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A Guide to DEAP Version 2.1: A Data Envelopment Analysis (Computer) Program

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A Data Envelopment Analysis (Computer) Program

by

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ABSTRACT

This paper describes a computer program which has been written to conduct data envelopment analyses (DEA) for the purpose of calculating efficiencies in production. The methods implemented in the program are based upon the work of Rolf Fare, Shawna Grosskopf and their associates. Three principal options are available in the computer program. The first involves the standard CRS and VRS DEA models (that involve the calculation of technical and scale efficiencies) which are outlined in Fare, Grosskopf and Lovell (1994). The second option considers the extension of these models to account for cost and allocative efficiencies. These methods are also outlined in Fare et al (1994). The third option considers the application of Malmquist DEA methods to panel data to calculate indices of total factor productivity (TFP) change; technological change; technical efficiency change and scale efficiency change. These latter methods are discussed in Fare, Grosskopf, Norris and Zhang (1994). All methods are available in either an input or an output orientation (with the exception of the cost efficiencies option).

1. INTRODUCTION

This guide describes a computer program which has been written to conduct data envelopment analyses (DEA). DEA involves the use of linear programming methods to construct a non-parametric piecewise surface (or frontier) over the data, so as to be able to calculate efficiencies relative to this surface. The computer program can consider a variety of models. The three principal options are:

1. Standard CRS and VRS DEA models that involve the calculation of technical and scale efficiencies (where applicable). These methods are outlined in Fare, Grosskopf and Lovell (1994).

2. The extension of the above models to account for cost and allocative efficiencies. These methods are also outlined in Fare et al (1994).

3. The application of Malmquist DEA methods to panel data to calculate indices of total factor productivity (TFP) change; technological change; technical efficiency change and scale efficiency change. These methods are discussed in Fare, Grosskopf, Norris and Zhang (1994).

All methods are available in either an input or an output orientation (with the exception of the cost efficiencies option). The output from the program includes, where applicable, technical, scale, allocative and cost efficiency estimates; residual slacks; peers; TFP and technological change indices.

The paper is divided into sections. Section 2 provides a brief introduction to efficiency measurement concepts developed by Farrell (1957); Fare, Grosskopf and Lovell (1985, 1994) and others. Section 3 outlines how these ideas may be empirically implemented using linear programming methods (DEA). Section 4 describes the computer program, DEAP, and section 5 provides some illustrations of how to use the program. Final concluding points are made in Section 6. An appendix is added which summarises important technical aspects of program use

2. EFFICIENCY MEASUREMENT CONCEPTS

The primary purpose of this section is to outline a number of commonly used efficiency measures and to discuss how they may be calculated relative to an efficient technology, which is generally represented by some form of frontier function. Frontiers have been

estimated using many different methods over the past 40 years. The two principal methods are:

1. data envelopment analysis (DEA) and

2. stochastic frontiers,

which involve mathematical programming and econometric methods, respectively. This paper and the DEAP computer program are concerned with the use of DEA methods. The computer program FRONTIER can be used to estimate frontiers using stochastic frontier methods. For more information on FRONTIER see Coelli (1992, 1994).

The discussion in this section provides a very brief introduction to modern efficiency measurement. A more detailed treatment is provided by Fare, Grosskopf and Lovell (1985, 1994) and Lovell (1993). Modern efficiency measurement begins with Farrell (1957) who drew upon the work of Debreu (1951) and Koopmans (1951) to define a simple measure of firm efficiency which could account for multiple inputs. He proposed that the efficiency of a firm consists of two components: *technical efficiency*, which reflects the ability of a firm to obtain maximal output from a given set of inputs, and *allocative efficiency*, which reflects the ability of a firm to use the inputs in optimal proportions, given their respective prices. These two measures are then combined to provide a measure of total *economic efficiency*.[1]

The following discussion begins with Farrell's original ideas which were illustrated in input/input space and hence had an input-reducing focus. These are usually termed *input-orientated* measures.

2.1 Input-Orientated Measures

Farrell illustrated his ideas using a simple example involving firms which use two inputs $(x_1$ and $x_2)$ to produce a single output (y), under the assumption of constant returns to scale.[2] Knowledge of the unit isoquant of the *fully efficient firm*,[3] represented by SS′

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[1] Some of Farrell's terminology differed from that which is used here. He used the term *price efficiency* instead of *allocative efficiency* and the term *overall efficiency* instead of *economic efficiency*. The terminology used in the present document conforms with that which has been used most often in recent literature.
[2] The constant returns to scale assumption allows one to represent the technology using a unit isoquant. Furthermore, Farrell also discussed the extension of his method so as to accommodate more than two inputs, multiple outputs, and non-constant returns to scale.

in Figure 1, permits the measurement of technical efficiency. If a given firm uses quantities of inputs, defined by the point P, to produce a unit of output, the technical inefficiency of that firm could be represented by the distance QP, which is the amount by which all inputs could be proportionally reduced without a reduction in output. This is usually expressed in percentage terms by the ratio QP/0P, which represents the percentage by which all inputs could be reduced. The technical efficiency (TE) of a firm is most commonly measured by the ratio

$$TE_I = 0Q/0P, \tag{1}$$

which is equal to one minus QP/0P.[4] It will take a value between zero and one, and hence provides an indicator of the degree of technical inefficiency of the firm. A value of one indicates the firm is fully technically efficient. For example, the point Q is technically efficient because it lies on the efficient isoquant.

Figure 1

Technical and Allocative Efficiencies
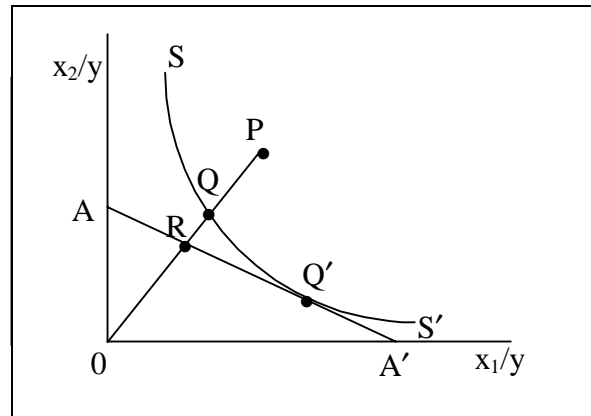
If the input price ratio, represented by the line AA′ in Figure 1, is also known, allocative efficiency may also be calculated. The *allocative efficiency (AE)* of the firm operating at P is defined to be the ratio

$$AE_I = 0R/0Q, \tag{2}$$

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[3] The production function of the fully efficient firm is not known in practice, and thus must be estimated from observations on a sample of firms in the industry concerned. In this paper we use DEA to estimate this frontier.
[4] The subscript "I" is used on the TE measure to show that it is an input-orientated measure. Output-orientated measures will be defined shortly.

since the distance RQ represents the reduction in production costs that would occur if production were to occur at the allocatively (and technically) efficient point Q′, instead of at the technically efficient, but allocatively inefficient, point Q.[5]

The total *economic efficiency (EE)* is defined to be the ratio
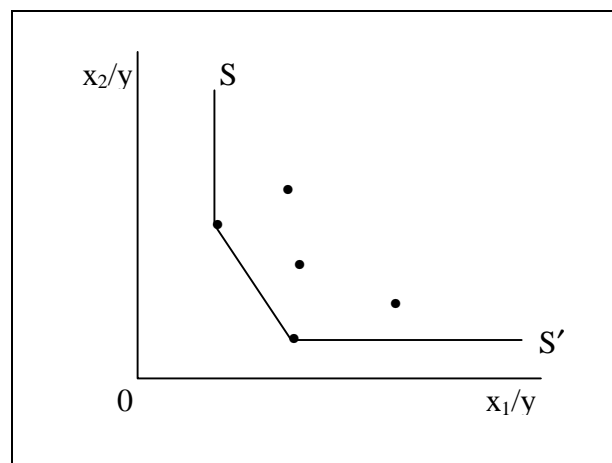
$$EE_I = 0R/0P, \tag{3}$$

where the distance RP can also be interpreted in terms of a cost reduction. Note that the product of technical and allocative efficiency provides the overall economic efficiency

$$TE_I \times AE_I = (0Q/0P) \times (0R/0Q) = (0R/0P) = EE_I. \tag{4}$$

Note that all three measures are bounded by zero and one.

Figure 2

Piecewise Linear Convex Isoquant

These efficiency measures assume the production function of the fully efficient firm is known. In practice this is not the case, and the efficient isoquant must be estimated from the sample data. Farrell suggested the use of either (a) a non-parametric piecewise-linear convex isoquant constructed such that no observed point should lie to the left or below it (refer to Figure 2), or (b) a parametric function, such as the Cobb-Douglas form, fitted to the data, again such that no observed point should lie to the left or below it. Farrell provided an illustration of his methods using agricultural data for

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[5] One could illustrate this by drawing two isocost lines through Q and Q′. Irrespective of the slope of these two parallel lines (which is determined by the input price ratio) the ratio RQ/0Q would represent the percentage reduction in costs associated with movement from Q to Q′.

the 48 continental states of the US.
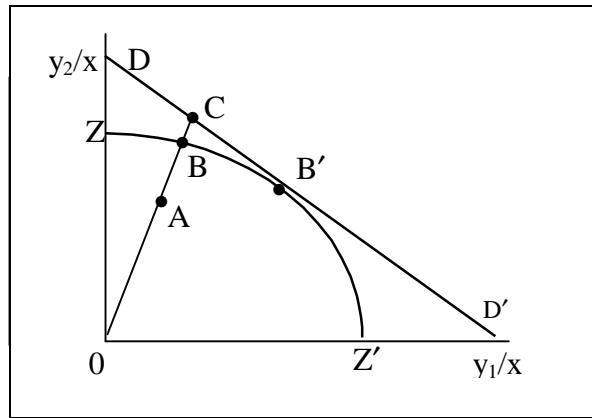
2.2 Output-Orientated Measures

The above input-orientated technical efficiency measure addresses the question: "By how much can input quantities be proportionally reduced without changing the output quantities produced?". One could alternatively ask the question ": "By how much can output quantities be proportionally expanded without altering the input quantities used?". This is an output-orientated measure as opposed to the input-oriented measure discussed above. The difference between the output- and input-orientated measures can be illustrated using a simple example involving one input and one output. This is depicted in Figure 3(a) where we have a decreasing returns to scale technology represented by f(x), and an inefficient firm operating at the point P. The Farrell input-orientated measure of TE would be equal to the ratio AB/AP, while the output-orientated measure of TE would be CP/CD. The output- and input-orientated measures will only provide equivalent measures of technical efficiency when constant returns to scale exist, but will be unequal when increasing or decreasing returns to scale are present (Fare and Lovell 1978). The constant returns to scale case is depicted in Figure 3(b) where we observe that AB/AP=CP/CD, for any inefficient point P we care to choose.

