2 Neuron Model and Network Architectures

2

Objectives

In Chapter 1 we presented a simplified description of biological neurons and neural networks. Now we will introduce our simplified mathematical model of the neuron and will explain how these artificial neurons can be interconnected to form a variety of network architectures. We will also illustrate the basic operation of these networks through some simple examples. The concepts and notation introduced in this chapter will be used throughout this book.

This chapter does not cover all of the architectures that will be used in this book, but it does present the basic building blocks. More complex architectures will be introduced and discussed as they are needed in later chapters. Even so, a lot of detail is presented here. Please note that it is not necessary for the reader to memorize all of the material in this chapter on a first reading. Instead, treat it as a sample to get you started and a resource to which you can return.

Theory and Examples

Notation

Neural networks are so new that standard mathematical notation and architectural representations for them have not yet been firmly established. In addition, papers and books on neural networks have come from many diverse fields, including engineering, physics, psychology and mathematics, and many authors tend to use vocabulary peculiar to their specialty. As a result, many books and papers in this field are difficult to read, and concepts are made to seem more complex than they actually are. This is a shame, as it has prevented the spread of important new ideas. It has also led to more than one "reinvention of the wheel."

In this book we have tried to use standard notation where possible, to be clear and to keep matters simple without sacrificing rigor. In particular, we have tried to define practical conventions and use them consistently.

Figures, mathematical equations and text discussing both figures and mathematical equations will use the following notation:

Scalars — small *italic* letters: a,b,c

Vectors — small **bold** nonitalic letters: $\mathbf{a,b,c}$

Matrices — capital **BOLD** nonitalic letters: $\mathbf{A,B,C}$

Additional notation concerning the network architectures will be introduced as you read this chapter. A complete list of the notation that we use throughout the book is given in Appendix B, so you can look there if you have a question.

Neuron Model

Single-Input Neuron

Weight
Bias
Net Input
Transfer Function

A single-input neuron is shown in Figure 2.1. The scalar input p is multiplied by the scalar *weight* w to form wp, one of the terms that is sent to the summer. The other input, 1, is multiplied by a *bias* b and then passed to the summer. The summer output n, often referred to as the *net input*, goes into a *transfer function* f, which produces the scalar neuron output a. (Some authors use the term "activation function" rather than *transfer function* and "offset" rather than *bias*.)

If we relate this simple model back to the biological neuron that we discussed in Chapter 1, the weight w corresponds to the strength of a synapse, the cell body is represented by the summation and the transfer function, and the neuron output a represents the signal on the axon.

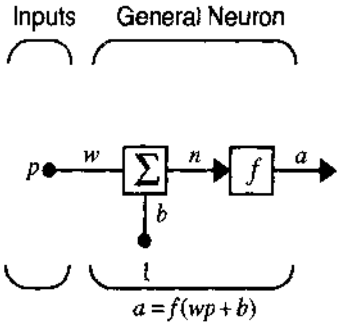

Figure 2.1 Single-Input Neuron

The neuron output is calculated as

$$a = f(wp + b) \ .$$

If, for instance, $w = 3$, $p = 2$ and $b = -1.5$, then

$$a = f(3(2) - 1.5) = f(4.5)$$

The actual output depends on the particular transfer function that is chosen. We will discuss transfer functions in the next section.

The bias is much like a weight, except that it has a constant input of 1. However, if you do not want to have a bias in a particular neuron, it can be omitted. We will see examples of this in Chapters 3, 7 and 14.

Note that w and b are both *adjustable* scalar parameters of the neuron. Typically the transfer function is chosen by the designer and then the parameters w and b will be adjusted by some learning rule so that the neuron input/output relationship meets some specific goal (see Chapter 4 for an introduction to learning rules). As described in the following section, we have different transfer functions for different purposes.

Transfer Functions

The transfer function in Figure 2.1 may be a linear or a nonlinear function of n. A particular transfer function is chosen to satisfy some specification of the problem that the neuron is attempting to solve.

A variety of transfer functions have been included in this book. Three of the most commonly used functions are discussed below.

Hard Limit Transfer Function

The *hard limit transfer function*, shown on the left side of Figure 2.2, sets the output of the neuron to 0 if the function argument is less than 0, or 1 if its argument is greater than or equal to 0. We will use this function to create neurons that classify inputs into two distinct categories. It will be used extensively in Chapter 4.

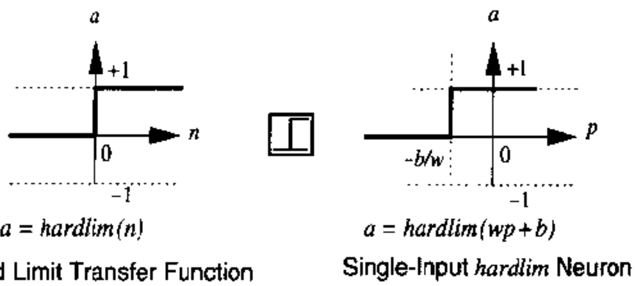

Figure 2.2 Hard Limit Transfer Function

The graph on the right side of Figure 2.2 illustrates the input/output characteristic of a single-input neuron that uses a hard limit transfer function. Here we can see the effect of the weight and the bias. Note that an icon for the hard limit transfer function is shown between the two figures. Such icons will replace the general f in network diagrams to show the particular transfer function that is being used.

Linear Transfer Function

The output of a *linear transfer function* is equal to its input:

$$a = n, \tag{2.1}$$

as illustrated in Figure 2.3.

Neurons with this transfer function are used in the ADALINE networks, which are discussed in Chapter 10.

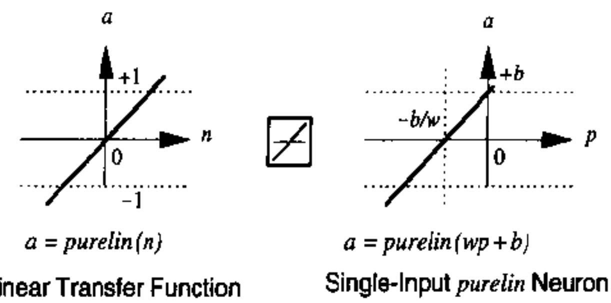

Figure 2.3 Linear Transfer Function

The output (a) versus input (p) characteristic of a single-input linear neuron with a bias is shown on the right of Figure 2.3.

