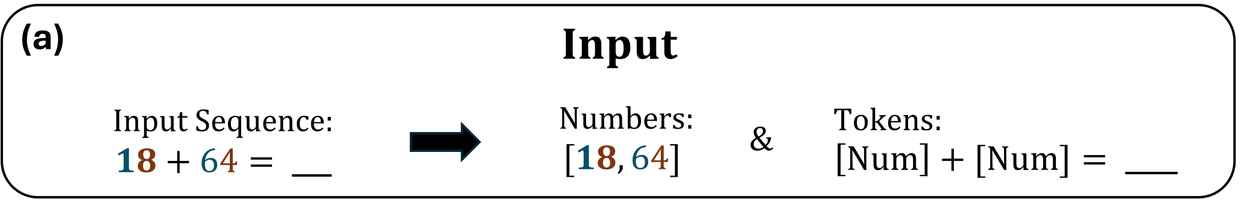
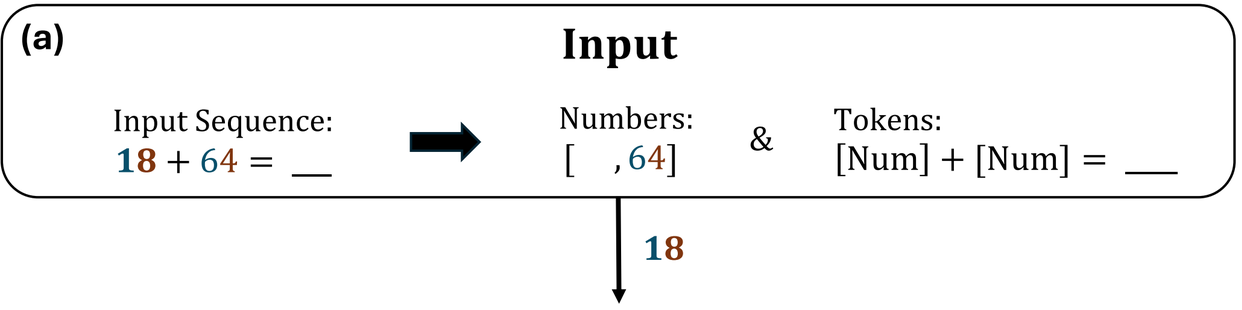
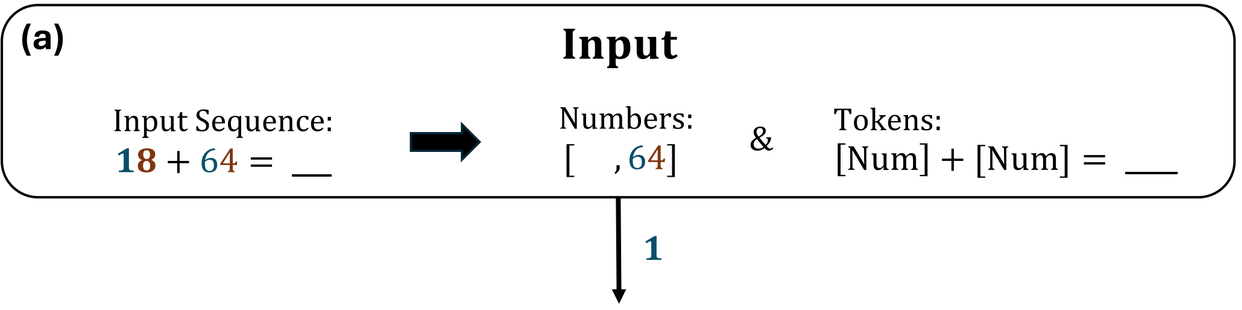
(a)

