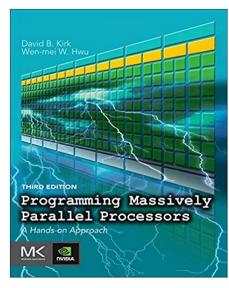


#### Introduction to CUDA

(3) Memory And Data Locality

#### Reference

- CUDA C Programming Guide,
  - https://docs.nvidia.com/cuda/cuda-c-programmingguide/index.html
- Programming Massively Parallel Processors,
  - A Hands-on Approach
  - Third Edition



#### Content

- Memory Access Efficiency
- Matrix Multiplication

#### Importance of Memory Access Efficiency

```
for(int blurRow = -BLUR_SIZE; blurRow < BLUR_SIZE+1; ++blurRow) {
   for(int blurCol = -BLUR_SIZE; blurCol < BLUR_SIZE+1; ++blurCol) {
    int curRow = Row + blurRow;
    int curCol = Col + blurCol;
        // Verify we have a valid image pixel
        if(curRow > -1 && curRow < h && curCol > -1 && curCol < w) {
        pixVal += in[curRow * w + curCol];
        pixels++; // Keep track of number of pixels in the avg
        }
    }
}</pre>
```

**FIGURE 4.1:** The most executed part of the image blurring kernel in Fig. 3.8.

#### Compute-to-global-memory-access Ratio

- Global memory bandwidth is 1TB/s, with 4 bytes in each single precision float value,
- No more than 1000/4 = 250 G single-precision ops can be loaded in one second.
- If Compute-to-global-memory-access ratio is 1, no more than 250GFLOPS can be achieved, only 2% of 12TFLOPS of recent device.
- The ratio need to be improved to 48 or higher.

# Matrix Multiplication Example A Simple Host Version in C

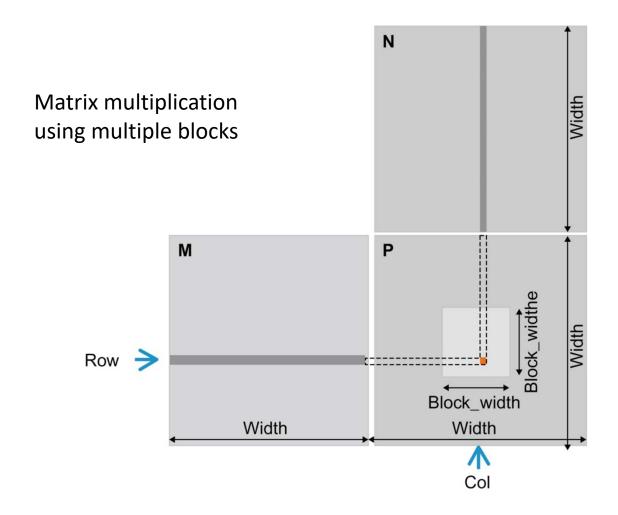
```
// Matrix multiplication on the (CPU) host in single precision
void MatrixMulOnHost(float* M, float* N, float* P, int Width)
  for (int i = 0; i < Width; ++i)
     for (int j = 0; j < Width; ++j) {
        float sum = 0;
        for (int k = 0; k < Width; ++k) {
           float a = M[i * Width + k];
           float b = N[k * Width + j];
           sum += a * b;
        P[i * Width + j] = sum;
                                                               P
                                       M
                                                WIDTH
                                                                       WIDTH
```

#### Initial matrix multiplication version

```
__global___ void MatrixMulKernel(float* M, float* N, float* P,
    int Width) {
    // Calculate the row index of the P element and M
    int Row = blockIdx.v*blockDim.v+threadIdx.v;
    // Calculate the column index of P and N
    int Col = blockIdx.x*blockDim.x+threadIdx.x;
    if ((Row < Width) && (Col < Width)) {
        float Pvalue = 0;
        // each thread computes one element of the block sub-matrix
        for (int k = 0; k < Width; ++k) {
            Pvalue _ += M[Row*Width+k]*N[k*Width+Col];
        }
        P[Row*Width+Col] = Pvalue;
    }
}</pre>
```

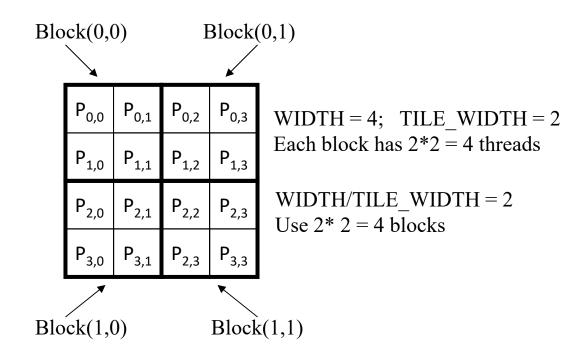
A simple matrix multiplication kernel using one thread to compute one P element.

#### Initial matrix multiplication version



### Kernel Function - A Small Example

- Have each 2D thread block to compute a (TILE\_WIDTH)<sup>2</sup> submatrix (tile) of the result matrix
  - Each has (TILE\_WIDTH)<sup>2</sup> threads
- Generate a 2D Grid of (WIDTH/TILE\_WIDTH)<sup>2</sup> blocks



# A Slightly Bigger Example (TILE\_WIDTH = 2)

P <sub>0,0</sub>	P <sub>0,1</sub>	P <sub>0,2</sub>	P <sub>0,3</sub>	P <sub>0,4</sub>	P <sub>0,5</sub>	P <sub>0,6</sub>	P <sub>0,7</sub>
P <sub>1,0</sub>	P <sub>1,1</sub>	P <sub>1,2</sub>	P <sub>1,3</sub>	P <sub>1,4</sub>	P <sub>1,5</sub>	P <sub>1,6</sub>	P <sub>1,7</sub>
P <sub>2,0</sub>	P <sub>2,1</sub>	P <sub>2,2</sub>	P <sub>2,3</sub>	P <sub>2,4</sub>	P <sub>2,5</sub>	P <sub>2,6</sub>	P <sub>2,7</sub>
P <sub>3,0</sub>	P <sub>3,1</sub>	P <sub>3,2</sub>	P <sub>3,3</sub>	P <sub>3,4</sub>	P <sub>3,5</sub>	P <sub>3,6</sub>	P <sub>3,7</sub>
P <sub>4,0</sub>	P <sub>4,1</sub>	P <sub>4,2</sub>	P <sub>4,3</sub>	P <sub>4,4</sub>	P <sub>4,5</sub>	P <sub>4,6</sub>	P <sub>4,7</sub>
						P <sub>4,6</sub>	
P <sub>5,0</sub>	P <sub>5,1</sub>	P <sub>5,2</sub>	P <sub>5,3</sub>	P <sub>5,4</sub>	P <sub>5,5</sub>		P <sub>5,7</sub>