Log-Sigmoid Transfer Function

The *log-sigmoid transfer function* is shown in Figure 2.4.

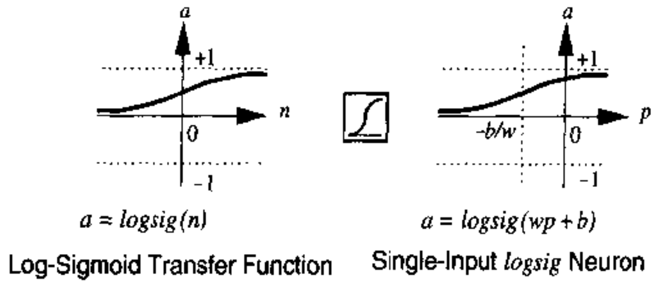

Figure 2.4 Log-Sigmoid Transfer Function

This transfer function takes the input (which may have any value between plus and minus infinity) and squashes the output into the range 0 to 1, according to the expression:

$$a = \frac{1}{1 + e^{-n}}. \tag{2.2}$$

The log-sigmoid transfer function is commonly used in multilayer networks that are trained using the backpropagation algorithm, in part because this function is differentiable (see Chapter 11).

Most of the transfer functions used in this book are summarized in Table 2.1. Of course, you can define other transfer functions in addition to those shown in Table 2.1 if you wish.

To experiment with a single-input neuron, use the Neural Network Design Demonstration One-Input Neuron `nnd2n1`.

Name	Input/Output Relation	Icon	MATLAB Function
Hard Limit	$a = 0 \quad n < 0$ $a = 1 \quad n \geq 0$		hardlim
Symmetrical Hard Limit	$a = -1 \quad n < 0$ $a = +1 \quad n \geq 0$		hardlims
Linear	$a = n$		purelin
Saturating Linear	$a = 0 \quad n < 0$ $a = n \quad 0 \leq n \leq 1$ $a = 1 \quad n > 1$		satlin
Symmetric Saturating Linear	$a = -1 \quad n < -1$ $a = n \quad -1 \leq n \leq 1$ $a = 1 \quad n > 1$		satlins
Log-Sigmoid	$a = \frac{1}{1 + e^{-n}}$		logsig
Hyperbolic Tangent Sigmoid	$a = \frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}$		tansig
Positive Linear	$a = 0 \quad n < 0$ $a = n \quad 0 \leq n$		poslin
Competitive	$a = 1$ neuron with max n $a = 0$ all other neurons	C	compet

Table 2.1 Transfer Functions

Multiple-Input Neuron

Weight Matrix

Typically, a neuron has more than one input. A neuron with R inputs is shown in Figure 2.5. The individual inputs $p_1, p_2, \ldots, p_R$ are each weighted by corresponding elements $w_{1,1}, w_{1,2}, \ldots, w_{1,R}$ of the *weight matrix* $\mathbf{W}$.

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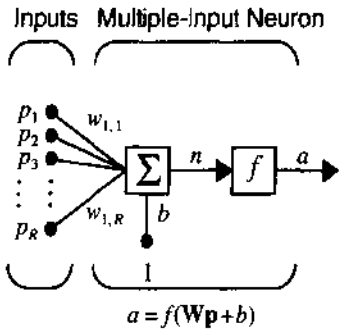


Figure 2.5 Multiple-Input Neuron

The neuron has a bias b, which is summed with the weighted inputs to form the net input n:

$$n = w_{1,1}p_1 + w_{1,2}p_2 + \cdots + w_{1,R}p_R + b. \tag{2.3}$$

This expression can be written in matrix form:

$$n = \mathbf{Wp} + b, \tag{2.4}$$

where the matrix $\mathbf{W}$ for the single neuron case has only one row.

Now the neuron output can be written as

$$a = f(\mathbf{Wp} + b). \tag{2.5}$$

Fortunately, neural networks can often be described with matrices. This kind of matrix expression will be used throughout the book. Don't be concerned if you are rusty with matrix and vector operations. We will review these topics in Chapters 5 and 6, and we will provide many examples and solved problems that will spell out the procedures.

Weight Indices

We have adopted a particular convention in assigning the indices of the elements of the weight matrix. The first index indicates the particular neuron destination for that weight. The second index indicates the source of the signal fed to the neuron. Thus, the indices in $w_{1,2}$ say that this weight represents the connection *to* the first (and only) neuron *from* the second source. Of course, this convention is more useful if there is more than one neuron, as will be the case later in this chapter.

We would like to draw networks with several neurons, each having several inputs. Further, we would like to have more than one layer of neurons. You can imagine how complex such a network might appear if all the lines were drawn. It would take a lot of ink, could hardly be read, and the mass of detail might obscure the main features. Thus, we will use an *abbreviated notation*. A multiple-input neuron using this notation is shown in Figure 2.6.

Abbreviated Notation

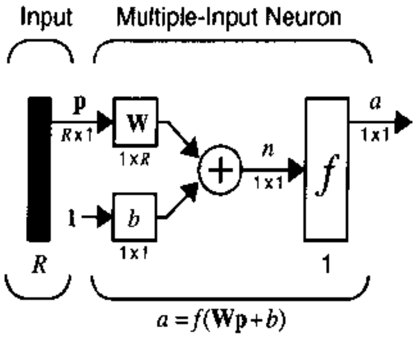


Figure 2.6 Neuron with R Inputs, Abbreviated Notation

As shown in Figure 2.6, the input vector $\mathbf{p}$ is represented by the solid vertical bar at the left. The dimensions of $\mathbf{p}$ are displayed below the variable as $R \times 1$, indicating that the input is a single vector of R elements. These inputs go to the weight matrix $\mathbf{W}$, which has R columns but only one row in this single neuron case. A constant 1 enters the neuron as an input and is multiplied by a scalar bias b. The net input to the transfer function f is n, which is the sum of the bias b and the product $\mathbf{Wp}$. The neuron's output a is a scalar in this case. If we had more than one neuron, the network output would be a vector.