One can consider output-orientated measures further by considering the case where production involves two outputs (y_1 and y_2) and a single input (x_1). Again, if we assume constant returns to scale, we can represent the technology by a unit production possibility curve in two dimensions. This example is depicted in Figure 4 where the line ZZ′ is the unit production possibility curve and the point A corresponds to an inefficient firm. Note that the inefficient point, A, lies *below* the curve in this case because ZZ′ represents the upper bound of production possibilities.

Figure 3

Input- and Output-Orientated Technical Efficiency Measures

and Returns to Scale
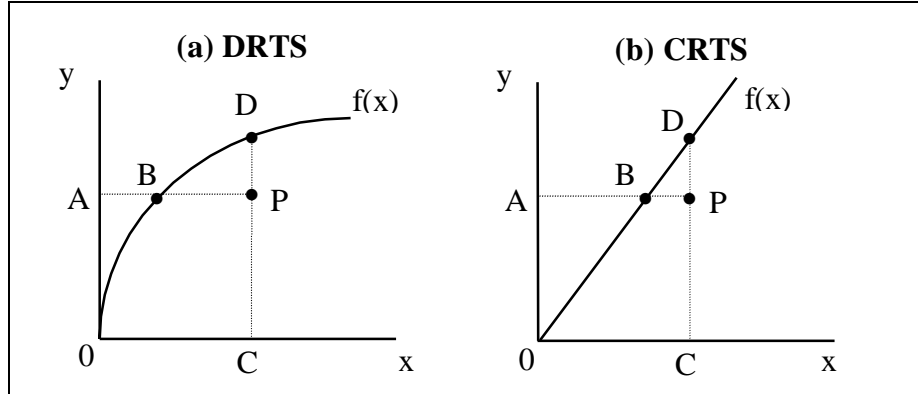
Figure 4

Technical and Allocative Efficiencies from an

Output Orientation

The Farrell output-orientated efficiency measures would be defined as follows. In Figure 4 the distance AB represents technical inefficiency. That is, the amount by which outputs could be increased without requiring extra inputs. Hence a measure of output-orientated technical efficiency is the ratio

$$TE_O = 0A/0B. \tag{7}$$

If we have price information then we can draw the isorevenue line DD′, and define the allocative efficiency to be

$$AE_O = 0B/0C \qquad\qquad\qquad\qquad (8)$$

which has a revenue increasing interpretation (similar to the cost reducing interpretation of allocative inefficiency in the input-orientated case). Furthermore, one can define overall economic efficiency as the product of these two measures

$$EE_O = (0A/0C) = (0A/0B) \times (0B/0C) = TE_O \times AE_O. \qquad\qquad (9)$$

Again, all of these three measures are bounded by zero and one.

Before we conclude this section, two quick points should be made regarding the six efficiency measures that we have defined:

1) All of them are measured along a ray from the origin to the observed production point. Hence they hold the relative proportions of inputs (or outputs) constant. One advantage of these *radial* efficiency measures is that they are *units invariant*. That is, changing the units of measurement (e.g. measuring quantity of labour in person hours instead of person years) will not change the value of the efficiency measure. A non-radial measure, such as the shortest distance from the production point to the production surface, may be argued for, but this measure will not be invariant to the units of measurement chosen. Changing the units of measurement in this case could result in the identification of a different "nearest" point. This issue will be discussed further when we come to consider the treatment of slacks in DEA.

2) The Farrell input- and output-orientated technical efficiency measures can be shown to be equal to the input and output distance functions discussed in Shepherd (1970). For more on this see Lovell (1993, p10). This observation becomes important when we discuss the use of DEA methods in calculating Malmquist indices of TFP change.

3. Data Envelopment Analysis (DEA)

Data envelopment analysis (DEA) is the non-parametric mathematical programming approach to frontier estimation. The discussion of DEA models presented here is brief, with relatively little technical detail. More detailed reviews of the methodology are presented by Seiford and Thrall (1990), Lovell (1993), Ali and Seiford (1993), Lovell (1994), Charnes et al (1995) and Seiford (1996).

The piecewise-linear convex hull approach to frontier estimation, proposed by Farrell

(1957), was considered by only a handful of authors in the two decades following Farrell's paper. Authors such as Boles (1966) and Afriat (1972) suggested mathematical programming methods which could achieve the task, but the method did not receive wide attention until a the paper by Charnes, Cooper and Rhodes (1978) which coined the term *data envelopment analysis* (DEA). There has since been a large number of papers which have extended and applied the DEA methodology.

Charnes, Cooper and Rhodes (1978) proposed a model which had an input orientation and assumed constant returns to scale (CRS).[6] Subsequent papers have considered alternative sets of assumptions, such as Banker, Charnes and Cooper (1984) who proposed a variable returns to scale (VRS) model. The following discussion of DEA begins with a description of the input-orientated CRS model in section 3.1, because this model was the first to be widely applied.

3.1 The Constant Returns to Scale Model (CRS)

We shall begin by defining some notation. Assume there is data on K inputs and M outputs on each of N firms or DMU's as they tend to be called in the DEA literature.[7] For the i-th DMU these are represented by the vectors x_i and y_i, respectively. The K×N input matrix, X, and the M×N output matrix, Y, represent the data of all N DMU's. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all observed points lie on or below the production frontier. For the simple example of an industry where one output is produced using two inputs, it can be visualised as a number of intersecting planes forming a tight fitting cover over a scatter of points in three-dimensional space. Given the CRS assumption, this can also be represented by a unit isoquant in input/input space (refer to Figure 2).

The best way to introduce DEA is via the *ratio* form. For each DMU we would like to obtain a measure of the ratio of all outputs over all inputs, such as $u'y_i/v'x_i$, where u is an M×1 vector of output weights and v is a K×1 vector of input weights. To select optimal weights we specify the mathematical programming problem:

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[6] At this point we will begin to use CRS to refer to constant returns to scale rather than CRTS. Most economics texts use the latter, while most DEA papers use the former.

[7] DMU stands for "decision making unit". It is a more appropriate term than "firm" when, for example, a bank is studying the performance of its branches or an education district is studying the performance of its schools.

$$\max_{u,v} (u'y_i/v'x_i),$$

$$\text{st} \quad u'y_j/v'x_j \leq 1, \ j=1,2,...,N,$$

$$u, v \geq 0. \tag{10}$$

This involves finding values for u and v, such that the efficiency measure of the i-th DMU is maximised, subject to the constraint that all efficiency measures must be less than or equal to one. One problem with this particular ratio formulation is that it has an infinite number of solutions.[8] To avoid this one can impose the constraint $v'x_i = 1$, which provides:

$$\max_{\mu,\nu} (\mu'y_i),$$

$$\text{st} \quad \nu'x_i = 1,$$

$$\mu'y_j - \nu'x_j \leq 0, \ j=1,2,...,N,$$

$$\mu, \nu \geq 0, \tag{11}$$

where the notation change from u and v to μ and ν reflects the transformation. This form is known as the *multiplier* form of the linear programming problem.

Using the duality in linear programming, one can derive an equivalent *envelopment* form of this problem:

$$\min_{\theta,\lambda} \theta,$$

$$\text{st} \quad -y_i + Y\lambda \geq 0,$$

$$\theta x_i - X\lambda \geq 0,$$

$$\lambda \geq 0, \tag{12}$$

where θ is a scalar and λ is a N×1 vector of constants. This envelopment form involves fewer constraints than the multiplier form (K+M < N+1), and hence is generally the preferred form to solve.[9] The value of θ obtained will be the efficiency score for the i-th DMU. It will satisfy $\theta \leq 1$, with a value of 1 indicating a point on the

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[8] That is, if (u*,v*) is a solution, then (αu*,αv*) is another solution, etc.
[9] The forms defined by equations 10 and 11 are introduced here for expository purposes. They are not used again in the remainder of this paper. The multiplier form has, however, been estimated in a

frontier and hence a technically efficient DMU, according to the Farrell (1957) definition. Note that the linear programming problem must be solved N times, once for each DMU in the sample. A value of θ is then obtained for each DMU.

Slacks

The piecewise linear form of the non-parametric frontier in DEA can cause a few difficulties in efficiency measurement. The problem arises because of the sections of the piecewise linear frontier which run parallel to the axes (refer Figure 2) which do not occur in most parametric functions (refer Figure 1). To illustrate the problem, refer to Figure 5 where the DMU's using input combinations C and D are the two efficient DMU's which define the frontier, and DMU's A and B are inefficient DMU's. The Farrell (1957) measure of technical efficiency gives the efficiency of DMU's A and B as OA′/OA and OB′/OB, respectively. However, it is questionable as to whether the point A′ is an efficient point since one could reduce the amount of input x_2 used (by the amount CA′) and still produce the same output. This is known as *input slack* in the literature.[10] Once one considers a case involving more inputs and/or multiple outputs, the diagrams are no longer as simple, and the possibility of the related concept of *output slack* also occurs.[11] Thus it could be argued that both the Farrell measure of technical efficiency (θ) and any non-zero input or output slacks should be reported to provide an accurate indication of technical efficiency of a DMU in a DEA analysis.[12] Note that for the i-th DMU the output slacks will be equal to zero only if $Y\lambda-y_i=0$, while the input slacks will be equal to zero only if $\theta x_i-X\lambda=0$ (for the given optimal values of θ and λ).

number of studies. The μ and ν weights can be interpreted as normalised shadow prices.

[10] Some authors use the term *input excess*.

[11] Output slack is illustrated later in these notes (see Figure 4.8).

[12] Koopman's (1951) definition of technical efficiency was stricter than the Farrell (1957) definition. The former is equivalent to stating that a firm is only technically efficient if it operates on the frontier and furthermore that all associated slacks are zero.

