Assessment Schedule - 2016

Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

One	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	Using a triangular distribution: f(x) 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.083 0.25 0.25 0.083 0.25 0.25 0.25 0.083 0.333	Either $P(X < 2)$ OR P(X > 6) correctly calculated.	Required probability is calculated.	
(b)(i)	Sketch shows roughly symmetric bell shape, centred around 33 with "bell" between around 0 and 65. e.g.	Sketch demonstrates correct understanding of key features of the normal distribution.		
(ii)	Normal distribution $\mu = 32.5$, $\sigma = 10.8$ $P(X > 40) = 0.244$ $P(both shoppers) = 0.244^2 = 0.06$ (or 0.059) Assuming that the amounts of time for each shopper are independent.	Correct probability for an individual shopper. OR Incorrect probability of one shopper, consistently squared and with correct independence statement, in context.	Correct probability of both shoppers spending more than 40 minutes, with assumption of independence stated in context.	

(iii)	Using the sketch above, which combines answers from (b)(i) and (b)(ii), for the new model to give a probability estimate for $P(X > 40)$ to be 31.1% – a larger value than the old model – the standard deviation for the new model must be larger so that the distribution is more stretched out and the resulting area under the curve above 40 is larger. Note: Calculations are not needed to support the explanation, however, accept use of inverse processes to determine the standard deviation of the new model.		Discusses why the standard deviation for the new model is larger than the standard deviation for the old model. OR	Discusses why the standard deviation for the new model is larger than the standard deviation. for the old model. AND
(iv)	A very small proportion of shoppers are modelled to spend less than 0 minutes at the supermarket, which is not possible. There should be a minimum time to spend at the supermarket, based on the time to walk from the entrance to the checkout, and the time spent at the checkout, which is not set when using the normal distribution as a model as the normal distribution is unbounded.	Identifies a potential limitation with the use of the normal distribution but does not link the feature with the context or vice versa e.g. can't have negative times.	Describes a potential limitation with the use of the normal distribution and links this feature to the context.	Describes a potential limitation with the use of the normal distribution and links this feature to the context.

NØ	N1	N2	A3	A4	M5	М6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	l of u	2 of u	3 of u	1 of r	2 of r	1 of t (with minor omission or error)	1 of t

Two	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i) (a)(ii)	Binomial distribution $n = 8, p = 0.14$ $P(X < 3) = P(X \le 2) = 0.9109$ Binomial because:	Correct probability calculated for (a)(i).	Correct probability calculated for (a)(i). AND Model identified as binomial and	
	 fixed number of trials (8 employees) fixed probability of success (14% unavailable) only two outcomes (available or not available) independent events (one employee being unavailable should not affect another employee being unavailable). 		justified with at least two conditions linked to the context.	
(b)(i)	Completed probability distribution table. N	Probability distribution table correctly completed.		
(b)(ii)	E(N) $= 0 \times 0.11 + 1 \times 0.14 + 2 \times 0.23 + 3 \times 0.16 + 4 \times 0.15 + 5 \times 0.15 + 6 \times 0.06$ $= 2.79$ So, according to this model, the mean number of collectable items gained by shoppers per purchase is 2.79. This assumes that no more than 6 collectable items are gained by shoppers per purchase, since no information was given about the distribution of amounts above \$350. This assumes that the amount customers spend does not change over the time of the promotion, nor that the amount customers spend changes because of the promotion.	Mean correctly calculated.	Mean correctly calculated. AND One assumption with model given.	
(b)(iii)	When a collectable item is given for every \$25 spent, each \$50 segment is split into two parts. The first part gets twice as many items, but the second part gets twice as many, plus 1. E.g. for \$50 – \$100 which was previously 1 item, becomes 2 items for the \$50 – \$75 and 3 items for \$75 – \$100. Therefore the expected number of collectable items will be more than double.		Identifies the mean number of collectable items gained by shoppers under the new promotion would not double. AND Supports this with some evidence from the distribution(s).	Identifies the mean number of collectable items gained by shoppers under the new promotion would not double. AND Supports this with statistical reasoning that shows a full understanding.

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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	l of u	2 of u	3 of u	1 of r	2 of r	1 of t (with minor omission or error)	1 of t

Three	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	Poisson distribution $\lambda = 1.3$ (per 5 minutes) $P(X > 2) = 1 - P(X \le 2)$ = 1 - 0.857 = 0.143	Probability correctly calculated.		
(ii)	Between 6 am and 10 pm $P(X \ge 1) = 0.94$ $P(X = 0) = 0.06$ $e^{-\lambda} = 0.06$ $\lambda = 2.81 \text{ (accept use of GC instead of formula)}$ The mean number of shoppers who arrive at the supermarket per 5 minutes between 6 am and 10 pm each day is 2.81. This is over double the rate between 10 pm and 6 am ($\lambda = 1.3$). Accept method that compares probabilities in the two models i.e. $P(X = 0) - 0.273 \text{ compared to 0.06 or P}(X \ge 1) - 0.727 \text{ compared to 0.94 and then correctly concludes that mean is higher for 6 am to 10 pm.}$	$\lambda = 2.81$, no statement made. OR Attempted to compare means by considering suitable probabilities	λ correctly calculated / obtained for 6 am to 10 pm. AND compared to λ for 10 pm to 6 am. OR Correctly compared probabilities and reached correct conclusion.	
(iii)	Possible factors: Day of the week – likely to have more shoppers during the weekend, which would change the mean rate used in the model.	Possible factor identified.	Possible factor identified. AND Effect on model described.	
(b)(i)	The standard deviation of ratings for the 'after' survey is smaller than the standard deviation of ratings for the 'before' survey, so the 'after' survey has less variation in rating scores. For supporting evidence, accept standard deviations estimated from graph, or calculated using formula, or describing the visual distributions (e.g. 'after' has over 90% of results on scores 4 and 5).	The ratings for the 'after' survey are identified as having less variation. BUT Insufficient supporting evidence is provided.	The ratings for the 'after' survey are identified as having less variation. AND Supporting evidence is provided. OR A clear argument of why a Poisson	The ratings for the 'after' survey are identified as having less variation. AND Supporting evidence is provided. AND
(ii)	It would not be appropriate to use the Poisson distribution to model the ratings for the 'before' survey. The rating scores are not rates – they are not a comparison of a count to a measured variable. Additionally, the ratings have to be between 0 and 5 – they are upper bounded. Note: Accept other valid arguments for why a Poisson distribution would not be a suitable model.		distribution would not be a suitable model for the ratings of the 'before' survey is presented.	A clear argument of why a Poisson distribution would not be a suitable model for the ratings of the 'before' survey is presented.

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No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	l of u	2 of u	3 of u	1 of r	2 of r	1 of t (with minor omission or error)	l of t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0–7	8-13	14-18	19-24