Input

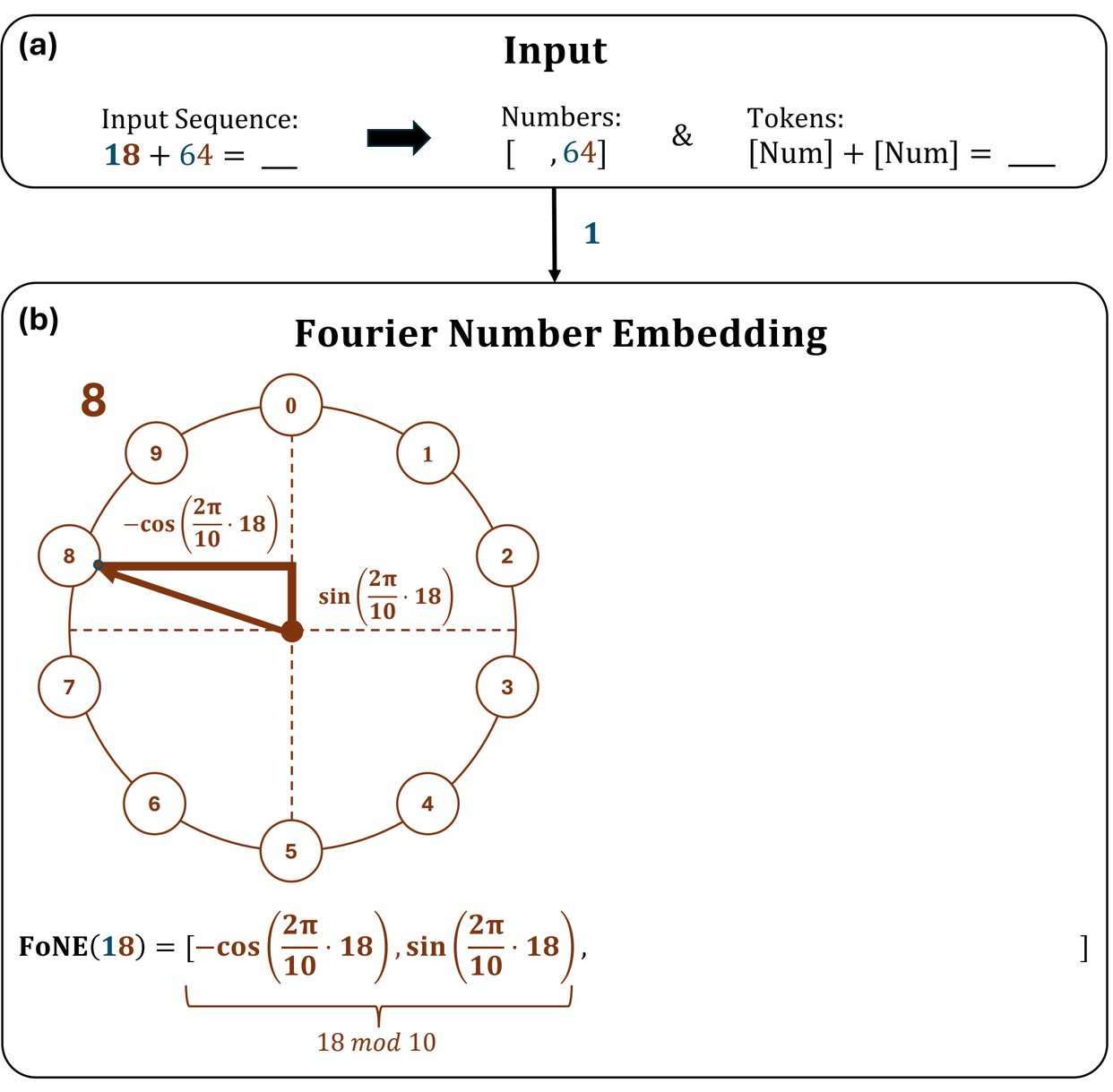
Input Sequence:
$18 + 64 = __$


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[18, 64]
&
Tokens:
[Num] + [Num] = ___


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[, 64]
&
Tokens:
[Num] + [Num] = ___
18



8


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[, 64]
&
Tokens:
[Num] + [Num] = ___
1
(b)
Fourier Number Embedding
8
0
1
2
3
4
5
6
7
8
9
$-\cos\left(\frac{2\pi}{10}\cdot 18\right)$
$\sin\left(\frac{2\pi}{10}\cdot 18\right)$
$\mathbf{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right),$
$]$
$18 \bmod 10$

(a)