Figure 5

Efficiency Measurement and Input Slacks
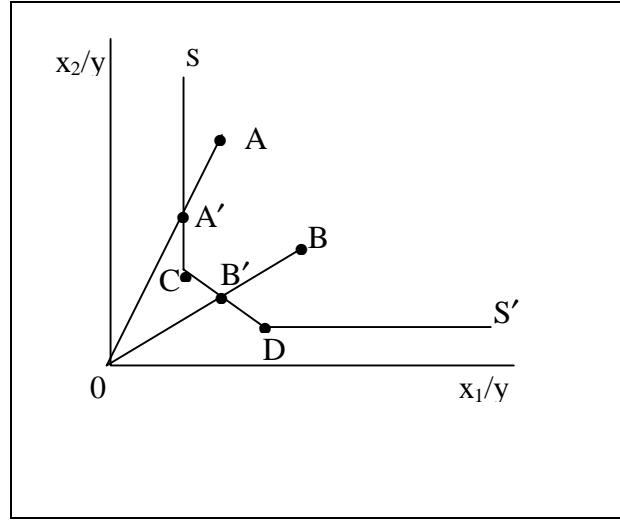
In Figure 5 the input slack associated with the point A′ is CA′ of input x_2. In cases when there are more inputs and outputs than considered in this simple example, the identification of the "nearest" efficient frontier point (such as C), and hence the subsequent calculation of slacks, is not a trivial task. Some authors (see Ali and Seiford 1993) have suggested the solution of a second-stage linear programming problem to move to an efficient frontier point by MAXIMISING the sum of slacks required to move from an inefficient frontier point (such as A′ in Figure 5) to an efficient frontier point (such as point C). This second stage linear programming problem may be defined by:

$$\min_{\lambda, OS, IS} -(M1'OS + K1'IS),$$

$$\text{st} \quad -y_i + Y\lambda - OS = 0,$$

$$\theta x_i - X\lambda - IS = 0,$$

$$\lambda \geq 0, \, OS \geq 0, \, IS \geq 0, \tag{13}$$

where OS is an M×1 vector of output slacks, IS is a K×1 vector of input slacks, and M1 and K1 are M×1 and K×1 vectors of ones, respectively. Note that in this second-stage linear program, θ is not a variable, its value is taken from the first-stage results. Furthermore, note that this second-stage linear program must also be solved for each

of the N DMU's involved.[13]

There are two major problems associated with this second stage LP. The first and most obvious problem is that the sum of slacks is MAXIMISED rather than MINIMISED. Hence it will identify not the NEAREST efficient point but the FURTHEST efficient point. The second major problem associated with the above second-stage approach is that it is not invariant to units of measurement. The alteration of the units of measurement, say for a fertiliser input from kilograms to tonnes (while leaving other units of measurement unchanged), could result in the identification of different efficient boundary points and hence different slack and lambda measures.[14]

Note, however, that these two issues are not a problem in the simple example presented in Figure 5 because there is only one efficient point to choose from on the vertical facet. However, if slack occurs in 2 or more dimensions (which it often does) then the above mentioned problems can come into play.

As a result of this problem, many studies simply solve the first-stage linear program (equation 12) for the values of the Farrell radial technical efficiency measures (θ) for each DMU and ignore the slacks completely, or they report both the radial Farrell technical efficiency score (θ) and the residual slacks, which may be calculated as $OS = -y_i + Y\lambda$ and $IS = \theta x_i - X\lambda$. However, this approach is not without problems either because these residual slacks may not always provide all (Koopmans) slacks (e.g., when a number of observations appear on the vertical section of the frontier in Figure 5.5) and hence may not always identify the nearest (Koopmans) efficient point for each DMU.

In the DEAP software we give the user three choices regarding the treatment of slacks. These are:

1. One-stage DEA, in which we conduct the LP in equation 12 and calculate slacks residually;

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[13] This method is used by all the popular DEA software such as Warwick DEA and IDEAS.
[14] Charnes, Cooper, Rousseau and Semple (1987) suggest a units invariant model where the unit worth of a slack is made inversely proportional to the quantity of that input or output used by the i-th firm. This does solve the immediate problem, but does create another, in that there is no obvious reason for the slacks to be weighted in this way.

2. Two-stage DEA, where we conduct the LP's in equations 12 and 13; and

3. Multi-stage DEA, where we conduct a sequence of radial LP's to identify the efficient projected point.

The multi-stage DEA method is more computationally demanding that the other two methods(see Coelli 1997 for details). However, the benefits of the approach are that it identifies efficient projected points which have input and output mixes which are as similar as possible to those of the inefficient points, and that it is also invariant to units of measurement. Hence we would recommend the use of the multi-stage method over the other two alternatives.

Having devoted a number of pages of this manual to the issue of slacks we would like to conclude by observing that the importance of slacks can be overstated. Slacks may be viewed as being an artefact of the frontier construction method chosen (DEA) and the use of finite sample sizes. If an infinite sample size were available and/or if an alternative frontier construction method was used, which involved a smooth function surface, the slack issue would disappear. In addition to this observation it also seems quite reasonable to accept the arguments of Ferrier and Lovell (1990) that slacks may essentially be viewed as allocative inefficiency. Hence we believe that an analysis of technical efficiency can reasonably concentrate upon the radial efficiency score provided in the first stage DEA LP (refer to equation 12). However if one insists on identifying Koopmans-efficient projected points then we would strongly recommend the use of the multi-stage method in preference to the two-stage method for the reasons outlined above.[15]

Example 1

We will illustrate CRS input-orientated DEA using a simple example involving five observations on DMU's (firms) which use two inputs to produce a single output. The data is as follows:

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[15] However we have also included the 2-stage option in our software because it is the method used in other popular DEA software packages such as Warwick DEA and IDEAS.

Table 1

Example Data for CRS DEA

DMU	y	x_1	x_2	x_1/y	x_2/y
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

The input/output ratios for this example are plotted in Figure 6, along with the DEA frontier corresponding to equation 12. You should keep in mind, however, that this DEA frontier is the result of running five linear programming problems - one for each of the five DMU's. For example, for DMU 3 we could rewrite equation 12 as

$$\min_{\theta,\lambda} \theta,$$

st $-y_3 + (y_1\lambda_1 + y_2\lambda_2 + y_3\lambda_3 + y_4\lambda_4 + y_5\lambda_5) \geq 0,$

   $\theta x_{13} - (x_{11}\lambda_1 + x_{12}\lambda_2 + x_{13}\lambda_3 + x_{14}\lambda_4 + x_{15}\lambda_5) \geq 0,$

   $\theta x_{23} - (x_{21}\lambda_1 + x_{22}\lambda_2 + x_{23}\lambda_3 + x_{24}\lambda_4 + x_{25}\lambda_5) \geq 0,$

   $\lambda \geq 0,$ (14)

where $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)'$.

The values of θ and λ which provide a minimum value for θ are listed in row 3 of Table 2. We note that the TE$_I$ of DMU 3 is 0.833. That is, DMU 1 should be able to reduce the consumption of all inputs by 16.7% without reducing output. This implies production at the point denoted 3′ in Figure 6. This projected point, 3′, lies on a line joining points 2 and 5. DMU 2 and DMU 5 are therefore usually referred to as the *peers* of DMU 3. They define where the relevant part of the frontier is (i.e. relevant to DMU 3) and hence define efficient production for DMU 3. Point 3′ is a linear combination of points 2 and 5, where the weights in this linear combination are the λ's

in row 3 of Table 2.

Figure 6

CRS Input-Orientated DEA Example
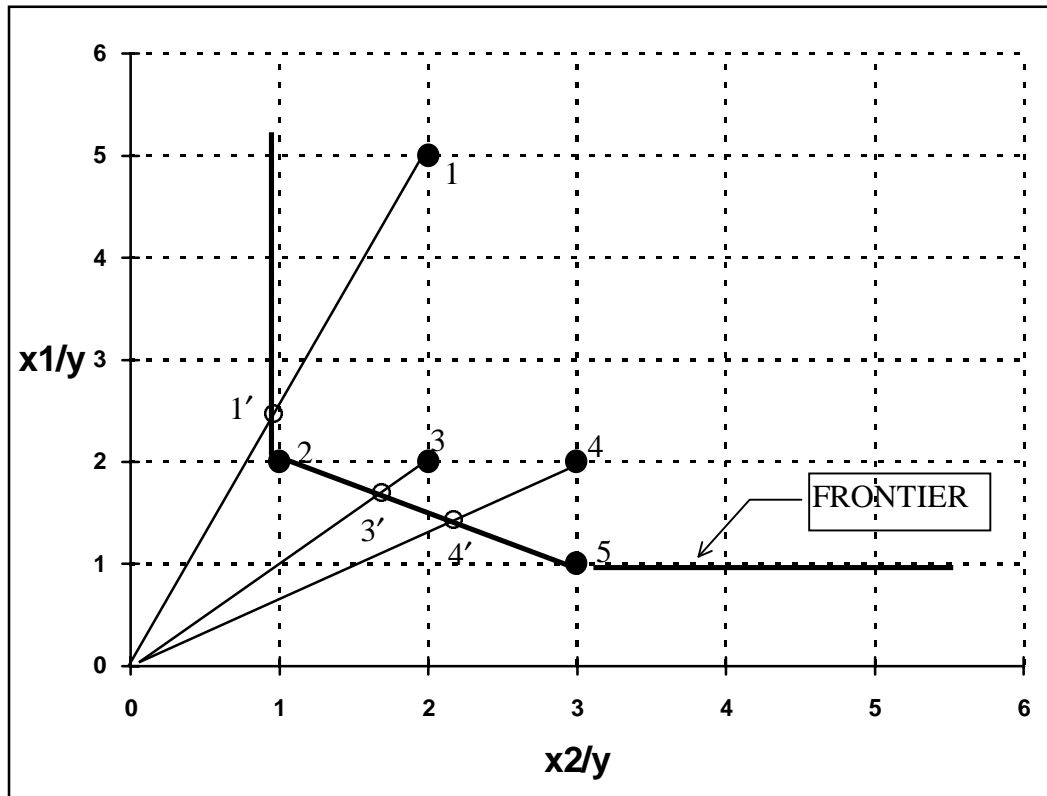
Table 2

CRS Input-Orientated DEA Results

DMU	θ	λ_1	λ_2	λ_3	λ_4	λ_5	IS_1	IS_2	OS
1	0.5	-	0.5	-	-	-	-	0.5	-
2	1.0	-	1.0	-	-	-	-	-	-
3	0.833	-	1.0	-	-	0.5	-	-	-
4	0.714	-	0.214	-	-	0.286	-	-	-
5	1.0	-	-	-	-	1.0	-	-	-

Many DEA studies also talk about *targets* as well as peers. The targets of DMU 3 are

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the coordinates of the efficient projection point 3′. These are equal to 0.833×(2,2) = (1.666,1.666). Thus DMU 3 should aim to produce its 3 units of output with 3×(1.666,1.666) = (5,5) units of the two inputs.

One could go through a similar discussion of the other two inefficient DMU's. DMU 4 has $TE_I = 0.714$ and has the same peers as DMU 3. DMU 1 has $TE_I = 0.5$ and has DMU 2 as its peer. You will also note that the projected point for DMU 1 (1′) lies upon part of the frontier which is parallel to the x_2 axis. Thus it does not represent an efficient point (according to Koopman's definition) because we could decrease the use of the input x_2 by 0.5 units (thus producing at the point 2) and still produce the same output. Thus DMU 1 is said to be radially inefficient in input usage by a factor of 50% plus it has (non-radial) input slack of 0.5 units of x_2. The targets of DMU 1 would therefore be to reduce usage of both inputs by 50% and also to reduce the use of x_2 by a further 0.5 units. This would result in targets of (x_1=1,x_2=2). That is, the coordinates of point 2.

A quick glance at Table 2 shows that DMU's 2 and 5 have TE_I values of 1.0 and that their peers are themselves. This is as one would expect for the efficient points which define the frontier.

3.2 The Variable Returns to Scale Model (VRS) and Scale Efficiencies

The CRS assumption is only appropriate when all DMU's are operating at an optimal scale (i.e one corresponding to the flat portion of the LRAC curve). Imperfect competition, constraints on finance, etc. may cause a DMU to be not operating at optimal scale. Banker, Charnes and Cooper(1984) suggested an extension of the CRS DEA model to account for variable returns to scale (VRS) situations. The use of the CRS specification when not all DMU's are operating at the optimal scale, will result in measures of TE which are confounded by *scale efficiencies* (SE). The use of the VRS specification will permit the calculation of TE devoid of these SE effects.

