equal to

$$\mathbf{F}_{g_{21}} = \boldsymbol{\psi}_1 m_2, \tag{1.3}$$

where

$$\boldsymbol{\psi}_1 = -\hat{\mathbf{R}}\,\frac{Gm_1}{R^2} \quad \text{(N/kg)}. \tag{1.4}$$

In Eq. (1.4) $\hat{\mathbf{R}}$ is a unit vector that points in the radial direction away from object m_1, and therefore $-\hat{\mathbf{R}}$ points toward m_1. The force due to $\boldsymbol{\psi}_1$ acting on a mass m_2, for example, is obtained from the combination of Eqs. (1.3) and (1.4) with $R = R_{12}$ and $\hat{\mathbf{R}} = \hat{\mathbf{R}}_{12}$. The field concept may be generalized by defining the gravitational field $\boldsymbol{\psi}$ at any point in space such that, when a test mass m is placed at that point, the force $\mathbf{F}_g$ acting on it is related to $\boldsymbol{\psi}$ by

$$\boldsymbol{\psi} = \frac{\mathbf{F}_g}{m}. \tag{1.5}$$

The force $\mathbf{F}_g$ may be due to a single mass or a collection of many masses.

1-3.2 Electric Fields

The electromagnetic force consists of an electrical component $\mathbf{F}_e$ and a magnetic component $\mathbf{F}_m$. The electrical force $\mathbf{F}_e$ is similar to the gravitational force, but with two major differences. First, ***the source of the electrical field is electric charge, not mass.*** Second, even though both types of fields vary inversely as the square of the distance from their respective sources, electric charges may have positive or negative polarity, resulting in a force that may be attractive or repulsive.

We know from atomic physics that all matter contains a mixture of neutrons, positively charged protons, and negatively charged electrons, with the fundamental quantity of charge being that of a single electron, usually denoted by the letter e. The unit by which electric charge is measured is the coulomb (C), named in honor of the eighteenth-century French scientist Charles Augustin de Coulomb (1736–1806). The magnitude of e is

$$\boxed{e = 1.6 \times 10^{-19} \quad \text{(C)}.} \tag{1.6}$$

The charge of a single electron is $q_e = -e$ and that of a proton is equal in magnitude but opposite in polarity: $q_p = e$.

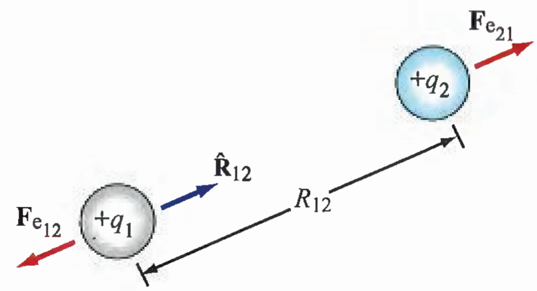

Figure 1-5: Electric forces on two positive point charges in free space.

Coulomb's experiments demonstrated that:

(1) *two like charges repel one another, whereas two charges of opposite polarity attract,*

(2) *the force acts along the line joining the charges, and*

(3) *its strength is proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance between them.*

These properties constitute what today is called ***Coulomb's law***, which can be expressed mathematically as

$$\boxed{\mathbf{F}_{e_{21}} = \hat{\mathbf{R}}_{12}\,\frac{q_1 q_2}{4\pi\varepsilon_0 R_{12}^2} \quad \text{(N)} \quad \text{(in free space)},} \tag{1.7}$$

where $\mathbf{F}_{e_{21}}$ is the ***electrical force*** acting on charge q_2 due to charge q_1 when both are in ***free space*** (vacuum), R_{12} is the distance between the two charges, $\hat{\mathbf{R}}_{12}$ is a unit vector pointing from charge q_1 to charge q_2 (Fig.1-5), and ε_0 is a universal constant called the ***electrical permittivity of free space*** [$\varepsilon_0 = 8.854 \times 10^{-12}$ farad per meter (F/m)]. The two charges are assumed to be isolated from all other charges. The force $\mathbf{F}_{e_{12}}$ acting on charge q_1 due to charge q_2 is equal to force $\mathbf{F}_{e_{21}}$ in magnitude, but opposite in direction: $\mathbf{F}_{e_{12}} = -\mathbf{F}_{e_{21}}$.