Input

Input Sequence:
$18 + 64 = __$

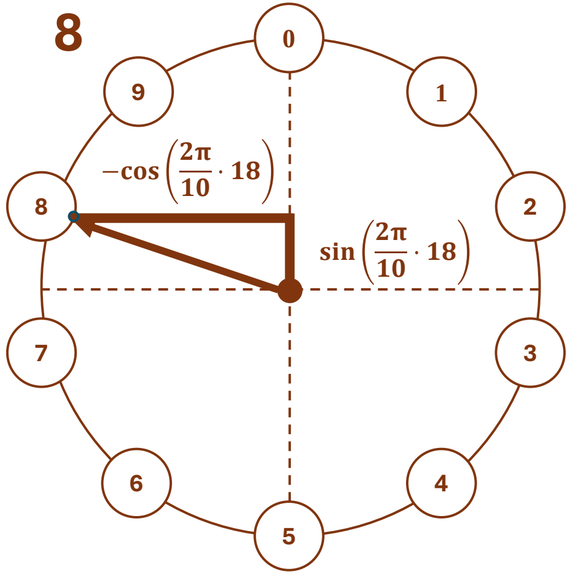
Numbers:
[, 64]

&

Tokens:
[Num] + [Num] = ___

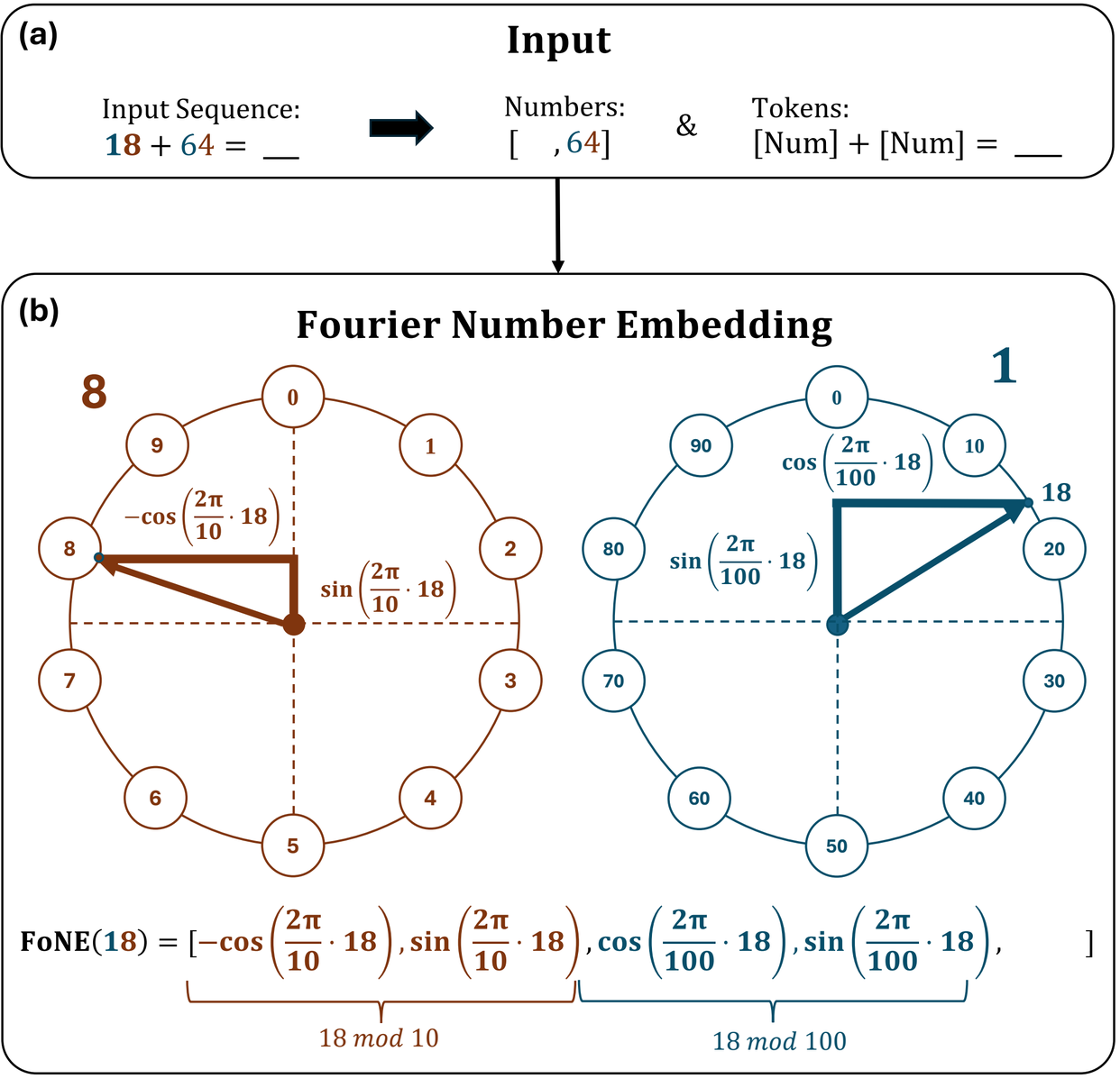
(b)