WIDTH = 8; TILE\_WIDTH = 2 Each block has 2\*2 = 4 threads

WIDTH/TILE\_WIDTH = 4 Use 4\* 4 = 16 blocks

# A Slightly Bigger Example (cont.) (TILE\_WIDTH = 4)

P <sub>0,0</sub>	P <sub>0,1</sub>	P <sub>0,2</sub>	P <sub>0,3</sub>	P <sub>0,4</sub>	P <sub>0,5</sub>	P <sub>0,6</sub>	P <sub>0,7</sub>
P <sub>1,0</sub>	P <sub>1,1</sub>	P <sub>1,2</sub>	P <sub>1,3</sub>	P <sub>1,4</sub>	P <sub>1,5</sub>	P <sub>1,6</sub>	P <sub>1,7</sub>
P <sub>2,0</sub>	P <sub>2,1</sub>	P <sub>2,2</sub>	P <sub>2,3</sub>	P <sub>2,4</sub>	P <sub>2,5</sub>	P <sub>2,6</sub>	P <sub>2,7</sub>
P <sub>3,0</sub>	P <sub>3,1</sub>	P <sub>3,2</sub>	P <sub>3,3</sub>	P <sub>3,4</sub>	P <sub>3,5</sub>	P <sub>3,6</sub>	P <sub>3,7</sub>
P <sub>4,0</sub>	P <sub>4,1</sub>	P <sub>4,2</sub>	P <sub>4,3</sub>	P <sub>4,4</sub>	P <sub>4,5</sub>	P <sub>4,6</sub>	P <sub>4,7</sub>
						P <sub>4,6</sub>	
P <sub>5,0</sub>	P <sub>5,1</sub>	P <sub>5,2</sub>	P <sub>5,3</sub>	P <sub>5,4</sub>	P <sub>5,5</sub>		P <sub>5,7</sub>

WIDTH = 8; TILE\_WIDTH = 4 Each block has 4\*4 = 16 threads

WIDTH/TILE\_WIDTH = 2 Use 2\* 2 = 4 blocks

## Work for Block (0,0)

blockDim.x

blockDim.y

Col = 
$$0 * 2 + \text{threadIdx.x}$$
  
Row =  $0 * 2 + \text{threadIdx.y}$ 

blockldx.x

blockIdx.y

Row = 0

Row = 1

				$\downarrow$	$\downarrow$		
M <sub>0,8</sub>	M <sub>e,1</sub>	M <sub>e,2</sub>	М <sub>е,э</sub>	<b>⊅</b>	₽0,	P <sub>0,2</sub>	P <sub>0,3</sub>
M <sub>1,8</sub>	M <sub>1,1</sub>	M <sub>1,2</sub>	M <sub>1,3</sub>	▼	▼	P <sub>1,2</sub>	
M <sub>2,0</sub>	M <sub>2,1</sub>	M <sub>2,2</sub>	M <sub>2,3</sub>	P <sub>2,0</sub>	P <sub>2,1</sub>	P <sub>2,2</sub>	P <sub>2,3</sub>
M <sub>3.0</sub>	M <sub>3.1</sub>	M <sub>3.2</sub>	M <sub>3 3</sub>	P <sub>3.0</sub>	P <sub>3 1</sub>	P <sub>3 2</sub>	P <sub>3 3</sub>

Col	Col
<b>0</b>	$\longrightarrow$

N<sub>0,2</sub>

 $N_{1,2}$ 

N<sub>2,2</sub> |

، N<sub>3,2</sub> ا

 $N_{0,3}$ 

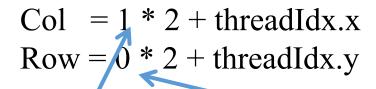
N<sub>1,3</sub>

N<sub>2,3</sub>

 $N_{3,3}$ 

### Work for Block (0,1)

Col = 3Col = 2



blockIdx.x

#### blockIdx.y

Row = 0

Row = 1

M <sub>e,e</sub>	M <sub>e,1</sub>	M <sub>e,2</sub>	М <sub>е,э</sub>	P <sub>e,8</sub>	P <sub>0,1</sub>	P	P <sub>0,3</sub>
M <sub>1,8</sub>	M <sub>1,1</sub>	M <sub>1,2</sub>	M <sub>1,3</sub>	P <sub>0,1</sub>	P <sub>1,1</sub>	P ▼ 1,2	P <sub>1,3</sub>
M <sub>2,0</sub>	M <sub>2,1</sub>	M <sub>2,2</sub>	M <sub>2,3</sub>	P <sub>2,0</sub>	P <sub>2,1</sub>	P <sub>2,2</sub>	P <sub>2,3</sub>
M <sub>3,0</sub>	M <sub>3,1</sub>	M <sub>3,2</sub>	M <sub>3,3</sub>	P <sub>3,0</sub>	P <sub>3,1</sub>	P <sub>3,2</sub>	P <sub>3,3</sub>

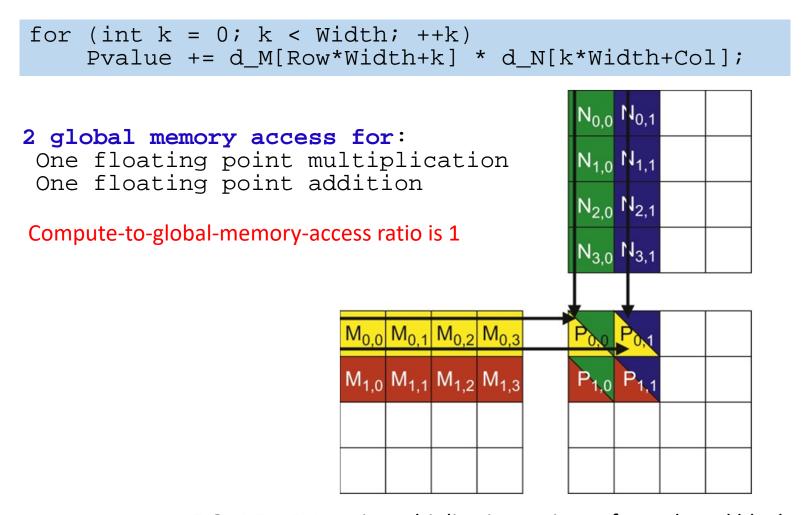
 $N_{0,0}$ 

 $N_{1,0}$ 

 $N_{2,0} \mid N_{2,1}$ 

 $N_{3,0} \mid N_{3,1}$ 

## Walk through the for loop



**FIGURE 4.5:** Matrix multiplication actions of one thread block.

# How about performance on a device with 150 GB/s memory bandwidth?

 All threads access global memory for their input matrix elements.

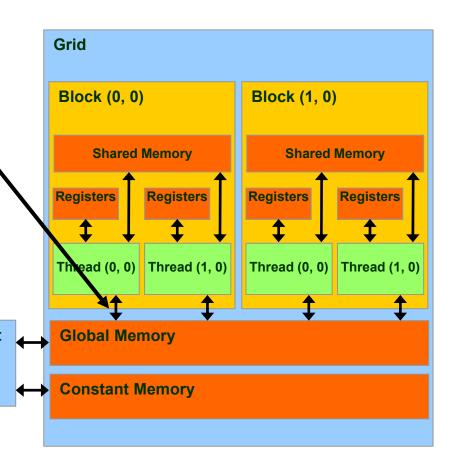
> Two memory accesses (8 bytes) per floating point multiply-add

4B/s of memory bandwidth/PLOPS

 150 GB/s limits the code at 37.5 GFLOPS

 The actual code runs at about 25 GFLOPS

 Need to drastically cut down memory accesses to get closer to the peak of more than 1,000 GFLOPS