The expression given by Eq. (1.7) for the electrical force is analogous to that given by Eq. (1.2) for the gravitational force, and we can extend the analogy further by defining the existence of an ***electric field intensity*** $\mathbf{E}$ due to any charge q as

$$\boxed{\mathbf{E} = \hat{\mathbf{R}}\,\frac{q}{4\pi\varepsilon_0 R^2} \quad \text{(V/m)} \quad \text{(in free space)},} \tag{1.8}$$

where R is the distance between the charge and the observation point, and $\hat{\mathbf{R}}$ is the radial unit vector pointing away from the charge. Figure 1-6 depicts the electric-field lines due to a positive charge. For reasons that will become apparent in later chapters, the unit for $\mathbf{E}$ is volt per meter (V/m).

If any point charge q' is present in an electric field $\mathbf{E}$ (due to other charges), the point charge will experience a force acting on it equal to $\mathbf{F}_e = q'\mathbf{E}$.

Electric charge exhibits two important properties. The first is encapsulated by the ***law of conservation of electric charge***, which states that ***the (net) electric charge can neither be created nor destroyed.*** If a volume contains n_p protons and n_e electrons, then its total charge is

$$q = n_p e - n_e e = (n_p - n_e)e \qquad \text{(C)}. \tag{1.9}$$

Even if some of the protons were to combine with an equal number of electrons to produce neutrons or other elementary particles, the net charge q remains unchanged. In matter, the quantum mechanical laws governing the behavior of the protons inside the atom's nucleus and the electrons outside it do not allow them to combine.

The second important property of electric charge is embodied by the ***principle of linear superposition***, which states that ***the total vector electric field at a point in space due to a system of point charges is equal to the vector sum of the electric fields at that point due to the individual charges.*** This seemingly simple concept will allow us in future chapters to compute the electric field due to complex distributions of charge without having to be concerned with the forces acting on each individual charge due to the fields by all of the other charges.

The expression given by Eq. (1.8) describes the field induced by an electric charge residing in free space. Let us now consider what happens when we place a positive point charge in a material composed of atoms. In the absence of the point charge, the material is electrically neutral, with each atom having a positively charged nucleus surrounded by a cloud of electrons of equal but opposite polarity. Hence, at any point in the material not occupied by an atom the electric field $\mathbf{E}$ is zero. Upon placing a point charge in the material, as shown in Fig. 1-7, the atoms experience forces that cause them to become distorted. The center of symmetry of the electron cloud is altered with respect to the nucleus, with one pole of the atom becoming positively charged relative to the other pole. Such a polarized atom is called an ***electric dipole***, and the distortion process is called ***polarization***. The degree of polarization depends on the distance between the atom and the isolated point charge, and the orientation of the dipole is such that the axis connecting its two poles is directed toward the point charge, as illustrated schematically in Fig. 1-7. The net result of this polarization process is that the electric fields of the dipoles of the atoms (or molecules) tend to counteract the field due to the point charge. Consequently, the electric field at any point in the material is different from the field that would have been induced by the point charge in the absence of the material. To extend Eq. (1.8) from the free-space case to any medium, we replace the permittivity of free space ε_0 with ε, where ε is the permittivity of the material in which the electric field is measured and is

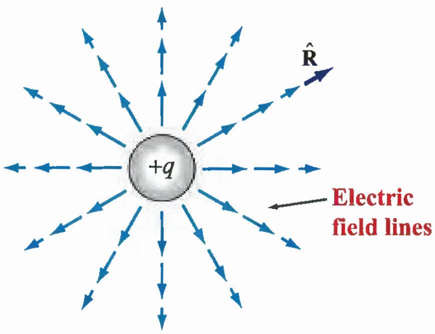

Figure 1-6: Electric field $\mathbf{E}$ due to charge q.

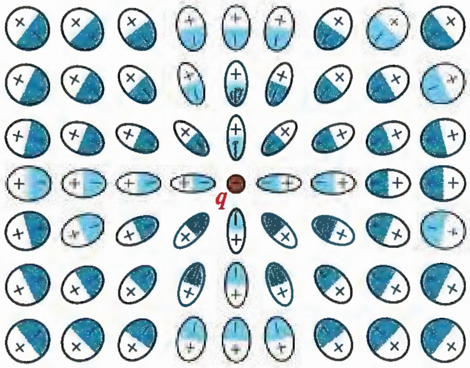

Figure 1-7: Polarization of the atoms of a dielectric material by a positive charge q.

therefore characteristic of that particular material. Thus,

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \qquad \text{(V/m)}. \qquad (1.10)$$

Often, ε is expressed in the form

$$\varepsilon = \varepsilon_r \varepsilon_0 \qquad \text{(F/m)}, \qquad (1.11)$$

where ε_r is a dimensionless quantity called the ***relative permittivity*** or ***dielectric constant*** of the material. For vacuum, $\varepsilon_r = 1$; for air near Earth's surface, $\varepsilon_r = 1.0006$; and the values of ε_r for materials that we will have occasion to use in this book are tabulated in Appendix B.