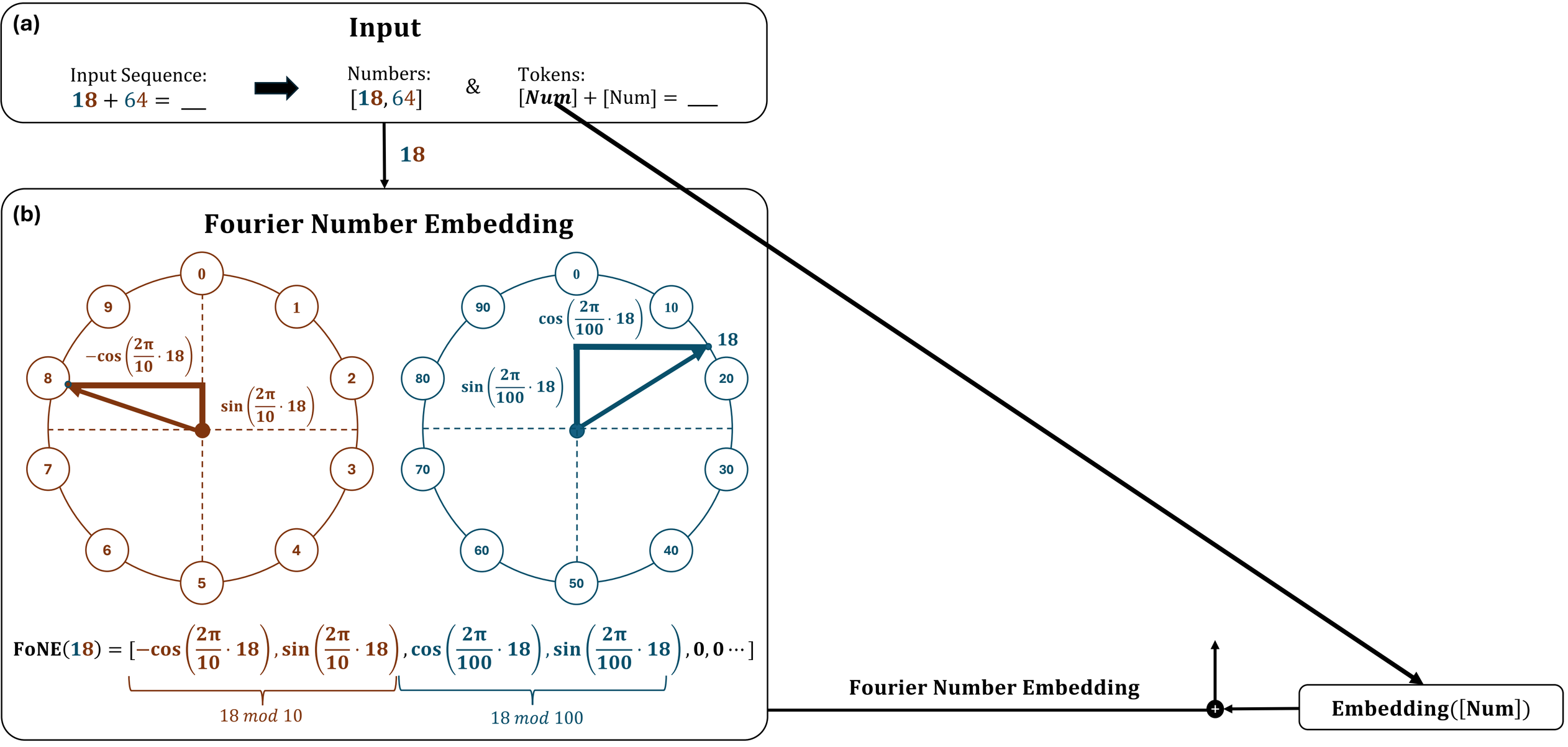
Fourier Number Embedding