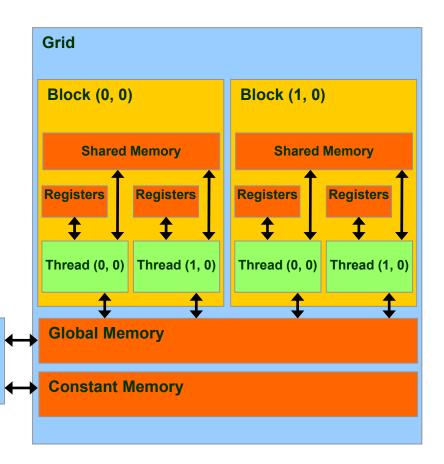
### Content

Memory Types

#### Programmer View of CUDA Memories

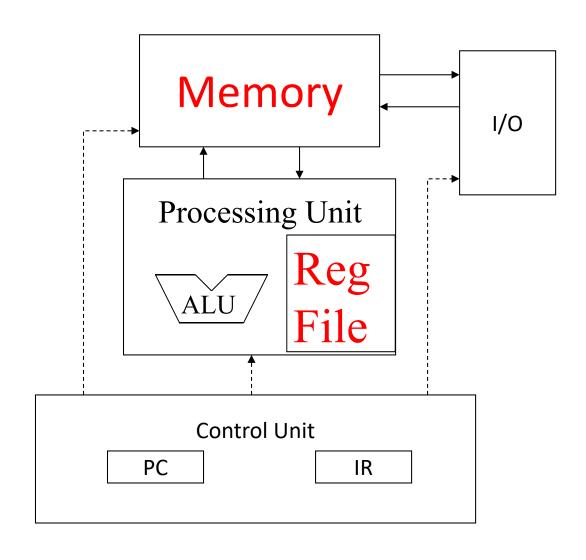
#### Each thread can:

- Read/write per-thread registers (~1 cycle)
- Read/write per-block shared memory (~5 cycles)
- Read/write per-grid global memory (~500 cycles)
- Read/only per-grid constant memory (~5 cycles with caching)



Host

#### The Von-Neumann Model



## Going back to the program

- Every instruction needs to be fetched from memory, decoded, then executed.
  - The decode stage typically accesses register file
- Instructions come in three flavors: Operate, Data transfer, and Program Control Flow.
- An example instruction cycle is the following:

Fetch | Decode | Execute | Memory

### Operate Instructions

• Example of an operate instruction: ADD R1, R2, R3

Instruction cycle for an operate instruction:
 Fetch | Decode | Execute | Memory

### Memory Access Instructions

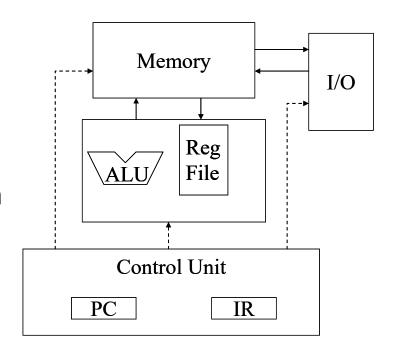
Examples of memory access instruction:

LDR R1, R2, #2 STR R1, R2, #2

Instruction cycle for an operate instruction:
 Fetch | Decode | Execute | Memory

### Registers vs Memory

- Registers are "free"
  - No additional memory access instruction
  - Very fast to use, however, there are very few of them
- Memory is expensive (slow), but very large



Variable decla	Memory	Scope	Lifetime	
Automatic variables	int LocalVar;	Register*	thread	kernel
deviceshared	int SharedVar;	shared	block	Kernel
device	int GlobalVar;	global	grid	application
deviceconstant	int ConstantVar;	constant	grid	application

- <u>device</u> is optional when used with <u>shared</u>, or <u>constant</u>
- Automatic variables
  - without any qualifier reside in a register
  - Private copy for each thread (do not exceed limit)
  - \*Except per-thread arrays that reside in global memory

Variable decla	Memory	Scope	Lifetime	
Automatic variables	int LocalVar;	Register*	thread	kernel
deviceshared	int SharedVar;	shared	block	Kernel
device	int GlobalVar;	global	grid	application
deviceconstant	int ConstantVar;	constant	grid	application

#### Shared variables

- All threads in a block see the same version
- Private copy for each block
- Efficient means for threads in block to collaborate
- Hold portion of global memory data that heavily used in kernel execution phase.

Variable decla	Memory	Scope	Lifetime	
Automatic variables	int LocalVar;	Register*	thread	kernel
deviceshared	int SharedVar;	shared	block	Kernel
device	int GlobalVar;	global	grid	application
deviceconstant	int ConstantVar;	constant	grid	application

#### Global variables

- Slow, but recent devices use cache to improve latency and throughput
- Can be used for threads to collaborate across blocks,
- Or pass information from one kernel invocation to another kernel invocation.

Variable decl	Memory	Scope	Lifetime	
Automatic variables	int LocalVar;	Register*	thread	kernel
deviceshared	int SharedVar;	shared	block	Kernel
device	int GlobalVar;	global	grid	application
deviceconstant_	int ConstantVar;	constant	grid	application

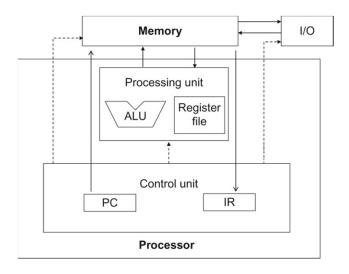
#### Constant variables

- Must outside any function body
- All threads in all grids see the same version
- Often used for variables that provide input values to kernel functions.

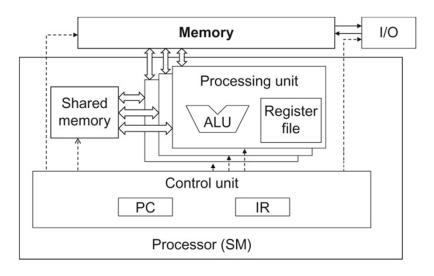
#### Content

- Tiling for reduced memory traffic
- Matrix-Matrix Multiplication using Shared Memory

# Registers vs Memory



von Neumann model



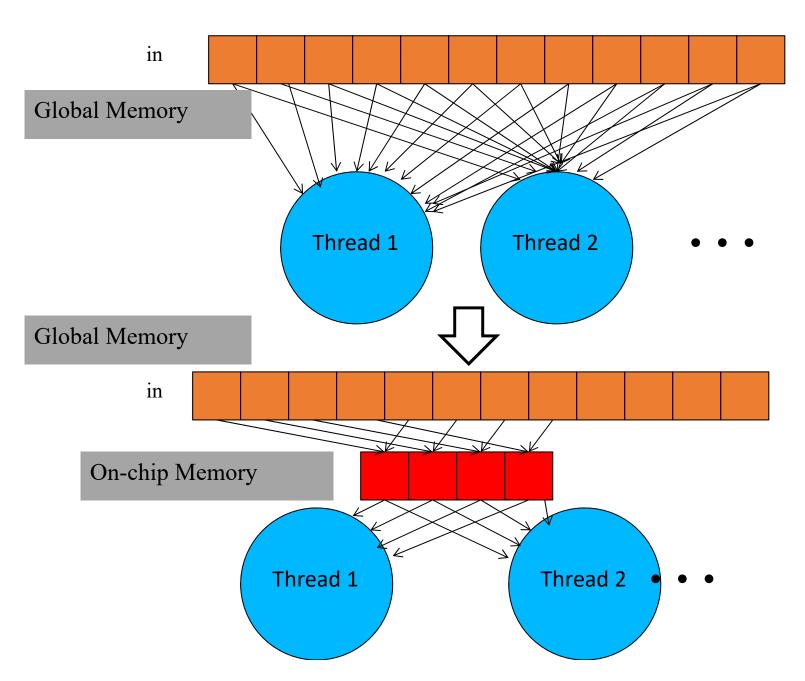
**CUDA** device SM

## A Common Programming Strategy

- Global memory resides in device memory (DRAM)
- A profitable way of performing computation on the device is to tile the input data to take advantage of fast shared memory:
  - Partition data into subsets (tiles) that fit into shared memory
  - Handle each data subset with one thread block by:
    - Loading the subset from global memory to shared memory, using multiple threads to exploit memory-level parallelism
    - Performing the computation on the subset from shared memory;
       each thread can efficiently multi-pass over any data element
    - Copying results from shared memory to global memory