The dimensions of the variables in these abbreviated notation figures will always be included, so that you can tell immediately if we are talking about a scalar, a vector or a matrix. You will not have to guess the kind of variable or its dimensions.

Note that the number of inputs to a network is set by the external specifications of the problem. If, for instance, you want to design a neural network that is to predict kite-flying conditions and the inputs are air temperature, wind velocity and humidity, then there would be three inputs to the network.

To experiment with a two-input neuron, use the Neural Network Design Demonstration Two-Input Neuron (`nnd2n2`).

Network Architectures

Commonly one neuron, even with many inputs, may not be sufficient. We might need five or ten, operating in parallel, in what we will call a "layer." This concept of a layer is discussed below.

2

A Layer of Neurons

Layer A single-*layer* network of S neurons is shown in Figure 2.7. Note that each of the R inputs is connected to each of the neurons and that the weight matrix now has S rows.

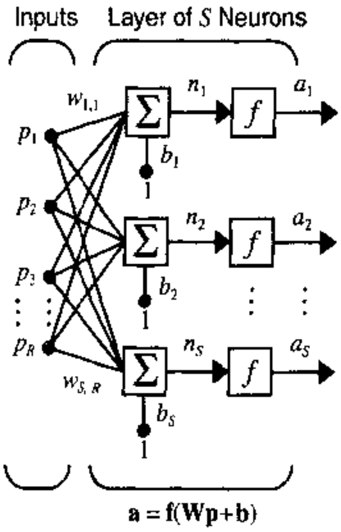


Figure 2.7 Layer of S Neurons

The layer includes the weight matrix, the summers, the bias vector $\mathbf{b}$, the transfer function boxes and the output vector $\mathbf{a}$. Some authors refer to the inputs as another layer, but we will not do that here.

Each element of the input vector $\mathbf{p}$ is connected to each neuron through the weight matrix $\mathbf{W}$. Each neuron has a bias b_i, a summer, a transfer function f and an output a_i. Taken together, the outputs form the output vector $\mathbf{a}$.

It is common for the number of inputs to a layer to be different from the number of neurons (i.e., $R \neq S$).

You might ask if all the neurons in a layer must have the same transfer function. The answer is no; you can define a single (composite) layer of neurons having different transfer functions by combining two of the networks shown above in parallel. Both networks would have the same inputs, and each network would create some of the outputs.

The input vector elements enter the network through the weight matrix $\mathbf{W}$:

$$\mathbf{W} = \begin{bmatrix} w_{1,1} & w_{1,2} & \cdots & w_{1,R} \\ w_{2,1} & w_{2,2} & \cdots & w_{2,R} \\ \vdots & \vdots & & \vdots \\ w_{S,1} & w_{S,2} & \cdots & w_{S,R} \end{bmatrix} . \tag{2.6}$$

As noted previously, the row indices of the elements of matrix $\mathbf{W}$ indicate the destination neuron associated with that weight, while the column indices indicate the source of the input for that weight. Thus, the indices in $w_{3,2}$ say that this weight represents the connection *to* the third neuron *from* the second source.

Fortunately, the S-neuron, R-input, one-layer network also can be drawn in abbreviated notation, as shown in Figure 2.8.

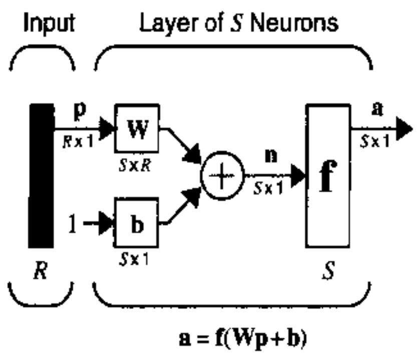


Figure 2.8 Layer of S Neurons, Abbreviated Notation

Here again, the symbols below the variables tell you that for this layer, $\mathbf{p}$ is a vector of length R, $\mathbf{W}$ is an $S \times R$ matrix, and $\mathbf{a}$ and $\mathbf{b}$ are vectors of length S. As defined previously, the layer includes the weight matrix, the summation and multiplication operations, the bias vector $\mathbf{b}$, the transfer function boxes and the output vector.

Multiple Layers of Neurons

Now consider a network with several layers. Each layer has its own weight matrix $\mathbf{W}$, its own bias vector $\mathbf{b}$, a net input vector $\mathbf{n}$ and an output vector $\mathbf{a}$. We need to introduce some additional notation to distinguish between these layers. We will use superscripts to identify the layers. Specifically, we

Layer Superscript

append the number of the layer as a *superscript* to the names for each of these variables. Thus, the weight matrix for the first layer is written as $\mathbf{W}^1$, and the weight matrix for the second layer is written as $\mathbf{W}^2$. This notation is used in the three-layer network shown in Figure 2.9.

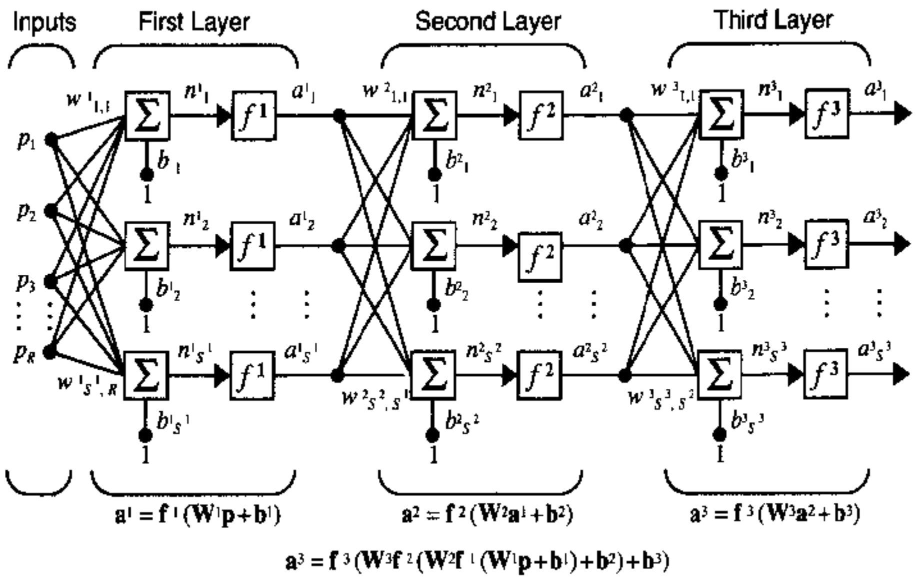


Figure 2.9 Three-Layer Network

As shown, there are R inputs, S^1 neurons in the first layer, S^2 neurons in the second layer, etc. As noted, different layers can have different numbers of neurons.