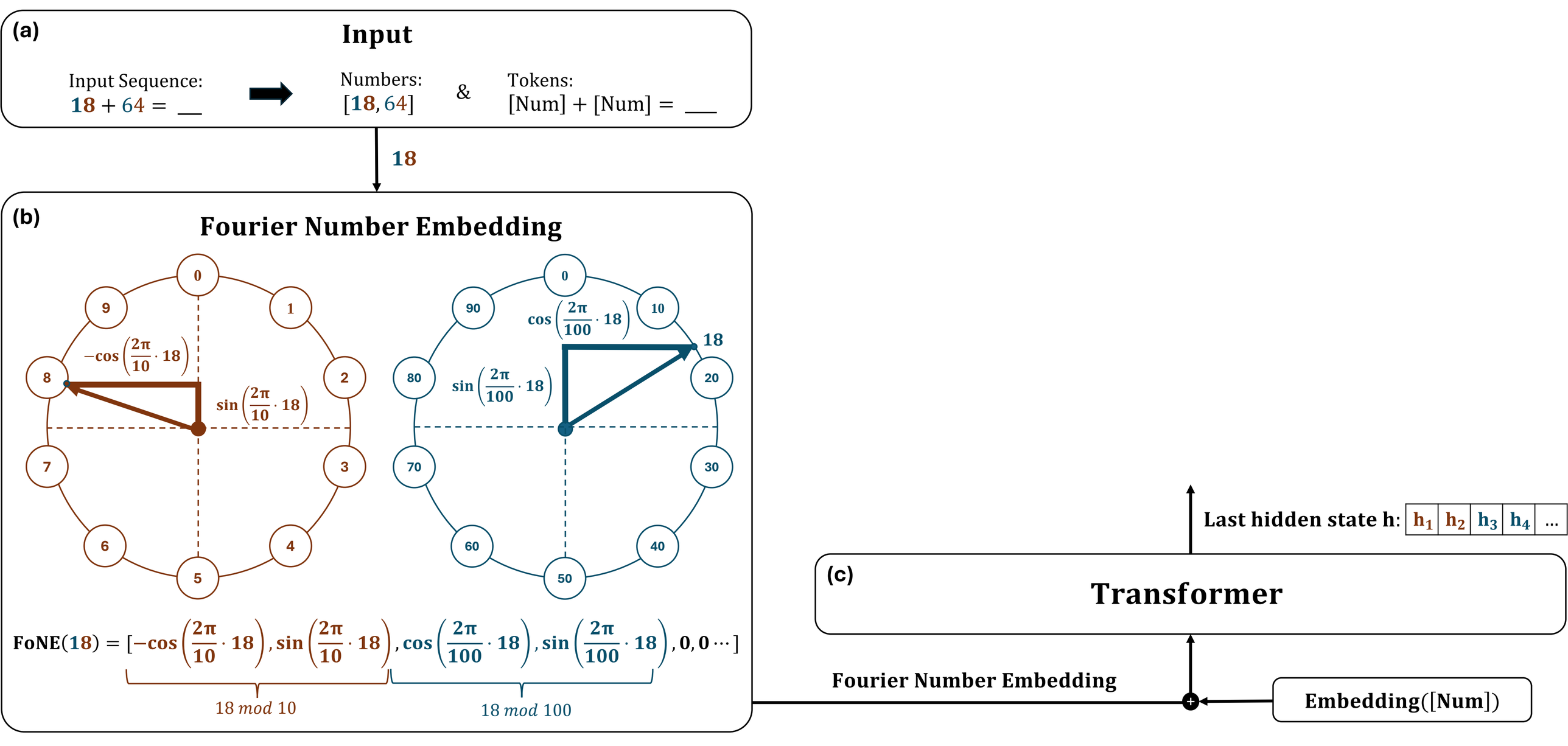


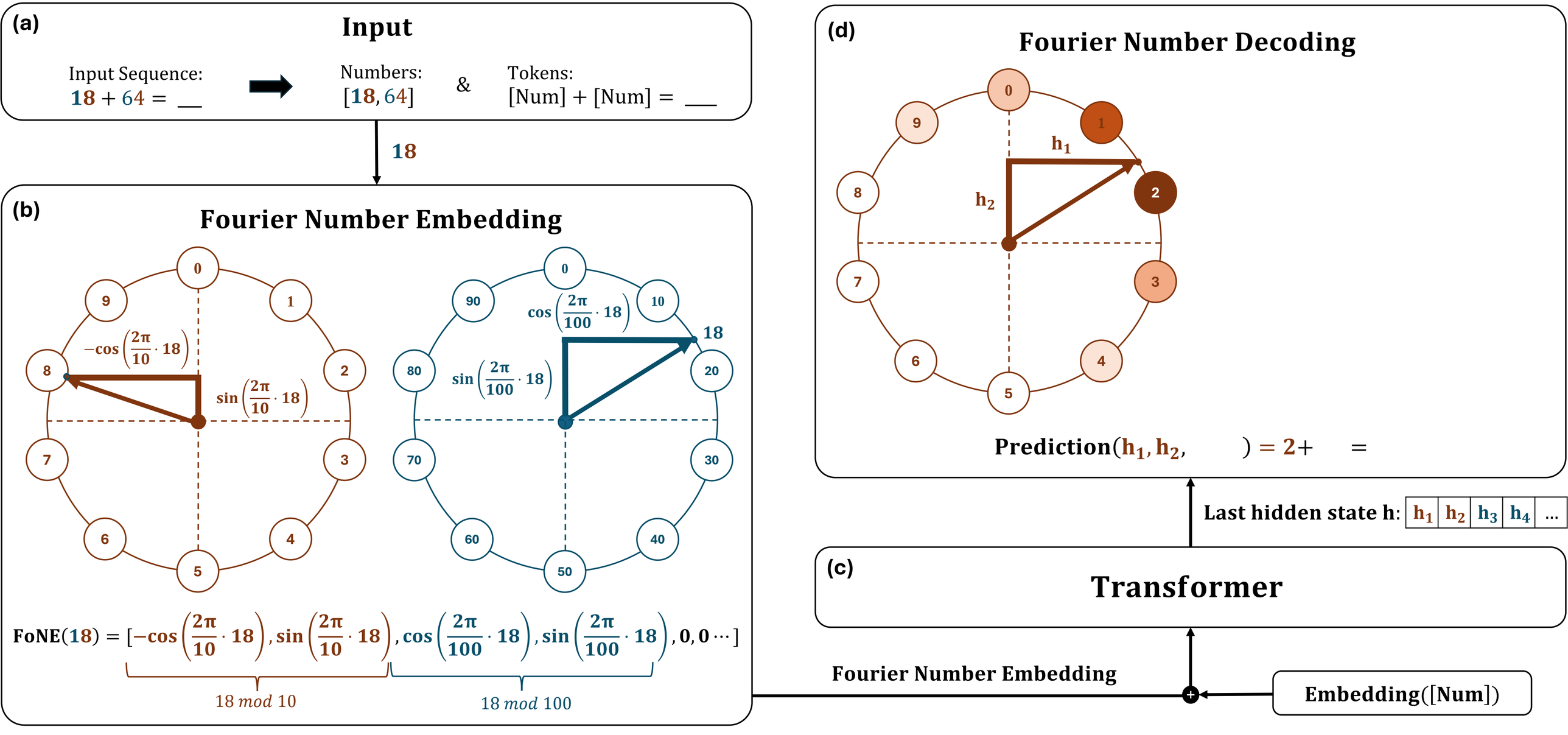
$$\text{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right), \;]$$

