

# Linear Algebra for Machine Learning in Python

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#### **Overview**

Introcution

Essential operations

Linear curve fitting

System dynamics, and dimensions

# Introcution

# Motvating linear algebra

#### **Matrices**

m, n is a matrix  $\mathbf{A}$ 

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}, a_{ij} \in \mathbb{R}.$$
 (1)

3

# **Essential operations**

### **Addition**

## Multiplication

#### Motivation of the determinant

### Computing determinants in two or three dimensions

The two dimensional case:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21} \tag{2}$$

(3)

Computing the determinant of a three dimensional matrix.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$(4)$$

7

Linear Algebra for Machine Learning in Python

—Essential operations

Computing determinants in two or three dimensions

Computing determinants in two or three dimensions. The two dimensional case:  $\begin{vmatrix} a_{11} & a_{21} \\ a_{21} & a_{21} \\ a_{21} & a_{21} & a_{21} \\ a_{22} & a_{21} & a_{21} \\ a_{21} & a_{22} & a_{21} \\ a_{22} & a_{22} \\ a_{21} & a_{22} & a_{22} \\ a_{22} & a_{22} \\ a_{22} & a_{22} \\ a_{23} & a_{22} \\ a_{22} & a_{23} \\ a_{23} & a_{23} \\ a_{24} & a_{22} \\ a_{22} & a_{23} \\ a_{23} & a_{24} \\ a_{24} & a_{22} \\ a_{24} & a_{24} \\ a_{25} & a_{25} \\ a_{25} & a_{$ 

(5)

Draw the sign pattern on the board:

The determinant can be expanded along any column as long as the sign pattern is respected.

#### **Determinants in n-dimensions**

$$\begin{vmatrix} a_{11} & a_{21} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & \dots & a_{2n} \\ \vdots & & \vdots \\ a_{m2} & \dots & a_{mn} \end{vmatrix} + a_{21} \begin{vmatrix} a_{21} & \dots & a_{2n} \\ \vdots & & \vdots \\ a_{m2} & \dots & a_{mn} \end{vmatrix}$$

$$-a_{m1}\begin{vmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \vdots & & \vdots \end{vmatrix}$$

# The Transpose

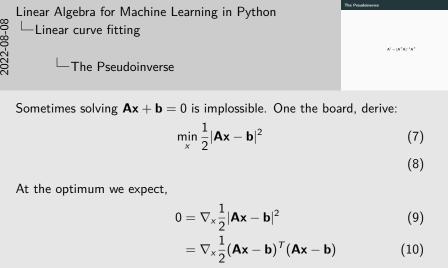
## The Inverse

# Linear curve fitting

### What is the best line connecting measurements?

#### The Pseudoinverse

$$\mathbf{A}^{\dagger} = (\mathbf{A}^{T} \mathbf{A})^{-1} \mathbf{A}^{T} \tag{6}$$



$$0 = \bigvee_{\mathbf{x}} \frac{1}{2} |\mathbf{A}\mathbf{x} - \mathbf{b}|^{2}$$

$$= \nabla_{\mathbf{x}} \frac{1}{2} (\mathbf{A}\mathbf{x} - \mathbf{b})^{T} (\mathbf{A}\mathbf{x} - \mathbf{b})$$

$$= (\mathbf{A}\mathbf{x} - \mathbf{b}) \mathbf{A}^{T}$$

$$= \mathbf{A}^{T} \mathbf{A}\mathbf{x} - \mathbf{A}^{T} \mathbf{b}$$

$$\mathbf{A}^{T} \mathbf{b} = \mathbf{A}^{T} \mathbf{A}\mathbf{x}$$

$$(13)$$

(14)

 $(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} = \mathbf{x}$ 

System dynamics, and dimensions

# Eigenvalues