Cambridge University Engineering Department
Engineering Tripos Part IIA
PROJECTS: Interim and Final Report Coversheet

TO BE COMPLETED BY THE STUDENT(S)

Project:	SA1 - Aircraft Wing Analysis
Title of report:	SA1 - Aircraft Wing Analysis Second Interim Report Group Report / ~~Individual Report~~ (delete as appropriate)

Name(s): (capitals)	crsID(s):	College(s):
SCHOEN CAO	zc282	Gonville & Caius
KAI TAN	ykt25	Wolfson

Declaration for: ~~Interim Report 1~~ / Interim Report 2 / ~~Final Report~~ (delete as appropriate)

I/we confirm that, except where indicated, the work contained in this report is my/our own original work.

Instructions to markers of Part IIA project reports:

Grading scheme

Grade	A / A*	B	C	D	E
Standard	Very Good / Excellent	Good	Acceptable	Minimum acceptable for Honours	Below Honours

Grade the reports on the scale A* to D by marking the appropriate Overall Assessment box, and provide feedback against as many of the criteria as are applicable (or add your own). Feedback is particularly important for work graded C-E. Students should be aware that different projects and reports will require different characteristics.

Penalties for lateness: Interim Reports: 3 marks per weekday; Final Reports: 0 marks awarded – late reports not accepted.

Overall assessment (circle grade)	**A***	**A**	**B**	**C**	**D**	**E**
Guideline standard	> 80%	70-80%	60-70%	50-60%	40-50%	< 40%

Marker:		Date:	

Delete (1) or (2) as appropriate (for marking in hard copy – different arrangements apply for feedback on Moodle):

(1) Feedback from the marker is provided on the report itself.

(2) Feedback from the marker is provided on second page of cover sheet.

	Typical Criteria	Feedback comments
Project Skills, Initiative, Originality	Appreciation of problem, and development of ideas	
	Competence in planning and record-keeping	
	Practical skill, theoretical work, programming	
	Evidence of originality, innovation, wider reading (with full referencing), or additional research	
	Initiative, and level of supervision required	
Report	Overall planning and layout, within set page limit	
	Clarity of introductory overview and conclusions	
	Logical account of work, clarity in discussion of main issues	
	Technical understanding, competence and accuracy	
	Quality of language, readability, full referencing of papers and other sources	
	Clarity of figures, graphs and tables, with captions and full referencing in text	

Project SA1 - Aircraft Wing Analysis Second Report

Schoen Cao (zc282), Kai Tan (ykt25)

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1 Exercise 1

Listing 1: ueintbit.m

```
function f = ueintbit(xa,ua,xb,ub)

    %Calculate for variables
    ubar = (ua+ub)/2;
    du = ub-ua;
    dx = xb-xa;

    %Solve for integral
    f = (ubar^5 + 5/6*ubar^3*(du)^2 + 1/16*ubar*(du)^4)*dx;
end
```

Listing 2: Script for Exercise 1

```
clear
close all

%Input variables
ReL = 2500;
x = linspace(0,1,101);
ue = 1;

%Iterate for theta/L
for i=1:length(x);
    theta(i) = sqrt(.45/ReL*(ue)^-6*ueintbit(0,ue,x(i),ue));
end

%Plot of Analytical Solution
hold on
plot(x,theta);
xlabel('Non-dimensional position, x/L');
ylabel('Non-dimensional momentum thickness, theta/L');
title('Non-dimensional Momentum Thickness against Non-dimensional Position')

%Calculate Blasius Solution
thetab = 0.664/ReL^.5 .* (x).^.5;

%Plot Blasius Solution
plot(x,thetab);
legend('Analytical','Blasius','location','Southeast');
hold off
```

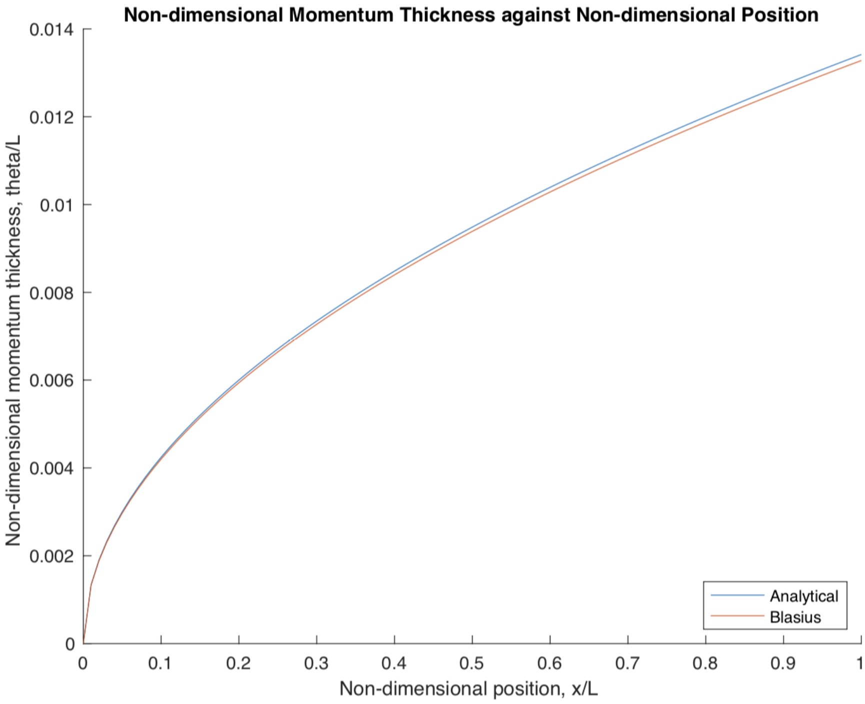

Figure 1: Plot of Non-Dimensional Momentum Thickness Against Non-Dimensional Position