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width)
{
    __shared__ float subTileM[TILE_WIDTH][TILE_WIDTH];
    __shared__ float subTileN[TILE_WIDTH][TILE_WIDTH];
```

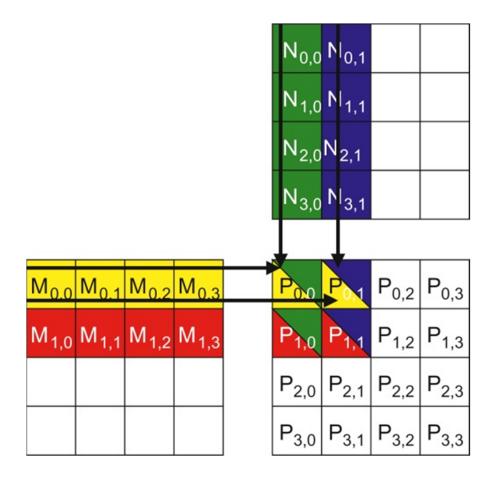
#### Shared Memory Blocking Basic Idea



### Outline of Technique

- Identify a tile of global data that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Have the multiple threads to access their data from the on-chip memory
- Move on to the next block/tile

## The small example



**FIGURE 4.9:** For brevity, we show  $M[y^*Width + x]$ ,  $N[y^*Width + x]$ ,  $P[y^*Width + x]$  as My,x, Ny,x Py,x.

## The small example

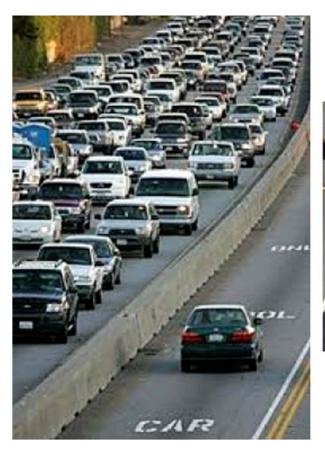
Memory access of Block(0,0)

#### Access order

thread <sub>0,0</sub>	M <sub>0,0</sub> * N <sub>0,0</sub>	M <sub>0,1</sub> *N <sub>1,0</sub>	M <sub>0,2</sub> * N <sub>2,0</sub>	M <sub>0,3</sub> * N <sub>3,0</sub>
thread <sub>0,1</sub>	$M_{0,0}$ * $N_{0,1}$	M <sub>0,1</sub> * N <sub>1,1</sub>	M <sub>0,2</sub> * N <sub>2,1</sub>	M <sub>0,3</sub> * N <sub>3,1</sub>
thread <sub>1,0</sub>	M <sub>1,0</sub> * N <sub>0,0</sub>	M <sub>1,1</sub> *N <sub>1,0</sub>	M <sub>1,2</sub> * N <sub>2,0</sub>	M <sub>1,3</sub> * N <sub>3,0</sub>
thread <sub>1,1</sub>	M <sub>1,0</sub> * N <sub>0,1</sub>	M <sub>1,1</sub> * N <sub>1,1</sub>	M <sub>1,2</sub> * N <sub>2,1</sub>	M <sub>1,3</sub> * N <sub>3,1</sub>

**Every M and N element is accessed twice.** 

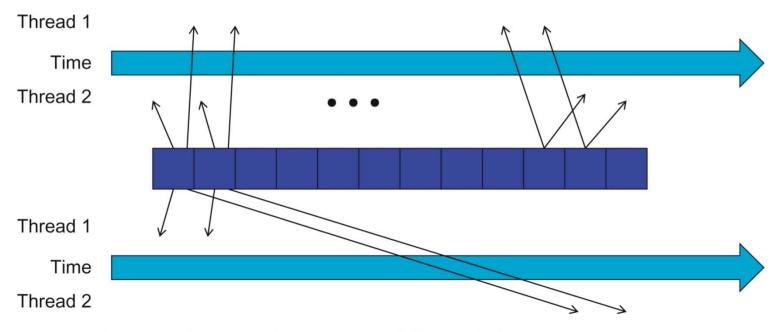
The potential reduction in global memory traffic is proportional to the dimension of blocks used.





**Tiled algorithm are highly similar to carpooling arrangement**. With extra effort.

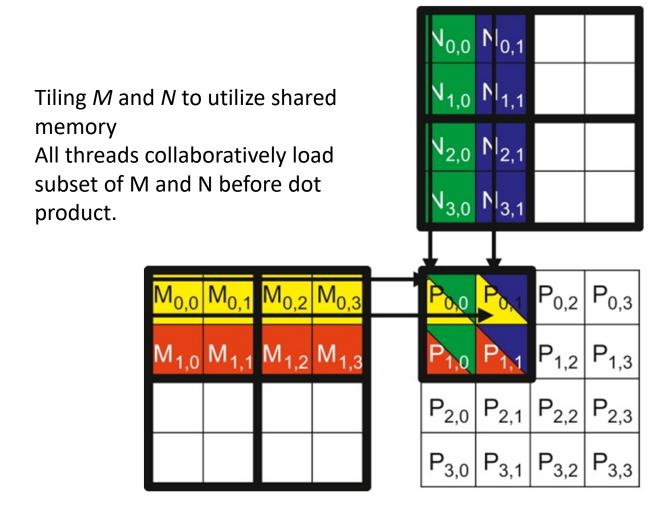




Bad — threads have very different timing

**FIGURE 4.13:** Tiled Algorithms require synchronization among threads.

# The small example



## The small example

	Phase 1			Phase 2		
thread <sub>0,0</sub>	$M_{0,0}$ $\downarrow$ $Mds_{0,0}$	$N_{0,0}$ $\downarrow$ $Nds_{0,0}$	PValue <sub>0,0</sub> += Mds <sub>0,0</sub> *Nds <sub>0,0</sub> + Mds <sub>0,1</sub> *Nds <sub>1,0</sub>	$M_{0,2}$ $\downarrow$ $Mds_{0,0}$	$N_{2,0}$ $\downarrow$ $Nds_{0,0}$	PValue <sub>0,0</sub> += Mds <sub>0,0</sub> *Nds <sub>0,0</sub> + Mds <sub>0,1</sub> *Nds <sub>1,0</sub>
thread <sub>0,1</sub>	$M_{0,1}$ $\downarrow$ $Mds_{0,1}$	$N_{0,1}$ $\downarrow$ $Nds_{1,0}$	PValue <sub>0,1</sub> += Mds <sub>0,0</sub> *Nds <sub>0,1</sub> + Mds <sub>0,1</sub> *Nds <sub>1,1</sub>	$M_{0,3} \downarrow \\ Mds_{0,1}$	$N_{2,1}$ $\downarrow$ $Nds_{0,1}$	PValue <sub>0,1</sub> += Mds <sub>0,0</sub> *Nds <sub>0,1</sub> +
thread <sub>1,0</sub>	$M_{1,0}$ $\downarrow$ $Mds_{1,0}$	<b>N</b> <sub>1,0</sub> ↓ Nds <sub>1,0</sub>	PValue <sub>1,0</sub> += Mds <sub>1,0</sub> *Nds <sub>0,0</sub> + Mds <sub>1,1</sub> *Nds <sub>1,0</sub>	$M_{1,2}$ $\downarrow$ $Mds_{1,0}$	<b>N</b> <sub>3,0</sub> ↓ Nds <sub>1,0</sub>	PValue <sub>1,0</sub> += Mds <sub>1,0</sub> *Nds <sub>0,0</sub> +
thread <sub>1,1</sub>	$M_{1,1}$ $\downarrow$ $Mds_{1,1}$	$N_{1,1}$ $\downarrow$ $Nds_{1,1}$	PValue <sub>1,1</sub> += Mds <sub>1,0</sub> *Nds <sub>0,1</sub> + Mds <sub>1,1</sub> *Nds <sub>1,1</sub>	$M_{1,3}$ $\downarrow$ $Mds_{1,1}$	<b>N</b> <sub>3,1</sub> ↓ Nds <sub>1,1</sub>	PValue <sub>1,1</sub> += Mds <sub>1,0</sub> *Nds <sub>0,1</sub> + Mds <sub>1,1</sub> *Nds <sub>1,1</sub>
	Mds <sub>1,1</sub>	Nds <sub>1,1</sub>	time	Mds <sub>1,1</sub>	Nds <sub>1,1</sub>	• IVIUS <sub>1,1</sub> IVUS <sub>1,1</sub>

Execution phases of a tiled matrix multiplication.

#### Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width)

    __shared__ float subTileM[TILE_WIDTH][TILE_WIDTH];

2. __shared__ float subTileN[TILE_WIDTH];
3. int bx = blockIdx.x; int by = blockIdx.y;
4. int tx = threadIdx.x; int ty = threadIdx.y;
    // Identify the row and column of the P element to work on
5. int Row = by * TILE_WIDTH + ty;
6. int Col = bx * TILE_WIDTH + tx;
7. float Pvalue = 0;
// Loop over the M and N tiles required to compute the P element
8. for (int m = 0; m < Width/TILE WIDTH; ++m) {
       // Collaborative loading of M and N tiles into shared memory
9.
       subTileM[ty][tx] = M[Row*Width + m*TILE WIDTH+tx];
10.
       subTileN[ty][tx] = N[(m*TILE WIDTH+ty)*Width+Col];
       __syncthreads();
11.
12.
       for (int k = 0; k < TILE_WIDTH; ++k)</pre>
           Pvalue += subTileM[ty][k] * subTileN[k][tx];
13.
       __syncthreads();
14.
15. }
16. P[Row*Width+Col] = Pvalue;
```