The CRS linear programming problem can be easily modified to account for VRS by adding the convexity constraint: N1′λ=1 to (12) to provide:

$$\min_{\theta,\lambda} \theta,$$

$$st \quad -y_i + Y\lambda \geq 0,$$

$$\theta x_i - X\lambda \geq 0,$$

$$N1'\lambda = 1$$

$$\lambda \geq 0, \tag{15}$$

where N1 is an N×1 vector of ones. This approach forms a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are greater than or equal to those obtained using the CRS model. The VRS specification has been the most commonly used specification in the 1990's.

Calculation of Scale Efficiencies

Many studies have decomposed the TE scores obtained from a CRS DEA into two components, one due to scale inefficiency and one due to "pure" technical inefficiency. This may be done by conducting both a CRS and a VRS DEA upon the same data. If there is a difference in the two TE scores for a particular DMU, then this indicates that the DMU has scale inefficiency, and that the scale inefficiency can be calculated from the difference between the VRS TE score and the CRS TE score.

Figure 7 attempts to illustrate this. In this figure we have a one-input one-output example and have drawn the CRS and VRS DEA frontiers. Under CRS the input-orientated technical inefficiency of the point P is the distance PP_C, while under VRS the technical inefficiency would only be PP_V. The difference between these two, $P_C P_V$, is put down to scale inefficiency. One can also express all of this in ratio efficiency measures as:

$$TE_{I,CRS} = AP_C/AP$$

$$TE_{I,VRS} = AP_V/AP$$

$$SE_I = AP_C/AP_V$$

where all of these measures will be bounded by zero and one. We also note that
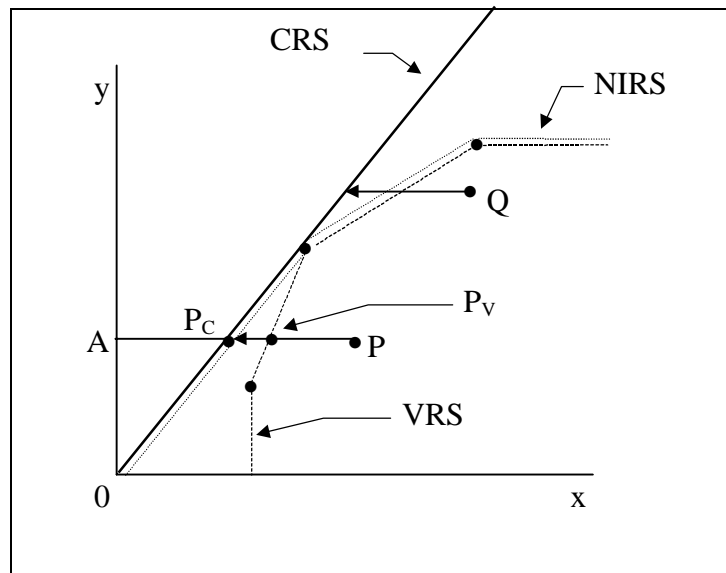
$$TE_{I,CRS} = TE_{I,VRS} \times SE_I$$

because

$$AP_C/AP = (AP_V/AP) \times (AP_C/AP_V).$$

That is, the CRS technical efficiency measure is decomposed into "pure" technical efficiency and scale efficiency.

Figure 7

Calculation of Scale Economies in DEA

One shortcoming of this measure of scale efficiency is that the value does not indicate whether the DMU is operating in an area of increasing or the decreasing returns to scale. This may be determined by running an addition DEA problem with non-increasing returns to scale (NIRS) imposed. This can be done by altering the DEA model in equation 15 by substituting the N1′λ=1 restriction with N1′$\lambda \leq 1$, to provide:

$$\min_{\theta, \lambda} \theta,$$

$$\text{st} \quad -y_i + Y\lambda \geq 0,$$

$$\theta x_i - X\lambda \geq 0,$$

$$N1'\lambda \leq 1$$

$$\lambda \geq 0, \tag{16}$$

The NIRS DEA frontier is also plotted in Figure 7. The nature of the scale inefficiencies (i.e. due to increasing or decreasing returns to scale) for a particular DMU can be determined by seeing whether the NIRS TE score is equal to the VRS TE score. If they are unequal (as will be the case for the point P in Figure 7) then increasing returns to scale exist for that DMU. If they are equal (as is the case for

point Q in Figure 7) then decreasing returns to scale apply. An example of this approach applied to international airlines is provided in BIE (1994).

Example 2

This is a simple numerical example involving five firms which produce a single output using a single input. The data are listed in Table 3 and the VRS and CRS input-orientated DEA results are listed in Table 4 and plotted in Figure 8. Given that we are using an input orientation, the efficiencies are measured horizontally across Figure 8. We observe that firm 3 is the only efficient firm (i.e., on the DEA frontier) when CRS is assumed but that firms 1, 3 and 5 are efficient when VRS is assumed.

The calculation of the various efficiency measures can be illustrated using firm 2 which is inefficient under both CRS and VRS technologies. The CRS technical efficiency (TE) is equal to 2/4=0.5; the VRS TE is 2.5/4=0.625 and the scale efficiency is equal to the ratio of the CRS TE to the VRS TE which is 0.5/0.625=0.8. We also observe that firm 2 is on the increasing returns to scale (IRS) portion of the VRS frontier.

Table 3

Example Data for VRS DEA

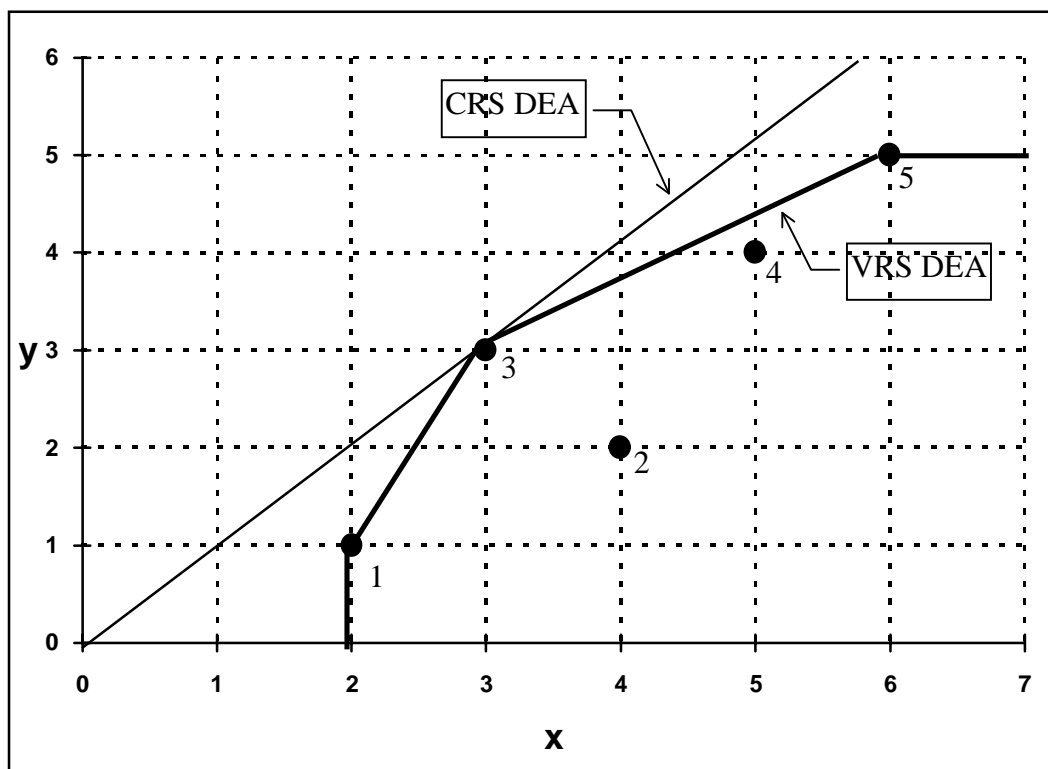| DMU	y	x
1	1	2
2	2	4
3	3	3
4	5	5
5	5	6

Table 4

VRS Input-Orientated DEA Results

DMU	CRS TE	VRS TE	SCALE	
1	0.500	1.000	0.500	irs
2	0.500	0.625	0.800	irs
3	1.000	1.000	1.000	-
4	0.800	0.900	0.889	drs
5	0.833	1.000	0.833	drs
mean	**0.727**	**0.905**	**0.804**	

Figure 8

VRS Input-Orientated DEA Example

3.3 Input and Output Orientations

In the preceding input-orientated models, discussed in sections 3.1 and 3.2, the method sought to identify technical inefficiency as a proportional reduction in input usage. This corresponds to Farrell's input-based measure of technical inefficiency. As discussed in section 2.2, it is also possible to measure technical inefficiency as a proportional increase in output production. The two measures provide the same value under CRS but are unequal when VRS is assumed (see Figure 3). Given that linear programming cannot suffer from such statistical problems as simultaneous equation bias, the choice of an appropriate orientation is not as crucial as it is in the econometric estimation case. In many studies the analysts have tended to select input-orientated models because many DMU's have particular orders to fill (e.g. electricity generation) and hence the input quantities appear to be the primary decision variables, although this argument may not be as strong in all industries. In some industries the DMUs may be given a fixed quantity of resources and asked to produce as much output as possible. In this case an output orientation would be more appropriate. Essentially one should select an orientation according to which quantities (inputs or outputs) the managers have most control over. Furthermore, in many instances you will observe that the choice of orientation will have only minor influences upon the scores obtained (e.g. see Coelli and Perelman 1996).

The output-orientated models are very similar to their input-orientated counterparts. Consider the example of the following output-orientated VRS model:

$$\max_{\phi,\lambda} \phi,$$

$$\text{st} \quad -\phi y_i + Y\lambda \geq 0,$$

$$x_i - X\lambda \geq 0,$$

$$N1'\lambda = 1$$

$$\lambda \geq 0, \tag{17}$$

where $1 \leq \phi < \infty$, and $\phi-1$ is the proportional increase in outputs that could be achieved by the i-th DMU, with input quantities held constant.[16] Note that $1/\phi$ defines a TE score which varies between zero and one (and that this is the output-orientated TE score

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[16] An output-oriented CRS model is defined in a similar way, but is not presented here for brevity.

reported by DEAP).

A two-output example of an output-orientated DEA could be represented by a piecewise linear production possibility curve, such as that depicted in Figure 8. Note that the observations lie *below* this curve, and that the sections of the curve which are at right angles to the axes will cause output slack to be calculated when a production point is projected onto those parts of the curve by a radial expansion in outputs. For example the point P is projected to the point P′ which is on the frontier but not on the *efficient frontier*, because the production of y_1 could be increased by the amount AP′ without using any more inputs. That is there is output slack in this case of AP′ in output y_1.