2 Exercise 2

Listing 3: Script for Exercise 2

```
clear
close all

%Input Conditions, change according to required by Exercise
Re = 1e6;
duedx = -.2;

%Initial Conditions and Discretisation steps
n = 101;
laminar = true;
x = linspace(0,1,n);
ue0 = 1;

%Initialise transition and separation indicators
int = 0;
ils = 0;

%Generate a matrix of ue values
for i = 1:n;
    ue(i) = duedx*x(i)+ue0;
end

%To initialise loop
i = 1;
while laminar && i < n; %laminar loop
    i = i+1;    %Increase interation counter

    %Solve for theta, Rethet, m, H, He
    theta(i) = sqrt(.45/Re*(ue(i))^-6*ueintbit(0,ue(1),x(i),ue(i)));
    Rethet = theta(i)*Re*(ue(i));
    m = -Re*(theta(i))^2*duedx;
    H = thwaites_lookup(m);
    He(i) = laminar_He(H);

    %Check for transition
    if log(Rethet) >= 18.4*He(i) - 21.74;
        laminar = false;    %Flow no longer laminar
        int = i;    %Save iteration when transition occurs
        display([x(i), Rethet/1000]);
    end
end
```

$d(u_e/U)/d(x/L)$	Re_L (10^6)	x/L	Re_θ (10^3)
-0.2	1	0.58	0.5666
-0.2	10	0.24	1.0802
-0.2	100	0.04	1.3498
0	10	0.37	1.2903
0	100	0.04	1.3416
0.2	100	0.04	1.3337

Table 1: Transition Locations and Corresponding Re_θ for Various Conditions

3 Exercise 3

Listing 4: Script for Exercise 3

```
clear
close all

%Input Conditions, change according to required by Exercise
Re = 1e6;
duedx = -.5;

%Initial Conditions and Discretisation steps
n = 101;
laminar = true;
x = linspace(0,1,n);
ue0 = 1;

%Initialise transition and separation indicators
int = 0;
ils = 0;

%Generate a matrix of ue values
for i = 1:n;
    ue(i) = duedx*x(i)+ue0;
end

%To initialise loop
i=1;
while laminar && i < n; %laminar loop
    i = i+1;    %Increase interation counter

    %Solve for theta, Rethet, m, H, He
    theta(i) = sqrt(.45/Re*(ue(i))^-6*ueintbit(0,ue(1),x(i),ue(i)));
    Rethet = theta(i)*Re*(ue(i));
    m = -Re*(theta(i))^2*duedx;
    H = thwaites_lookup(m);
    He(i) = laminar_He(H);

    %Check for transition
    if log(Rethet) >= 18.4*He(i) - 21.74;   %Transition condition
        laminar = false;    %Flow no longer laminar
        int = i;    %Save iteration where transition occurs

    %Check for separation
    elseif m >= 0.09;   %Separation Condition
        laminar = false;    %Flow no longer laminar
        ils = i;    %Save iteration where transition occurs
    end
end

if int ~= 0;
    disp(['Natural transition at ' num2str(x(int)) ' with Rethet ' num2str(Rethet)]);
elseif ils ~= 0;
    disp(['Laminar seperation at ' num2str(x(ils)) ' with Rethet ' num2str(Rethet)]);
end
```

Re_L	Separation Location (x/L)
10^4	0.25
10^5	0.25
10^6	0.25

Table 2: Separation Location for Various Flow Conditions

Listing 5: Script for Exercise 3 to Find Re_L at which Transition Supplants Laminar Separation

```
clear
close all

%Input Conditions, change according to required by Exercise
Re = linspace(1e6,1e7,9001);
duedx = -.5;

%Initial Conditions and Discretisation steps
n = 101;
x = linspace(0,1,n);
ue0 = 1;

%Generate a matrix of ue values
for i = 1:n;
    ue(i) = duedx*x(i)+ue0;
end

%To initialise loop
for k = 1:length(Re);
    i=1;

    %Initialise indicators at the start of every loop
    int = 0;
    ils = 0;

    %Reset laminar flag at the start of every loop
    laminar = true;

    %Laminar loop
    while laminar && i < n;
        i = i+1;    %Increase interation counter

        %Solve for theta, Rethet, m, H, He
        theta(i) = sqrt(.45/Re(k)*(ue(i))^-6*ueintbit(0,ue(1),x(i),ue(i)));
        Rethet = theta(i)*Re(k)*(ue(i));
        m = -Re(k)*(theta(i))^2*duedx;
        H = thwaites_lookup(m);
        He(i) = laminar_He(H);

            %Check for transition
            if log(Rethet) >= 18.4*He(i) - 21.74;    %Transition condition
                laminar = false;    %Flow no longer laminar
                int = i;    %Save iteration where transition occurs

            %Check for separation
            elseif m >= 0.09;    %Separation Condition
                laminar = false;    %Flow no longer laminar
                ils = i;    %Save iteration where transition occurs
            end
    end

    %Display Re at which transition will supplant laminar separation
    if int ~= 0; %If transition occurs before separation, ils = 0
        disp(['Re_L at which transition supplants laminar transion is '...
        ,num2str(Re(k))])
        break %break loop once required Re is found
    end
end
```