#### Loading an Input Tile 0

Accessing tile 0 2D indexing:

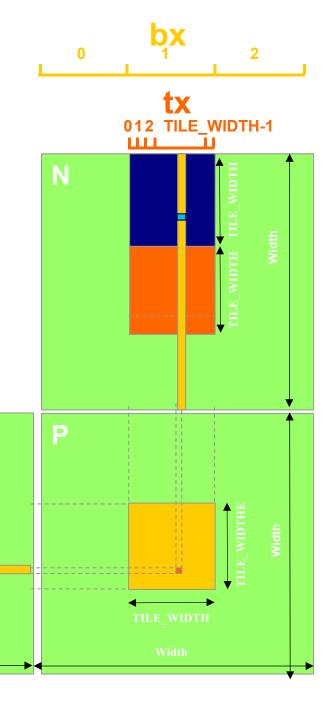
M[Row][tx] N[ty][Col]

0

2

TILE\_WIDTH-7

M



#### Loading an Input Tile 1

0 1 2

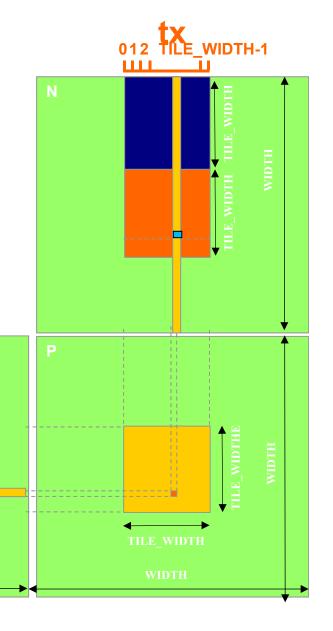
Accessing tile 1 in 2D indexing:

0

2

M[Row][1\*TILE\_WIDTH+tx]
N[1\*TILE\_WIDTH+ty][Col]

TILE WIDTH



#### Loading an Input Tile m

0

2

0 **bx** 2

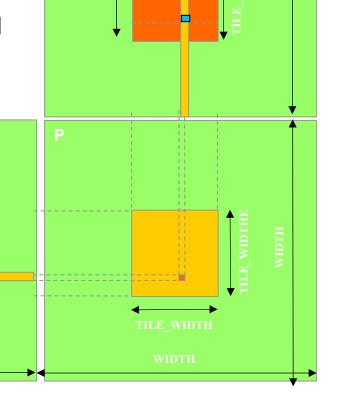
However, recall that M and N are dynamically allocated and can only use 1D indexing:

```
M[Row][m*TILE_WIDTH+tx]
M[Row*Width + m*TILE_WIDTH + tx]

N[m*TILE_WIDTH+ty][Col]
N[(m*TILE_WIDTH+ty) * Width + Col]
```

TILE WIDT

m



m

### Barrier Synchronization

- An API function call in CUDA
  - \_\_syncthreads()
- All threads in the same block must reach the \_\_syncthreads() before any can move on
- Best used to coordinate tiled algorithms
  - To ensure that all elements of a tile are loaded
  - To ensure that all elements of a tile are consumed

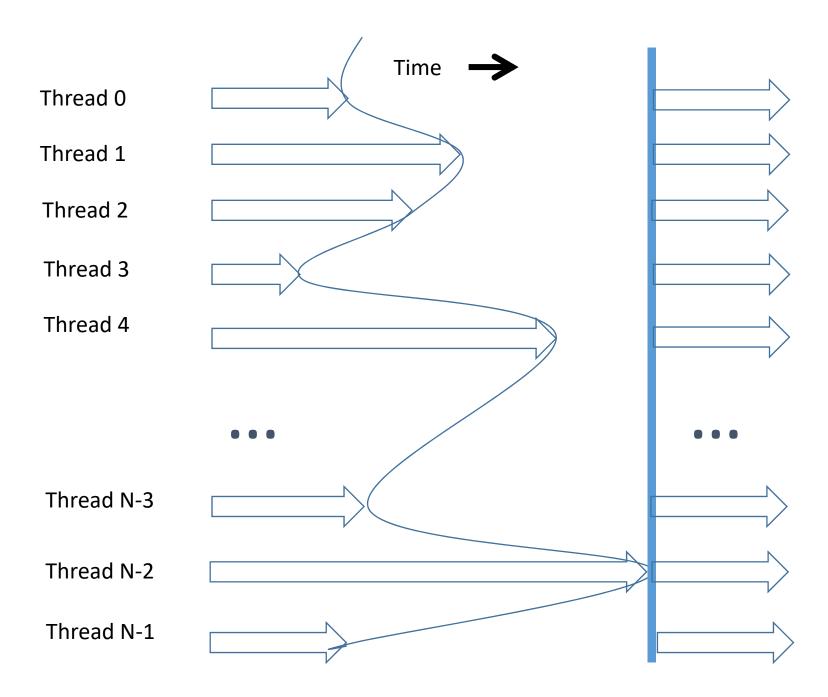
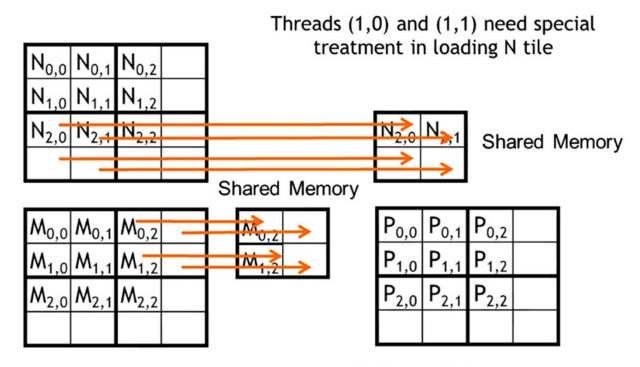


Figure 4.11 An example execution timing of barrier synchronization.

