Homework 7

Name: Yuan Li
ID: N19728558 netID yl6606
Email: foxerlee1@gmail.com

Part I: Written Exercises

1.

Answer:

For each v, we can calculate the result of $v^T Dv$.

$$v_1^T Dv_1 = 157$$
$$v_2^T Dv_2 = 84.1437$$
$$v_3^T Dv_3 = 44.25$$

So $v = [1\ 0\ 0\ 0]^T$ will maximize $v^T Dv$. There is no better unit vector v.

2.

Answer:

$$V^T v_1 = \begin{bmatrix} 0.999 \\ -0.001 \\ -0.001 \\ 0.000 \end{bmatrix}$$

$$V^T v_2 = \begin{bmatrix} -0.001 \\ 0.999 \\ 0.000 \\ 0.000 \end{bmatrix}$$

$$V^T v_3 = \begin{bmatrix} -0.001 \\ 0.000 \\ 1.001 \\ 0.000 \end{bmatrix}$$

$$V^T v_4 = \begin{bmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 1.001 \end{bmatrix}$$

3.

Answer:

Based on the Question-1 result, if we want to maximize the of $v^T Av$, we need to let the first element of v maximize. So $v = \begin{bmatrix} -0.477 \\ 0.476 \\ 0.561 \\ -0.480 \end{bmatrix}$ can maximize $v^T Av$.

4.

Answer:

Multiple constants only scale results without changing results relationship. So still $v = \begin{bmatrix} -0.477 \\ 0.476 \\ 0.561 \\ -0.480 \end{bmatrix}$ can maximize $v^T Av$.

5.

Answer:

Based on the previous discussion, We will choose $v = \begin{bmatrix} -0.477 \\ 0.476 \\ 0.561 \\ -0.480 \end{bmatrix}$ to maximize variance. So the variance should be:

$$\begin{aligned} variance_1 &= \frac{1}{N}|x^T v_1|^2 \\ &= 39.22833125 \end{aligned}$$

We can't find a better line. I choose the second column of V, and the variance is:

$$\begin{aligned} variance_2 &= \frac{1}{N}|x^T v_2|^2 \\ &= 3.9888855 \end{aligned}$$

The first feature can make the examples have the highest variance. We should choose the first feature.

6.

Answer:

$$\begin{aligned} proportion &= \frac{\sum_{i=1}^{2} \lambda_i}{\sum_{j=1}^{4} \lambda_j} \\ &= \frac{157 + 16}{157 + 16 + 4 + 0} \\ &= 0.9774 \end{aligned}$$

So **0.9774** original variance we retain have after projecting onto the top two principle components.