From this code, it was found that the minimum Re_L for transition to supplant laminar separation is 1.792×10^6

4 Exercise 4

Listing 6: thickdash.m

```
function dthickdx = thickdash(xmx0,thick);
    %import global variables
    global Re ue0 duedx

    %find theta and deltaE as defined
    theta = thick(1);
    deltaE = thick(2);

    %calculate He
    He = deltaE/theta;

    %calculate H
    if He >= 1.46;
        H = (11*He+15) / (48*He-59);
    else
        H = 2.803;
    end

    %calculate ue
    ue = ue0 + duedx * xmx0;

    %calculate Re_theta
    Rethet = Re * ue * theta;

    %calculate cf
    cf = 0.091448*((H-1)*Rethet)^(-.232)*exp(-1.260*H);

    %calculate cdiss
    cdiss = 0.010019*((H-1)*Rethet)^(-1/6);

    %calculate f
    dthickdx = zeros(2,1);
    dthickdx(1) = cf/2 - (H+2) / ue * duedx * theta;
    dthickdx(2) = cdiss - 3/ue * duedx * deltaE;
end
```

Listing 7: Script for Exercise 4

```
clear
close all

%Defining global variables value
global Re ue0 duedx

Re = 1e7;
ue0 = 1;
duedx = 0;

%Initial values of theta and delta_E
x0 = 0.01;
thick0(1) = 0.037*x0*(Re*x0)^(-1/5);
thick0(2) = 1.80 * thick0(1);

[delx thickhist] = ode45(@thickdash,[0 0.99],thick0);

for i = 1:length(delx);

    theta_7(i) = 0.037 * (x0+delx(i)) * (Re * (x0+delx(i)))^(-1/5);
    theta_9(i) = 0.023 * (x0+delx(i)) * (Re * (x0+delx(i)))^(-1/6);
end
```

```
hold on
plot(delx+x0,thickhist(:,1));
plot(delx+x0,theta_7);
plot(delx+x0,theta_9);
xlabel('non dimensional position x/L');
ylabel('non dimensional momentum thickness \theta/L');
legend('Differential equation','1/7^{th} Power Law Estimate','1/9^{th} Power Law ...
Estimate','location','Southeast');
hold off
```

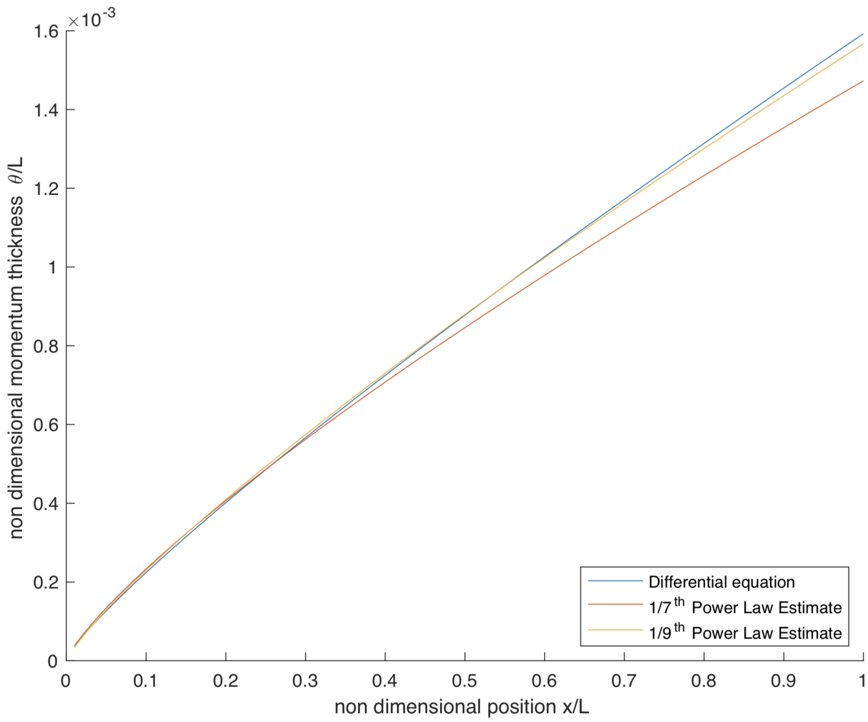

Figure 2: Non-Dimensional Momentum Thickness against Non-Dimensional Position for Various Solutions