One point that should be stressed is that *the output- and input-orientated models will estimate exactly the same frontier and therefore, by definition, identify the same set of DMU's as being efficient. It is only the efficiency measures associated with the inefficient DMU's that may differ between the two methods.* The two types of measures were illustrated in section 2 using Figure 3, where we observed that the two measures would provide equivalent values only under constant returns to scale.

Figure 8

Output-Orientated DEA
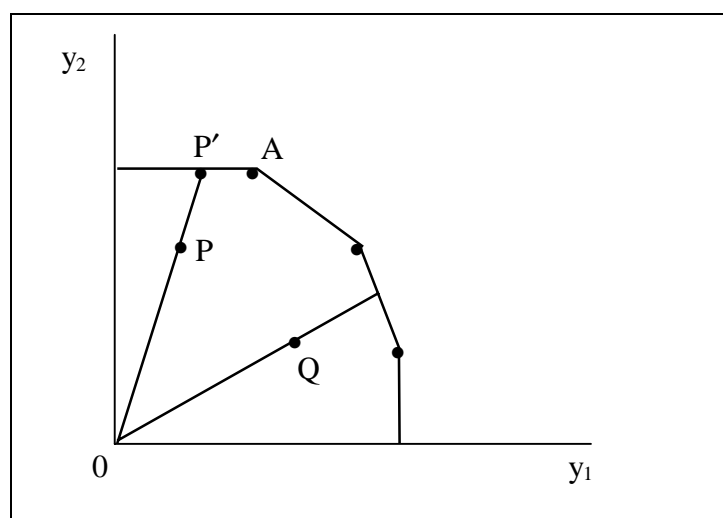
3.4 Price Information and Allocative Efficiency

If one has price information and is willing to consider a behavioural objective, such as cost minimisation or revenue maximisation, then one can measure both technical and allocative efficiencies. For the case of VRS cost minimisation, one would run the input-orientated DEA model set out in equation 15 to obtain technical efficiencies (TE). One would then run the following cost minimisation DEA

$$\min_{\lambda, xi*} \; w_i'x_i*,$$

$$\text{st} \qquad -y_i + Y\lambda \geq 0,$$

$$x_i* - X\lambda \geq 0,$$

$$N1'\lambda = 1$$

$$\lambda \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad\qquad (23)$$

where w_i is a vector of input prices for the i-th DMU and x_i* (which is calculated by the LP) is the cost-minimising vector of input quantities for the i-th DMU, given the input prices w_i and the output levels y_i. The total cost efficiency (CE) or economic efficiency of the i-th DMU would be calculated as

$$CE = w_i'x_i* / \, w_i'x_i.$$

That is, the ratio of minimum cost to observed cost. One can then use equation 4 to calculate the allocative efficiency residually as

$$AE = CE/TE.$$

Note that this procedure will include any slacks into the allocative efficiency measure. This is often justified on the grounds that slack reflects an inappropriate input mix (see Ferrier and Lovell, 1990, p235).

Note also that one can also consider revenue maximisation and allocative inefficiency in output mix selection in a similar manner. See Lovell (1993, p33) for a discussion of this. Note that this revenue efficiency model is not implemented in DEAP.

Example 3

In this example we take the data from Example 1 and add the information that all firms face the same prices which are 1 and 3 for inputs 1 and 2, respectively. Thus if we draw an isocost line with a slope of -1/3 onto Figure 6 which is tangential to the

isoquant we obtain Figure 9. From this diagram we observe that the firm 5 is the only cost efficient firm and that all other firms have some allocative efficiency to some degree. The various cost efficiencies and allocative efficiencies are listed in Table 5.

The calculation of these efficiencies can be illustrated using firm 3. We noted earlier that the technical efficiency of firm 3 is measured along the ray from the origin (0) to the point 3 and that it is equal to the ratio of the distance from 0 to the point 3′ over the distance from 0 to the point 3 and that this is equal to 0.833. The allocative efficiency is equal to the ratio of the distances 0 to 3″ over 0 to 3′ which is equal to 0.9. The cost efficiency is the ratio of distances 0 to 3″ over 0 to 3 which is equal to 0.75. We also note that 0.833×0.9=0.750.

Figure 9

CRS Cost Efficiency DEA Example

Table 5

CRS Cost Efficiency DEA Results

firm	technical	allocative	cost

	efficiency	efficiency	efficiency
1	0.500	0.706	0.353
2	1.000	0.857	0.857
3	0.833	0.900	0.750
4	0.714	0.933	0.667
5	1.000	1.000	1.000
mean	**0.810**	**0.879**	**0.725**

3.5 *Panel Data, DEA and the Malmquist Index*

When one has panel data, one may use DEA-like linear programs and a (input- or output-based) Malmquist TFP index to measure productivity change, and to decompose this productivity change into technical change and technical efficiency change.

Fare et al (1994) specifies an output-based Malmquist productivity change index[17] as:

$$m_o\left(y_{t+1}, x_{t+1}, y_t, x_t\right) = \left[\frac{d_o^t\left(x_{t+1}, y_{t+1}\right)}{d_o^t\left(x_t, y_t\right)} \times \frac{d_o^{t+1}\left(x_{t+1}, y_{t+1}\right)}{d_o^{t+1}\left(x_t, y_t\right)}\right]^{1/2}. \tag{24}$$

This represents the productivity of the production point $(x_{t+1},\ y_{t+1})$ relative to the production point $(x_t,\ y_t)$. A value greater than one will indicate positive TFP growth from period t to period t+1. This index is, in fact, the geometric mean of two output-based Malmquist TFP indices. One index uses period t technology and the other period t+1 technology. To calculate equation 24 we must calculate the four component distance functions, which will involve four LP problems (similar to those conducted in calculating Farrell technical efficiency (TE) measures).

We begin by assuming CRS technology (we conduct a further decomposition later to look at scale efficiency questions). The CRS output-orientated LP used to calculate $d_o^t(x_t,\ y_t)$ is identical to equation 17, except that the convexity (VRS) restriction has been removed and time subscripts have been included. That is,

---

[17] The subscript "o" has been introduced to remind us that these are output-orientated measures. Note that input-orientated Malmquist TFP indices can also be defined in a similar way to the output-orientated measures presented here (see Grosskopf, 1993, p183).

$$[d_o^t(x_t, y_t)]^{-1} = \max_{\phi, \lambda} \phi,$$

$$\text{st} \quad -\phi y_{it} + Y_t \lambda \geq 0,$$

$$x_{it} - X_t \lambda \geq 0,$$

$$\lambda \geq 0, \tag{25}$$

The remaining three LP problems are simple variants of this:

$$[d_o^{t+1}(x_{t+1}, y_{t+1})]^{-1} = \max_{\phi, \lambda} \phi,$$

$$\text{st} \quad -\phi y_{i,t+1} + Y_{t+1} \lambda \geq 0,$$

$$x_{i,t+1} - X_{t+1} \lambda \geq 0,$$

$$\lambda \geq 0, \tag{26}$$

$$[d_o^t(x_{t+1}, y_{t+1})]^{-1} = \max_{\phi, \lambda} \phi,$$

$$\text{st} \quad -\phi y_{i,t+1} + Y_t \lambda \geq 0,$$

$$x_{i,t+1} - X_t \lambda \geq 0,$$

$$\lambda \geq 0, \tag{27}$$

$$[d_o^{t+1}(x_t, y_t)]^{-1} = \max_{\phi, \lambda} \phi,$$

$$\text{st} \quad -\phi y_{it} + Y_{t+1} \lambda \geq 0,$$

$$x_{it} - X_{t+1} \lambda \geq 0,$$

$$\lambda \geq 0, \tag{28}$$

Note that in LP's 27 and 28, where production points are compared to technologies from different time periods, the ϕ parameter need not be ≥ 1, as it must be when calculating Farrell efficiencies. The point could lie above the feasible production set. This will most likely occur in LP 27 where a production point from period t+1 is compared to technology in period t. If technical progress has occurred, then a value of $\phi < 1$ is possible. Note that it could also possibly occur in LP 28 if technical regress has occurred, but this is less likely.

Some points to keep in mind are that the ϕ and λ's are likely to take different values in the above four LP's. Furthermore, note that the above four LP's must be calculated for each firm in the sample. Thus if you have 20 firms and 2 time periods you must

calculate 80 LP's. Note also that as you add extra time periods, you must calculate an extra three LP's for each firm (to construct a chained index). If you have T time periods, you must calculate (3T-2) LP's for each firm in the sample. Hence, if you have N firms, you will need to calculate N×(3T-2) LP's. For example, with N=20 firms and T=10 time periods, this would provide 20×(3×10-2) = 560 LP's.

Results on each and every firm for each and every adjacent pair of time periods can be tabulated, and/or summary measures across time and/or space can be presented.

Scale Efficiency

The above approach can be extended by decomposing the (CRS) technical efficiency change into scale efficiency and "pure" (VRS) technical efficiency components. This will involve calculating two additional LP's (when comparing two production points). These would involve repeating LP's 25 and 26 with the convexity restriction (N1′λ=1) added to each. That is, one would calculate the distance functions relative to a VRS (instead of a CRS) technology. One can then use the CRS and VRS values to calculate the scale efficiency effect residually, using the methods outlined in section 3.2. For the case of N firms and T time periods, this would increase the number of LP's from N×(3T-2) to N×(4T-2). See Fare et al (1994, p75) for more on scale efficiencies.

Example 4

In this example we take the data from Example 2 and add an extra year of data. This data is listed in Table 6 and is also plotted in Figure 9. Also plotted in Figure 9 are the CRS and VRS DEA frontiers for the two time periods. The various distances (or technical efficiencies) needed to calculate the Malmquist indices and the Malmquist indices themselves are listed in Table 10c in Section 5.4.
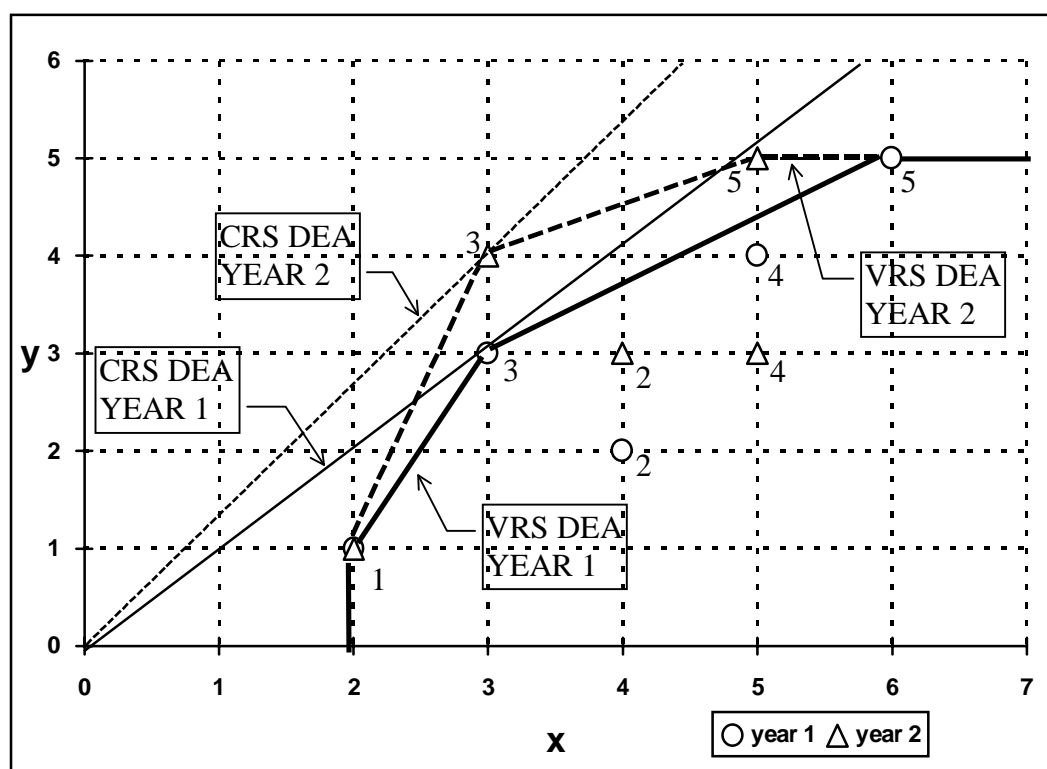
Table 6