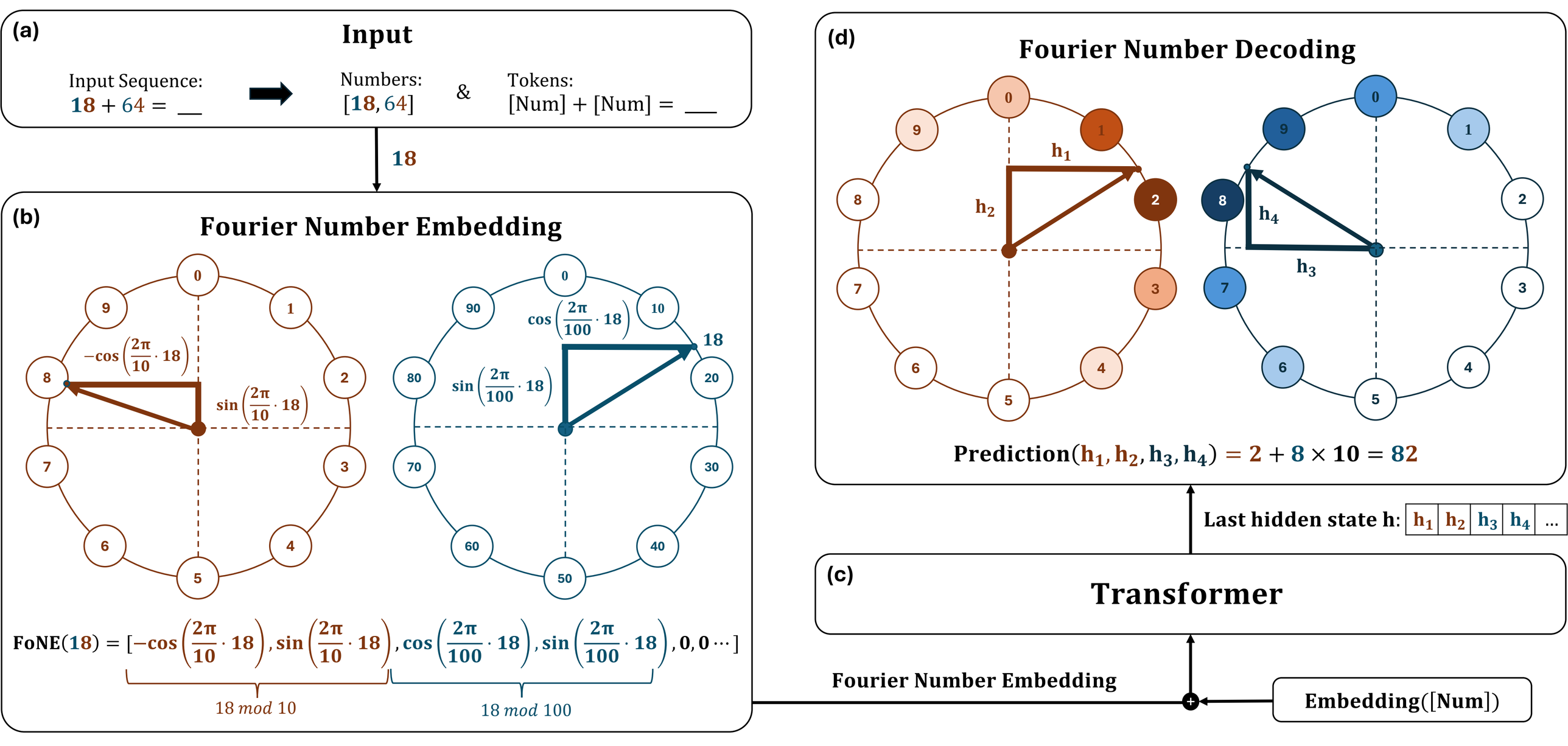
$18 \; mod \; 10$


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[, 64]
&
Tokens:
[Num] + [Num] = ___
(b)
Fourier Number Embedding
8
1
0
1
2
3
4
5
6
7
8
9
$-\cos\left(\frac{2\pi}{10}\cdot 18\right)$
$\sin\left(\frac{2\pi}{10}\cdot 18\right)$
0
10
20
30
40
50
60
70
80
90
18
$\cos\left(\frac{2\pi}{100}\cdot 18\right)$
$\sin\left(\frac{2\pi}{100}\cdot 18\right)$
$\mathbf{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right), \cos\left(\frac{2\pi}{100}\cdot 18\right), \sin\left(\frac{2\pi}{100}\cdot 18\right), \quad]$
18 mod 10
18 mod 100