5 Exercise 5

Listing 8: Script for Exercise 5

```
clear
close all

%Defining global variables value
global Re ue0 duedx

Re = 1e7;
ue0 = 1;
duedx = -.3;

%Boundary values of theta and delta_E
x0 = 0.01;
thick0(1) = 0.037*x0*(Re*x0)^(-1/5);
thick0(2) = 1.80 * thick0(1);

[delx thickhist] = ode45(@thickdash,[0 0.99],thick0);

%Calculate He
for i = 1:length(delx);
    He(i) = thickhist(i,2)/thickhist(i,1);
end

%Plotting duedx = -.6
clear He
duedx = -.6;
[delx thickhist] = ode45(@thickdash,[0 0.99],thick0);
for i = 1:length(delx);
    He(i) = thickhist(i,2)/thickhist(i,1);
end

%Plot theta and deltaE
figure(1);
hold on
plot(delx+x0,thickhist(:,1));
plot(delx+x0,thickhist(:,2));
xlabel('non dimensional position x/L');
ylabel('non dimensional thickness');
legend('\theta','\delta_E','location','Northwest');
title(['Re_L=',num2str(Re),' d(u_e/U)/d(x/L)=',num2str(duedx)]);
hold off

%He plot to be done command window and by changing the input variables
```

Listing 9: Command Window Scripts Used to Plot Manually

```
%Plot He
plot([0 1],[1.46 1.46],'--');    %Plot reference value
hold on
plot(delx+x0,He);   %Plot He, repeat for every run with different condition
xlabel('non dimensional position x/L');
ylabel('Energy shape factor H_E');
title(['Re_L=',num2str(Re)]);
plot(delx+x0,He);
xlabel('non dimensional position x/L');
ylabel('Energy shape factor H_E');
title(['Re_L=',num2str(Re)]);
legend('H_E = 1.46 Separation condition','d(u_e/U)/d(x/L) = -0.3',...
'd(u_e/U)/d(x/L) = -0.6','d(u_e/U)/d(x/L) = -0.9','location','Southwest');
```

By plotting the graphs for H_E again x/L using the command window, the turbulent separation for the various flow conditions can be found. This is done by running the code at various conditions, and plotting the H_E for each condition. The list of commands are shown in listing 9. The results are tabulated in Table 3 and Table 4. It should be noted that these values are read off the graph and are approximated values of the actual separation location.

$d(u_e/U)/d(x/L)$	Turbulent Separation Location x/L
-0.3	NA
-0.6	0.81
-0.9	0.54

Table 3: Turbulent Separation Location for Various Velocity Gradient at $Re_L = 10^7$

Re_L	Turbulent Separation Location x/L
10^6	0.73
10^7	0.81
10^8	0.90

Table 4: Turbulent Separation Location for Various Re_L at $d(u_e/U)/d(x/L) = -0.6$

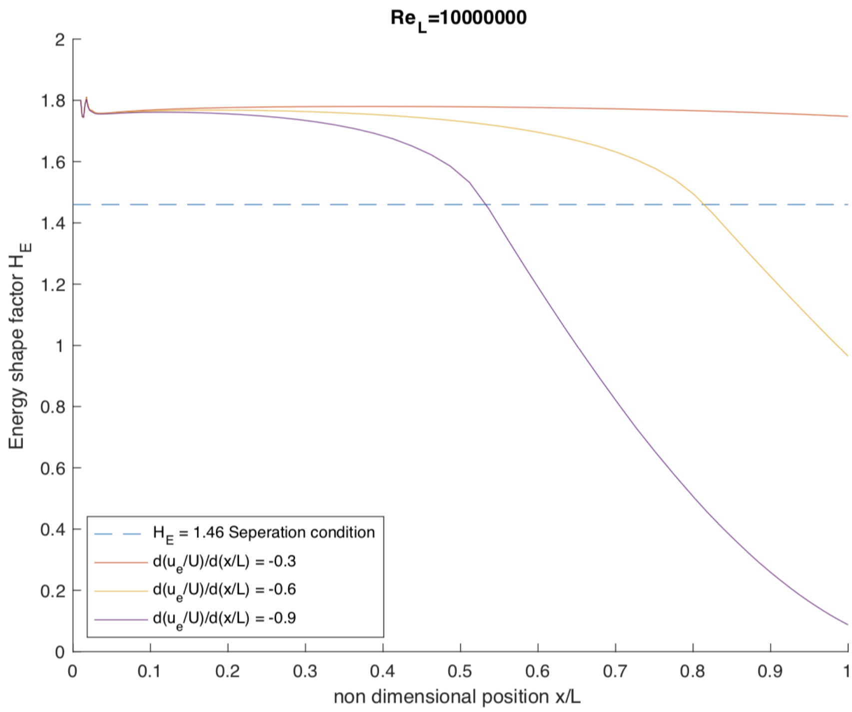

Figure 3: Plot of Turbulent Separation for Various Velocity Gradient at $Re_L = 10^7$