The outputs of layers one and two are the inputs for layers two and three. Thus layer 2 can be viewed as a one-layer network with $R = S^1$ inputs, $S = S^2$ neurons, and an $S^1 \times S^2$ weight matrix $\mathbf{W}^2$. The input to layer 2 is $\mathbf{a}^1$, and the output is $\mathbf{a}^2$.

Output Layer
Hidden Layers

A layer whose output is the network output is called an *output layer*. The other layers are called *hidden layers*. The network shown above has an output layer (layer 3) and two hidden layers (layers 1 and 2).

The same three-layer network discussed previously also can be drawn using our abbreviated notation, as shown in Figure 2.10.

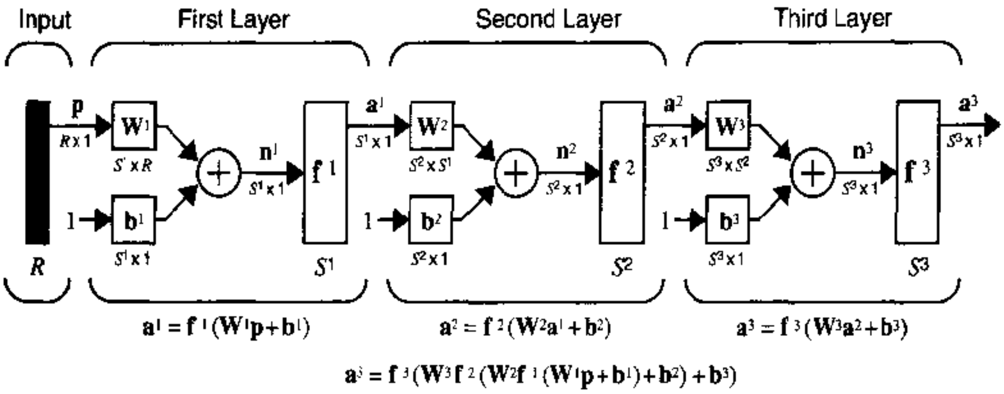


Figure 2.10 Three-Layer Network, Abbreviated Notation

Multilayer networks are more powerful than single-layer networks. For instance, a two-layer network having a sigmoid first layer and a linear second layer can be trained to approximate most functions arbitrarily well. Single-layer networks cannot do this.

At this point the number of choices to be made in specifying a network may look overwhelming, so let us consider this topic. The problem is not as bad as it looks. First, recall that the number of inputs to the network and the number of outputs from the network are defined by external problem specifications. So if there are four external variables to be used as inputs, there are four inputs to the network. Similarly, if there are to be seven outputs from the network, there must be seven neurons in the output layer. Finally, the desired characteristics of the output signal also help to select the transfer function for the output layer. If an output is to be either -1 or 1, then a symmetrical hard limit transfer function should be used. Thus, the architecture of a single-layer network is almost completely determined by problem specifications, including the specific number of inputs and outputs and the particular output signal characteristic.

Now, what if we have more than two layers? Here the external problem does not tell you directly the number of neurons required in the hidden layers. In fact, there are few problems for which one can predict the optimal number of neurons needed in a hidden layer. This problem is an active area of research. We will develop some feeling on this matter as we proceed to Chapter 11, Backpropagation.

As for the number of layers, most practical neural networks have just two or three layers. Four or more layers are used rarely.

We should say something about the use of biases. One can choose neurons with or without biases. The bias gives the network an extra variable, and so you might expect that networks with biases would be more powerful than those without, and that is true. Note, for instance, that a neuron without a bias will always have a net input n of zero when the network inputs $\mathbf{p}$ are zero. This may not be desirable and can be avoided by the use of a bias. The effect of the bias is discussed more fully in Chapters 3, 4 and 5.

In later chapters we will omit a bias in some examples or demonstrations. In some cases this is done simply to reduce the number of network parameters. With just two variables, we can plot system convergence in a two-dimensional plane. Three or more variables are difficult to display.

Recurrent Networks

Before we discuss recurrent networks, we need to introduce some simple building blocks. The first is the *delay* block, which is illustrated in Figure 2.11.

Delay

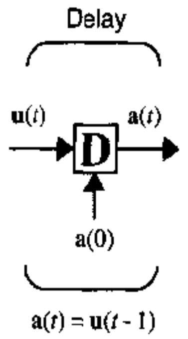


Figure 2.11 Delay Block

The delay output $\mathbf{a}(t)$ is computed from its input $\mathbf{u}(t)$ according to

$$\mathbf{a}(t) = \mathbf{u}(t-1). \tag{2.7}$$

Thus the output is the input delayed by one time step. (This assumes that time is updated in discrete steps and takes on only integer values.) Eq. (2.7) requires that the output be initialized at time $t = 0$. This initial condition is indicated in Figure 2.11 by the arrow coming into the bottom of the delay block.

Another related building block, which we will use for the continuous-time recurrent networks in Chapters 15–18, is the *integrator*, which is shown in Figure 2.12.

Integrator

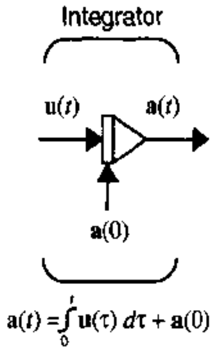


Figure 2.12 Integrator Block

The integrator output $\mathbf{a}(t)$ is computed from its input $\mathbf{u}(t)$ according to

$$\mathbf{a}(t) = \int_0^t \mathbf{u}(\tau)\, d\tau + \mathbf{a}(0) . \tag{2.8}$$

The initial condition $\mathbf{a}(0)$ is indicated by the arrow coming into the bottom of the integrator block.