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[18, 64]
&
Tokens:
[Num] + [Num] = ___
18
(b)
Fourier Number Embedding
0
1
2
3
4
5
6
7
8
9
$-\cos\left(\frac{2\pi}{10}\cdot 18\right)$
$\sin\left(\frac{2\pi}{10}\cdot 18\right)$
0
10
20
30
40
50
60
70
80
90
$\cos\left(\frac{2\pi}{100}\cdot 18\right)$
$\sin\left(\frac{2\pi}{100}\cdot 18\right)$
18
$\mathbf{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right), \cos\left(\frac{2\pi}{100}\cdot 18\right), \sin\left(\frac{2\pi}{100}\cdot 18\right), 0, 0 \cdots]$
18 mod 10
18 mod 100
Fourier Number Embedding
+
Embedding([Num])


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[18, 64]
&
Tokens:
[Num] + [Num] = ___
18
(b)
Fourier Number Embedding
0
1
2
3
4
5
6
7
8
9
$-\cos\left(\frac{2\pi}{10}\cdot 18\right)$
$\sin\left(\frac{2\pi}{10}\cdot 18\right)$
0
10
20
30
40
50
60
70
80
90
18
$\cos\left(\frac{2\pi}{100}\cdot 18\right)$
$\sin\left(\frac{2\pi}{100}\cdot 18\right)$
$\text{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right), \cos\left(\frac{2\pi}{100}\cdot 18\right), \sin\left(\frac{2\pi}{100}\cdot 18\right), 0, 0 \cdots]$
18 mod 10
18 mod 100
(c)
Transformer
Last hidden state h: h_1 h_2 h_3 h_4 ...
Fourier Number Embedding
Embedding([Num])


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[18, 64]
&
Tokens:
[Num] + [Num] = ___
18
(b)
Fourier Number Embedding
$-\cos\left(\frac{2\pi}{10}\cdot 18\right)$
$\sin\left(\frac{2\pi}{10}\cdot 18\right)$
$\cos\left(\frac{2\pi}{100}\cdot 18\right)$
$\sin\left(\frac{2\pi}{100}\cdot 18\right)$
18
$\text{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right), \cos\left(\frac{2\pi}{100}\cdot 18\right), \sin\left(\frac{2\pi}{100}\cdot 18\right), 0, 0 \cdots]$
18 mod 10
18 mod 100
(d)
Fourier Number Decoding
h_1
h_2
Prediction(h_1, h_2,) = 2+ =
Last hidden state h: h_1 h_2 h_3 h_4 ...
(c)
Transformer
Fourier Number Embedding
Embedding([Num])


(a)
Input
Input Sequence:
18 + 64 = __
Numbers:
[18, 64]
&
Tokens:
[Num] + [Num] = ___
18
(b)
Fourier Number Embedding
$-\cos\left(\frac{2\pi}{10}\cdot 18\right)$
$\sin\left(\frac{2\pi}{10}\cdot 18\right)$
$\cos\left(\frac{2\pi}{100}\cdot 18\right)$
$\sin\left(\frac{2\pi}{100}\cdot 18\right)$
18
$\text{FoNE}(18) = [-\cos\left(\frac{2\pi}{10}\cdot 18\right), \sin\left(\frac{2\pi}{10}\cdot 18\right), \cos\left(\frac{2\pi}{100}\cdot 18\right), \sin\left(\frac{2\pi}{100}\cdot 18\right), 0, 0 \cdots]$
18 mod 10
18 mod 100
(d)
Fourier Number Decoding
h_1
h_2
h_3
h_4
$\text{Prediction}(h_1, h_2, h_3, h_4) = 2 + 8 \times 10 = 82$
Last hidden state h: h_1 h_2 h_3 h_4 ...
(c)
Transformer
Fourier Number Embedding
Embedding([Num])