Thomas Fowley 15315353

ST003: Statistical Methods For Computer Science

Assignment 1

Question 1. A substitution cypher is derived from orderings of the first 10 letters of the alphabet. How many ways can the 10 letters be ordered if each letter appears exactly once and:

(a) There are no other restrictions?

10! Because 10 letters are available at first then multiplied, by one less each time due to that letter being used in the previous slot = ***3628800***

(b) The letters E and F must be next to each other (but in any order)?

EF must be next to each other therefore can be treated as 1 letter with 2! Since their order doesn't matter.
Therefore 9! * 2! (not 8! * 2! Since EF is treated as one letter) = ***725760***

(c) How many different letter arrangements can be formed from the letters BANANA?

Can arrange a1, a2, a3 in 3! different ways.
Can arrange n1, n2 in 2! different ways.
Can arrange b in 1! = 1 way.
Finally, can arrange all letters 6! (taking order into account)
Thus, the letters can be formed in 6! / (3! * 2! * 1) = ***60***

N.B Answer edited from explanation of a similar problem in the lecture notes.

(d) How many different letter arrangements can be formed by drawing 3 letters from ABCDE?

Since five unique letters are used and any three can be chosen, we can use (5 choose 3) to calculate the different letter arrangements.

5 choose 3 = ***10***

Question 2. A 6-sided die is rolled four times.

(a) How many outcome sequences are possible, where we say, for instance, that the outcome is 3, 4, 3, 1 if the first roll landed on 3, the second on 4, the third on 3, and the fourth on 1?

Every time a die is rolled, there are six possible outcomes and these six combinations must be multiplied together for each throw.

Thus, 6 * 6 * 6 * 6 = ***1296***

(b) How many of the possible outcome sequences contain exactly two 3's?

Two of the throws must be 3 and the other two must be non-3 = 5 * 5 * 1 * 1.
We then need to multiply this by the different permutations of having two 3s e.g. the orders in which the 3s can appear, which are: (33xy) (x33y) (xy33) (3xy3) (3x3y) (x3y3) Where x and y represent non-3 rolls.

Thus (5 * 5 * 1 * 1) * 6 = ***150***

(c) How many contain at least two 3's?

When two 3s are present is already calculated above (150).
Three 3s are calculated as 1*1*1*5 which represents the three 3s followed by another number (1-6 and not 3), this must then be multiplied by the different permutations of 3s in four rolls which are: (333x) (x333) (33x3) (3x33) Where x represents a non-3 roll.
Thus (5 * 1 * 1 * 1) * 4 = 20
Finally, there is only one permutation of four 3s: (3333)
These must all be added up to find the total outcomes with at least two 3s
150 + 20 + 1 = ***171***

Question 3. You are counting cards in a card game that uses two decks of cards. Each deck has 4 cards (the ace from each of 4 suits), so there are 8 cards total. Cards are only distinguishable based on their suit, not which deck they came from.

(a) In how many distinct ways can the 8 cards be ordered?

8! is the total number of orders however each card is duplicated, and their deck does not matter therefore 2! must be divided for each pair

Thus $8! / 2!^4$ = ***2520***

(b) You are dealt two cards. How many distinct pairs of cards can you be dealt? Note: the order of the two cards you are dealt does not matter.

The first card dealt can be any of the four suits.
The second card must be any other suit i.e. 3*4 pairs total.
However, since order doesn't matter, the number of pairs must be halved to avoid pairs such as AsAd and AdAs being counted as two different pairs.

(3 * 4) / 2 = ***6***

(c) You are dealt two cards. Cards with suits hearts and diamonds are considered "good" cards. How many ways can you get two "good" cards? Order does not matter.

The first card can be any of the two diamonds or two hearts = 4
The second card is then the two cards of the remaining suit not chosen = 2
However, since order doesn't matter, 2! must be divided to avoid pairs such as AdAh and AhAd being counted as two pairs.

Thus 6 / 2! = ***3***