Logic and Databases, COMP0009,

From a previous exam question.

Marks for each part of each question are indicated in square brackets.

Calculators are NOT permitted.

1. This question is about first-order tableaus.

   a. For each of these formulas state if the formula is an α-formula, β-formula, δ-formula, γ-formula, or a literal.

      1. $\neg P^2(x, y)$
         literal
      2. $\exists x(P^2(x, y) \vee P^2(y, x))$
         δ
      3. $\neg\exists x\forall y P'^2(x, y)$
         γ
      4. $\neg(\forall x P^2(x, x) \vee \neg\exists y P^2(x, y))$
         α

   [8 marks]

   b. Explain how to expand a δ-formula in a tableau.
   Pick a new constant c not occuring so far in the tableau. For $\exists x\phi(x)$ add a new node $\phi(c)$ at every leaf below the current node and for $\neg\forall x\phi(x)$ add new node $\neg\phi(c)$ at every leaf below the current node. Tick the current node.

   [7 marks]

   c. Describe a good method of scheduling the expansion of nodes in a tableau. In particular, say which nodes should be expanded first and how you should schedule the expansion of γ nodes.
   It is important that every possible expansion occurs eventually, else a tableau that could close might never close. Idea is to expand α, β and δ nodes first. Then, for all γ nodes put them in a queue and take them in turn. Make sure eventually all closed terms are used to expand a given γ node.

   [8 marks]

[Question 1 cont. over page]

d. For each of these formulas construct a tableau with the formula at the root and state whether the formula is satisfiable or not.

1. $(\forall x \forall y(P^2(x, y) \to \neg P^2(y, x)) \wedge \exists x P^2(x, x))$

$$
\begin{array}{c}
\forall x\forall y(P(xy) \to \neg P(yx)) \wedge \exists x P(xx) \\
| \\
\forall x\forall y(P(xy) \to P(yx)) \\
| \\
\exists x P(xx) \\
| \\
P(aa) \\
| \\
\forall y(P(ay) \to \neg P(ya)) \\
\diagup \qquad \diagdown \\
\underline{\neg P(aa)} \qquad \underline{\neg P(aa)}
\end{array}
$$

no

2. $(\exists x Q^1(x) \wedge \forall x \exists y P^2(x, y))$.

yes

3. $\exists x \forall y P^2(x, y) \wedge \neg \forall x \exists y P^2(y, x)$.

no

[10 marks]

[Total=33 marks]