Example Data for Malmquist DEA

DMU	year	y	x
1	1	1	2
2	1	2	4
3	1	3	3
4	1	5	5
5	1	5	6
1	2	1	2
2	2	3	4
3	2	4	3
4	2	5	5
5	2	5	5

Figure 8

VRS Input-Orientated DEA Example

4. The DEAP Computer Program

This section describes the use of the DEAP computer program. This program is written in Fortran (Lahey F77LEM/32) for IBM compatible PCs. It is a DOS program but can be easily run from WINDOWS using FILE MANAGER. The program involves a simple batch file system where the user creates a data file and a small file containing instructions. The user then starts the program by typing "DEAP" at the DOS prompt[18] and is then prompted for the name of the instruction file. The program then executes these instructions and produces an output file which can be read using a text editor, such as NOTEPAD or EDIT, or using a word processor, such as WORD or WORD PERFECT.

The execution of DEAP Version 2.0 on an IBM PC generally involves five files:

1) The executable file DEAP.EXE

2) The start-up file DEAP.000

3) A data file (for example, called TEST.DTA)

4) An instruction file (for example, called TEST.INS)

5) An output file (for example, called TEST.OUT).

The executable file and the start-up file is supplied on the disk. The start-up file, DEAP.000, is a file which stores key parameter values which the user may or may not need to alter.[19] The data and instruction files must be created by the user prior to execution. The output file is created by DEAP during execution. Examples of data, instruction and output files are listed in the next section.

Data file

The program requires that the data be listed in a text file[20] and expects the data to appear in a particular order. The data must be listed by observation (i.e., one row for each firm). There must be a column for each output and each input, with all outputs listed first and then all inputs listed (from left to right across the file). For example, if you have 40 observations on two outputs and two inputs there would be four columns

---

[18] The program can also be run by double-clicking on the DEAP.EXE file in FILE MANAGER in WINDOWS.

[19] At present this file only contains the value of a variable used to test inequalities with zero. This text file may be edited if the user wishes to alter this value.

of data (each of length 40) listed in the order: y1, y2, x1, x2.

If you choose the cost efficiencies option you will also need to supply price information for the inputs. These price columns should be listed to the right of the input data columns and appear in the same order. That is, if you have three outputs and two inputs, the order for the columns should be: y1, y2, y3, x1, x2, w1, w2, where w1 and w2 are input prices corresponding to input quantities x1 and x2.

If you choose the Malmquist option you will be dealing with panel data. For example, you may have 30 firms observed in each of 4 years. In this instance you must list all data for year 1 first, followed by the year 2 data listed in the same order (of firms) and so on. Note that the panel must be "balanced". That is, all firms must be observed in all time periods.

A data file can be produced using any number of computer packages. For example:

- using a text editor (such as DOS EDIT or NOTEPAD),

- using a word processor (such as WORD or WORD PERFECT) and saving the file in text format,

- using a spreadsheet (such as LOTUS or EXCEL) and printing to a file, or

- using a statistics package (such as SHAZAM or SAS) and writing data to a file.

Note that the data file should only contain numbers separated by spaces or tabs. It should not contain any column headings.

Instruction file

The instruction file is a text file which is usually constructed using a text editor or a word processor. The easiest way to create an instruction file is to make a copy of the DBLANK.INS file which is supplied with the program (by using the FILE/COPY menus in FILE MANAGER in WINDOWS or by using the COPY command at the DOS prompt). We then edit this file (using a text editor or word processor) and type in the relevant information. The best way to describe the structure of the instruction file is to provide a few examples. These are listed in the following section.

---

[20] All data, instruction and output files are (ASCII) text files.

Output file

As noted earlier, the output file is a text file which is produced by DEAP when an instruction file is executed. The output file can be read using a text editor, such as NOTEPAD or EDIT, or using a word processor, such as WORD or WORD PERFECT. The output may also be imported into a spreadsheet program, such as EXCEL or LOTUS, to allow further manipulation into tables and graphs for subsequent inclusion into report documents.

5. Examples

In this section we shall consider four examples:

1) An input-orientated CRS DEA involving five observations on one output and two inputs..

2) An input-orientated VRS DEA involving five observations on one output and one input.

3) A CRS cost efficiency DEA using the data from example (1) along with some input price data.

4) An output-orientated Malmquist DEA involving data on one output and one input for 5 firms observed over a three year period.

These examples correspond to the four examples discussed in Section 3.

5.1 Example 1: An Input-orientated CRS DEA Example

The text file EG1.DTA (refer to Table 7a) contains five observations on one output and two inputs. The output is listed in the first column and the inputs in the next two columns. This data is identical to that listed in Table 1.

The EG1.INS file is listed in Table 7b. The purpose of the majority of entries in the file should be self explanatory, due to the comments on the right-hand side of the file.[21] The first two lines of the file contain the name of the data file (EG1.DTA) and an output file name (here we have used EG1.OUT). Then on the next four lines we

---

[21]It should be mentioned that the comments in DBLANK.INS and DEAP.000 are not read by the program. Hence users may have instruction files which are made from scratch with a text editor and which contain no comments. This is not recommended, however, as it would be too easy to lose track of which input value belongs on which line.

specify the number of firms (5); number of time periods (1);[22] number of outputs (1); and number of inputs (2). On the final three lines we specify a "0" to indicate CRS; a "0" to indicate an input orientation; and a "0" to indicate that we wish to estimate a standard DEA model.[23]

Finally we type "DEAP" at the DOS prompt, and then type in the name of the instruction file (EG1.INS). The program will then take somewhere between a few seconds and a few minutes (depending upon the size of the model and the speed of your computer) to run the required LP problems and send the output to the file you have named (EG1.OUT). This file is reproduced in Table 7c. These results are identical to those presented in Table 2.

Table 7a - Listing of Data File EG1.DTA
_____
```
1 2 5
2 2 4
3 6 6
1 3 2
2 6 2
```
_____

Table 7b - Listing of Instruction File EG1.INS
_____
```
eg1.dta           DATA FILE NAME
eg1.out           OUTPUT FILE NAME
5                 NUMBER OF FIRMS
1                 NUMBER OF TIME PERIODS
1                 NUMBER OF OUTPUTS
2                 NUMBER OF INPUTS
0                 0=INPUT AND 1=OUTPUT ORIENTATED
0                 0=CRS AND 1=VRS
0                 0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA,
                     3=DEA(1-STAGE), 4=DEA(2-STAGE)
```
_____

Table 7c - Listing of Output File EG1.OUT
_____
```
Results from DEAP Version 2.1

Instruction file = eg1.ins
Data file        = eg1.dta
```

_____

[22] Note that the number of time periods will always be equal to 1 unless the Malmquist DEA option is selected.

[23] Note that by specifying "0" on the final line we are asking that slacks be calculated using the multi-stage method. If we wished the 1-stage or 2-stage methods to be used we would have used a "3" or a "4", respectively.

```
Input orientated DEA

Scale assumption: CRS

Slacks calculated using multi-stage method


EFFICIENCY SUMMARY:

 firm      te
    1   0.500
    2   1.000
    3   0.833
    4   0.714
    5   1.000

 mean   0.810


SUMMARY OF OUTPUT SLACKS:

 firm  output:          1
    1                0.000
    2                0.000
    3                0.000
    4                0.000
    5                0.000

 mean                0.000


SUMMARY OF INPUT SLACKS:

 firm  input:           1           2
    1                0.000       0.500
    2                0.000       0.000
    3                0.000       0.000
    4                0.000       0.000
    5                0.000       0.000

 mean                0.000       0.100


SUMMARY OF PEERS:

 firm  peers:
    1       2
    2       2
    3       5     2
    4       5     2
    5       5


SUMMARY OF PEER WEIGHTS:
   (in same order as above)

 firm  peer weights:
    1   0.500
    2   1.000
    3   0.500 1.000
    4   0.286 0.214
    5   1.000
```

```
PEER COUNT SUMMARY:
  (i.e., no. times each firm is a peer for another)

 firm  peer count:
   1        0
   2        3
   3        0
   4        0
   5        2


SUMMARY OF OUTPUT TARGETS:

 firm  output:        1
   1              1.000
   2              2.000
   3              3.000
   4              1.000
   5              2.000


SUMMARY OF INPUT TARGETS:

 firm  input:         1         2
   1              1.000     2.000
   2              2.000     4.000
   3              5.000     5.000
   4              2.143     1.429
   5              6.000     2.000


FIRM BY FIRM RESULTS:


Results for firm:      1
Technical efficiency = 0.500
PROJECTION SUMMARY:
  variable         original      radial        slack     projected
                     value      movement     movement        value
 output   1         1.000        0.000        0.000        1.000
 input    1         2.000       -1.000        0.000        1.000
 input    2         5.000       -2.500       -0.500        2.000
LISTING OF PEERS:
 peer    lambda weight
    2       0.500


Results for firm:      2
Technical efficiency = 1.000
PROJECTION SUMMARY:
  variable         original      radial        slack     projected
                     value      movement     movement        value
 output   1         2.000        0.000        0.000        2.000
 input    1         2.000        0.000        0.000        2.000
 input    2         4.000        0.000        0.000        4.000
LISTING OF PEERS:
 peer    lambda weight
    2       1.000


Results for firm:      3
```

```
Technical efficiency = 0.833
PROJECTION SUMMARY:
  variable          original        radial          slack         projected
                      value        movement        movement          value
 output    1          3.000          0.000          0.000          3.000
 input     1          6.000         -1.000          0.000          5.000
 input     2          6.000         -1.000          0.000          5.000
LISTING OF PEERS:
 peer    lambda weight
    5       0.500
    2       1.000


Results for firm:      4
Technical efficiency = 0.714
PROJECTION SUMMARY:
  variable          original        radial          slack         projected
                      value        movement        movement          value
 output    1          1.000          0.000          0.000          1.000
 input     1          3.000         -0.857          0.000          2.143
 input     2          2.000         -0.571          0.000          1.429
LISTING OF PEERS:
 peer    lambda weight
    5       0.286
    2       0.214


Results for firm:      5
Technical efficiency = 1.000
PROJECTION SUMMARY:
  variable          original        radial          slack         projected
                      value        movement        movement          value
 output    1          2.000          0.000          0.000          2.000
 input     1          6.000          0.000          0.000          6.000
 input     2          2.000          0.000          0.000          2.000
LISTING OF PEERS:
 peer    lambda weight
    5       1.000
```

5.2 Example 2: An Input-orientated VRS DEA Example

The text file EG2.DTA (refer to Table 8a) contains five observations on one output and one input. The output is listed in the first column and the input in the second column. This data is identical to that listed in Table 3.

