ST003: Statistical Methods for Computer Science

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Question 1. Say we roll a fair 6-sided die six times. Using the fact that each roll is an

independent random event, what is the probability that we roll:

(a) The sequence 1,1,2,2,3,3 ?

The probability for that exact sequence to occur would 1/6 for each roll since they are independent and there is only one exact result required for each roll, thus 1/6 * 1/6 *1/6 *1/6 *1/6 *1/6 = ***0.00002143***

(b) A three exactly 4 times?

For a 3 to be rolled times, four of the rolls must be a 3 (thus 1/6 each) and the other two rolls must not be a 3 (5/6 each) this becomes 1/6 * 1/6 *1/6 *1/6 * 5/6 * 5/6. However, we also have to multiply by the number of permutations that four 3s can occur in six rolls (6Choose4).

0.00053583676* 15 = ***0.00803755144***

(c) A single 1.

Using the same reasoning as before, 1/6 * 5/6 * 5/6 * 5/6* 5/6* 5/6, is the likelihood of the first roll being 1 and the rest as non 1s. We then multiply this by the number of permutations (6) since the one can be any of the six rolls. Thus, 0.06697959533 * 6 = ***0.40187757201***

(d) One or more 1's

The probability of one or more 1s is simply to calculate if we first find the probability of no 1s (5/6 * 5/6 * 5/6 * 5/6* 5/6* 5/6) and subtract from 1 to find the probability of any 1s occurring. Thus,
1 - 0.33489797668 = ***0.66510202332***

Question 2. Suppose one 6-sided and one 20-sided die are rolled. Let A be the event that the first die comes up 1 and B that the sum of the dice is 2. Are these events independent ? Explain using the formal definition of independence.

Independence is defined as $P(A \cap B) = P(A) * P(B)$.

From this definition, we can conclude that these events are dependant on each other since A is itself a subset of B due to the fact that for the sun of two rolls to equal 2, the first roll must equal 1.

Thus, $P(A \cap B) = P(A)$ and since it is impossible for P(B) to equal 1, as $P(A \cap B)$ does not equal $P(A) * P(B)$. The events are ***Dependant***.

Question 3. Say a hacker has a list of n distinct password candidates, only one of which will successfully log her into a secure system.

(a) If she tries passwords from the list uniformly at random, deleting those passwords that do not work, what is the probability that her first successful login will be (exactly) on her k-th try?

Since the hacker removes the passwords as she goes through the probability must start at (n-1)/n and both n-values must be decremented each time since the number of passwords gets smaller.

This gives a probability of (n-1)/n * (n-2)/(n-1) + …. Until we reach k removals (i.e. the correct password) 1/(n-k). Both sides can then be simplified using factorials: giving

((n-1)!/(n-k)!) / (n!/(n-k)!)

(b) When n = 6 and k = 3 what is the value of this probability ?

((6-1)!/(6-3)!) / (6!/(6-3)!) = ***0.16666666666***

(c) Now say the hacker tries passwords from the list at random, but does not delete previously tried passwords from the list. She stops after her first successful login attempt. What is the probability that her first successful login will be (exactly) on her k-th try?

The formula will be similar to the previous answer however the n values will not be decremented since the list of passwords remains the same size thus giving (n-1)/n * (n-1)/n. this will occur k-1 times since the kth attempt will be correct and instead of 1/(n-k) in the previous answer, we don't decrement by k (since passwords are not deleted) giving 1/n at the end.

= ***((n-1)/n)^(k-1)) * (1/n)***

(d) When n = 6 and k = 3 what is the value of this probability ?

Hint: use the fact that the outcome of each try is an independent random event (since passwords are selected uniformly at random at each attempt)

((6-1)/6)^(3-1)) * (1/6) ***= 0.11574074074***

Question 4. A website wants to detect if a visitor is a robot. They decide to deploy three CAPTCHA tests that are hard for robots and if the visitor fails in one of the tests, they are flagged as a possible robot. The probability that a human succeeds at a single test is 0.95, while a robot only succeeds with probability 0.3. Assume all tests are independent.

(a) If a visitor is actually a robot, what is the probability they get flagged?

The probability of a robot getting flagged can be calculated by subtracting the probability of a robot not getting flagged from 1. Since a robot succeeds each test with probability 0.3 , the probability of a robot succeeding each test (and thus not being flagged) is 0.3 * 0.3 * 0.3. The probability of a robot being flagged must then be 1 – 0.027 = ***0.973***

(b) If a visitor is human, what is the probability they get flagged?

The probability of a human not being flagged is 0.95 * 0.95 * 0.95 (0.95 for each test). Thus, the probability of a human being flagged is 1 - 0.857375 = ***0.142625***

(c) The fraction of visitors on the site that are robots is 1/10. Suppose a visitor gets flagged. What is the probability that visitor is a robot? Hint: use Bayes Rule

Bayes law states that : P(E|F) =(P(F|E)P(E)) / P(F)

From the question we can state that E is the event in which the visitor is a robot (prior), and F is the event in which a visitor is flagged (evidence).

P(E) = 0.1 //given in question ,

P(F) = 0.973(0.1) + 0.142625(0.9) = 2256625 // probability of robot flagged + human flagged

P(F|E) = 0.973 // calculated earlier, probability of a robot being flagged

Finally, we can find P(E|F) by using (P(F|E) * P(E)) / P(F) = (0.973 * 0.1)/ 0.2256625

Thus P(E/F) = ***0. 431174874***