Recurrent Network

We are now ready to introduce recurrent networks. A *recurrent network* is a network with feedback; some of its outputs are connected to its inputs. This is quite different from the networks that we have studied thus far, which were strictly feedforward with no backward connections. One type of discrete-time recurrent network is shown in Figure 2.13.

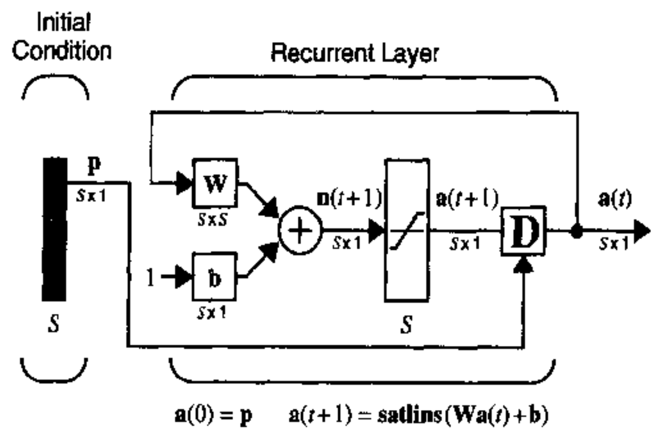


Figure 2.13 Recurrent Network

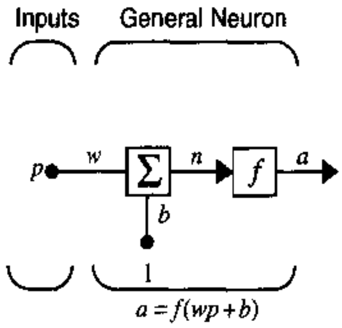
In this particular network the vector $\mathbf{p}$ supplies the initial conditions (i.e., $\mathbf{a}(0) = \mathbf{p}$). Then future outputs of the network are computed from previous outputs:

$$\mathbf{a}(1) = \mathbf{satlins}(\mathbf{Wa}(0) + \mathbf{b}), \mathbf{a}(2) = \mathbf{satlins}(\mathbf{Wa}(1) + \mathbf{b}), \ldots$$

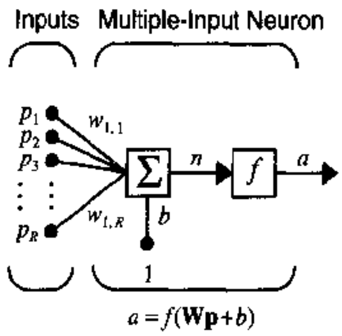
Recurrent networks are potentially more powerful than feedforward networks and can exhibit temporal behavior. These types of networks are discussed in Chapters 3 and 15–18.

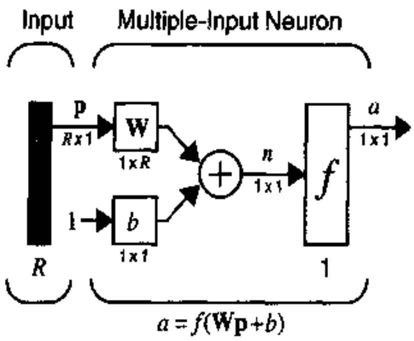
Summary of Results

Single-Input Neuron



Multiple-Input Neuron

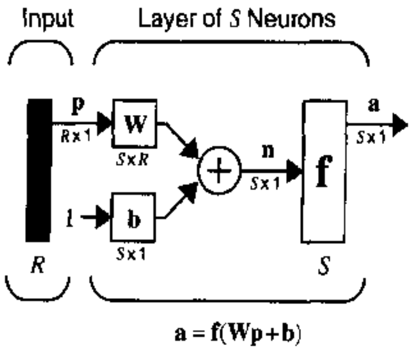




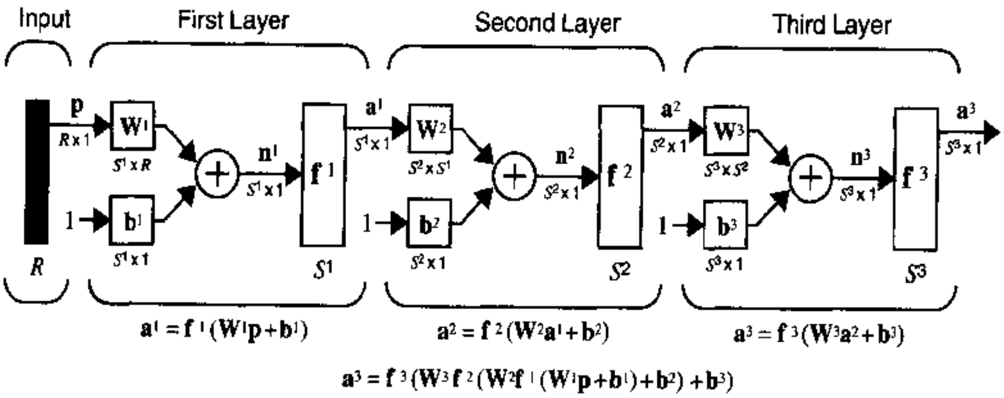
Transfer Functions

Name	Input/Output Relation	Icon	MATLAB Function
Hard Limit	$a = 0 \quad n < 0$ $a = 1 \quad n \geq 0$		hardlim
Symmetrical Hard Limit	$a = -1 \quad n < 0$ $a = +1 \quad n \geq 0$		hardlims
Linear	$a = n$		purelin
Saturating Linear	$a = 0 \quad n < 0$ $a = n \quad 0 \leq n \leq 1$ $a = 1 \quad n > 1$		satlin
Symmetric Saturating Linear	$a = -1 \quad n < -1$ $a = n \quad -1 \leq n \leq 1$ $a = 1 \quad n > 1$		satlins
Log-Sigmoid	$a = \frac{1}{1+e^{-n}}$		logsig
Hyperbolic Tangent Sigmoid	$a = \frac{e^{n}-e^{-n}}{e^{n}+e^{-n}}$		tansig
Positive Linear	$a = 0 \quad n < 0$ $a = n \quad 0 \leq n$		poslin
Competitive	$a = 1$ neuron with max n $a = 0$ all other neurons	C	compet

