

Binary Search

Implementation:

```
#include<stdio.h>
int binarysearch(int arr[],int n,int x)
{
    int i,left,right,mid;
    left = 0;
    right = n-1;

    while(left<=right)
    {
        mid = (left+right)/2;
        if(x==arr[mid])
        {
            return mid;
        }
        else if(x<arr[mid])
        {
            right = mid-1;
        }
        else
        {
            left = mid+1;
        }
    }
    return -1;
}
int main()
{
    int n,i,res,x;

    scanf("%d",&n); //how many values in the list?

    int arr[n];

    for(i=0; i<n; i++)
    {
        scanf("%d",&arr[i]);
    }
    scanf("%d",&x); //value that want to search

    res = binarysearch(arr,n,x);

    if(res==-1)
```

```

{
    printf("Value not found in the list\n");
}
else
{
    printf("Value found\nThe position of the value is %d\n",res);
}
return 0;
}

```

Analysis:

We start analyzing the Insertion Sort procedure with the time “cost” of each statement and the number of times each statement is executed.

pseudo code of binary search:

BINARY_SEARCH(A, x)		<u>Cost</u>	<u>Time</u>
1	n <- length(A)	c1	(1)
2	left <- 1		
3	right <- n		
4	while(left<=right)	c2	k
5	mid = (left+right)/2		
6	if x == A[mid]		
7	Return mid		
8	else if x<A[mid]		
9	Right <- mid - 1	c3	(1)
10	else		
11	Left <- mid + 1		
12	return -1		

The running time of the algorithm is the sum of running times for each statement executed; a statement that takes c_i steps to execute and executes n times will contribute $c_i * n$ times to the total running time.

Considering the algorithm of binary search,

$$T(n) = c1 + c2 * k + c3$$

As $c1, c2$ and $c3$ are some constant then this equation can be written as,

$$T(n) = c * k + c'$$

Imagine an array of having 16 elements as follows:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----

The value we want to search is 16 which is in the last position of the array.

According to the algorithm,

left = 0

right = 16

Entering the while loop the n will be divided as follows to find the value 16:

$$Mid = 8 \text{ (left + right)/2}$$

Now left = mid+1 = 9 and again, calculating the mid = 12 which is not equal to our desired value.

Again, Left = mid+1 = 13 and mid = 14 which is also not the searched value.

Again, left = mid+1 = 15 ; now mid = 15 which is also not the searched value.

Lastly, left = mid+1 = 16 ; now mid is also 16, which is the actual value we are looking for.

Therefore, we can say that if $n = 16$ we need 4 division of the array.

$$n \rightarrow \frac{n}{2} \rightarrow \frac{n}{2 * 2} \rightarrow \frac{n}{2 * 2 * 2} \rightarrow \frac{n}{2 * 2 * 2 * 2}$$

If we continue for k times then n will be divided for, $\frac{n}{2^k}$ times.

Let,

$$\frac{n}{2^k} = 1$$

$$2^k = n$$

$$k = \log_2 n$$

$$\text{Therefore, } T(n) = c * \log_2 n + c'$$

Time Complexity:

Worst Case: The time function we get from the analysis is $T(n) = c * \log_2 n + c'$

So the complexity of binary search in worst case is $O(\log n)$

Best Case:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----

$$n = 16$$

$$x = 8 \text{ (search value)}$$

In 1st iteration n will be divided for $\frac{n}{2}$ times.

Now mid = 8 which is the same as our search value. We need only one iteration to find the value.

So the complexity of binary search in best case is $O(1)$