The EG2.INS file is listed in Table 8b. The only changes relative to EG1.INS is that:

- the input and output file names are different;

- the number of inputs is reduced to 1; and

- there is a "1" entered on the 2nd last line to indicate that VRS is required.

The output file EG1.OUT is reproduced in Table 8c. These results are identical to those presented in Table 4.

Table 8a - Listing of Data File EG2.DTA

```
_____
1 2
2 4
3 3
4 5
5 6

_____
```

Table 8b - Listing of Instruction File EG2.INS

```
_____
eg2.dta           DATA FILE NAME
eg2.out           OUTPUT FILE NAME
5                 NUMBER OF FIRMS
1                 NUMBER OF TIME PERIODS
1                 NUMBER OF OUTPUTS
1                 NUMBER OF INPUTS
0                 0=INPUT AND 1=OUTPUT ORIENTATED
1                 0=CRS AND 1=VRS
0                 0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA,
                      3=DEA(1-STAGE), 4=DEA(2-STAGE)

_____
```

Table 8c - Listing of Output File EG2.OUT

```
_____
Results from DEAP Version 2.1

Instruction file = eg2.ins
Data file        = eg2.dta

Input orientated DEA

Scale assumption: VRS

Slacks calculated using multi-stage method


EFFICIENCY SUMMARY:

 firm  crste  vrste  scale

    1  0.500  1.000  0.500 irs
    2  0.500  0.625  0.800 irs
    3  1.000  1.000  1.000  -
    4  0.800  0.900  0.889 drs
    5  0.833  1.000  0.833 drs


 mean  0.727  0.905  0.804

Note: crste = technical efficiency from CRS DEA
      vrste = technical efficiency from VRS DEA
      scale = scale efficiency = crste/vrste


Note also that all subsequent tables refer to VRS results
```

```
SUMMARY OF OUTPUT SLACKS:

 firm  output:         1
   1                 0.000
   2                 0.000
   3                 0.000
   4                 0.000
   5                 0.000

 mean              0.000


SUMMARY OF INPUT SLACKS:

 firm  input:          1
   1                 0.000
   2                 0.000
   3                 0.000
   4                 0.000
   5                 0.000

 mean              0.000


SUMMARY OF PEERS:

 firm  peers:
   1       1
   2       1    3
   3       3
   4       3    5
   5       5


SUMMARY OF PEER WEIGHTS:
  (in same order as above)

 firm  peer weights:
   1   1.000
   2   0.500 0.500
   3   1.000
   4   0.500 0.500
   5   1.000


PEER COUNT SUMMARY:
  (i.e., no. times each firm is a peer for another)

 firm  peer count:
   1       1
   2       0
   3       2
   4       0
   5       1



SUMMARY OF OUTPUT TARGETS:

 firm  output:         1
   1                 1.000
   2                 2.000
   3                 3.000
   4                 4.000
```

```
        5                   5.000


   SUMMARY OF INPUT TARGETS:

    firm  input:           1
       1                 2.000
       2                 2.500
       3                 3.000
       4                 4.500
       5                 6.000



   FIRM BY FIRM RESULTS:


   Results for firm:      1
   Technical efficiency = 1.000
   Scale efficiency     = 0.500  (irs)
   PROJECTION SUMMARY:
     variable          original        radial         slack        projected
                         value        movement      movement          value
    output   1          1.000          0.000         0.000          1.000
    input    1          2.000          0.000         0.000          2.000
   LISTING OF PEERS:
    peer   lambda weight
       1      1.000


   Results for firm:      2
   Technical efficiency = 0.625
   Scale efficiency     = 0.800  (irs)
   PROJECTION SUMMARY:
     variable          original        radial         slack        projected
                         value        movement      movement          value
    output   1          2.000          0.000         0.000          2.000
    input    1          4.000         -1.500         0.000          2.500
   LISTING OF PEERS:
    peer   lambda weight
       1      0.500
       3      0.500


   Results for firm:      3
   Technical efficiency = 1.000
   Scale efficiency     = 1.000  (crs)
   PROJECTION SUMMARY:
     variable          original        radial         slack        projected
                         value        movement      movement          value
    output   1          3.000          0.000         0.000          3.000
    input    1          3.000          0.000         0.000          3.000
   LISTING OF PEERS:
    peer   lambda weight
       3      1.000


   Results for firm:      4
   Technical efficiency = 0.900
   Scale efficiency     = 0.889  (drs)
   PROJECTION SUMMARY:
     variable          original        radial         slack        projected
                         value        movement      movement          value
    output   1          4.000          0.000         0.000          4.000
```

```
 input    1           5.000         -0.500        0.000          4.500
LISTING OF PEERS:
 peer   lambda weight
    3      0.500
    5      0.500


Results for firm:     5
Technical efficiency = 1.000
Scale efficiency     = 0.833  (drs)
PROJECTION SUMMARY:
  variable          original       radial          slack      projected
                       value     movement       movement          value
 output    1           5.000        0.000          0.000          5.000
 input     1           6.000        0.000          0.000          6.000
LISTING OF PEERS:
 peer   lambda weight
    5      1.000
```
_____

5.3 Example 3: A Cost Efficiency DEA Example

The text file EG3.DTA (refer to Table 9a) contains the same data as EG1.DTA. That is, five observations on one output and two inputs. The output is listed in the first column and the inputs in the next two columns. In addition to this, two columns containing input price data are listed to the right of these. In this particular example we assume all firms face the same prices and that these prices are 1 and 3 for inputs 1 and 2, respectively.

The EG3.INS file is listed in Table 9b. The only changes relative to EG1.INS is that:

- the input and output file names are different;

- there is a "1" entered on the last line to indicate that a cost efficiency DEA is required.

The output file EG3.OUT is reproduced in Table 9c. The results in the efficiencies summary table are identical to those listed in EG1.OUT with the exception that allocative and cost efficiencies are now also listed. Furthermore, a table of input quantities for minimum cost production are now also listed.

Table 9a - Listing of Data File EG3.DTA

_____
```
1 2 5 1 3
2 2 4 1 3
3 6 6 1 3
1 3 2 1 3
2 6 2 1 3
```

Table 9b - Listing of Instruction File EG3.INS

```
eg3.dta           DATA FILE NAME
eg3.out           OUTPUT FILE NAME
5                 NUMBER OF FIRMS
1                 NUMBER OF TIME PERIODS
1                 NUMBER OF OUTPUTS
2                 NUMBER OF INPUTS
0                 0=INPUT AND 1=OUTPUT ORIENTATED
0                 0=CRS AND 1=VRS
1                 0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA,
                    3=DEA(1-STAGE), 4=DEA(2-STAGE)
```

Table 9c - Listing of Output File EG3.OUT

```
Results from DEAP Version 2.1

Instruction file = eg3.ins
Data file        = eg3.dta

Cost efficiency DEA

Scale assumption: CRS


EFFICIENCY SUMMARY:

 firm      te      ae      ce
    1   0.500   0.706   0.353
    2   1.000   0.857   0.857
    3   0.833   0.900   0.750
    4   0.714   0.933   0.667
    5   1.000   1.000   1.000

 mean  0.810   0.879   0.725

Note: te = technical efficiency
      ae = allocative efficiency = ce/te
      ce = cost efficiency



SUMMARY OF COST MINIMISING INPUT QUANTITIES:

 firm  input:          1          2
    1                3.000      1.000
    2                6.000      2.000
    3                9.000      3.000
    4                3.000      1.000
    5                6.000      2.000
```

5.4 Example 4: A Malmquist DEA Example

The text file EG4.DTA (refer to Table 10a) contains observations on five firms over a three year period. These firms produce one output using one input. Data for year 1 is listed in the first five rows, year 2 data is in the second five rows and year 3 in the final five rows. Note that the year 1 and 2 data is identical to that listed in Table 6.

The EG4.INS file is listed in Table 10b. The only changes relative to EG1.INS is that:

- the input and output file names are different;

- the number of time periods is now 3;

- the number of inputs is reduced to 1; and

- there is a "2" entered on the last line to indicate that Malmquist DEA is required.

Note that the VRS/CRS option has no influence on the Malmquist DEA because both are used to calculate the various distances used to construct the Malmquist indices.

The output file EG4.OUT is reproduced in Table 10c. The output begins with a listing of the distances (or technical efficiencies) needed for the Malmquist calculations. Four distances are calculated for each firm in each year. These are relative to:

1. the previous periods CRS DEA frontier;

2. the current periods CRS DEA frontier;

3. the next periods CRS DEA frontier; and

4. the current periods VRS frontier.

Following this the Malmquist indices are presented. All indices are relative to the previous year. Hence the output begins with year 2. Five indices are presented for each firm in each year. These are:

1. technical efficiency change (relative to a CRS technology);

2. technological change;

3. pure technical efficiency change (i.e., relative to a VRS technology);

4. scale efficiency change; and

5. total factor productivity (TFP) change.

Following this we present summary tables for the different time periods (over all firms) and for the different firms (over all time periods). Note that all indices are equal to one for time period 3. This is because in the example data set used (see Table 8a) the data for year 3 is identical to the year 2 data.