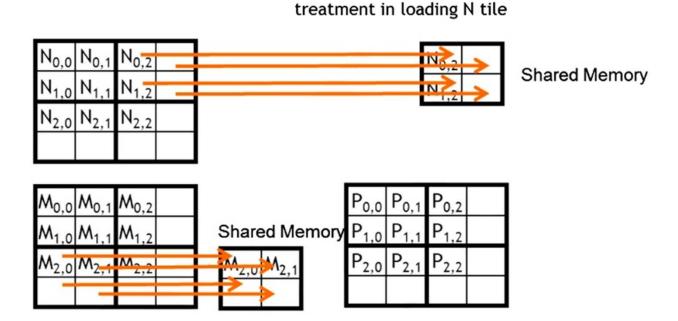
### **Boundary Checks**



Threads (0,1) and (1,1) need special treatment in loading M tile

**FIGURE 4.18:** Loading input matrix elements that are close to the edge–phase 1 of Block0,0.

## **Boundary Checks**



Threads (0,1) and (1,1) need special

Threads (1,0) and (1,1) need special treatment in loading M tile

FIGURE 4.19: Loading input elements during phase 0 of block1,0.

#### **Boundary Checks**

```
// Loop over the M and N tiles required to compute P element
 8. for (int ph = 0; ph < ceil(Width/(float)TILE WIDTH); ++ph) {
       // Collaborative loading of M and N tiles into shared memory
       if ((Row< Width) && (ph*TILE WIDTH+tx)< Width)
 9.
            Mds[ty][tx] = M[Row*Width + ph*TILE WIDTH + tx];
        if ((ph*TILE WIDTH+ty) < Width && Col < Width)
10.
            Nds[ty][tx] = N[(ph*TILE WIDTH + ty)*Width + Col];
11.
       syncthreads();
12.
   for (int k = 0; k < TILE WIDTH; ++k) {
       Pvalue += Mds[ty][k] * Nds[k][tx];
13.
       __syncthreads();
14.
     if ((Row<Width) && (Col<Width)P[Row*Width + Col] = Pvalue;
15.
```

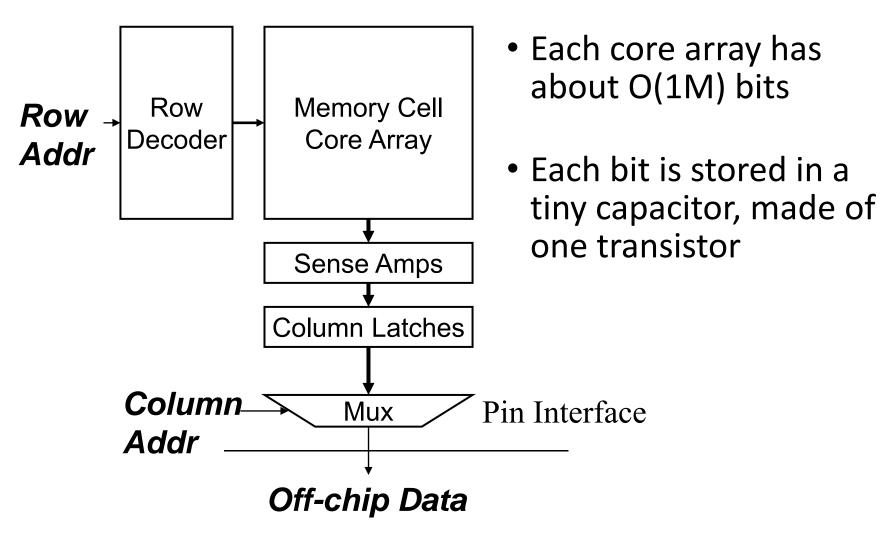
FIGURE 4.20: Tiled matrix multiplication kernel with boundary condition checks.

