1.a.i) True

ii) False

iii) False

iv) True

v) False

vi) True

vii) False

iii) True

ix) True

x) True

xi) False

xii) True

1.b)
```
SELECT COUNT(c.custID), state
FROM Orders as o LEFT JOIN Customers as c
GROUP BY state
WHERE 0 < ( SELECT SUM(quantity)
            FROM lineitem as li LEFT JOIN items as i
            WHERE i.description = 'umbrella' AND o.orderID = li.orderID)
HAVING COUNT(c.custID) > 0
```

1.c.i) TRUE

1.c.ii) TRUE

1.c.iii) FALSE

1.c.iv) TRUE

1.c.v) TRUE

2.a.1) $\forall x \forall y (P^2(x,y) \to \neg P^2(y,x)) \land \exists x P^2(x,x)$ ✓ Not satisfiable

$\forall x \forall y (P^2(x,y) \to \neg P^2(y,x))$ ✓

$\exists x P^2(x,x)$ ✓

$P^2(a,a)$

$\forall y (P^2(a,y) \to \neg P^2(y,a))$ ✓

$P^2(a,a) \to \neg P(a,a)$ ✓

$\neg P^2(a,a) \qquad \neg P(a,a)$

$\times \qquad \times$

2.a.2) $\exists x Q(x) \land \forall x \exists y P(x,y)$ ✓ Satisfiable

$\exists x Q(x)$ ✓

$\forall x \exists y P(x,y)$

$Q(a)$

$\exists y P(a,y)$ ✓

$P(a,b)$

2.a.3) $\exists x \forall y P^2(x,y) \wedge \neg \forall x \exists y P^2(y,x)$ Not Satisfiable

$\exists x \forall y P^2(x,y)$ ✓

$\neg \forall x \exists y P^2(y,x)$ ✓

$\neg \exists y P^2(y,a)$ ✓

$\forall y P^2(b,y)$ ✓

$P(b,a)$

$\neg P(b,a)$

×

2.a.4) $\exists x \exists y \exists z (P^2(x,y) \wedge P^2(y,z) \wedge \neg P^2(x,z))$ ✓ Satisfiable

$\exists y \exists z (P^2(a,y) \wedge P^2(y,z) \wedge \neg P^2(a,z))$ ✓

$\exists z (P^2(a,b) \wedge P^2(b,z) \wedge \neg P^2(a,z))$ ✓

$P^2(a,b) \wedge P^2(b,c) \wedge \neg P^2(a,c)$ ✓

$P^2(a,b)$

$P^2(b,c) \wedge \neg P^2(a,c)$ ✓

$P^2(b,c)$

$P^2(a,c)$

2.b.1) $\neg \exists x P(x)$

$= \forall x \neg P(x)$

2.b.2) $\neg \exists x \exists y \exists z (Q(x,y) \rightarrow Q(z,y))$

$= \forall x \forall y \forall z \neg (Q(x,y) \rightarrow Q(z,y))$

2.b.3) $(\forall x P(x) \rightarrow \exists y \exists z (Q(y,z) \vee Q(z,y))$

$= \forall x \exists x \exists z (P(x) \rightarrow (Q(y,z) \vee Q(z,y)))$

2.b.4) $(\forall x (P(x) \rightarrow P(f(x))) \wedge \neg \exists x Q(x, f(x)))$

$= \forall x_1 \forall x_2 \left((P(x_1) \rightarrow P(f(x_1))) \wedge \neg Q(x_2, f(x_2)) \right)$

2.c) $\exists x_0 \exists x_1 \cdots \exists x_n \left(x_1 \neq x_0 \wedge x_2 \neq x_1 \wedge x_2 \neq x_0 \wedge \cdots \wedge x_0 \neq x_{n-1} \wedge E(x_0, x_1) \wedge E(x_1, x_2) \wedge \cdots \wedge E(x_{n-1}, x_0) \right)$

2.d) Assume contradiction that Σ exists for any G

$G \models \Sigma \leftrightarrow G$ has cycles of $n \geq 3$

ϕ_n = cycle of lenght $< n$ exists

$G \models \Sigma \cup \{\neg \phi_n, n = 1, 2 \ldots\}$ by compactness is unsatisfiable

However, it is satisfiable with cycle length $n+1$

$\downarrow$

contradiction, Σ does not exist

3.a.1) $\neg F d$

3.a.2) $\phi \wedge \phi' \Leftrightarrow \neg(\neg\phi \vee \neg\phi')$

$(\neg P y) \wedge F(y \wedge m)$

3.a.3) $\phi \to \phi' \Leftrightarrow \neg\phi \vee \phi'$

$\neg l \longrightarrow F(b)$

3.b.1) $\Diamond p \wedge \Diamond(\neg p) \wedge \Box(p \to \Diamond p)$

satisfiable $W: \{w_0, w_1\}$
$R: \{(w_0, w_0), (w_0, w_1)\}$
$v(p): \{w_0\}$

3.b.2) $\underline{\underline{\Box p}} \wedge \Diamond(\Diamond\neg q \wedge \underline{\underline{\Box(\neg p \vee q)}})$

not satisfiable

3.b.3) $\Box(p \to \Diamond(\neg p)) \wedge \Box(\neg p \to \Diamond p)$

satisfiable $W: \{w_0\}$
$R: \{\}$
$v(p): \{\}$

3.b.4) $\Box(p \to \Diamond(\neg p)) \wedge \Box(\neg p \to \Diamond p) \wedge \Diamond(\Box p)$

not satisfiable

3.c.i) $\{y\}$ satisfies

3.c.ii) $\{x\}$ satisfies

3.c.iii) $\{x, z\}$ satisfies