Table 10a - Listing of Data File EG4.DTA

```
1 2
2 4
3 3
4 5
5 6
1 2
3 4
4 3
3 5
5 5
1 2
3 4
4 3
3 5
5 5
```

Table 10b - Listing of Instruction File EG4.INS

```
eg4.dta          DATA FILE NAME
eg4.out          OUTPUT FILE NAME
5                NUMBER OF FIRMS
3                NUMBER OF TIME PERIODS
1                NUMBER OF OUTPUTS
1                NUMBER OF INPUTS
1                0=INPUT AND 1=OUTPUT ORIENTATED
0                0=CRS AND 1=VRS
2                0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA,
                      3=DEA(1-STAGE), 4=DEA(2-STAGE)
```

Table 10c - Listing of Output File EG4.OUT

```
Results from DEAP Version 2.1

Instruction file = eg4.ins
Data file        = eg4.dta

Output orientated Malmquist DEA


 DISTANCES SUMMARY


 year =      1

   firm     crs te rel to tech in yr      vrs
```

```
   no.      ***********************        te
           t-1         t         t+1

     1    0.000       0.500       0.375     1.000
     2    0.000       0.500       0.375     0.545
     3    0.000       1.000       0.750     1.000
     4    0.000       0.800       0.600     0.923
     5    0.000       0.833       0.625     1.000

 mean    0.000       0.727       0.545     0.894

 year =    2

 firm     crs te rel to tech in yr      vrs
  no.     ***********************        te
           t-1         t         t+1

     1    0.500       0.375       0.375     1.000
     2    0.750       0.563       0.563     0.667
     3    1.333       1.000       1.000     1.000
     4    0.600       0.450       0.450     0.600
     5    1.000       0.750       0.750     1.000

 mean    0.837       0.628       0.628     0.853

 year =    3

 firm     crs te rel to tech in yr      vrs
  no.     ***********************        te
           t-1         t         t+1

     1    0.375       0.375       0.000     1.000
     2    0.563       0.563       0.000     0.667
     3    1.000       1.000       0.000     1.000
     4    0.450       0.450       0.000     0.600
     5    0.750       0.750       0.000     1.000

 mean    0.628       0.628       0.000     0.853
```

[Note that t-1 in year 1 and t+1 in the final year are not defined]

```
 MALMQUIST INDEX SUMMARY

 year =    2

  firm   effch  techch   pech    sech   tfpch

     1   0.750   1.333   1.000   0.750   1.000
     2   1.125   1.333   1.222   0.920   1.500
     3   1.000   1.333   1.000   1.000   1.333
     4   0.562   1.333   0.650   0.865   0.750
     5   0.900   1.333   1.000   0.900   1.200

 mean   0.844   1.333   0.955   0.883   1.125

 year =    3

  firm   effch  techch   pech    sech   tfpch

     1   1.000   1.000   1.000   1.000   1.000
     2   1.000   1.000   1.000   1.000   1.000
     3   1.000   1.000   1.000   1.000   1.000
     4   1.000   1.000   1.000   1.000   1.000
```

```
    5    1.000    1.000    1.000    1.000    1.000

mean     1.000    1.000    1.000    1.000    1.000


MALMQUIST INDEX SUMMARY OF ANNUAL MEANS

 year    effch   techch    pech     sech    tfpch

    2    0.844    1.333    0.955    0.883    1.125
    3    1.000    1.000    1.000    1.000    1.000

mean     0.918    1.155    0.977    0.940    1.061


MALMQUIST INDEX SUMMARY OF FIRM MEANS

 firm    effch   techch    pech     sech    tfpch

    1    0.866    1.155    1.000    0.866    1.000
    2    1.061    1.155    1.106    0.959    1.225
    3    1.000    1.155    1.000    1.000    1.155
    4    0.750    1.155    0.806    0.930    0.866
    5    0.949    1.155    1.000    0.949    1.095

mean     0.918    1.155    0.977    0.940    1.061

[Note that all Malmquist index averages are geometric means]
```
_____

6. Concluding Comments

We plan to revise the DEAP computer program in 1997. This revision will involve the inclusion of additional model options and the construction of a Windows interface. Please contact me by email at **tcoelli@metz.une.edu.au** if you have found any bugs or if you have any suggestions on how the program may be improved. All users of the DOS version will be notified when the Windows version is available.

REFERENCES

Afriat, S.N. (1972), "Efficiency Estimation of Production Functions", *International Economic Review*, 13, 568-598.

Ali, A.I. and L.M. Seiford (1993), "The Mathematical Programming Approach to Efficiency Analysis", in Fried, H.O., C.A.K. Lovell and S.S. Schmidt (Eds*), The Measurement of Productive Efficiency*, Oxford University Press, New York, 120-159.

Banker, R.D., Charnes, A. and Cooper, W.W. (1984), "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis", *Management Science*, 30, 1078-1092.

BIE (1994), *International Performance Indicators: Aviation*, research report #59, BIE, Canberra.

Boles, J.N. (1966), "Efficiency Squared - Efficient Computation of Efficiency Indexes", *Proceedings of the 39th Annual Meeting of the Western Farm Economic Association*, pp 137-142.

Charnes, A., W.W. Cooper, A.Y. Lewin and L.M. Seiford (1995), "Data Envelopment Analysis: Theory, Methodology and Applications", Kluwer.

Charnes, A., W.W. Cooper and E. Rhodes (1978), "Measuring the Efficiency of Decision Making Units", *European Journal of Operations Research*, 2, 429-444.

Charnes, A., W.W. Cooper, J. Rousseau and J. Semple (1987), "Data Envelopment Analysis and Axiomatic Notions of Efficiency and Reference Sets", Research Report CCS 558, Centre for Cybernetic Studies, The University of Texas at Austin.

Coelli, T.J. (1992), "A Computer Program for Frontier Production Function Estimation: FRONTIER, Version 2.0", *Economics Letters*, 39, 29-32.

Coelli, T.J. (1994), *A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation*, mimeo, Department of Econometrics, University of New England, Armidale.

Coelli, T.J. (1997), *A Multi-Stage Methodology for the Solution of Orientated DEA Models*, mimeo, Centre for Efficiency and Productivity Analysis, University of New England, Armidale.

Coelli, T.J. and S. Perelman (1996), "A Comparison of Parametric and Non-parametric Distance Functions: With Application to European Railways", *CREPP Discussion Paper*, University of Liege, Liege.

Debreu, G. (1951), "The Coefficient of Resource Utilisation", *Econometrica*, 19, 273-292.

Fare, R., S. Grosskopf, and C.A.K. Lovell (1985), *The Measurement of Efficiency of Production*, Boston, Kluwer.

Fare, R., S. Grosskopf, and C.A.K. Lovell (1994), *Production Frontiers*, Cambridge University Press.

Fare, R. and C.A.K. Lovell (1978), "Measuring the Technical Efficiency of

Production", *Journal of Economic Theory*, 19, 150-162.

Fare, R., S. Grosskopf, M. Norris and Z. Zhang (1994), "Productivity Growth, Technical Progress, and Efficiency Changes in Industrialised Countries", *American Economic Review*, 84, 66-83.

Farrell, M.J. (1957), "The Measurement of Productive Efficiency", *Journal of the Royal Statistical Society*, A CXX, Part 3, 253-290.

Ferrier, G.D. and C.A.K. Lovell (1990), "Measuring Cost Efficiency in Banking: Econometric and Linear Programming Evidence", *Journal of Econometrics*, 46, 229-245.

Ganley, J.A and J.S. Cubbin (1992), *Public Sector Efficiency Measurement: Applications of Data Envelopment Analysis*, North Holland, Amsterdam.

Grosskopf, S. (1993), "Efficiency and Productivity", in Fried, H.O., C.A.K. Lovell and S.S. Schmidt (Eds*), The Measurement of Productive Efficiency*, Oxford University Press, New York, 160-194.

Koopmans, T.C. (1951), "An Analysis of Production as an Efficient Combination of Activities", in T.C. Koopmans, Ed., *Activity Analysis of Production and Allocation*, Cowles Commission for Research in Economics, Monograph No. 13, Wiley, New York.

Lovell, C.A.K. (1993), "Production Frontiers and Productive Efficiency", in Fried, H.O., C.A.K. Lovell and S.S. Schmidt (Eds*), The Measurement of Productive Efficiency*, Oxford University Press, New York, 3-67.

Lovell, C.A.K. (1994), "Linear Programming Approaches to the Measurement and Analysis of Productive Efficiency", *Top*, 2, 175-248.

Norman, M. and B. Stoker (1991), *Data Envelopment Analysis: An Assessment of Performance,* Wiley.

Shepherd, R.W. (1970), *Theory of Cost and Production Functions*, Princeton, Princeton University Press.

Seiford, L.M. (1996), "Data Envelopment Analysis: The Evolution of the State of the Art (1978-1995)", Journal of Productivity Analysis, 7, 99-138.

Seiford, L.M. and R.M. Thrall (1990), "Recent Developments in DEA: The Mathematical Approach to Frontier Analysis", *Journal of Econometrics*, 46, 7-38.

APPENDIX

This appendix simply provides a collection of technical and semi-technical points regarding the DEAP computer program.

- Computer requirements: The program is written for an IBM compatible computer. A minimum configuration would be a 386 with a math-coprocessor and 4 MB RAM running DOS 5.0 and/or Windows 3.1 (or higher).

- Size limits: The program has been written using dynamic array allocations so the size of the DEA problem that can be handled is essentially only limited by the memory you have available on your computer. The only limits which may come into play relate to the listing of information on slacks and peers where the output tables are limited to 99 columns (I am yet to see a DEA analysis with dimensions which get even half way to these limits). It should also be noted that the slacks are reported in fixed-width fields. Hence if a slack value is larger than 10^8 it will not be reported and a string of stars (************) will appear instead. This can be avoided by scaling your data prior to estimating the DEA problem. For example, if coal input in an analysis of power station efficiency is expressed in tonnes, you may choose to change the units of measurement to thousand or million tonne units.

- Scaling: The program scales all quantity data by dividing by the sample means before running the DEA. The slack information is then scaled back to original units before it is reported.

- The technical efficiency estimates reported in output orientated models are inverted so they vary between 0 and 1.

- All program files and data, instruction and output files must be stored in the same directory. The program is not able to access files in directories other than the directory that the program resides in.

- The program can sometimes report that "no solutions satisfy the constraints given". This can often be corrected by editing the DEAP.000 file and increasing the size of the EPS parameter.

- The program can also occasionally cycle indefinitely (i.e., run for more than 5 minutes). To stop the program running press ctrl/c. Then to avoid this problem one can also try increasing the size of the EPS parameter. Also note that cycling will often occur if you have zeros in your data and you use the multi-stage DEA method.

- For further information on other CEPA software and publications, refer to the CEPA web site: **http://www.une.edu.au/econometrics/cepa.htm**

Tips on using DEAP in File Manager in Windows 3.1:

1. Use the FILE/CREATE DIRECTORY menu items to create a DEAP directory on your hard drive.

2. Use drag-and-drop to copy the DEAP program files from your floppy disk to the DEAP directory.

3. Use the FILE/ASSOCIATE menu items to associate your favourite Windows text editor (e.g., NOTEPAD) with the file extensions ".INS", ".DTA" and ".OUT". This will allow you to edit these files (or view them) by simply double-clicking on the file names. You only need to set the associations once and then Windows will store this association information.

4. To check that the associations have worked you can double-click on EG1.INS and EG1.DTA to look at their contents.

5. To practice running DEAP double-click on the DEAP.EXE file name. The program will then ask for an instruction file name. Type in EG1.INS (and hit the RETURN key). DEAP will only take a few seconds to complete this small DEA example. To look at the output file (EG1.OUT) you then simply double-click on the EG1.OUT file name.[24]

---

[24] If you are using Windows 3.1 you may first need to click on the hard drive icon (probably your C: drive) at the top of the File Manager window. This will refresh your window and therefore display the name of the newly created file (EG1.OUT).