#### Content

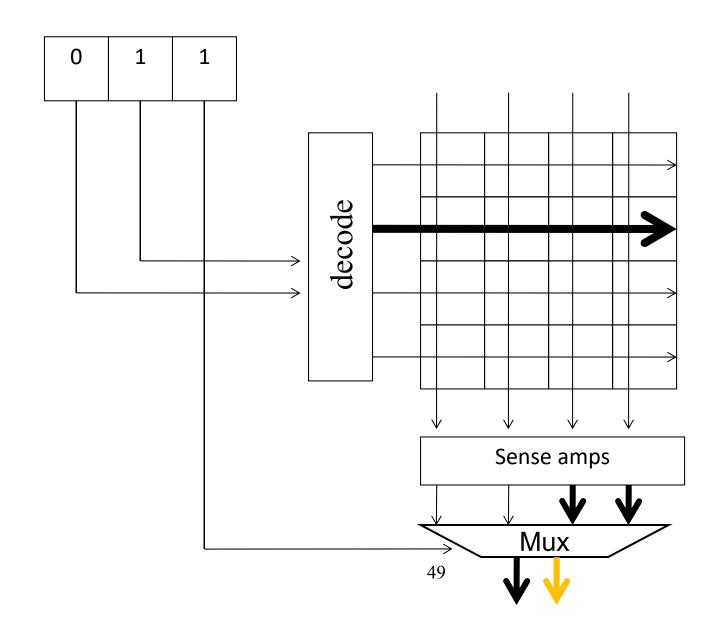
• Global Memory (DRAM) Bandwidth



#### DRAM Bank Organization

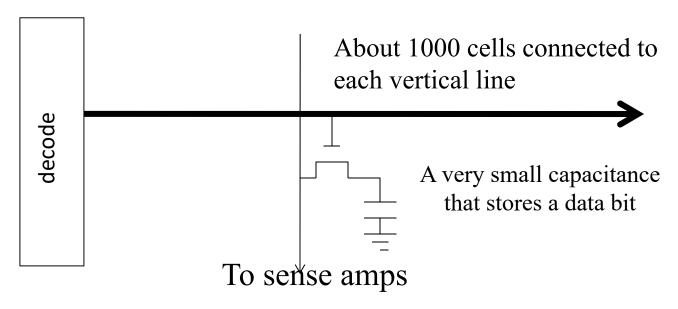


## A very small DRAM Bank

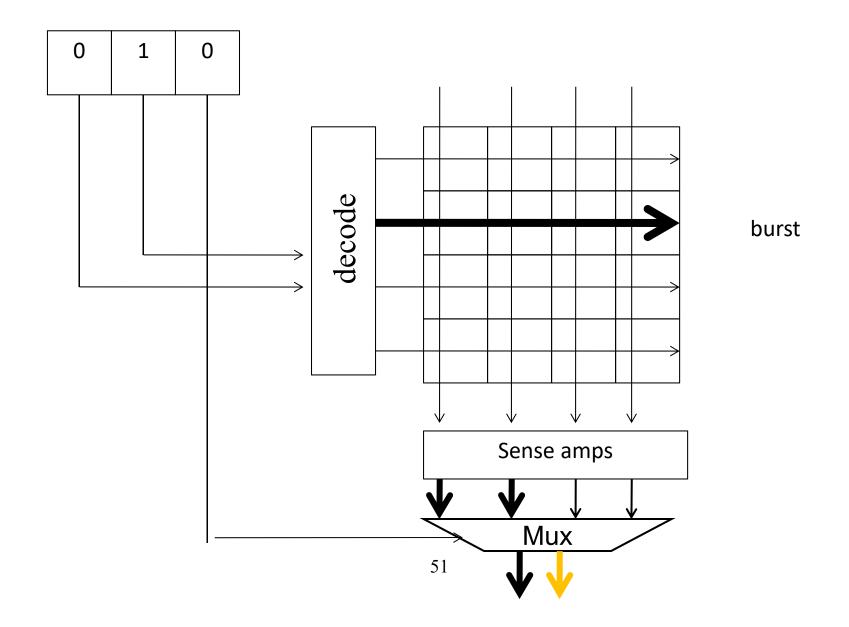


#### DRAM core arrays are slow.

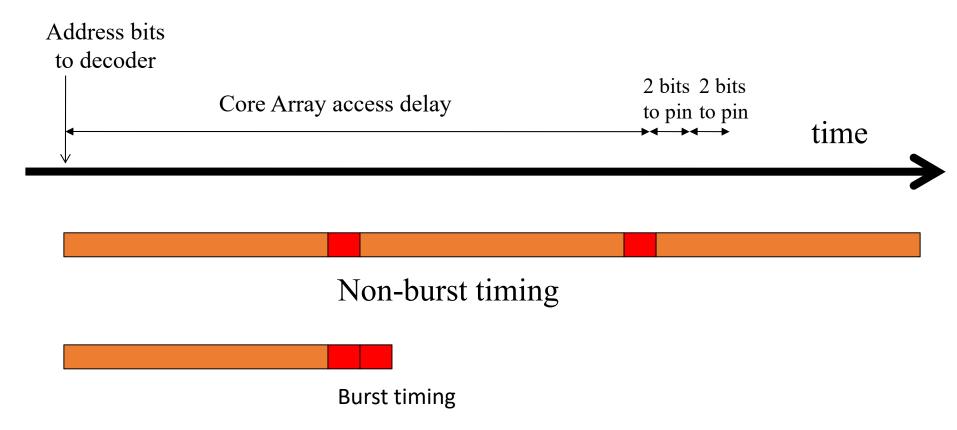
- Reading from a cell in the core array is a very slow process
  - 10s of nanoseconds v.s. sub-nanosecond clock cycle
  - Like determine the flavor of cup of coffee by smelling far away



#### DRAM Bursting (burst size = 4 bits)

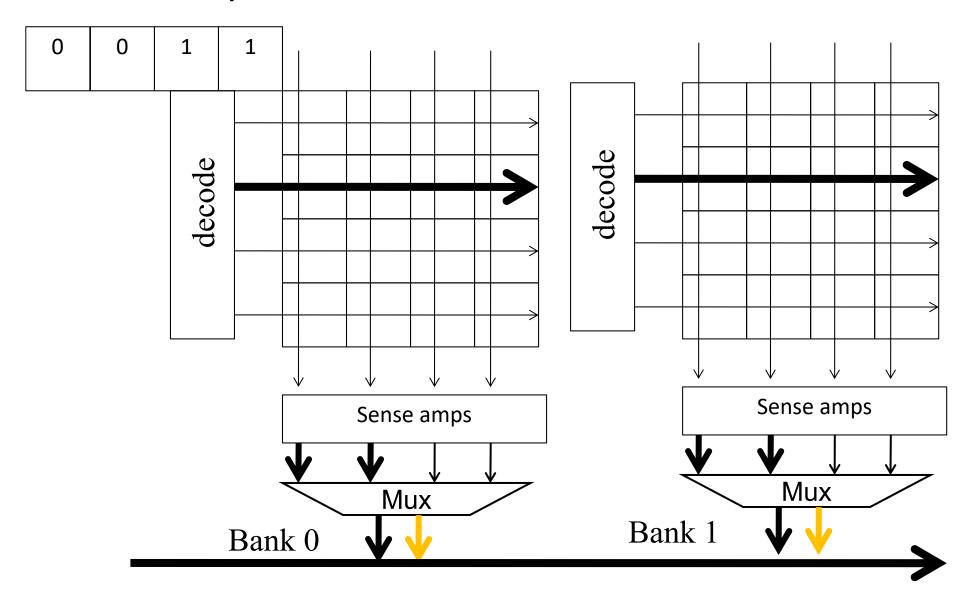


#### DRAM Bursting for the 8x2 Bank

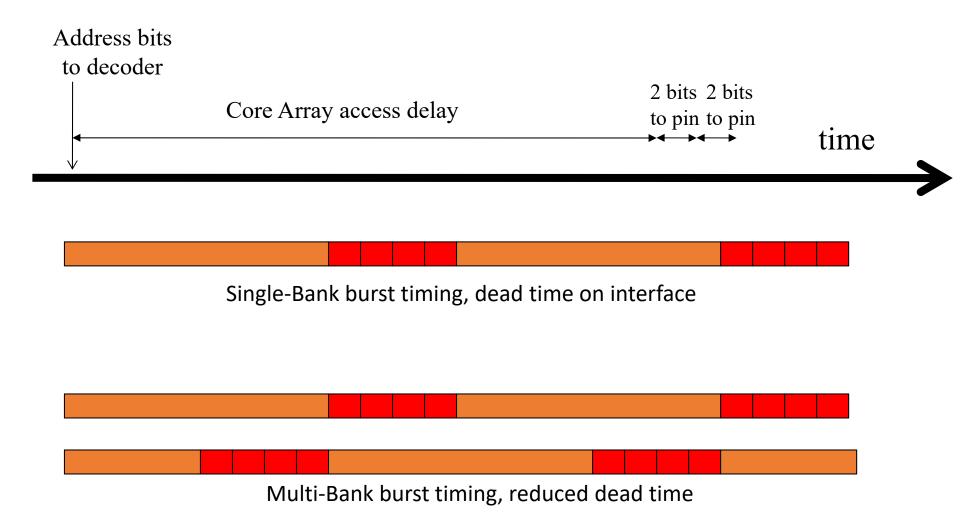


Modern DRAM systems are designed to be always accessed in burst mode. Burst bytes are transferred but discarded when accesses are not to sequential locations.

## Multiple DRAM Banks



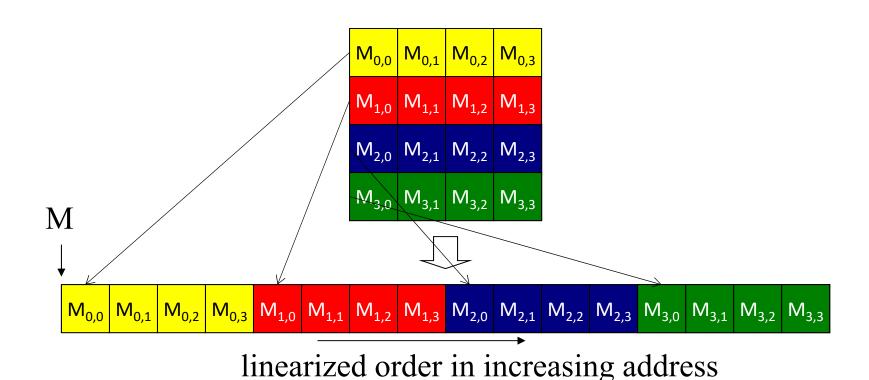
#### DRAM Bursting for the 8x2 Bank



#### Coalesce

- Take advantages of the fact that threads in a warp execute the same instruct at any given time.
- Modern DRAM systems are designed to be always accessed in burst mode.
- The favorable access pattern is achieved When all threads in a warp access consecutive global memory locations.
- In this case, the hardware combines, or coalesces, all these accesses into a consolidated access to consecutive DRAM locations (burst).