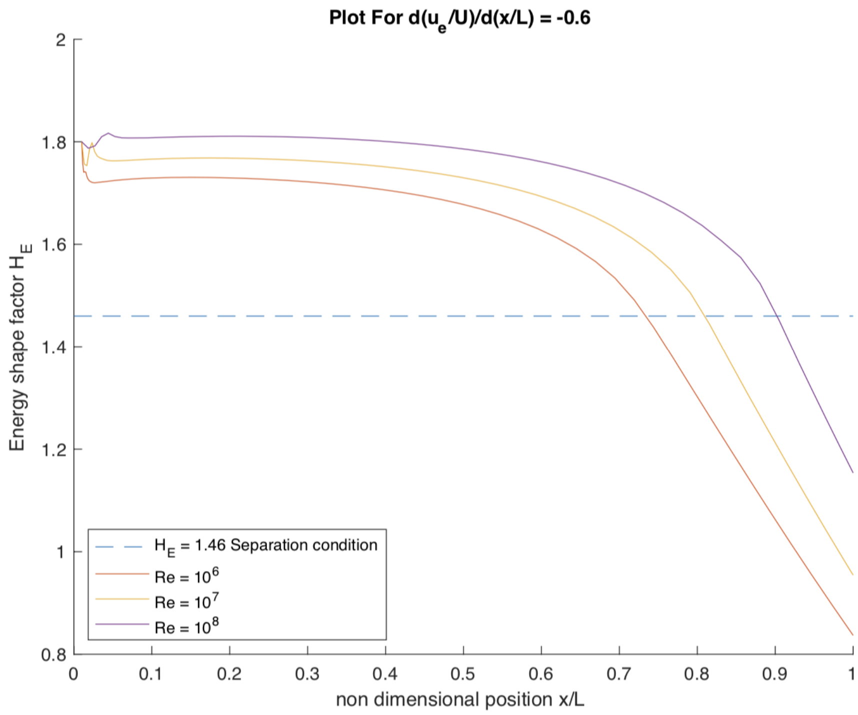

Figure 4: Plot of Turbulent Separation Location for Various Re_L at $d(u_e/U)/d(x/L) = -0.6$

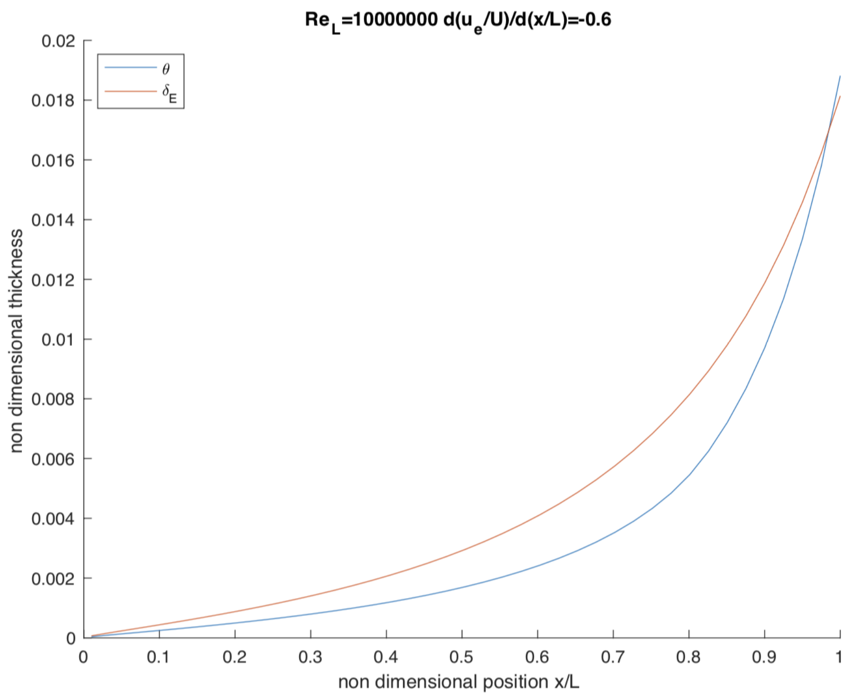

Figure 5: Plot of $\frac{\delta_E}{L}$ and $\frac{\theta}{L}$ Against Non-Dimensional Position