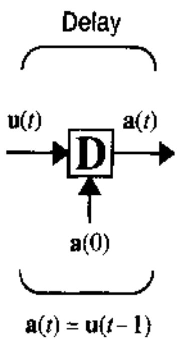
Layer of Neurons



Three Layers of Neurons



Delay



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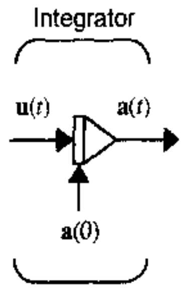
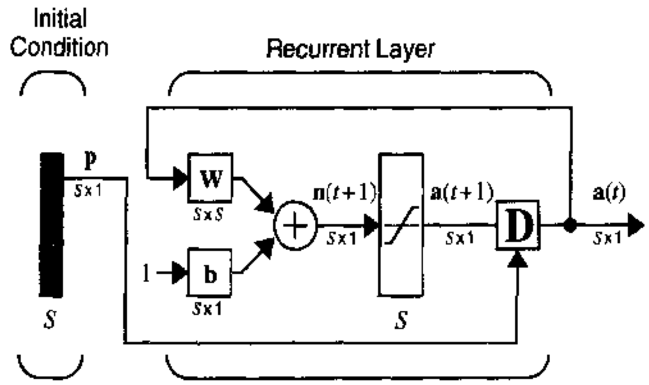
Integrator



$$\mathbf{a}(t) = \int_0^t \mathbf{u}(\tau)\, d\tau + \mathbf{a}(0)$$

Recurrent Network



$\mathbf{a}(0) = \mathbf{p}$ $\quad \mathbf{a}(t+1) = \mathbf{satlins}(\mathbf{W}\mathbf{a}(t) + \mathbf{b})$

How to Pick an Architecture

Problem specifications help define the network in the following ways:

1. Number of network inputs = number of problem inputs
2. Number of neurons in output layer = number of problem outputs
3. Output layer transfer function choice at least partly determined by problem specification of the outputs

Solved Problems

P2.1 The input to a single-input neuron is 2.0, its weight is 2.3 and its bias is -3.

i. What is the net input to the transfer function?

ii. What is the neuron output?

i. The net input is given by:

$$n = wp + b = (2.3)(2) + (-3) = 1.6$$

ii. The output cannot be determined because the transfer function is not specified.

P2.2 What is the output of the neuron of P2.1 if it has the following transfer functions?

i. Hard limit

ii. Linear

iii. Log-sigmoid

i. For the hard limit transfer function:

$$a = hardlim(1.6) = 1.0$$

ii. For the linear transfer function:

$$a = purelin(1.6) = 1.6$$

iii. For the log-sigmoid transfer function:

$$a = logsig(1.6) = \frac{1}{1 + e^{-1.6}} = 0.8320$$

Verify this result using MATLAB and the function `logsig`, which is in the MININNET directory (see Appendix B).

P2.3 Given a two-input neuron with the following parameters: $b = 1.2$, $\mathbf{W} = \begin{bmatrix} 3 & 2 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} -5 & 6 \end{bmatrix}^T$, calculate the neuron output for the following transfer functions:

i. A symmetrical hard limit transfer function

ii. A saturating linear transfer function

iii. A hyperbolic tangent sigmoid (tansig) transfer function

First calculate the net input n:

$$n = \mathbf{W}\mathbf{p} + b = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} -5 \\ 6 \end{bmatrix} + (1.2) = -1.8 .$$

2

Now find the outputs for each of the transfer functions.

i. $a = hardlims(-1.8) = -1$

ii. $a = satlin(-1.8) = 0$

iii. $a = tansig(-1.8) = -0.9468$

P2.4 A single-layer neural network is to have six inputs and two outputs. The outputs are to be limited to and continuous over the range 0 to 1. What can you tell about the network architecture? Specifically:

i. How many neurons are required?

ii. What are the dimensions of the weight matrix?

iii. What kind of transfer functions could be used?

iv. Is a bias required?

The problem specifications allow you to say the following about the network.

i. Two neurons, one for each output, are required.

ii. The weight matrix has two rows corresponding to the two neurons and six columns corresponding to the six inputs. (The product $\mathbf{W}\mathbf{p}$ is a two-element vector.)

iii. Of the transfer functions we have discussed, the $logsig$ transfer function would be most appropriate.

iv. Not enough information is given to determine if a bias is required.

Epilogue

This chapter has introduced a simple artificial neuron and has illustrated how different neural networks can be created by connecting groups of neurons in various ways. One of the main objectives of this chapter has been to introduce our basic notation. As the networks are discussed in more detail in later chapters, you may wish to return to Chapter 2 to refresh your memory of the appropriate notation.

This chapter was not meant to be a complete presentation of the networks we have discussed here. That will be done in the chapters that follow. We will begin in Chapter 3, which will present a simple example that uses some of the networks described in this chapter, and will give you an opportunity to see these networks in action. The networks demonstrated in Chapter 3 are representative of the types of networks that are covered in the remainder of this text.

Exercises

E2.1 The input to a single input neuron is 2.0, its weight is 1.3 and its bias is 3.0. What possible kinds of transfer function, from Table 2.1, could this neuron have, if its output is:

i. 1.6

ii. 1.0

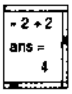
iii. 0.9963

iv. -1.0

E2.2 Consider a single-input neuron with a bias. We would like the output to be -1 for inputs less than 3 and +1 for inputs greater than or equal to 3.

i. What kind of a transfer function is required?

ii. What bias would you suggest? Is your bias in any way related to the input weight? If yes, how?



iii. Summarize your network by naming the transfer function and stating the bias and the weight. Draw a diagram of the network. Verify the network performance using MATLAB.

E2.3 Given a two-input neuron with the following weight matrix and input vector: $\mathbf{W} = \begin{bmatrix} 3 & 2 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} -5 & 7 \end{bmatrix}^T$, we would like to have an output of 0.5. Do you suppose that there is a combination of bias and transfer function that might allow this?

i. Is there a transfer function from Table 2.1 that will do the job if the bias is zero?

ii. Is there a bias that will do the job if the linear transfer function is used? If yes, what is it?

iii. Is there a bias that will do the job if a log-sigmoid transfer function is used? Again, if yes, what is it?

iv. Is there a bias that will do the job if a symmetrical hard limit transfer function is used? Again, if yes, what is it?

