ST003: Statistical Methods for Computer Science

Assignment 4 Thomas Fowley

Question 1. Consider an experiment where we roll two 6-sided dice. Let random variable Y be the sum of the values rolled. The sample space is {(1, 1), (1, 2),(1, 3), . . .,(6, 6)} and recall that a random event is a subset of the sample space.

(a) What random event corresponds to Y = 2?

Y=2 corresponds to the event of the dice rolls value being equal to 2. There is only one element in this subset: {(1,1)} i.e. both the dice landed on 1 to make two.

(b) What event corresponds to Y = 3?

Y=3 can be found using the same logic as part (a), the subset of S where the dice roll values equal 3 i.e. {(1,2), (2,1)}

(c) What event corresponds to Y = 4?

Y=4 is the subset of S where the dice rolls equal 4 i.e. {(1,3), (3,1),(2,2)}

(d) Now let X be the indicator random variable associated with the event {(1, 1),(2, 2),(3, 3)}. What is the probabilities that X = 1?

X = 1 if one of the elements of X occurs. The probability of this is 3 / (6*6) (The number of elements in X/The number of elements in S) = ***0.083***

Question 2. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 3 times.

(a) What are the possible values of X?

The four possible outcomes are: {(no heads),(one heads),(two heads),(three heads)}

This corresponds to -3, -1, 1 , 3 Since we subtract 1 for each tail and add 1 for each head (starting from 0)

(b) What is P(X = −3) ?

The Probability of three tails is ½ * ½ * ½ = ***0.125***

(c) What is P(X = −1) ?

The probability of one head and two tails is ½ * ½ * ½ however this must be multiplied by (3choose1) in order to get all permutations ({(HTT), (THT), (TTH)}) = ***0.375***

(d) If the coin is assumed fair, calculate the PMF and CDF of X and plot a sketch of both.

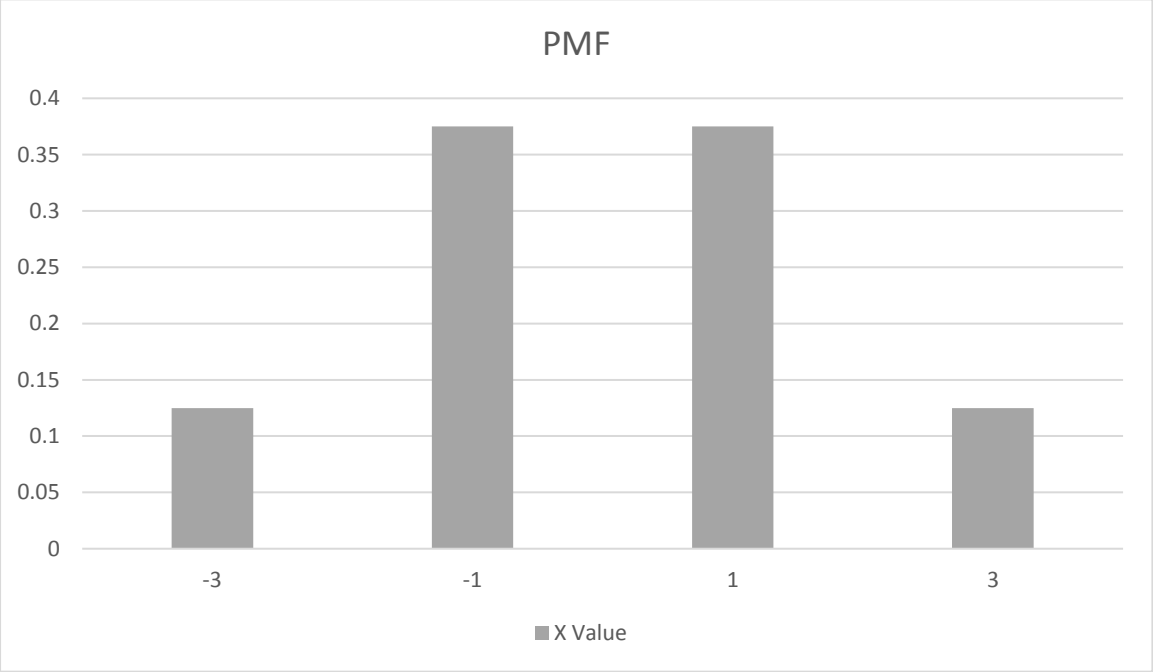
P(X = −3) = 0.125 // found in part (b)

P(X = −1) = 0.375 // found in part (c)

P(X = 1) = 0.375 // the same probability as P(X = −1)

P(X = 3) =0.125 // the same probability as P(X = −3)

PMF)