6 Exercise 6

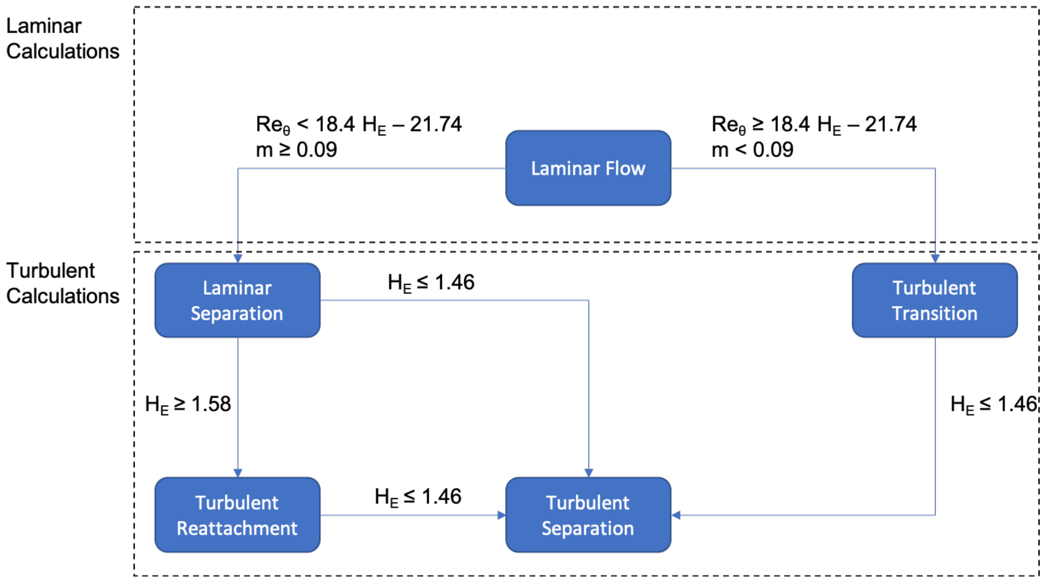

Figure 6: Flow Chart of Development of Flow

Listing 10: Script for Exercise 6

```
clear
close all

%Defining global variables value
global Re ue0 duedx

%Define simulation conditions
Re = 1e5;
duedx = -0.25;
ue0 = 1;

%Iteration setting & initial conditions
n = 101;
laminar = true;
x = linspace(0,1,n);

%%initialsing indicators
int = 0;     %natural transition
ils = 0;     %laminar seperation
itr = 0;     %turbulent reattachment
its = 0;     %turbulent seperation

%generating ue matrix
for i = 1:n;
    ue(i) = duedx*x(i)+ue0;
end

%initialising i
i = 1;
while laminar && i < n; %laminar loop
    i = i+1;     %Iteration counter
    theta(i) = sqrt(.45/Re*(ue(i))^-6*ueintbit(0,ue(1),x(i),ue(i)));
    Rethet = theta(i)*Re*(ue(i));
    m = -Re*(theta(i))^2*duedx;
    H = thwaites_lookup(m);
    He(i) = laminar_He(H);

    %Check for Transition
```

```
    if log(Rethet) >= 18.4*He(i) - 21.74;   %laminar check
        laminar = false;    %laminar flag & end loop
        int = i;    %set pointer
        disp(['Turbulent Transition at x/L = ' num2str(x(int)) ' at Re_L '...
        num2str(Re)]);

    %Check for Laminar Separation
    elseif m >= 0.09;
        laminar = false;    %laminar flag & end loop
        ils = i;
        He(i) = 1.51509;    %set He to separated value
        disp(['Laminar Separation at x/L = ' num2str(x(ils)) ' at Re_L '...
        num2str(Re)])
    end
end

%Value for He for Laminar Flow
He(1) = 1.57258;

%Calculate deltaE matrix
deltaE = He.*theta;

%Turbulent Loop after separation or transition
while its == 0 && i < n;
    thick0(1) = theta(i);   %y matrix, value at elemental plate's start
    thick0(2) = deltaE(i);
    i = i+1;
    [delx thickhist] = ode45(@thickdash,[0,x(i)-x(i-1)],thick0);
    theta(i) = thickhist(length(delx),1);
    deltaE(i) = thickhist(length(delx),2);
    He(i) = deltaE(i)/theta(i);

    %Check for turbulent reattachment
    if ils > 0 && He(i) >= 1.58 && itr == 0;
        itr = i;
        disp(['Turbulent Reattachmemt at x/L = ' num2str(x(itr)) ...
        ' at Re_L ' num2str(Re)]);
    end

    %Check for turbulent separation
    if He(i) <= 1.46;   %turbulent seperation check
        its = i;
        H=2.803;    %H at seperation
        disp(['Turbulent Separation at x/L= ' num2str(x(its)) ...
        ' at Re_L ' num2str(Re)]);
    end
end

while i < n;    %final loop
    theta(i+1) = theta(i)*(ue(i)/ue(i+1))^(H+2);    %theta for cf=0
    i = i+1;
    He(i) = He (its);   %H assumed to remain constant since He is constant
end

%Plot graph using command window as with Ex 5.
```

As mentioned in Exercise 5, the graphs for Exercise 6 is done is plotted manually using the command window using similar commands.

Listing 11: Script for Exercise 6 to Find Critical Velocity Gradient

```
clear
close all

%Defining global variables value
global Re ue0 duedx

%Define simulation conditions
Re = 1e5;
duedxtest = linspace(-0.55,-0.25,31); %Create an array for test gradient
ue0 = 1;

%Iteration setting & initial conditions
n = 101;
laminar = true;
x = linspace(0,1,n);

for k = 1:length(duedxtest); %loop for various velocity gradient conditions
    duedx = duedxtest(k);

    %generating ue matrix
    for i = 1:n;
        ue(i) = duedx*x(i)+ue0;
    end

    %initialising i
    i = 1;
    laminar = true;
    int = 0;    %natural transition
    ils = 0;    %laminar seperation
    itr = 0;    %turbulent reattachment
    its = 0;    %turbulent seperation

    while laminar && i < n; %laminar loop
        i = i+1;    %increase i counter
        theta(i) = sqrt(.45/Re*(ue(i))^-6*ueintbit(0,ue(1),x(i),ue(i)));
        Rethet = theta(i)*Re*(ue(i));
        m = -Re*(theta(i))^2*duedx;
        H = thwaites_lookup(m);
        He(i) = laminar_He(H);

        %Check for Transition
        if log(Rethet) >= 18.4*He(i) - 21.74;
            laminar = false;    %laminar flag & end loop
            int = i;    %set pointer

        %Check for Laminar Separation
        elseif m >= 0.09;
            laminar = false;    %also end laminar loop & turbulent formula
            ils = i;
            He(i) = 1.51509;    %set He to separated value
        end
    end

    %Value for He for Laminar Flow
    He(1) = 1.57258;

    %Calculate deltaE matrix
    deltaE = He.*theta;

    %Turbulent Loop
    while its == 0 && i < n;
        thick0(1) = theta(i);    %y matrix, value at elemental plate's start
```

```
            thick0(2) = deltaE(i);
            i = i+1;
            [delx thickhist] = ode45(@thickdash,[0,x(i)-x(i-1)],thick0);
            theta(i) = thickhist(length(delx),1);
            deltaE(i) = thickhist(length(delx),2);
            He(i) = deltaE(i)/theta(i);

            %Check for turbulent reattachment
            if ils > 0 && He(i) >= 1.58 && itr == 0;
                itr = i;

            end

            %Check for turbulent separation
            if He(i) <= 1.46;    %turbulent seperation check
                its = i;
                H=2.803;     %H at seperation
            end
            if i==101 && its ~=0
                disp(['Critical Velocity Gradient is ' num2str(duedx)]);
                break %break loop if turbulent separation occurs at x/L = 1.
            end

        end
end
```