In addition to the electric field intensity **E**, we will often find it convenient to also use a related quantity called the ***electric flux density*** **D**, given by

$$\boxed{\mathbf{D} = \varepsilon \mathbf{E} \qquad \text{(C/m}^2\text{)}, \qquad (1.12)}$$

with unit of coulomb per square meter (C/m²). These two electric quantities, **E** and **D**, constitute one of two fundamental pairs of electromagnetic fields. The second pair consists of the magnetic fields discussed next.

1-3.3 Magnetic Fields

As early as 800 B.C., the Greeks discovered that certain kinds of stones exhibit a force that attracts pieces of iron. These stones are now called ***magnetite*** (Fe_3O_4) and the phenomenon they exhibit is known as ***magnetism***. In the thirteenth century, French scientists discovered that when a needle was placed on the surface of a spherical natural magnet, the needle oriented itself along different directions for different locations on the magnet. By mapping the directions indicated by the needle, it was determined that the magnetic force formed magnetic-field lines that encircled the sphere and appeared to pass through two points diametrically opposite to each other. These points, called the ***north and south poles*** of the magnet, were found to exist for every magnet, regardless of its shape. The magnetic-field pattern of a bar magnet is displayed in Fig. 1-8. It was also observed that like poles of different magnets repel each other and unlike poles attract each other. This attraction–repulsion property is similar to the electric force between electric charges, except for one important difference: ***electric charges can be isolated, but magnetic poles always exist in pairs.*** If a permanent magnet is cut into small pieces, no matter how small each piece is, it will always have a north and a south pole.

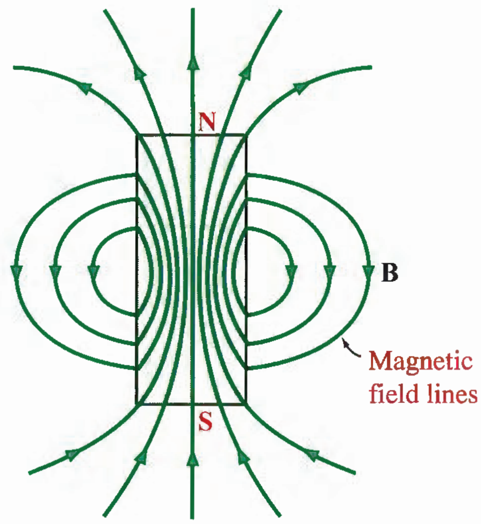

Figure 1-8: Pattern of magnetic field lines around a bar magnet.

The magnetic lines surrounding a magnet represent the ***magnetic flux density*** **B**. A magnetic field not only exists around permanent magnets but can also be created by electric current. This connection between electricity and magnetism was discovered in 1819 by the Danish scientist Hans Oersted (1777–1851), who found that an electric current in a wire caused a compass needle placed in its vicinity to deflect and that the needle turned so that its direction was always perpendicular to the wire and to the radial line connecting the wire to the needle. From these observations, he deduced that the current-carrying wire induced a magnetic field that formed closed circular loops around the wire (Fig. 1-9). Shortly after Oersted's discovery, French scientists Jean Baptiste Biot and Felix Savart developed an expression that relates the magnetic flux density **B** at a point in space to the current I in the conductor. Application of their formulation, known today as the ***Biot–Savart law***, to the situation depicted in Fig. 1-9 for a very long wire residing in free space leads to the result that the ***magnetic flux density*** **B** induced by a constant current I flowing in the z-direction is given by

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} \qquad \text{(T)}, \qquad (1.13)$$

where r is the radial distance from the current and $\hat{\boldsymbol{\phi}}$ is an azimuthal unit vector expressing the fact that the magnetic field direction is tangential to the circle surrounding the current (Fig. 1-9). The magnetic field is measured in tesla (T), named