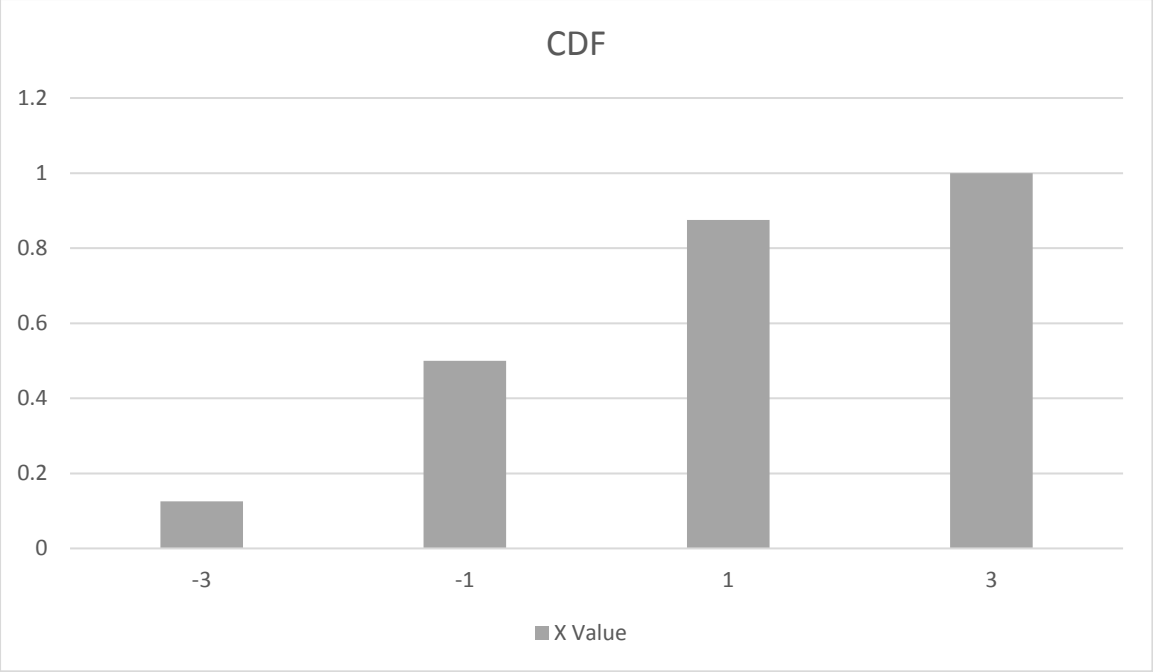
CDF)

P(X <= -3) = 0.125 // only P(X = -3) is used, no others added

P(X <= -1) = 0.500 // P(X = -1) + P(X = -3)

P(X <= 1) = 0.875 // P(X = -1) + P(X = -3) + P(X = 1)

P(X <= 3) = 1.000 // P(X = -1) + P(X = -3) + P(X = 1) + P(X = 3)



Question 3. Four 6-sided dice are rolled. The dice are fair, so each one has equal probability of producing a value in {1, 2, 3, 4, 5, 6}. Let X =the minimum of the four values

rolled. (It is fine if more than one of the dies has the minimal value.)

(a) What is P(X ≥ 1) ?

Since it is Impossible for a die to have a roll of less than one, P(X>=1) = ***1***

(b) What is P(X ≥ 2) ?

For the Minimum value to be a 2, all dice must not be 1s i.e. 5/6^4 = ***0.482***

(b) What is the CDF of X i.e. P(X ≤ k) for all values of k ?

Values of k are {1, 2, 3, 4, 5, 6}. // Taken from question

P(X ≤ 1) = 1-P(X >=2) = 0.518

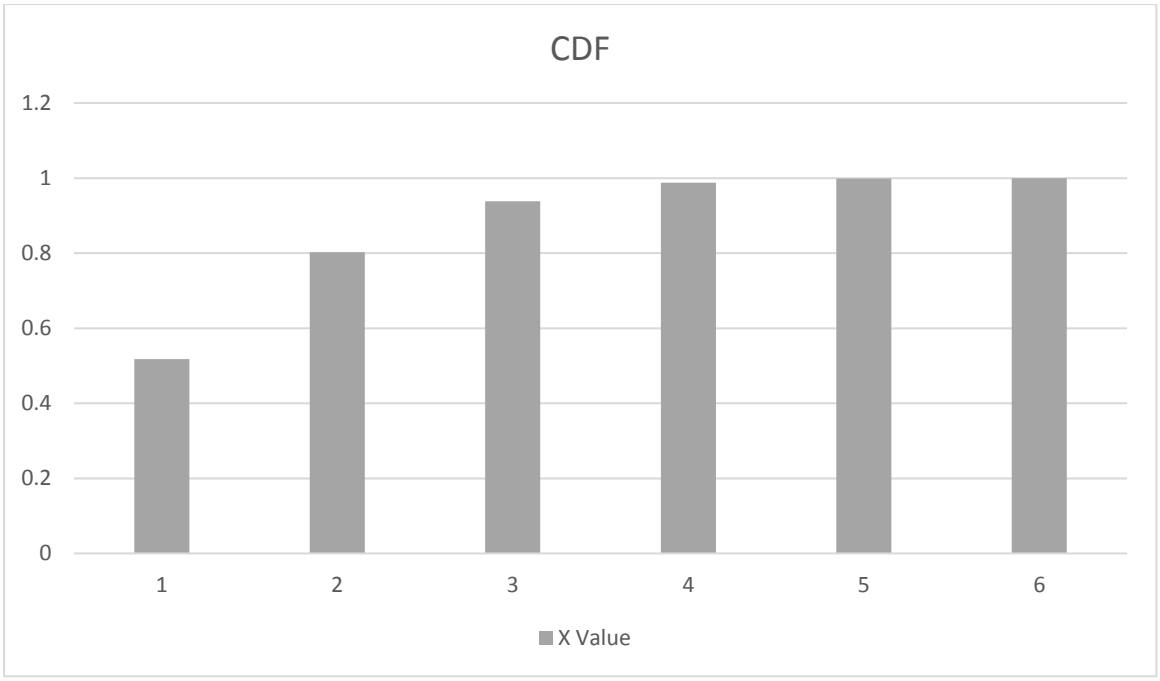
P(X ≤ 2) = 1-P(X >=3) = 0.803

P(X ≤ 3) = 1-P(X >=4) = 0.938

P(X ≤ 4) = 1-P(X >=5) = 0.988

P(X ≤ 5) = 1-P(X >=6) = 0.9992

P(X ≤ 6) = 1


CDF
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
X Value