Critical Velocity Gradient is found to be -0.51 (2 significant figures)

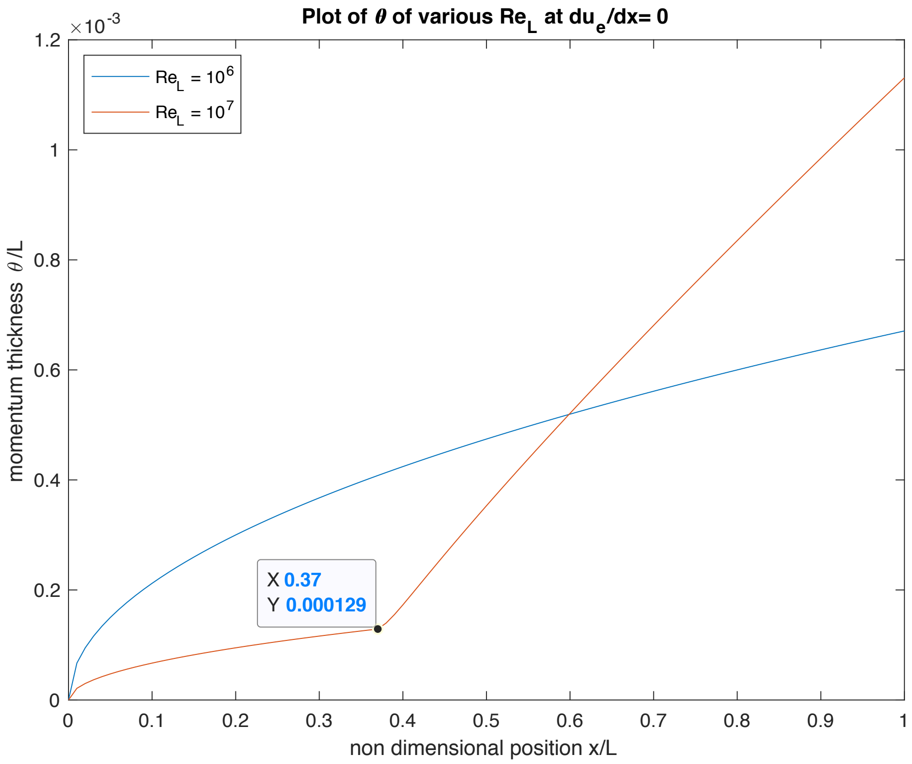

Figure 7: Plot of $\frac{\theta}{L}$ Against $\frac{x}{L}$ for Various Re and d(ue/U)/d(x/L) = 0

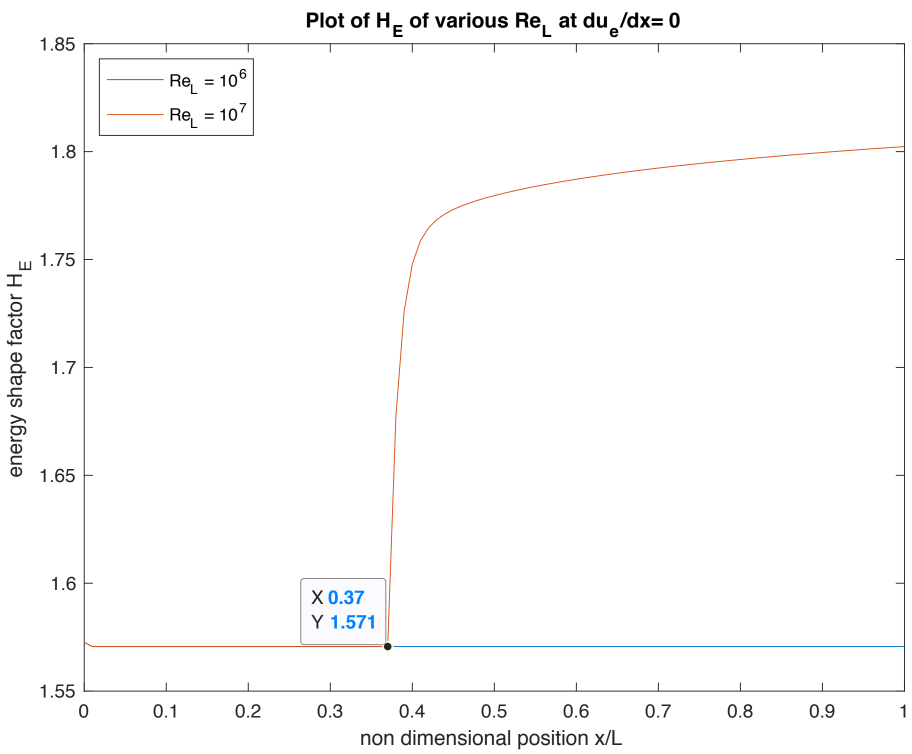

Figure 8: Plot of H_E Against $\frac{x}{L}$ for Various Re and d(ue/U)/d(x/L) = 0