E2.4 A two-layer neural network is to have four inputs and six outputs. The range of the outputs is to be continuous between 0 and 1. What can you tell about the network architecture? Specifically:

i. How many neurons are required in each layer?

ii. What are the dimensions of the first-layer and second-layer weight matrices?

iii. What kinds of transfer functions can be used in each layer?

iv. Are biases required in either layer?

3 An Illustrative Example

Objectives

Think of this chapter as a preview of coming attractions. We will take a simple pattern recognition problem and show how it can be solved using three different neural network architectures. It will be an opportunity to see how the architectures described in the previous chapter can be used to solve a practical (although extremely oversimplified) problem. Do not expect to completely understand these three networks after reading this chapter. We present tbem simply to give you a taste of what can be done with neural networks, and to demonstrate that there are many different types of networks that can be used to solve a given problem.

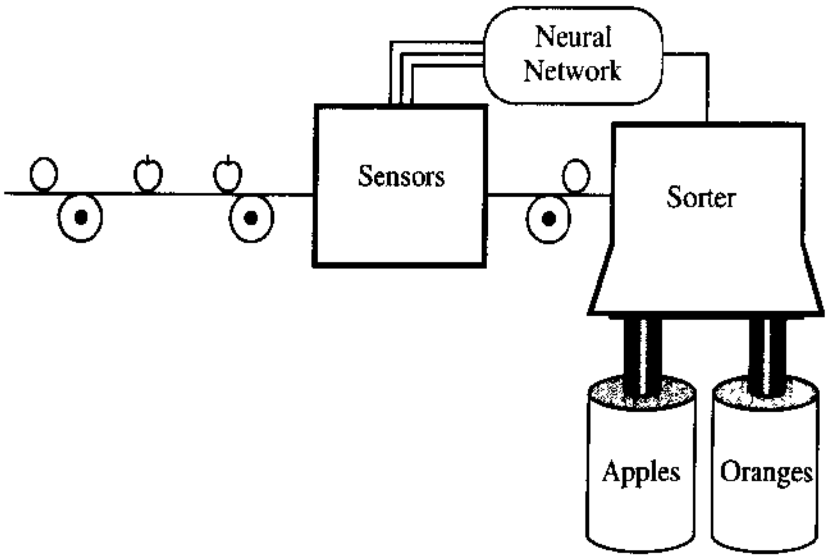
The three networks presented in this chapter are representative of the types of networks discussed in the remaining chapters: feedforward networks (represented here by the perceptron), competitive networks (represented here by the Hamming network) and recurrent associative memory networks (represented here by the Hopfield network).

Theory and Examples

Problem Statement

A produce dealer has a warehouse that stores a variety of fruits and vegetables. When fruit is brought to the warehouse, various types of fruit may be mixed together. The dealer wants a machine that will sort the fruit according to type. There is a conveyer belt on which the fruit is loaded. This conveyer passes through a set of sensors, which measure three properties of the fruit: *shape*, *texture* and *weight*. These sensors are somewhat primitive. The shape sensor will output a 1 if the fruit is approximately round and a -1 if it is more elliptical. The texture sensor will output a 1 if the surface of the fruit is smooth and a -1 if it is rough. The weight sensor will output a 1 if the fruit is more than one pound and a -1 if it is less than one pound.

The three sensor outputs will then be input to a neural network. The purpose of the network is to decide which kind of fruit is on the conveyor, so that the fruit can be directed to the correct storage bin. To make the problem even simpler, let's assume that there are only two kinds of fruit on the conveyor: apples and oranges.



As each fruit passes through the sensors it can be represented by a three-dimensional vector. The first element of the vector will represent shape, the second element will represent texture and the third element will represent weight:

$$\mathbf{p} = \begin{bmatrix} shape \\ texture \\ weight \end{bmatrix}. \quad (3.1)$$

Therefore, a prototype orange would be represented by

$$\mathbf{p}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \quad (3.2)$$

and a prototype apple would be represented by

$$\mathbf{p}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}. \quad (3.3)$$

The neural network will receive one three-dimensional input vector for each fruit on the conveyer and must make a decision as to whether the fruit is an *orange* $(\mathbf{p}_1)$ or an *apple* $(\mathbf{p}_2)$.

Now that we have defined this simple (trivial?) pattern recognition problem, let's look briefly at three different neural networks that could be used to solve it. The simplicity of our problem will facilitate our understanding of the operation of the networks.

Perceptron

The first network we will discuss is the perceptron. Figure 3.1 illustrates a single-layer perceptron with a symmetric hard limit transfer function *hardlims*.

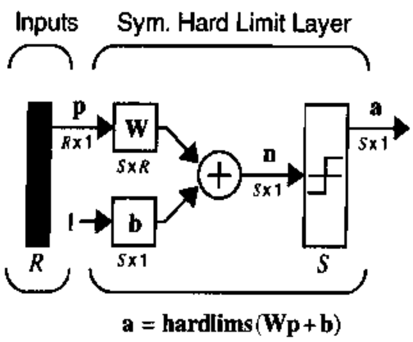


Figure 3.1 Single-Layer Perceptron

Two-Input Case

Before we use the perceptron to solve the orange and apple recognition problem (which will require a three-input perceptron, i.e., $R = 3$), it is useful to investigate the capabilities of a two-input/single-neuron perceptron ($R = 2$), which can be easily analyzed graphically. The two-input perceptron is shown in Figure 3.2.