# Placing a 2D C array into linear memory space (review)

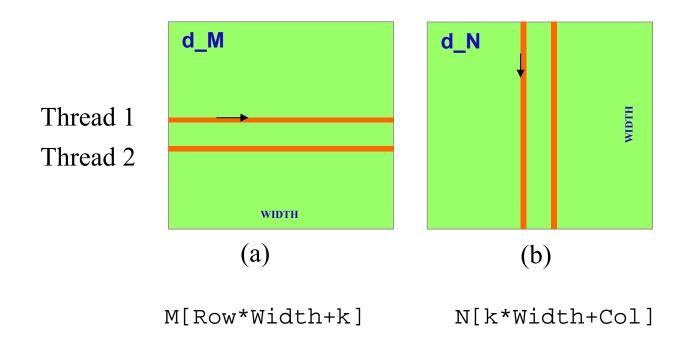


## A Simple Matrix Multiplication Kernel (review)

```
_global___ void MatrixMulKernel(float* M, float* N, float* P, int Width)
// Calculate the row index of the P element and M
int Row = blockIdx.y * blockDim.y + threadIdx.y;
// Calculate the column index of P and N
int Col = blockIdx.x * blockDim.x + threadIdx.x;
if ((Row < Width) && (Col < Width)) {
  float Pvalue = 0;
   // each thread computes one element of the block sub-matrix
  for (int k = 0; k < Width; ++k)
     Pvalue += M[Row*Width+k] * N[k*Width+Col];
  P[Row*Width+Col] = Pvalue;
```

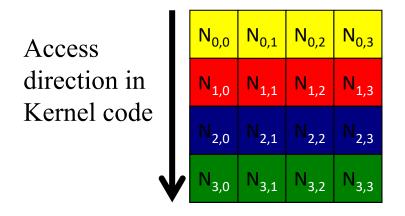
#### Two Access Patterns

For M, threads in a warp read adjacent rows; For N, threads in a warp read adjacent columns;

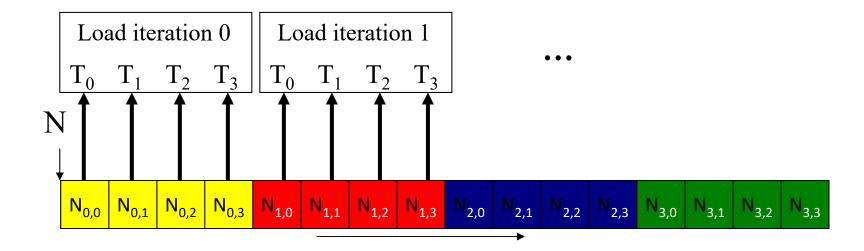


k is loop counter in the inner product loop of the kernel code

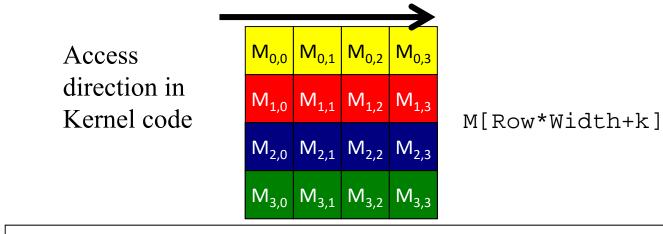
#### N accesses are coalesced.

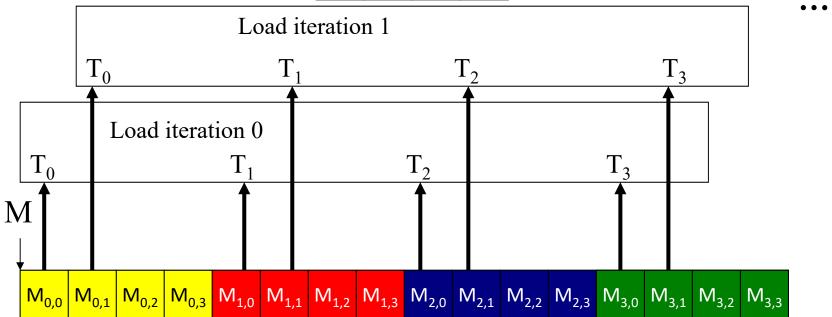


N[k\*Width+Col]

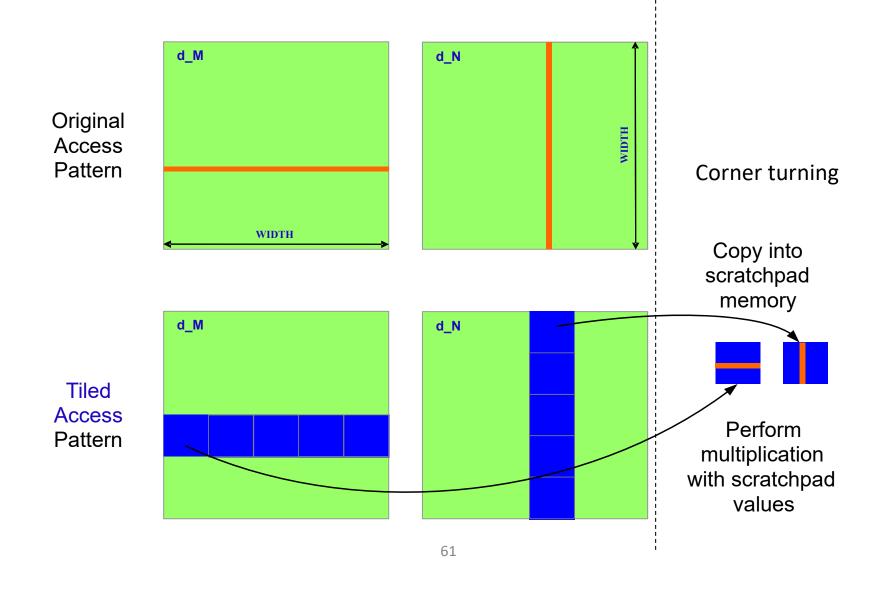


#### M accesses are not coalesced.





Use shared memory to enable coalescing in tiled matrix multiplication



#### Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width)
1. __shared__ float subTileM[TILE_WIDTH];
2. __shared__ float subTileN[TILE_WIDTH];
3. int bx = blockIdx.x; int by = blockIdx.y;
4. int tx = threadIdx.x; int ty = threadIdx.y;
    // Identify the row and column of the P element to work on
5. int Row = by * TILE_WIDTH + ty;
6. int Col = bx * TILE_WIDTH + tx;
7. float Pvalue = 0;
    // Loop over the M and N tiles required to compute the P element
8. for (int m = 0; m < Width/TILE_WIDTH; ++m) {
       // Collaborative loading of M and N tiles into shared memory
9.
       subTileM[? ][? ] = M[
                                                      ];
10.
       subTileN[? ][? ] = N[
                                                      ];
       __syncthreads();
11.
12.
      for (int k = 0; k < TILE WIDTH; ++k)
           Pvalue += subTileM[ty][k] * subTileN[k][tx];
13.
14.
       synchthreads();
15. }
16. P[Row*Width+Col] = Pvalue;
                                         62
```