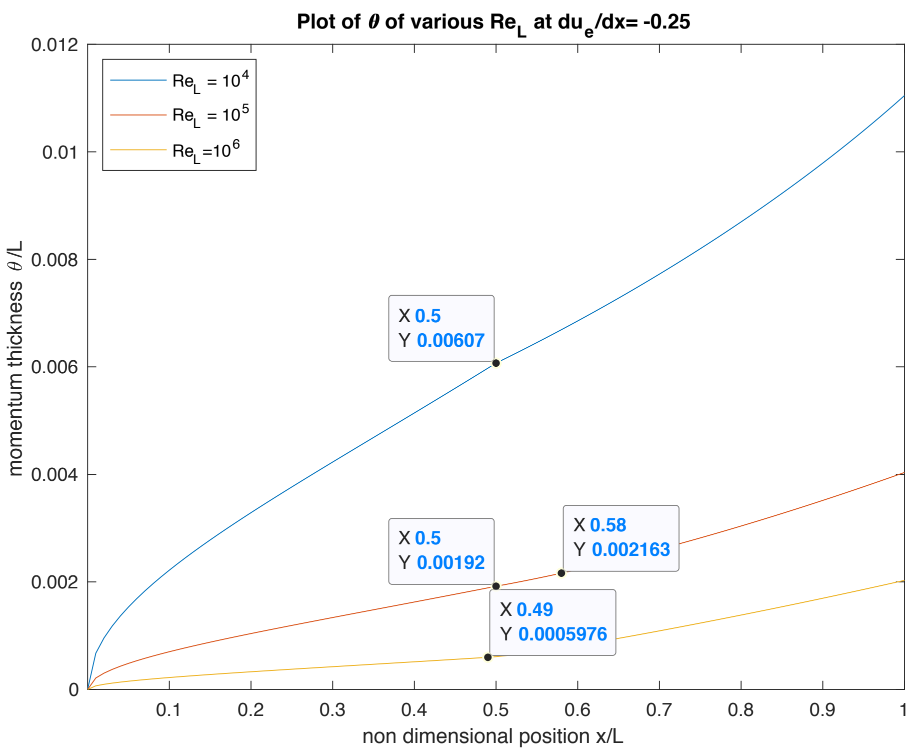

Figure 9: PPlot of $\frac{\theta}{L}$ Against $\frac{x}{L}$ for Various Re and d(ue/U)/d(x/L) = -0.25

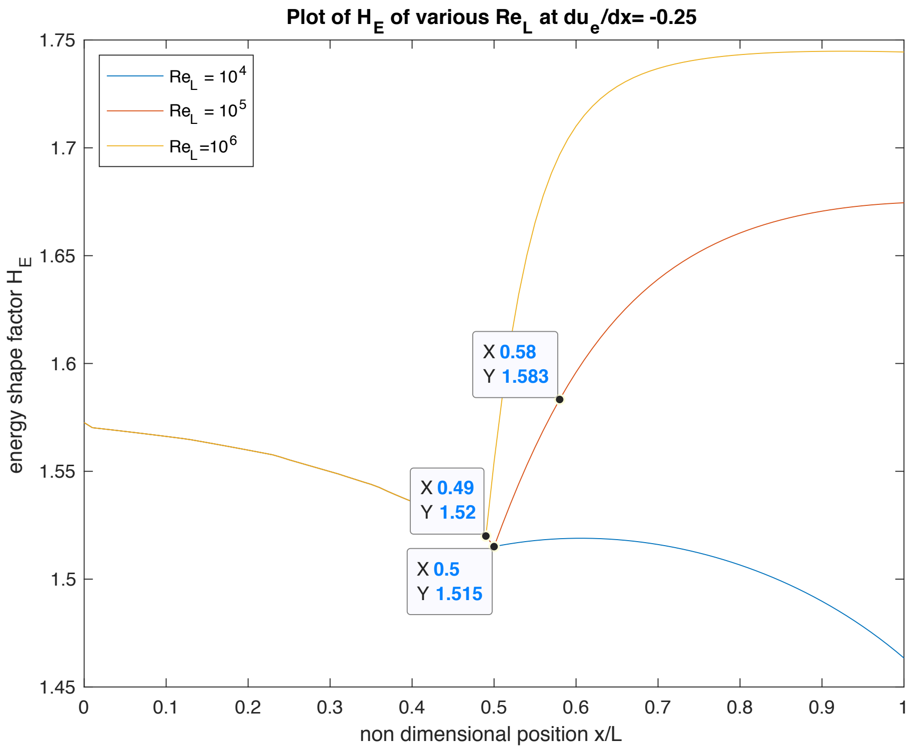

Figure 10: Plot of H_E Against $\frac{x}{L}$ for Various Re and d(ue/U)/d(x/L) = 0