ST003: Statistical Methods for Computer Science

Assignment 2

Question 1. A 6-sided die is rolled three times

(a) How many elements are there in the same space?

Sample space S is the set of all possible outcomes from rolling a six-sided die three times, therefore the number of elements is 6 * 6 * 6 = ***216***

(b) Out of the possible sets of outcomes, calculate in how many at one least 2 rolled. Using this, calculate what the probability is that at least one 2 is rolled.

Sample space S is 216 as previously stated,
Event E is the subset of all results with at least one 3,
Event Ec is the subset of results not containing 3 i.e. 5 * 5 * 5 = 125
Thus, E is 216 – 125 = 91
The probability of at least one 3, P(E) = E/S = 91/216 = ***0.42***

(c) Write a small MATLAB simulation of this experiment and confirm that the observed probability that at least one 2 is rolled matches your calculation in (a).

```
Count = 0;
sampleSize = 9999
for i = 1:sampleSize
        dieRolls = randi (6, 1, 3);
        if (dieRolls(1) == 2 || dieRolls(2) == 2 || dieRolls(3) == 2)
                Count = Count + 1;
        end
end
```

(d) What is the probability that the sum of the die rolls is 17?

If three dice are rolled, the only possible way for them to add up to 17 is if a 6,6, and 5 are rolled.

There are three possible combinations of this [(6,6,5), (6,5,6), (5,6,6)] and since we calculated that S is 216, P(E) = 3/216 = ***0.0138***

(e) What is the probability that the sum of the three die rolls is 12 given that the first roll was a 1?

If three dice are rolled and the first roll is 1, the only possible way for them to add up to 12 is if the other two rolls are 5 and 6. There are only two possible combinations of this [(5,6), (6,5)], thus the probability is 2/36 = ***0.055***

Question 2. I roll a 6-sided die. If it comes up a 1 then I throw a six-sided die and otherwise a 20-sided die.

(a) What is the probability that the second throw comes up a 5?

Scenario one: 1 is rolled on the first die (six-sided) and then a five is rolled on the second die (six-sided)

Scenario two: 2-6 is rolled on the first die (six-sided) and then a five is rolled on the second die (20-sided)

The probability of Scenario one is 1/6 * 1/6. The probability of Scenario two is 5/6 * 1/20

Thus, the probability of a five being rolled on the second die is both scenarios added i.e. 1/36 + 1/24

= ***0.0694***

(b) What is the probability that the second throw comes up a 15? Hint: use marginalisation

In order for the second roll to be 15, the first roll must be 2-5 (not 1) and the second roll must be 15 (obviously!).

Thus, the probability = (5/6 * 1/20) = 1/24 = ***0.0417***

Question 3. At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however, that a new piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, use Bayes Rule to calculate how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect has the characteristic.

Bayes law states that : P(E|F) =(P(F|E)P(E)) / P(F)

From the question we can state that E is the event in which the suspect is guilty (prior), and F is the event in which the suspect has the character trait (evidence).

P(E) = 0.6 //given in question ,

P(F/E) = 1 // The question states that the guilty person has the characteristic

Finally, we can find P(F) by using P(F) = P(F|E)P(E) + P(F|Ec)P(Ec) = 1 * 0.6 + 0.2 * 0.4 = 0.68

Thus P(E/F) = (1 * 0.6) / 0.68 = ***0.8823***

Question 4. Your cell phone is constantly trying to keep track of where you are. At any given point in time, for all nearby locations, your phone stores a probability that you are in that location. Right now your phone believes that you are in one of 16 different locations arranged in a grid with the following probabilities (see the figure on the left): Your phone connects to a known cell tower and records two bars of signal. For each grid location L you know the probability of observing two bars from this particular tower, given that the cell phone is in location L (see the figure on the right). Example: the highlighted cell on the left figure means that you believed there was a 0.05 probability that the user was in the bottom right grid cell prior to observing the cell tower signal. The highlighted cell on the right figure means that you think the probability of observing two bars, given the user was in the bottom right grid cell, is 0.75. For each of the 16 location positions, calculate the new probability that the user is in each location given the cell tower observation. Write a program to calculate the probabilities. Report the probabilities of all 16 cells and write a short explanation of your program

To calculate the new probability, we must use Bayes' law since we are given P(E) as prior belief and P(F/E) as P(Observe two bars/ Location) for each cell. With this information we can calculate P(F) by inputting the values into Bayes' law. We use marginalisation on each cell

P(F) = P(F/E1) ….. P(F/E16) = 0.504

The P(Fcell / E) can then be calculated for each cell by multiplying the cell value with its probability and dividing by) .504

0.0744 0.1885 0.0744 0.00496

0.00496 0.1488 0.0942 0.0744

0.00099 0.00496 0.1488 0.0942

0.00099 0.00099 0.0099 0.0744