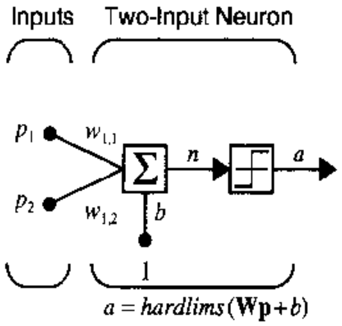


Figure 3.2 Two-Input/Single-Neuron Perceptron

Single-neuron perceptrons can classify input vectors into two categories. For example, for a two-input perceptron, if $w_{1,1} = -1$ and $w_{1,2} = 1$ then

$$a = hardlims(n) = hardlims(\begin{bmatrix} -1 & 1 \end{bmatrix}\mathbf{p} + b). \tag{3.4}$$

Therefore, if the inner product of the weight matrix (a single row vector in this case) with the input vector is greater than or equal to $-b$, the output will be 1. If the inner product of the weight vector and the input is less than $-b$, the output will be -1. This divides the input space into two parts. Figure 3.3 illustrates this for the case where $b = -1$. The blue line in the figure represents all points for which the net input n is equal to 0:

$$n = \begin{bmatrix} -1 & 1 \end{bmatrix}\mathbf{p} - 1 = 0. \tag{3.5}$$

Notice that this decision boundary will always be orthogonal to the weight matrix, and the position of the boundary can be shifted by changing b. (In the general case, $\mathbf{W}$ is a matrix consisting of a number of row vectors, each of which will be used in an equation like Eq. (3.5). There will be one boundary for each row of $\mathbf{W}$. See Chapter 4 for more on this topic.) The shaded region contains all input vectors for which the output of the network will be 1. The output will be -1 for all other input vectors.

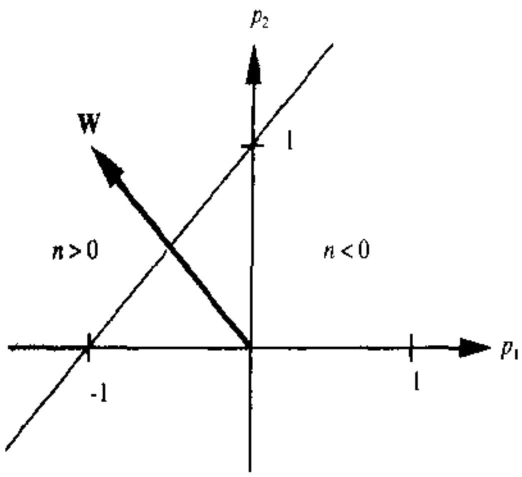


Figure 3.3 Perceptron Decision Boundary

The key property of the single-neuron perceptron, therefore, is that it can separate input vectors into two categories. The decision boundary between the categories is determined by the equation

$$\mathbf{Wp} + b = 0. \tag{3.6}$$

Because the boundary must be linear, the single-layer perceptron can only be used to recognize patterns that are linearly separable (can be separated by a linear boundary). These concepts will be discussed in more detail in Chapter 4.

Pattern Recognition Example

Now consider the apple and orange pattern recognition problem. Because there are only two categories, we can use a single-neuron perceptron. The vector inputs are three-dimensional ($R = 3$), therefore the perceptron equation will be

$$a = hardlims\left(\begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + b \right). \tag{3.7}$$

We want to choose the bias b and the elements of the weight matrix so that the perceptron will be able to distinguish between apples and oranges. For example, we may want the output of the perceptron to be 1 when an apple is input and -1 when an orange is input. Using the concept illustrated in Figure 3.3, let's find a linear boundary that can separate oranges and apples. The two prototype vectors (recall Eq. (3.2) and Eq. (3.3)) are shown in Figure 3.4. From this figure we can see that the linear boundary that divides these two vectors symmetrically is the p_1, p_3 plane.

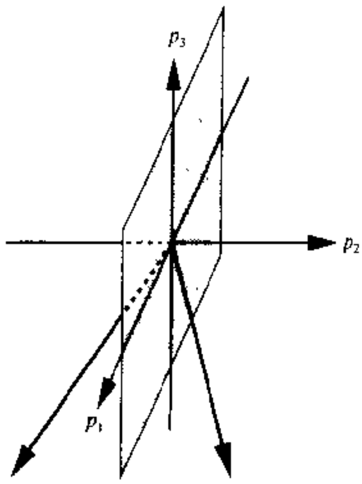


Figure 3.4 Prototype Vectors

The p_1, p_3 plane, which will be our decision boundary, can be described by the equation

$$p_2 = 0, \tag{3.8}$$

or

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + 0 = 0. \tag{3.9}$$

Therefore the weight matrix and bias will be

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \; b = 0. \tag{3.10}$$

The weight matrix is orthogonal to the decision boundary and points toward the region that contains the prototype pattern $\mathbf{p}_2$ (*apple*) for which we want the perceptron to produce an output of 1. The bias is 0 because the decision boundary passes through the origin.

Now let's test the operation of our perceptron pattern classifier. It classifies perfect apples and oranges correctly since

Orange:

$$a = hardlims\left(\begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\-1\\-1\end{bmatrix} + 0\right) = -1\,(orange)\,, \tag{3.11}$$

Apple:

$$a = hardlims\left(\begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix} + 0\right) = 1\,(apple)\,. \tag{3.12}$$

But what happens if we put a not-so-perfect orange into the classifier? Let's say that an orange with an elliptical shape is passed through the sensors. The input vector would then be

$$\mathbf{p} = \begin{bmatrix}-1\\-1\\-1\end{bmatrix}. \tag{3.13}$$

The response of the network would be

$$a = hardlims\left(\begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix}-1\\-1\\-1\end{bmatrix} + 0\right) = -1\,(orange)\,. \tag{3.14}$$

In fact, any input vector that is closer to the orange prototype vector than to the apple prototype vector (in Euclidean distance) will be classified as an orange (and vice versa).

To experiment with the perceptron network and the apple/orange classification problem, use the Neural Network Design Demonstration Perceptron Classification (`nnd3pc`).

This example has demonstrated some of the features of the perceptron network, but by no means have we exhausted our investigation of perceptrons. This network, and variations on it, will be examined in Chapters 4 through 12. Let's consider some of these future topics.

In the apple/orange example we were able to design a network graphically, by choosing a decision boundary that clearly separated the patterns. What about practical problems, with high dimensional input spaces? In Chapters 4, 7, 10 and 11 we will introduce learning algorithms that can be used to train networks to solve complex problems by using a set of examples of proper network behavior.